$\boxed{\text{GDsearch all}}$

m: the number of variants n: the number of individuals.

k: the number of traits.

$$\text{variants} = V = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}_{m\times n} \quad V_{ij}$$

$$\text{trait} = D = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}_{n\times k} \quad D_{j1}$$

$$V1D1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad V1D0 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad V0D1 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \\ 2 \end{bmatrix} \quad V0D0 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$V1D1 = V@D$$

$$V1D0 = V@(J_{n\times k} - D) = V@J_{n\times k} - V@D = V@J_{n\times k} - V1D1$$

$$V0D1 = (J_{m\times n} - V)@D = J_{m\times n}@D - V@D = J_{m\times n}@D - V1D1$$

$$V0D0 = (J_{m\times n} - V)@(J_{n\times k} - D) = J_{m\times n}@(J_{n\times k} - D) - V@(J_{n\times k} - D) = J_{m\times n}@J_{n\times k} - J_{m\times n}@D - V1D0$$

$$V@J_{n\times k} = \begin{bmatrix} V_{11} & V_{12} & \cdots & V_{ij} \\ \vdots & & & \\ V_{m1} & & \cdots & V_{mj} \end{bmatrix}_{m\times n} @ \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & & 1 \end{bmatrix}_{n\times k}$$

$$= \begin{bmatrix} \sum_{j=1}^{n} V_{ij} & \sum_{j=1}^{n} V_{ij} \\ \vdots & \\ \sum_{j=1}^{n} V_{mj} & \sum_{j=1}^{n} V_{mj} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} V_{ij} \\ \vdots \\ \sum_{j=1}^{n} V_{mj} \end{bmatrix}_{m\times 1} \odot \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & 1 \\ & & \vdots \\ 1 & & \end{bmatrix}_{m\times k}$$

$$J_{m\times n}@D = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \\ 1 & \cdots & 1 \end{bmatrix}_{m\times n} @ \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1k} \\ \vdots & & & \vdots \\ D_{n1} & D_{n2} & & D_{nk} \end{bmatrix}_{n\times k}$$

$$= \begin{bmatrix} \sum_{j=1}^{n} D_{j1} & \cdots & \sum_{j=1}^{n} D_{jk} \\ \vdots & & \\ \sum_{j=1}^{n} D_{j1} & \cdots & \sum_{j=1}^{n} D_{jk} \end{bmatrix}_{m\times k}$$

SGD search

$$\text{variants} = V = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}_{m \times n} \quad V_{ij}$$

$$\text{trait} = D = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}_{n \times k} \quad D_{j1}$$

$\boxed{\text{step 1: when variants} = 1}$

Method1 (old) : loop + mask

snp1: $index1 = [1, 2]$ $\quad V1 = [1, 1]$ $\quad BP\_V1 = [0, 0]$

snp1 :	V1D1 = 0	V1D0 = 2	V0D1 = 0	V0D0 = 0
snp2 :	V1D1 = 0	V1D0 = 1	V0D1 = 0	V0D0 = 0
snp3 :	V1D1 = 1	V1D0 = 0	V0D1 = 0	V0D0 = 0
snp4 :	V1D1 = 0	V1D0 = 1	V0D1 = 0	V0D0 = 0
snp5 :	V1D1 = 0	V1D0 = 1	V0D1 = 0	V0D0 = 0

Method2 : loop + matrix operation

$$\forall i \in (1, \cdots, m)$$

$$W_i = V_i$$

$$V1D1_i = V_i \ @ \ (D * W_i')$$

$A(B+C) = AB + AC$

$$V1D0_i = V_i \ @ \ ((J_n - D) * W_i')$$

$$V0D1_i = (J_{1\times n} - V_i) \ @ \ (D * W_i')$$

$$V0D0_i = (J_{1\times n} - V_i) \ @ \ ((J_n - D) * W_i')$$

$$J_{m\times n} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}_{m\times n}$$

$$J_n = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{n\times 1}$$

$$J_{1\times n} = [1 \ \cdots \ 1]_{1\times n}$$

$$V0D1_i = J_{1\times n} \ @ \ (D * W_i') - V_i \ @ \ (D * W_i')$$

$$= J_{1\times n} \ @ \ (D * W_i') - V1D1_i$$

$$= \sum_j D_j W_{ij}' - V1D1_i$$

$$V0D0_i = J_{1\times n} \ @ \ ((J_m - D) * W_i') - V_i \ @ \ ((J_m - D) * W_i')$$

$$= J_{1\times n} \ @ \ ((J_m - D) * W_i') - V1D0_i$$

$$= \left(\sum_j (J_m - D)_j * W_{ij}'\right) - V1D0_i$$

$$\text{variants} = V = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}_{m\times n} \quad (i,j)$$

$$\text{trait} = D = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}_{n\times 1} \quad (j,1)$$

$\boxed{\text{Step2: when variants} = 0}$

Method1 (old): loop + mask

snp1

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \quad V1D1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad V1D0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad V0D1 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \\ 2 \end{bmatrix} \quad V0D0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

snp2

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad V1D1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad V1D0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad V0D1 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \\ 2 \end{bmatrix} \quad V0D0 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Method2 : loop + matrix_operation

$\forall i \in (1, \cdots, m)$

$J_{1\times n} = [1 \cdots 1]_{1\times n}$

$$W_i = J_{1\times n} - V_i$$

$$V1D1_i = V \,@\, (W_i' * D)$$

$$V1D0_i = V \,@\, ((J_n - D) * W_i')$$

$$V0D1_i = (J_{m\times n} - V) \,@\, (W_i' * D)$$

$$= J_{m\times n} \,@\, (W_i' * D) - V \,@\, (W_i' * D)$$

$$= J_{m\times n} \,@\, (W_i' * D) - V1D1_i$$

$$V0D0_i = (J_{m\times n} - V) \,@\, ((1-D) * W_i')$$

$$= J_{m\times n} \,@\, ((1-D) * W_i') - V \,@\, ((1-D) * W_i')$$

$$= J_{m\times n} \,@\, ((1-D) * W_i') - V1D0_i$$

Variant $= 1$, $V' = V_i$

$J = J_{1\times n}$

$W_i = V_i$

Variant $= 0$, $V' = V$

$J = J_{m\times n}$

$W_i = J_{1\times n} - V_i$

Method2 when variant $=1$ $\quad V' = V_i \quad J = J_{1\times n} \quad W_i = V_i$

when variant $=0$ $\quad V' = V \quad J = J_{m\times n} \quad W_i = J_{1\times n} - V_i$

$$V1D1_i = V' @ (D * W_i')$$
$$V1D0_i = V' @ ((J_n - D) * W_i') \qquad \forall i \in (1, m)$$

$$V0D1_i = J @ (D * W_i') - V1D1_i$$
$$V0D0_i = J @ ((J_n - D) * W_i') - V1D0_i$$