

WEEK 10 - Python Application in Chemical Engineering

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Things you can do in python

- Process simulate
- Automation of chemical data analysis
- Chemical reaction and kinetic modelling
- Data mining
- Process control and monitoring

Linear Algebra 1

```
In [8]: # import libraries (https://docs.scipy.org/doc/scipy/reference/linalg.html)
import numpy as np
import scipy as sc
```

$$\begin{matrix} x + 3y + 5z = 10 \\ 2x + 5y + z = 8 \\ 2x + 3y + 8z = 3 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 5 & 1 \\ 2 & 3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 5 & 1 \\ 2 & 3 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 8 \\ 3 \end{bmatrix}$$

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```
In [10]: # define the coefficient matrix A
a = np.array([[1, 3, 5], [2, 5, 1], [2, 3, 8]])
print(a)

[[1 3 5]
 [2 5 1]
 [2 3 8]]

In [12]: # Define the right hand side vector to b
b = np.array([10], [8], [3])
print (b)

[[10]
 [ 8]
 [ 3]]

In [16]: # Calculate the inverse of A and multiply by b to find out the solution further
c = sc.linalg.inv(a).dot(b)
print (c)

[[-5.28]
 [ 5.16]
 [ 0.76]]

In [18]: # solve function to find the solution
d = sc.linalg.solve(a, b)
print (d)

[[-5.28]
 [ 5.16]
 [ 0.76]]
```

Linear Algebra Execution

```
In [23]: a = np.array([[13, 2], [1, 2]])
print (a)

[[13  2]
 [ 1  2]]

In [37]: b = np.array([1], [0])
print (b)

[[1]
 [0]]

In [39]: # solve function to find the solution
d = sc.linalg.solve(a, b) # longer version but dont need to import for every little function
print (d)

[[ 0.5 ]
 [-0.25]]

In [47]: solution = solve(a, b)
solution

Out[47]: array([[ 0.5 ],
                [-0.25]])

In [43]: # ways to resolve
from scipy.linalg import solve

In [45]: solution = solve (a, b) # shorter function but need to import for every little function available in library
solution

Out[45]: array([[ 0.5 ],
                [-0.25]])
```

Solving ODE using Runge-Kutta method

The Runge–Kutta method [\[ edit \]](#)

The most widely known member of the Runge–Kutta family is generally referred to as “RK4”, the “classic Runge–Kutta method” or simply as “the Runge–Kutta method”.

Let an [initial value problem](#) be specified as follows:

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0.$$

Here  $y$  is an unknown function (scalar or vector) of time  $t$ , which we would like to approximate; we are told that  $\frac{dy}{dt}$ , the rate at which  $y$  changes, is a function of  $t$  and of  $y$  itself. At the initial time  $t_0$  the corresponding  $y$  value is  $y_0$ . The function  $f$  and the initial conditions  $t_0, y_0$  are given.

Now pick a step-size  $h > 0$  and define

$$y_{n+1} = y_n + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4),$$
$$t_{n+1} = t_n + h$$

for  $n = 0, 1, 2, 3, \dots$  using<sup>[2]</sup>

$$k_1 = f(t_n, y_n),$$
$$k_2 = f\left(t_n + \frac{h}{2}, y_n + h \frac{k_1}{2}\right),$$
$$k_3 = f\left(t_n + \frac{h}{2}, y_n + h \frac{k_2}{2}\right),$$
$$k_4 = f(t_n + h, y_n + h k_3).$$

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$$k_3 = f\left(t_n + \frac{h}{2}, y_n + h \frac{k_2}{2}\right),$$
$$k_4 = f(t_n + h, y_n + h k_3).$$

```
# differential eq: dy/dx = (x-y)/2 given: x0, y0, h(step size)

In [81]: # def differential equation
def dydx(x, y):
    return (x-y)/2

In [91]: def rungeKutta(x0, y0, x, h):
    n = int((x-x0)/h)
    y = y0
    for i in range(1, n+1):
        k1 = h * dydx(x0, y)
        k2 = h * dydx(x0 + 0.5*h, y+0.5*k1)
        k3 = h * dydx(x0 + 0.5*h, y+0.5*k2)
        k4 = h * dydx(x0 + h, y + k1)
        # update value of y
        y = y + (1/6)*(k1 + 2*k2 + 2*k3 + k4)
        # update value of x
        x0 = x0 + h
    return y

In [93]: # driver method
x0 = 0
y = 1
x = 5
h = 0.2
print ("The value of y at x is:", rungeKutta(x0, y, x, h))

The value of y at x is: 3.246794758464962
```

Pressure profile in vessel (simulation)

Mass balance in the vessel:

$$\dot{n}_{in} \rightarrow \boxed{\frac{dn}{dt}} \rightarrow \dot{n}_{out}$$

$$\frac{dn}{dt} = \dot{n}_{in} - \dot{n}_{out}$$

$$PV = nRT$$

$$\frac{d\left(\frac{PV}{RT}\right)}{dt} = \dot{n}_{in} - \dot{n}_{out}$$

$$\frac{V}{RT} \frac{dP}{dt} = \dot{n}_{in} - \dot{n}_{out}$$

$$\frac{dP}{dt} = \frac{\dot{n}_{in} - \dot{n}_{out}}{\frac{V}{RT}}$$

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$$\frac{dP}{dt} = \frac{\dot{n}_{in} - \dot{n}_{out}}{\frac{V}{RT}}$$

```
In [96]: # import library
import matplotlib.pyplot as plt

In [100]: # parameter values for original pressure
V = 1154 #m^3
R = 8.314 # J/(mol*K)
T = 120 # degree celcius

mass_in = 473220 # kg/h
MW_in = 56.6
mass_out = 28742 # kg/h
MW_out = 40.4

In [105]: # conversion and calculation
Tk = T + 273.15 # convert to Kelvin
mol_in = mass_in/MW_in/60 # kmol/min
mol_out = mass_out/MW_out/60 # kmol/min

In [107]: # Initial condition
P0 = 1830 #kPa

In [109]: # differential equation using ideal gas
def dpdt(p, t):
    dpdt = (mol_in - mol_out)/(V/(R*Tk))
    return dpdt

In [111]: # time to response is 40 minutes
start = 0
end = 30
t = np.linspace(start, end, end)
Pinitial = np.linspace(P0, P0, end)

In [115]: # integrate the differential equation over 30 minutes to compute the pressure
p = sc.integrate.odeint(dpdt, P0, t)
```

In [125]: # Plot the result

```
In [127]: # zombie apocalypse modeling
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
plt.ion()
plt.rcParams['figure.figsize'] = 10, 8

# P = 0 # birth rate
d = 0.0001 # natural death percent (per day)
B = 0.0095 # transmission percent (per day)
G = 0.0001 # resurrect percent (per day)
A = 0.0001 # destroy percent (per day)

# solve the system dy/dt = f(y, t)
def f(y, t):
    S1 = y[0]
    Z1 = y[1]
    R1 = y[2]
    # the model equations (see Munz et al. 2009)
    f0 = P - B*S1*Z1 - d*S1
    f1 = B*S1*Z1 + G*R1 - A*S1*Z1
    f2 = d*S1 + A*S1*Z1 - G*R1
    return (f0, f1, f2)

# initial conditions
S0 = 500 # initial population
Z0 = 0 # initial zombie population
R0 = 0 # initial death population
y0 = [S0, Z0, R0] # initial condition vector
t = np.linspace(0, 5, 1000) # time grid

# solve the ODEs
soln = odeint(f, y0, t)
S = soln[:, 0]
Z = soln[:, 1]
R = soln[:, 2]

# plot results
plt.figure()
plt.plot(t, S, label='Living')
plt.plot(t, Z, label='Zombies')
plt.xlabel('Days from outbreak')
plt.ylabel('Population')
plt.title('Zombie Apocalypse - No Init. Dead Pop.; No New Births.')
plt.legend(loc=0)

# change the initial conditions
R0 = 0.01*S0 # 1% of initial pop is dead
y0 = [S0, Z0, R0]

# solve the ODEs
soln = odeint(f, y0, t)
S = soln[:, 0]
Z = soln[:, 1]
R = soln[:, 2]

plt.figure()
plt.plot(t, S, label='Living')
plt.plot(t, Z, label='Zombies')
plt.xlabel('Days from outbreak')
plt.ylabel('Population')
plt.title('Zombie Apocalypse - 1% Init. Pop. is Dead; No New Births.')
plt.legend(loc=0)

# change the initial conditions
R0 = 0.01*S0 # 1% of initial pop is dead
P = 10 # 10 new births daily
y0 = [S0, Z0, R0]

# solve the ODEs
soln = odeint(f, y0, t)
S = soln[:, 0]
Z = soln[:, 1]
R = soln[:, 2]

plt.figure()
plt.plot(t, S, label='Living')
plt.plot(t, Z, label='Zombies')
plt.xlabel('Days from outbreak')
plt.ylabel('Population')
plt.title('Zombie Apocalypse - 1% Init. Pop. is Dead; 10 Daily Births')
plt.legend(loc=0)

Out[127]: <matplotlib.legend.Legend at 0x2074b7f9b60>
```

