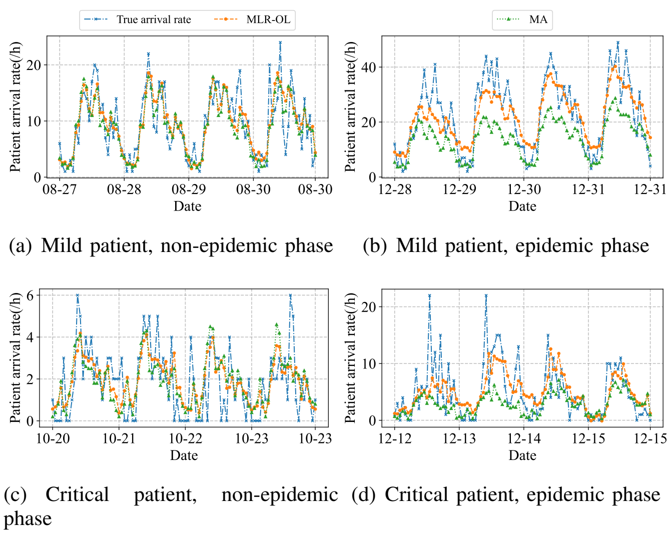
Dynamic Physician Allocation for Coping with Time-Varying Multi-Level Patient Demand in the Emergency Department Online Appendix

APPENDIX I
PREDICTION PERFORMANCE OF MRL-OL AND MA MODELS

Fig. 1 visualizes true arrival rates and prediction results for selected dates (including both epidemic and non-epidemic periods), further validating the advantages of the MLR-OL model in capturing the complex trends of patient arrival rates.



(a) Mild patient, non-epidemic phase (b) Mild patient, epidemic phase

(c) Critical patient, non-epidemic phase (d) Critical patient, epidemic phase

Fig. 1. Comparison of arrival rate prediction performance of MLR-OL and MA models

APPENDIX II
RAW DATA ANALYSIS

A. Analysis of Patient Arrival Rate

To analyze whether the patient arrival rate in each period follows an exponential distribution with stable parameters, we theoretically need to conduct statistical tests for each hour of each day. However, due to limited data volume, high-precision daily testing is not feasible. Considering the weekly cyclical fluctuations in hospital patient arrival rates, we conduct statistical analysis on the patient arrival intervals for each period of the week, with a period length of one hour, thereby obtaining patient inter-arrival time data for different periods.

The Kolmogorov-Smirnov (KS) test was used to examine whether the arrival intervals per hour follow an exponential distribution. The null hypothesis states that the arrival intervals per hour follow an exponential distribution, whereas the alternative hypothesis states that they do not. Based on the non-epidemic and epidemic phases, consulting room categories (resuscitation room and general room), day-of-week information, and hour-of-day, a total of 672 sets of patient arrival rate data were obtained (2 phases, 2 consulting rooms, 7 days, 24 hours). The test results show that the p-values of 539 sets of data are greater than the 0.05 significance leve, failing to reject the null hypothesis, indicating that the arrival intervals in these intervals have a high confidence level of following an exponential distribution. The corresponding p-values, test statistics, and degrees of freedom (df) are detailed in tables I and II. The remaining 143 sets of data are mainly concentrated in periods with lower patient arrival rates during the epidemic phase, where the sample size is small, and a significant number of arrival intervals are close to one hour. Since the Kolmogorov-Smirnov (KS) test is less effective with insufficient sample size or when a large proportion of data fall in the tail region, it may fail to accurately identify the distribution characteristics of these data.

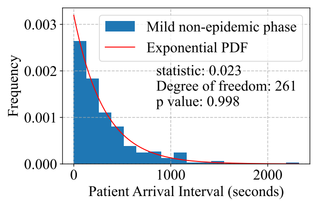
TABLE I
KS TEST PARAMETERS FOR PATIENT ARRIVAL INTERVALS IN SOME PERIODS OF GENERAL ROOM

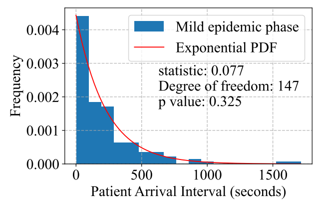
Period	Non-Epidemic Phase			Epidemic Phase		
	Statistic	DF	p Value	Statistic	DF	p Value
Mon 12-13	0.023	261	0.998	0.077	147	0.325
Mon 18-19	0.07	200	0.263	0.124	115	0.052
Wed 13-14	0.046	259	0.615	0.074	165	0.306
Fri 16-17	0.06	227	0.378	0.08	133	0.333
Sun 11-12	0.034	211	0.958	0.072	161	0.349

TABLE II
KS TEST PARAMETERS FOR PATIENT ARRIVAL INTERVALS IN SOME PERIODS OF RESUSCITATION ROOM

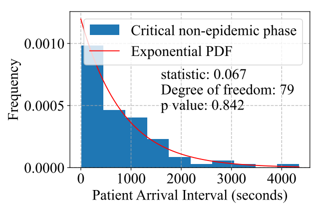
Period	Non-Epidemic Phase			Epidemic Phase		
	Statistic	DF	p Value	Statistic	DF	p Value
Mon 12-13	0.067	79	0.842	0.073	40	0.968
Mon 18-19	0.139	40	0.374	0.092	36	0.881
Wed 13-14	0.145	59	0.147	0.109	40	0.679
Fri 16-17	0.088	46	0.832	0.166	25	0.428
Sun 11-12	0.14	55	0.202	0.108	41	0.674

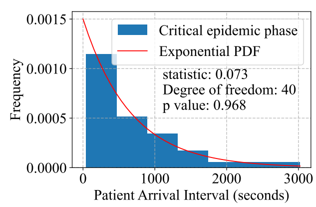
To further validate the fitting results, Fig. 2 presents the histograms and fitted probability density curves of patient arrival intervals from 12:00 to 13:00 on Monday in the general room and resuscitation room during the non-epidemic and epidemic phases. The histograms are largely consistent with the theoretical distribution curves, indicating that the exponential distribution can adequately describe the distribution characteristics of patient arrival intervals in most cases. This result further supports the conclusions of the KS test.





(a) Mild patients, non-epidemic phase (b) Mild patients, epidemic phase





(c) Critical patients, non-epidemic phase (d) Critical patients, epidemic phase

Fig. 2. Histograms of patient arrival intervals from 12:00 to 13:00 on Monday.

B. Physician Shift Plan

TABLE III
PHYSICIAN SHIFT PLANS FOR BOTH NON-PANDEMIC AND PANDEMIC PHASES INSTANCES

Instance	Date	0–7	7–11	11–13	13–17	17–21	21–22	22–24
11-20	2022/7/1∼11/30	3	5	5	5	5	4	3
21	2022/12/25	4	11	8	11	8	8	6
22	2022/12/26	5	12	9	12	9	9	6
23	2022/12/28	5	13	10	13	10	10	6
24	2022/12/29	5	15	12	15	12	12	5
25	2022/12/30	5	14	11	14	11	11	5
26	2023/1/15	4	9	6	9	6	6	4
27	2023/1/17	4	9	6	9	6	6	4
28	2023/1/18	3	9	6	9	6	6	4
29	2023/1/24	3	7	4	7	4	4	3
30	2023/1/25	3	7	4	7	4	4	3

C. Calculation Method of Physicians' Newly Arrived Patient Service Rate

To estimate the physician service rate, we need to count the number of newly arrived patients served by each physician in each time period. Since physician shifts usually start on the hour or half-hour, we start from 0:00 and count the number of newly arrived patients served by physicians in half-hour intervals, such as 0:00–0:30, 0:30–1:00, etc. In shorter time periods (e.g., half an hour), situations may arise where a physician is on duty but has not served any patients. To ensure data accuracy, we need to clearly define the start and end times of physician shifts. Physicians in the ED are divided into regular duty physicians and temporary support physicians, and their shift times need to be determined separately: for regular duty physicians, they usually work a full shift (e.g., morning, afternoon, or night shift), and their shift start times are determined according to the shift schedule; for temporary support physicians, most are temporarily scheduled from other consulting rooms, and their duty time in the current consulting room is usually less than 1 hour, so the time period during which they provide services to patients is used as their shift start and end times.

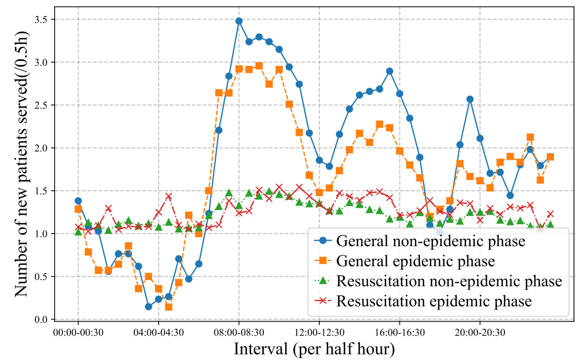


Fig. 3. Number of newly arrived patients served by physicians

Based on the above method, we counted the number of newly arrived patients served by the general and resuscitation rooms during non-epidemic and epidemic phases, as shown in Fig. 3. From Fig. 3, we can observe that the newly arrived patient service rate of the general room exhibits three peaks, corresponding to the start times of the morning, afternoon, and night shifts. In practice, this can be explained by the fact that at the beginning of each physician's shift, the proportion of newly arrived patients served is highest because reflux patients have not yet accumulated; as time progresses, the number of reflux patients gradually increases, and the service rate decreases accordingly. In contrast, the rescue room does not show significant peak periods, but the number of patients served during the daytime is significantly higher than at night, which may be related to the urgency of conditions in the rescue room. To reflect the average work efficiency of physicians during their shifts, we use the mean number of newly arrived patients served per hour as the estimated value of the physician service rate.

APPENDIX III
HEURISTIC POLICY UNDER THE GENERALIZED $c\mu$ RULE

The generalized $c\mu$ rule depends on the definition of its cost function. In (1), we define the cost function and determine the actions taken by the policy under different states according to the generalized $c\mu$ rule. If the policy in each time period is determined by the generalized $c\mu$ rule, we can obtain the policy set $\pi_{gc\mu}$ under the generalized $c\mu$ rule.

$$
\begin{aligned}
\pi_{gc\mu}(q_0, q_1) &= \begin{cases} \min\{q_0, n\}, & c_0'\mu_0 \geq c_1'\mu_1 \\ m - \min\{q_1, n\}, & otherwise \end{cases} \\
c_i' &= \begin{cases} c_i, & q_i \leq \bar{q}_i \\ c_i + p_i & q_i > \bar{q}_i \end{cases}, \; i = 0, 1
\end{aligned} \tag{1}
$$

APPENDIX IV
STATE-BY-STATE POLICY ITERATION METHOD

This method starts from a given policy π, for each state s, each possible action a is evaluated to generate a new policy π'. If π' is better than π, then π is updated to π'. This process is repeated for each state, and iterations continue until the policy remains unchanged between two consecutive iterations. Since the policy reward decreases after each update, the algorithm is guaranteed to converge to a local optimum. The specific process is shown in Algorithm 1.

Algorithm 1 State-by-State Policy Iteration Method

```
Step 0: Input: π_0, t, w_t, k ← 1, K
Step 1: Policy Improvement
π_k ← π_{k−1}
for s ∈ S do
   for a ∈ A do
      π′ ← π_k, π′(s) ← a
      if R_t(w_t, π′) < R_t(w_t, π_k) then
         π_k(s) ← a
      end if
   end for
end for
Step 2: Termination Condition
if π_k = π_{k−1} or k ≥ K then
   return π_k
end if
k ← k + 1, return to Step 1
```