INFO 6105
Data Science Engineering Methods and Tools

Lecture 8
Classification Trees

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$$y_i \in \{\mathcal{C}_1, \ldots, \mathcal{C}_K\}.$$

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  - criterion for splitting nodes. Instead of minimizing the RSS, we minimize a classification loss function.
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- We predict the response by majority vote, i.e., pick the most common class in every region.

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- The class probability distribution in region R_j is:

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Classification losses

- Misclassification rate:

$$\sum_{j=1}^{|T|} q_j \sum_{i:x_j \in R_j} \mathbf{1}(y_i \neq \hat{y}_{R_j})$$

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- The Gini index

$$\sum_{j=1}^{|T|} q_j \sum_{k=1}^{K} \hat{p}_{jk} \left(1 - \hat{p}_{jk}\right)$$

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- It is typical to use the Gini index or cross-entropy for growing the tree, while using the misclassification rate when pruning the tree.

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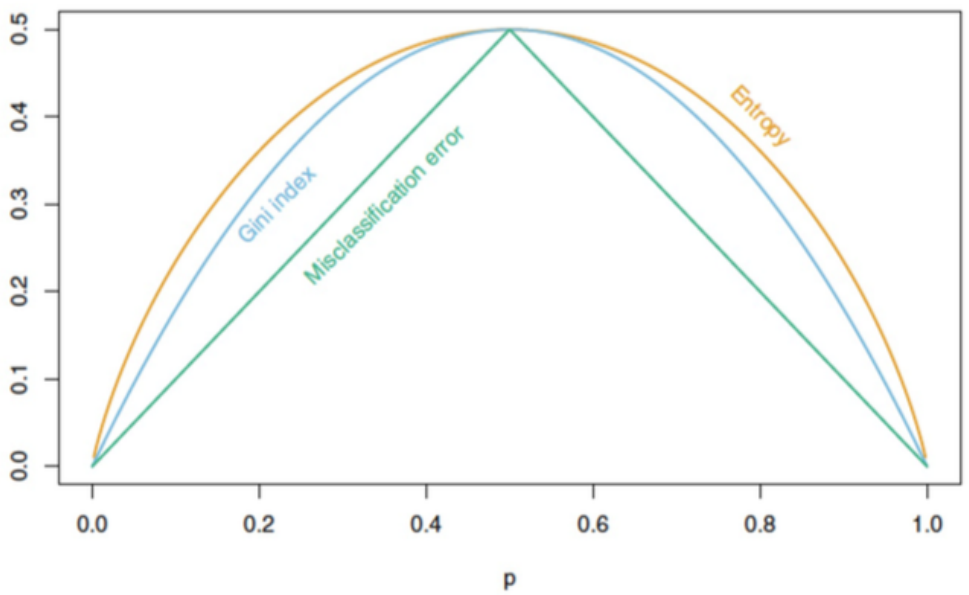

Example

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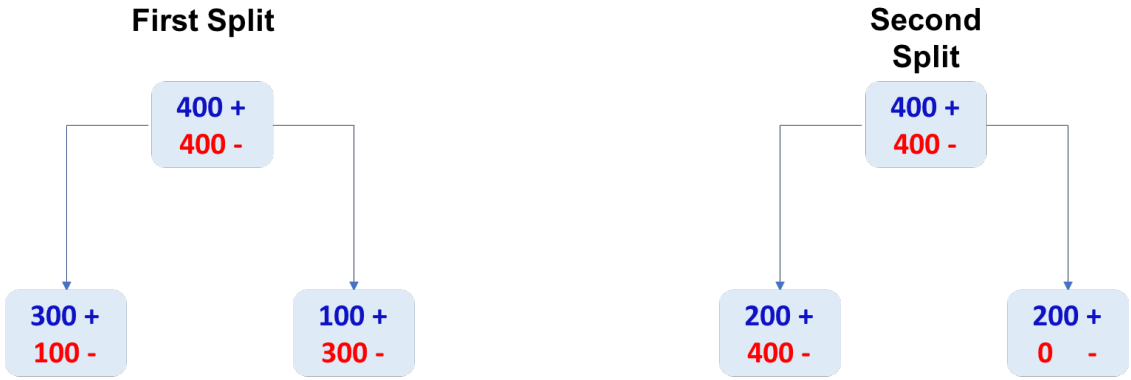

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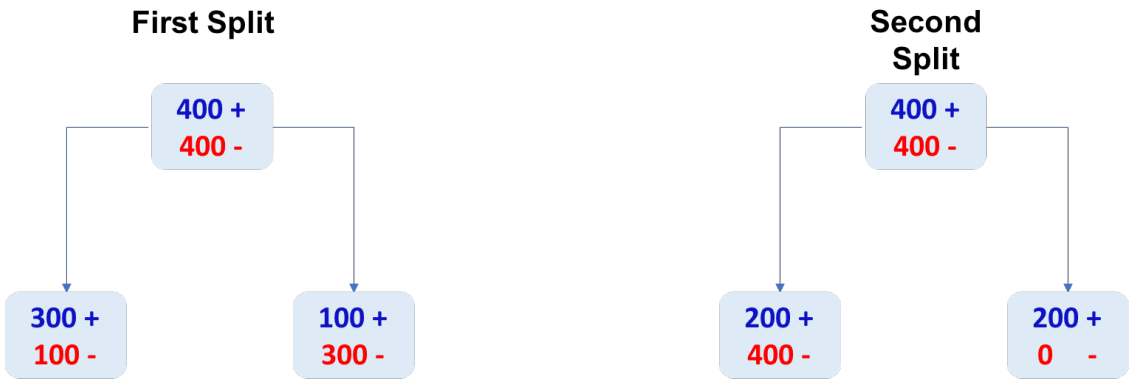

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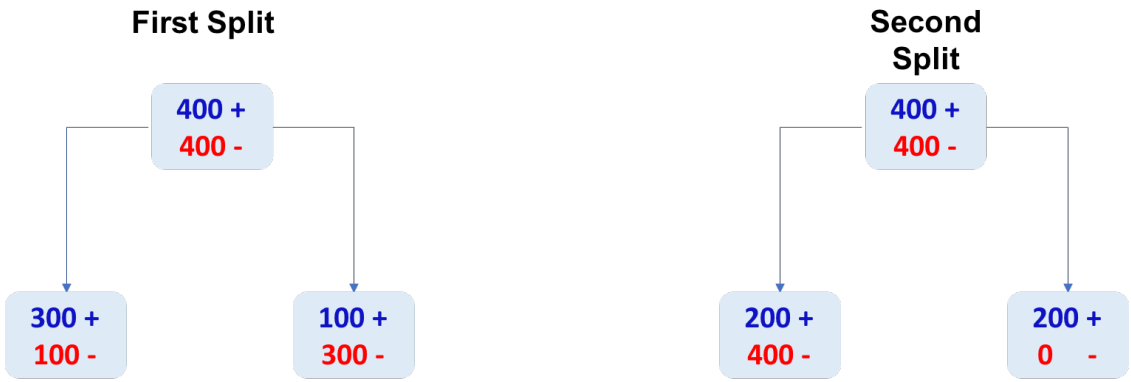

- Both splits produce a misclassification rate of 0.25, but the second split produces a pure node and is probably preferable.
- Both the Gini index and cross-entropy are lower for the second split. For this reason, either the Gini index or cross-entropy are typically used when growing the tree.

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- We select at each step the splitting variable j and the split point s that maximizes $\Delta(j,s)$ or, equivalently, that minimizes the average impurity

$$\frac{N_{t_L}}{N_t}Q_{t_R} + \frac{N_{t_L}}{N_t}Q_{t_R}$$

Categorical predictors

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  - We order the predictor values according to the proportion falling in positive class.
  - Then we split this predictor as if it were an ordered predictor.
  - One can show this gives the optimal split, in terms of cross-entropy or Gini index, among all possible $2^{q-1} - 1$ splits.

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- the number of partitions grows exponentially in q,
- the more choices we have, the more likely we can find a good one for the data at hand
- This can lead to severe overfitting if q is large, and such variables should be avoided

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- Unfortunately, trees generally do not have the same level of predictive accuracy as some of the other modern regression and classification approaches.