Machine Learning and Data Sciences

INFO 6105, Fall 2019

QUIZ 2, SATURDAY OCT 11, 2019

Answer the questions in the spaces provided on the question sheets. If you run out of space for an answer, continue on the back of the page. Write your name below and in the top right corner of every page after the first.

Name: ____________________

1. (Total: 15 points)

   For a classification problem, we are given predicted probability scores as

i	y_i(True Class)	$h_\beta(x_i)$(Score)
1	1	0.90
2	1	0.80
3	0	0.75
4	1	0.70
5	1	0.65
6	0	0.60
7	1	0.60
8	0	0.55
9	1	0.45
10	1	0.40
11	0	0.30
12	0	0.25
13	1	0.15
14	0	0.10
15	0	0.05

   (a) (1 point) What is the accuracy for Threshold = 0.5?

   Solution:

$$\begin{aligned} \text{Accuracy} = &\ \text{fraction of observations that are classified correctly} \\ = &\ \frac{9}{15} = 0.6 \end{aligned}$$

   (b) (2 points) What is TP for Threshold = 0.5?

Solution:

$$\begin{aligned} \text{TP Rate} &= \text{fraction of positive observations that are classified correctly} \\ &= \frac{5}{8} = 0.625 \end{aligned}$$

(c) (2 points) What is FP for Threshold = 0.5?

Solution:

$$\begin{aligned} \text{FP Rate} &= \text{fraction of negative observations that are misclassified} \\ &= \frac{2}{7} = 0.286 \end{aligned}$$

(d) (5 points) Estimate AUC.

Solution: Remember that AUC can be approximated as the probability that the classifier will rank a randomly positive example higher than a randomly chosen negative example. That is,

$$\begin{aligned} \text{AUC} &\approx P\big(\text{score}(X^+) \geq \text{score}(X^-)\big) \\ &\approx \frac{U}{|N_+| \cdot |N_-|} \end{aligned}$$

where

- N_+ is the set of positive examples
- N_- is the set of negative examples
- $U = \sum_{X_i \in N_+} \sum_{X_j \in N_-} [\text{score}(X_i) > \text{score}(X_j)]$

Hence,

$$\begin{aligned} \text{AUC} &\approx \frac{U}{|N_+| \cdot |N_-|} \\ &= \frac{7+7+6+6+5+4+4+2}{8*7} = \frac{41}{56} = 0.73 \end{aligned}$$

(e) (5 points) Estimate Log Loss.

Solution: Remember that the log loss is given by

$$-\frac{1}{n} \sum_i [y_i \log(\hat{p}_i) + (1 - y_i) \log(1 - \hat{p}_i)]$$

where y_i is the actual response and $\hat{p}_i$ i the predicted score.
Thus, the log loss is

$$-\frac{1}{15} \times (-9.09) = 0.606$$

Please note that the logarithm used is the natural logarithm (base-e).