CSE 586A Problem Set 3

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1. Part a

1.1 Problem Description

The goal of this part is to write a function that computes a mean point set and aligns all other face point sets to it using a similarity transformation model for this following form

$$\Phi(x, y) = s \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

Where $s > 0$ denotes scale, θ is a rotation angle and t_x and t_y are the x and y components for the translation respectively.

1.2 Solution

I follow these steps to solve this problem and compute a mean point set.

(1) Set the mean to the first point set $x^{\mu} - x^1$

(2) Align all x^i to x^{μ} by minimizing

$$E(s, \theta, t_x, t_y) = \sum_{j=1}^{M} \| \begin{pmatrix} x^{\mu} \\ y^{\mu} \end{pmatrix} - s \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x_j^i \\ y_j^i \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \end{pmatrix} \|^2$$

To obtain { $\hat{x}^1, \ldots, \hat{x}^M$ }.

(3) Calculate the new mean as

$$x^{\mu} = \frac{1}{N} \sum_{i=1}^{N} \hat{x}^i$$

(4) Align x^{μ} to x^1, and use it as the new x^{μ}

(5) Goto (2) unless x^{μ} has converged.

In order to check whether x^{μ} has converged, I need to compute the values of energy function. The method that I use to compute the energy is mentioned before.
First, I compute the energy for every single point in every point set through the

$$E(s, \theta, t_x, t_y) = \sum_{j=1}^{M} \| \begin{pmatrix} x^{\mu} \\ y^{\mu} \end{pmatrix} - s \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x_j^i \\ y_j^i \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \end{pmatrix} \|^2$$

Then I add all single energy together, and get the total energy for this problem.

$$E_{total} = \sum_{j=1}^{M} E(s, \theta, t_x, t_y)$$

Then I draw the plot of energy function. When the energy has converged, iterations that is implemented to align all points to can stop.

1.3 Experimental Details

At first, I am not sure when will converge, so I set the initial iterations number is 10, which means that align process will implement 10 times. I compute the energy every time I align all points to , so I get 10 energy values, and then I plot these energy value out to check whether has converged.

1.4 Experimental Results

After aligning all other point sets to a mean point set, I draw all faces in one figure. The result is shown in Figure 1.

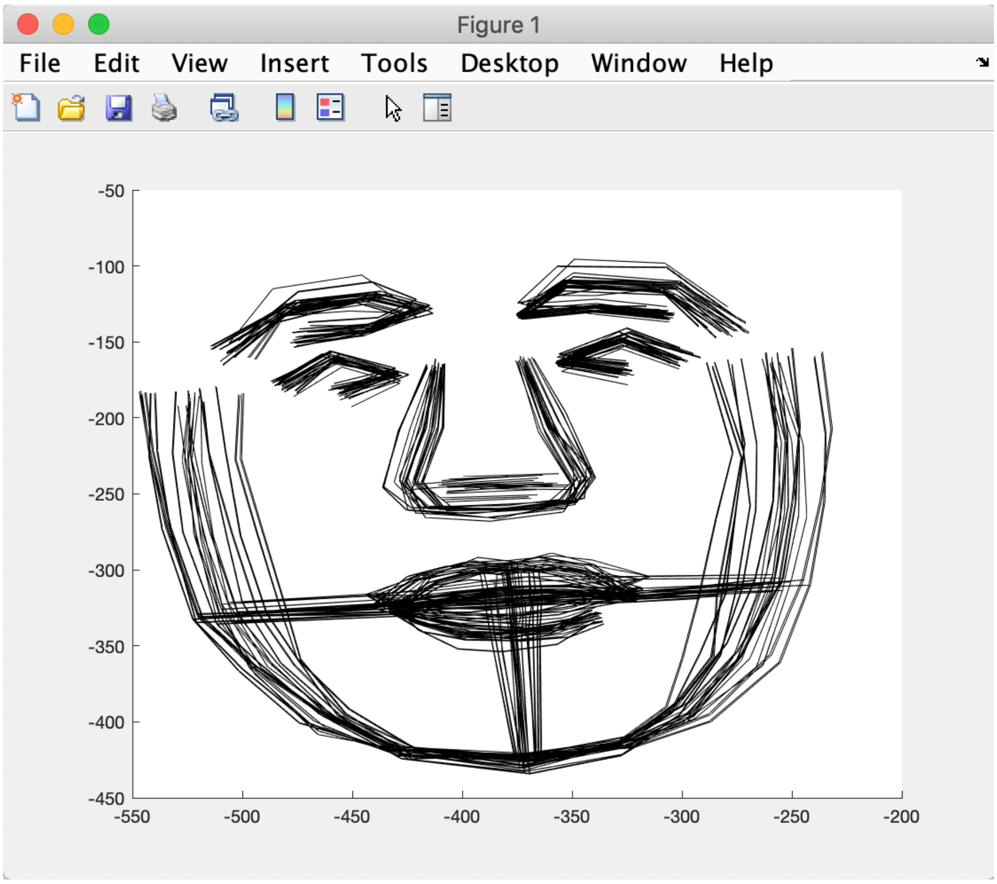

Figure 1. Aligned Faces

I plot the energy function to check whether the value of mean point sets has converged. The plot of energy function is shown in Figure 2.

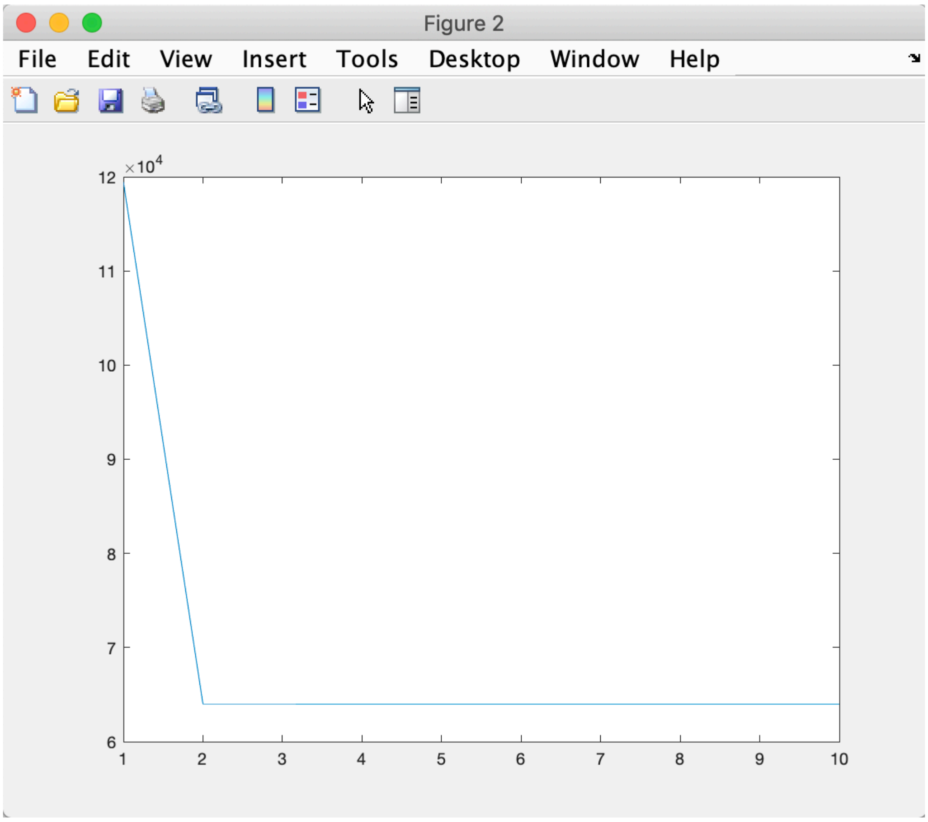

Figure 2. Energy Function

1.5 Discussion

From the Figure 1, it is clear that points from all point sets are almost aligned together, which means that faces are nearly overlapped. From the Figure 2, it is clear that energy function converges after the first iteration, so x^{μ} converges after the first iteration. Therefore, I can stop updating the mean point set and aligning all point sets to this mean point set after the first iteration. Although I set the iteration number is 10, it is no need to update the mean point set 10 times, because the x^{μ} has converged before the second iteration.

2. Part b

2.1 Problem Description

Write a function to extract the three most important shape variations using principal component analysis. Show the results by plotting shape variations with respect to these three principle components independently by varying between two standard deviations around the mean.

2.2 Solution

Step 1: Get original Data

The original data that I use is provided by the Prof. Zhang. All data are put in the folder, dat.

Step 2: Subtract the mean and re-center the data

I compute the mean point set, and the method is same with the part a.

(1) Set the mean to the first point set $x^{\mu} - x^{1}$

(2) Align all x^{i} to x^{μ} by minimizing

$$E(s,\theta,t_x,t_y) = \sum_{j=1}^{M} \| \begin{pmatrix} x^{\mu} \\ y^{\mu} \end{pmatrix} - s \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x_j^i \\ y_j^i \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \end{pmatrix} \|^2$$

To obtain { $\hat{x}^1, \ldots, \hat{x}^M$ }.

(3) Calculate the new mean as

$$x^{\mu} = \frac{1}{N} \sum_{i=1}^{N} \hat{x}^i$$

(4) Align x^{μ} to x^{1}, and use it as the new x^{μ}

(5) Goto 2 unless x^{μ} has converged.

I align all point sets to this mean point set, and the data type of original point sets and mean point set is 68×2 double. I reshape the type of every original point sets and mean point set to 136×1 double. After that, I need to "re-center" the data so that the mean is zero. The method to accomplish it is to subtract the mean $\hat{\mu}$ from each sample vector $\hat{x}_i$. I center these original data sets at mean point set with the form

$$x_{center} = x^i - x^{\mu}$$

Let X be the $m \times n$ matrix whose ith column is $\hat{x}_i - \hat{\mu}$.

$$X = | \hat{x}_1 - \hat{\mu} | \ldots | \hat{x}_n - \hat{\mu} |$$

Then I combine these point set together, and get the input data with 21×136 double.

Step 3: Calculate the covariance matrix

To apply principle component analysis, I need to compute the covariance matrix. I use the cov function to compute it. Since the data is 2 dimensional, the covariance matrix will be 2×2.

$$S = \frac{1}{n-1} X X^T$$

Step 4: Calculate the EigenVector and EigenValue of the covariance matrix
After that, I use eigs function to compute the EigenVector and EigenValue of the covariance matrix.

Step 5: Choose components and calculate the standard deviation
Because the result of EigenValue has been sorted in a descend order, I select the first value, the second one and the third one from the array of EigenValue, and get corresponding EigenVector. When I get the value of selected EigenValues and corresponding EigenVectors, I compute the standard deviation and varying its value with this following form:

$$sd = n * \sqrt{EigenValue} * EigenVector$$

Required by the problem, principle components have to vary between two standard deviations around the means, so I set n is equal to 2,0,-2.

Step 6. Derive the new data set
After I calculate the value of sd, I derive the new data set from the mean point set following the form.

$$x_{new} = sd + \mu$$

After that, I reshape the data type of new point set to 68×2 double, and then draw the faces based on the new point set.

2.3 Experimental Details
After I select three principle components, I compute the standard deviation. In order to show result face by plotting shape variations, I need to change values of principle components by varying between two standard deviations around the mean. Therefore, I show three result faces by compute mean plus the two times of standard deviations, mean and mean subtract two times of standard deviations.

I use function eigs to compute the EigenVector and EigenValue. At first, I choose the function eig to compute EigenVector and EigenValue, but because I need to select three largest values from EigenVector and EigenValue, I sort EigenValue in the descend order, and then I select the first, second and third values. However, I cannot find the corresponding EigenVector after I sort the EigenValue. In order to solve this problem, I choose to use eigs function.

To compute the EigenVector and EigenValue, I need to input all point sets together. The type of data in every point set is 68×2 double, and then after I align this point set to the mean point sets, I convert it to 136×1 double. Because there are total 21 point sets, the type of input data is 21×136 double.

2.4 Experimental Results

I select the first principle component, compute standard deviations and draw result faces. The results are shown in Figure 3, Figure 4 and Figure 5 respectively.

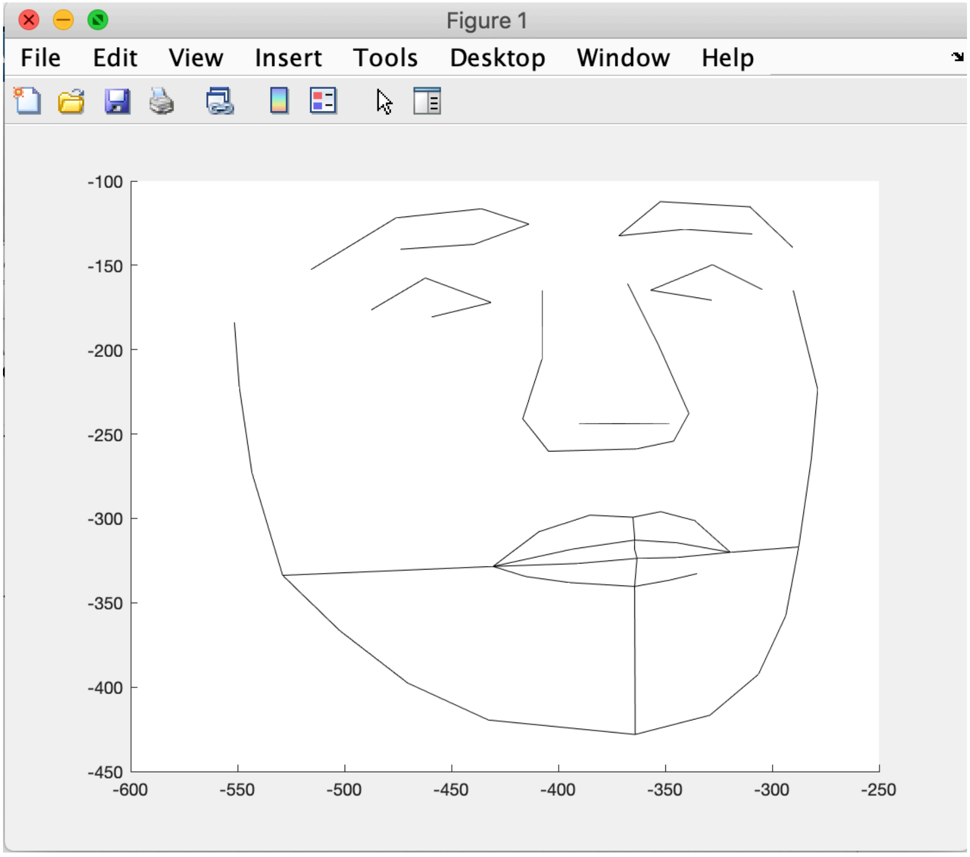

Figure 3. The first component with 2 standard deviation

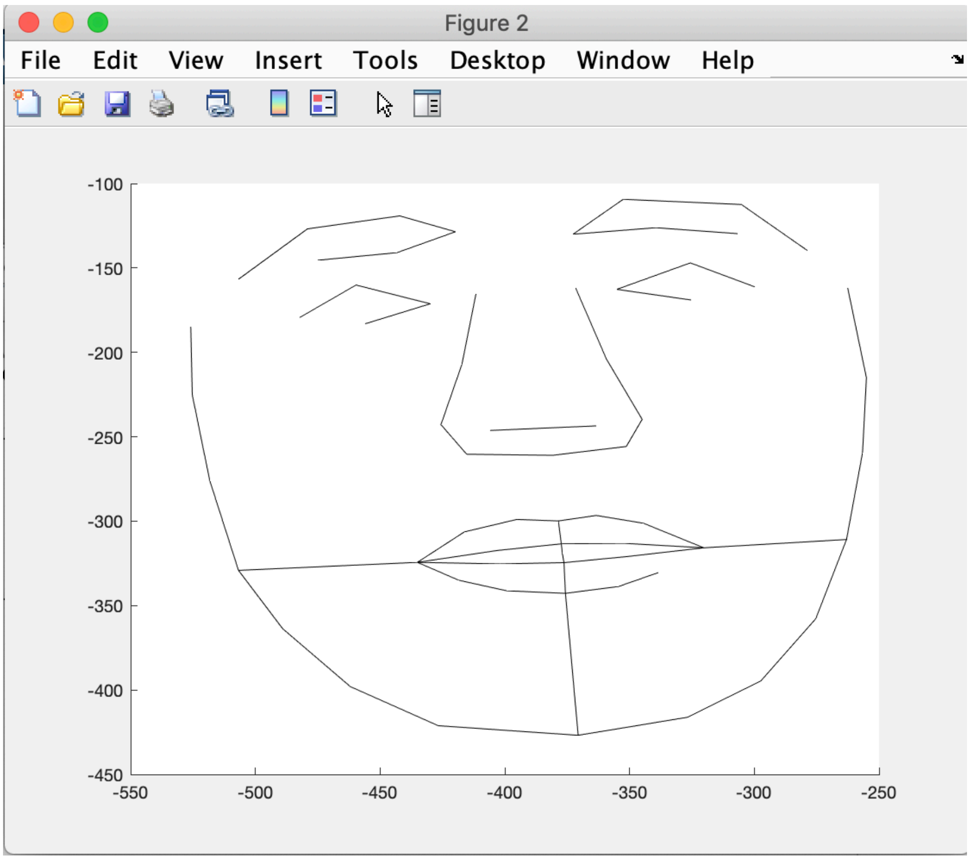

Figure 4. The first component with 0 standard deviation

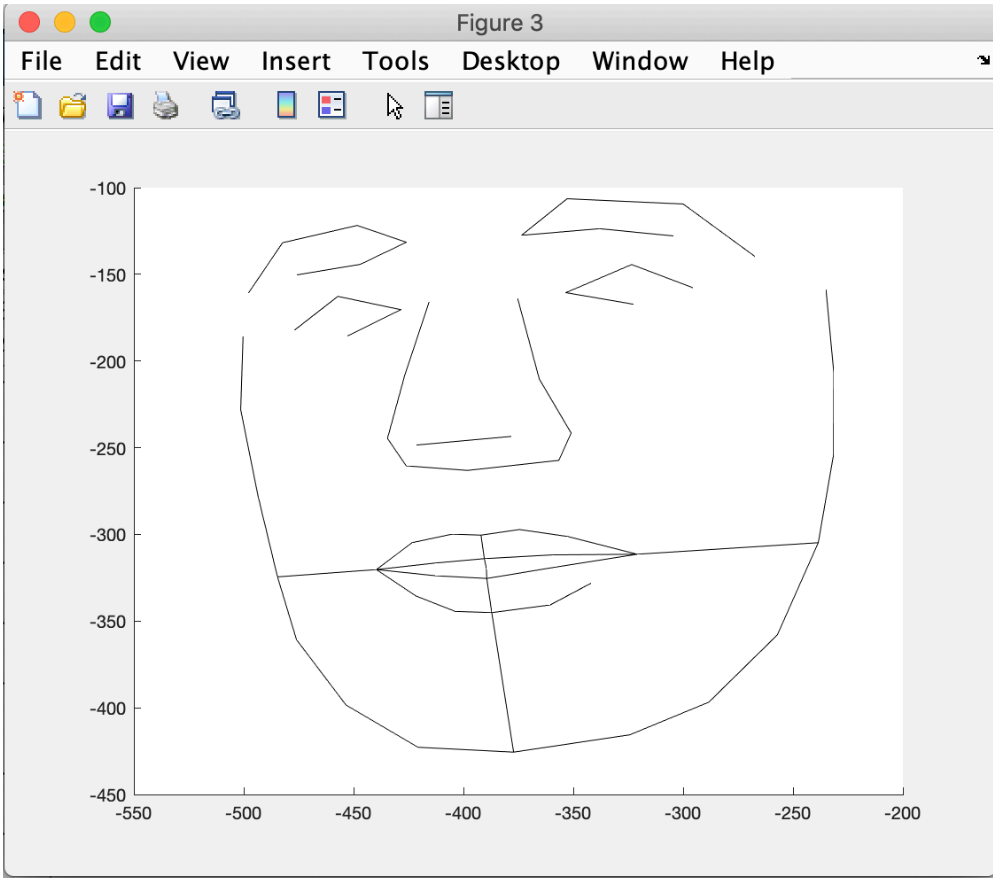

Figure 5. The first component with -2 standard deviation

Then I select the second principle component, compute standard deviations and draw result faces. The results are shown in Figure 6, Figure 7 and Figure 8 respectively.

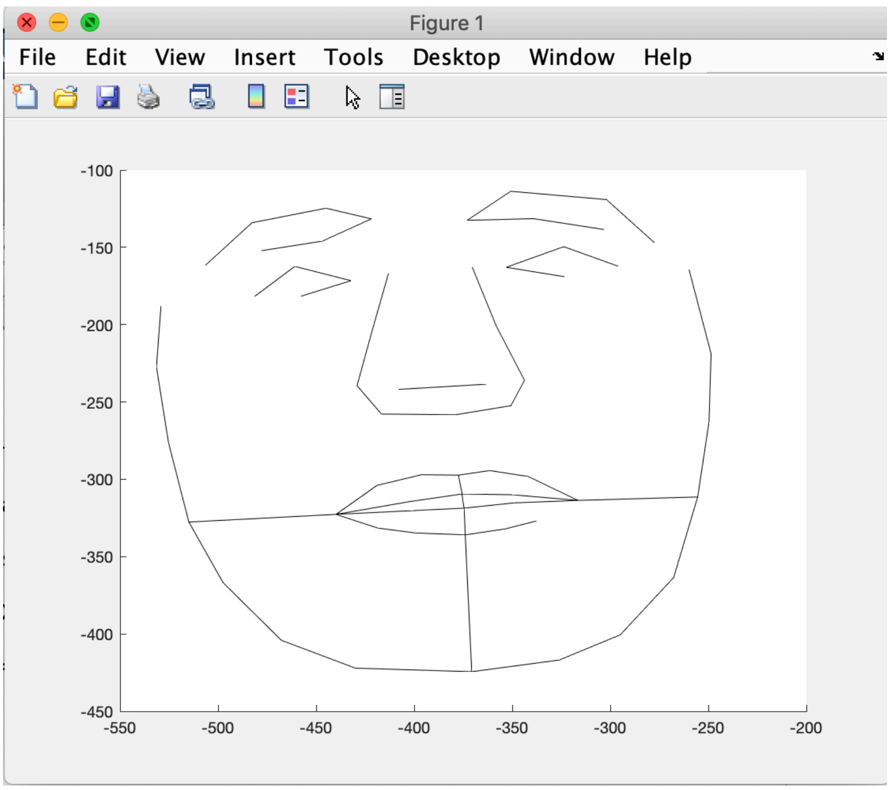

Figure 6. The second component with 2 standard deviation

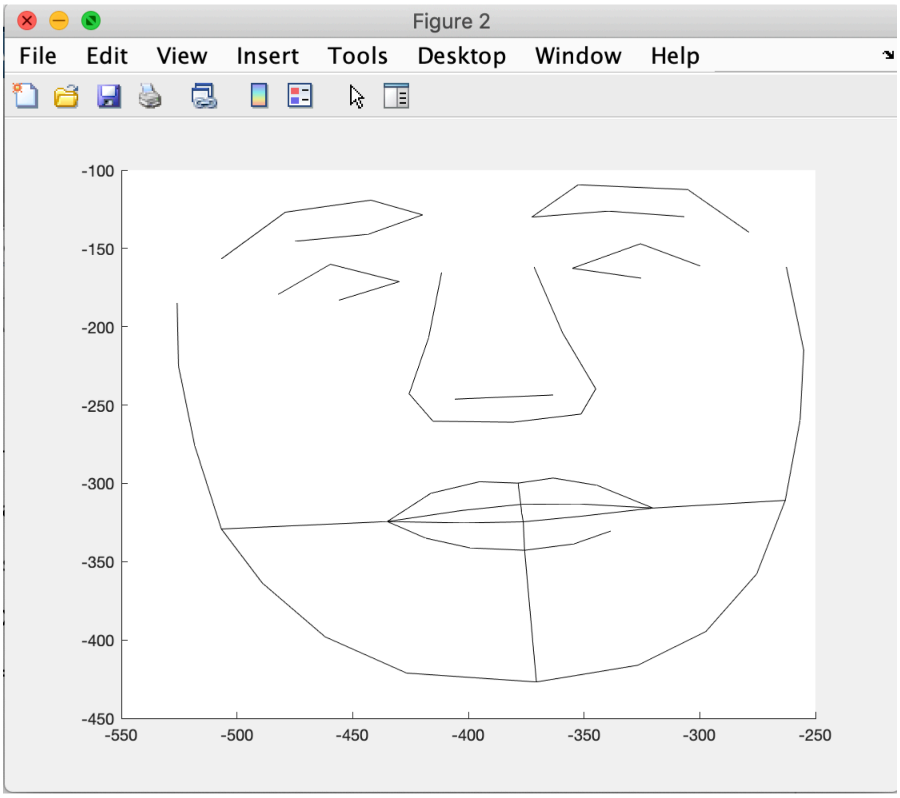

Figure 7. The second component with 0 standard deviation

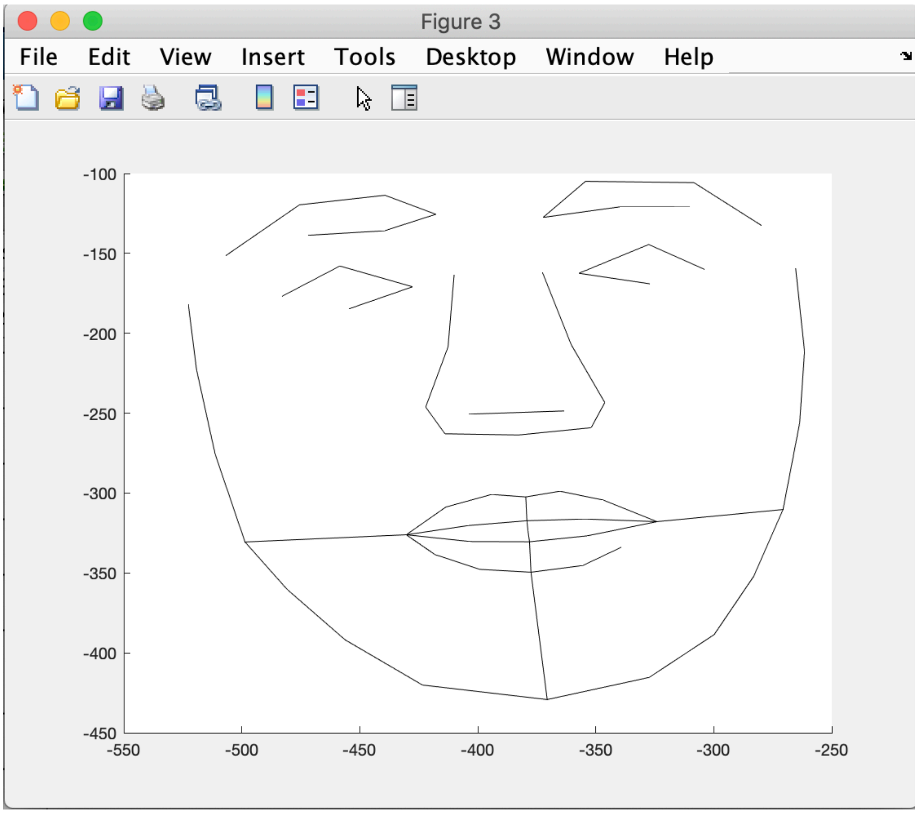

Figure 8. The second component with -2 standard deviation

Finally, I select the third principle component, compute standard deviations and draw result faces. The results are shown in Figure 9, Figure 10 and Figure 11 respectively.

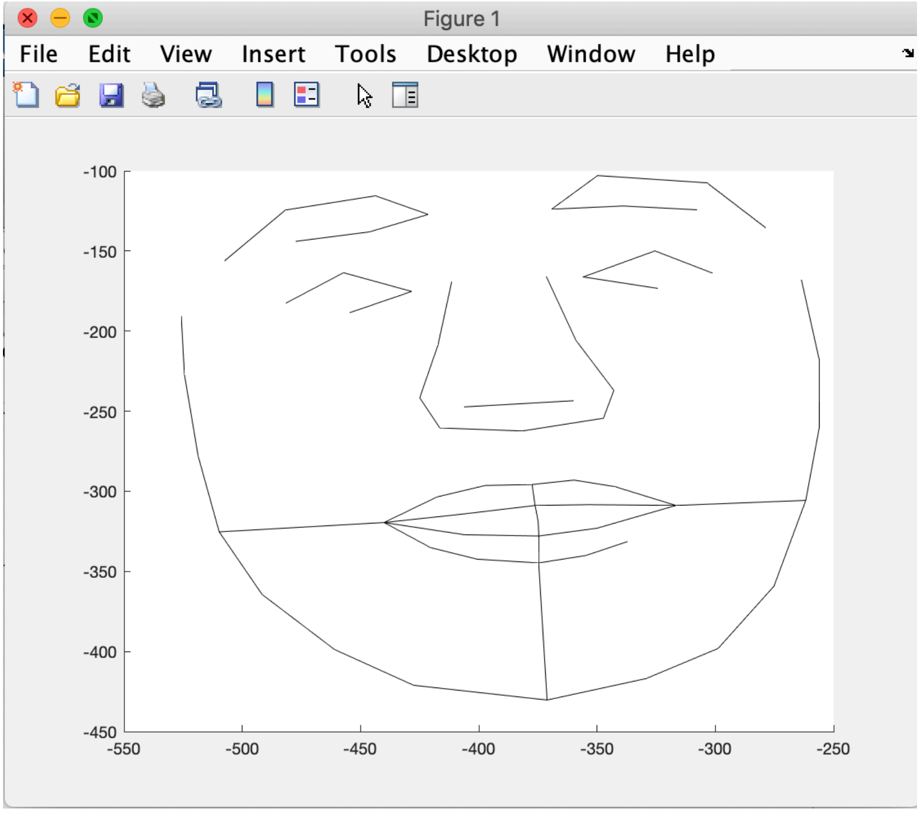

Figure 9. The third component with 2 standard deviation

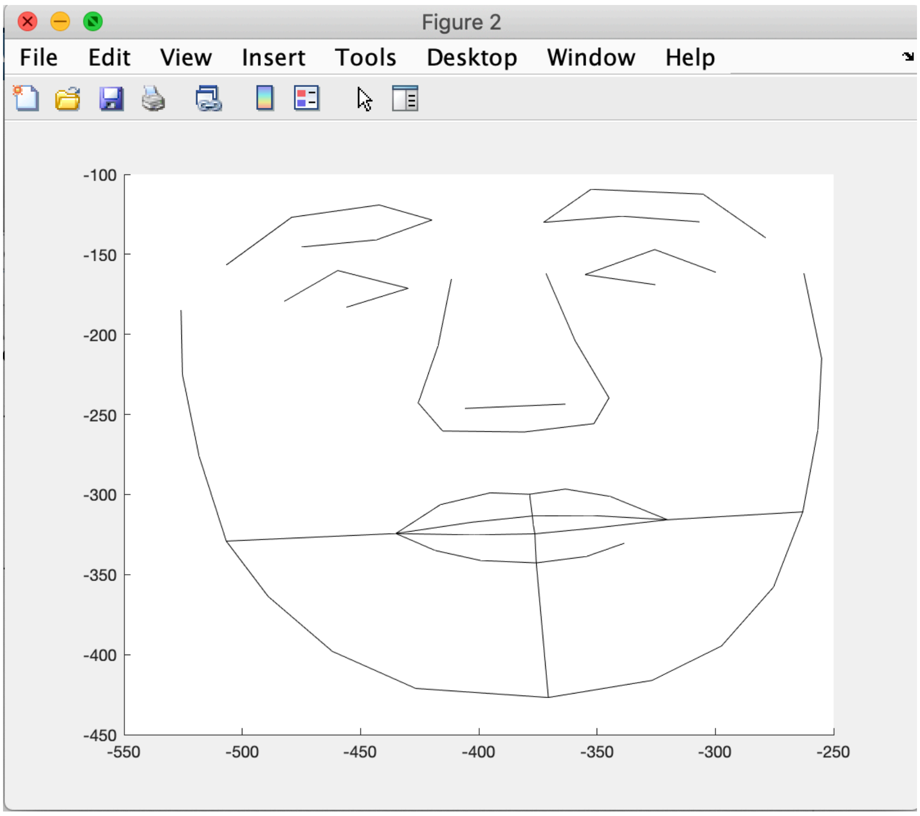

Figure 10. The third component with 0 standard deviation

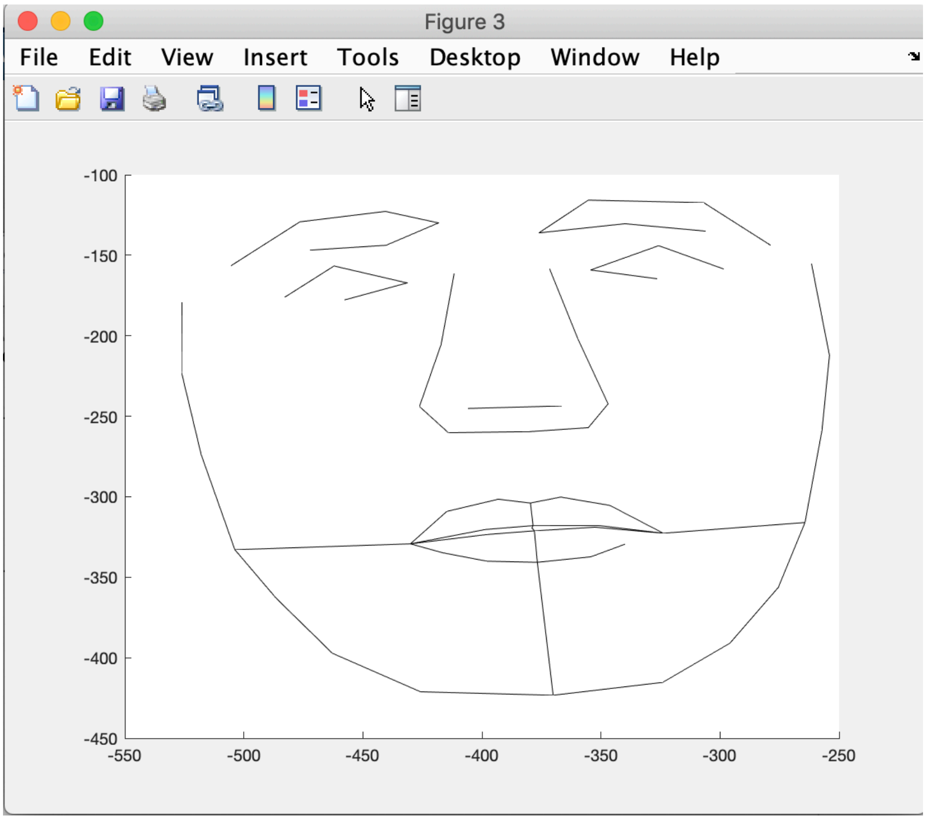

Figure 11. The third component with -2 standard deviation

2.5 Discussion

Through comparing three faces drawn from the same principle component with varied standard deviations, I can find the difference among these three faces. If the differences are not obvious, or it is a little hard to find the differences, I can use a larger value of standard deviations, and then the differences will be more obvious.