1. Introduction

Continuous signal $x(t)$ can be modeled as a polynomial curve of degree $p-1$ in additive noise

$$x(t) = \theta_1 + \theta_2 t + \ldots + \theta_p t^{p-1} + w(t)$$

$w(t)$ is represented as white Gaussian noise.

I assume that I can perform polynomial curve fitting based on samples $\{x(t)\}_{n=0}^{N-1}$, then I can extract these parameters from the model.

$$x = [x(t_0), \ldots, x(t_{N-1})]^T$$

$$w = [w(t_0), \ldots, w(t_{N-1})]^T$$

$$\theta = [\theta_1, \ldots, \theta_p]^T$$

$$H = \begin{bmatrix} 1 & t_0 & . & t_0^{p-1} \\ 1 & t_1 & . & t_1^{p-1} \\ . & . & . & . \\ 1 & t_{N-1} & . & t_{N-1}^{p-1} \end{bmatrix}_{N\times p}$$

Therefore, the signal $x(t)$ can be modeled mathematically as followed.

$$\begin{pmatrix} x(t_0) \\ x(t_1) \\ . \\ x(t_{N-1}) \end{pmatrix} = \begin{bmatrix} 1 & t_0 & . & t_0^{p-1} \\ 1 & t_1 & . & t_1^{p-1} \\ . & . & . & . \\ 1 & t_{N-1} & . & t_{N-1}^{p-1} \end{bmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ . \\ \theta_p \end{pmatrix} + \begin{pmatrix} w(t_0) \\ w(t_1) \\ . \\ w(t_{N-1}) \end{pmatrix}$$

For the polynomial models, the matrix H has a special form, and this matrix is called Vandermonde matrix.

2. Method

Since original data are not provided, I generated my own original data samples through a common way before I designed and created polynomial models to fit it, I. The ideas and methods that I used in this experiment are borrowed from problem 5 in assignment 3.

I generated noiseless data samples from the function $y = cos(2\pi x)$, and then I created white Gaussian noise term $w(t)$ which will be added to the original data samples. In detail, I generated a random sample using the model $y = cos(2\pi x) + w(t)$. The noise term can calculated from Gaussian distribution with zero mean and 0.2 covariance.The number of data points vary from part to part. To present properties of the polynomial curve fitting function, I try to explore it based on three problem. The first problem is that how the number of dimension will influence the performance of fitting function; the next one is that whether the number of data points which can used to fit will influence the performance of polynomial function. The third one is that whether regularizer is a satisfying method to avoid overfitting and how the regularizers will make contributions to the performance of polynomial function.

After figuring out tasks in the project, I try to design my experiment and split it to three parts to solve these three problems respectively. In the first part, I will change the dimension of fitting function without changing other parameters; I try to only change the number of data that I used to generate fitting curve in the next part; finally, regularizers are applied and implemented in the fitting models and the performance will be compared with the model without regularizer to make sure whether regularizers will help the model avoid overfitting.

3. Experiment

3.1 Dimension Exploration

For the first part, I changed the dimension of fitting models, which means that after I generated original data points, I tried various polynomial curve models with different dimensions.

(1) Polynomial model with dimension 1

The first model that I tried to fit these data points is the function with dimension 1, which is linear model. $y_1 = \theta_1 + \theta_2 x$. The result is shown in Figure 1.

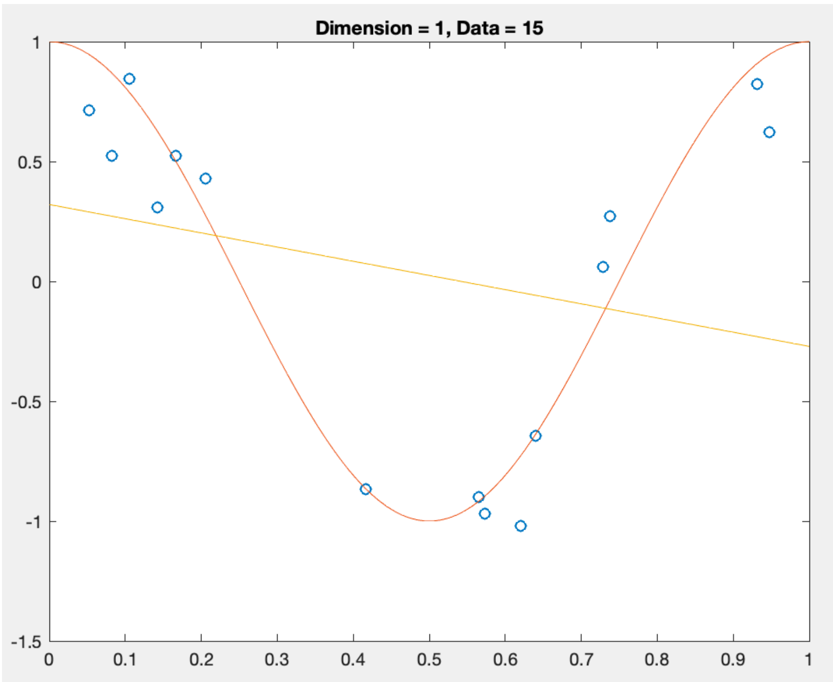

Figure 1

(2) Polynomial model with dimension 2

The second model that I tired to fit these data points is the function with dimension 2, which can be presented as followed. $y_2 = \theta_1 + \theta_2 x + \theta_3 x^2$. The result is shown in Figure 2.

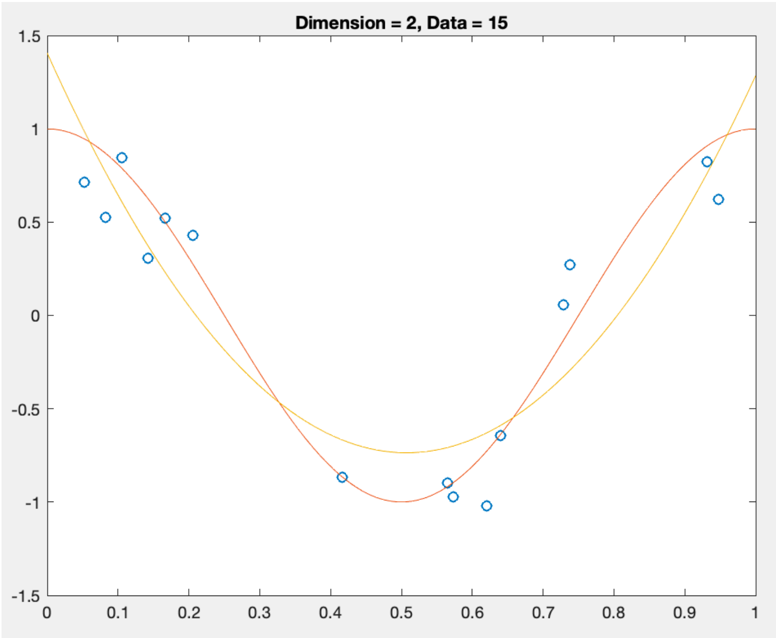

Figure 2

(3) Polynomial model with dimension 3

The third model that I tried to fit these data point is the function with dimension 3, which can be presented as followed. $y_3 = \theta_1 + \theta_2 x + \theta_3 x^2 + \theta_4 x^3$. The result is shown in Figure 3.

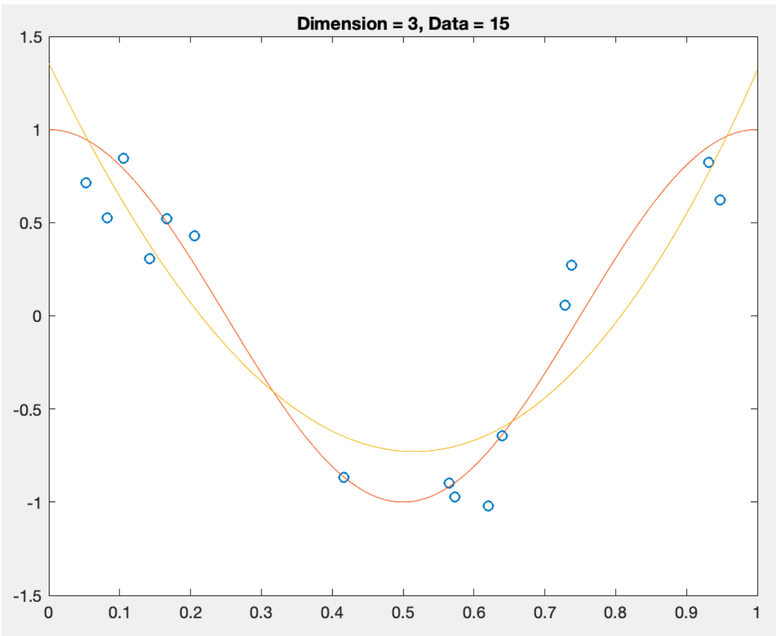

Figure 3

(4) Polynomial model with dimension 5

The final model that I tried to fit these data point is the function with dimension 8, which can be presented as followed. $y_4 = \theta_1 + \theta_2 x + \theta_3 x^2 + \theta_4 x^3 + \theta_5 x^4 + \theta_6 x^5$. The result is shown in Figure 4.

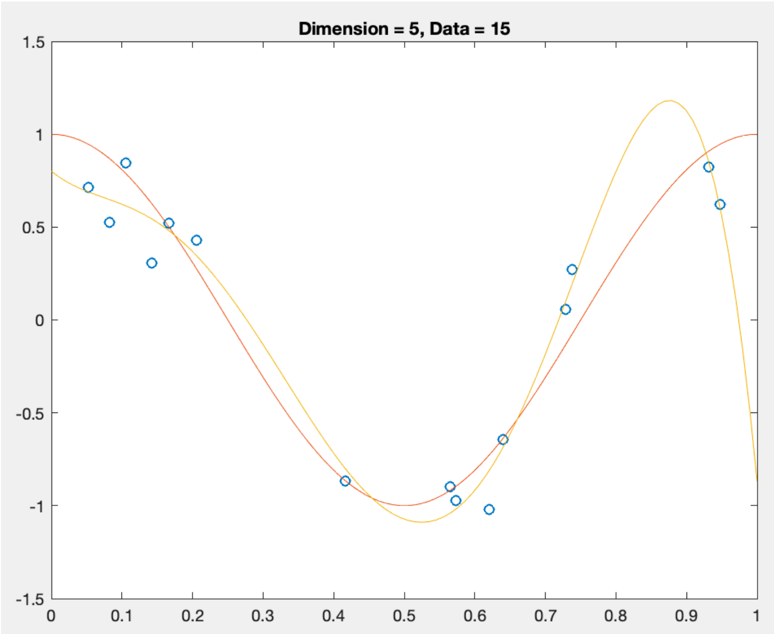

Figure 4

(5) Polynomial model with dimension 9

The final model that I tried to fit these data point is the function with dimension 8, which can be presented as followed.

$y_4 = \theta_1 + \theta_2 x + \theta_3 x^2 + \theta_4 x^3 + \theta_5 x^4 + \theta_6 x^5 + \theta_7 x^6 + \theta_8 x^7 + \theta_9 x^8$. The result is shown in Figure 5.

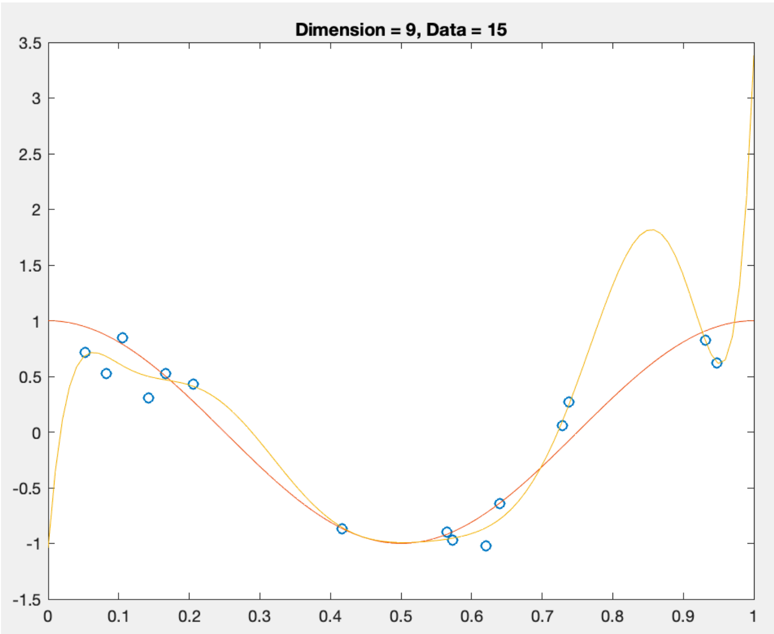

Figure 5

3.2 Data Number Exploration
In this part, I change the number of data which are applied to fit the polynomial model. I tried the number is 5, 10, 50 and 100 respectively.

(1) Data Number 10
The number of data points in the first part is only 5, and then I fit model through these data points. The result is shown in Figure 6.

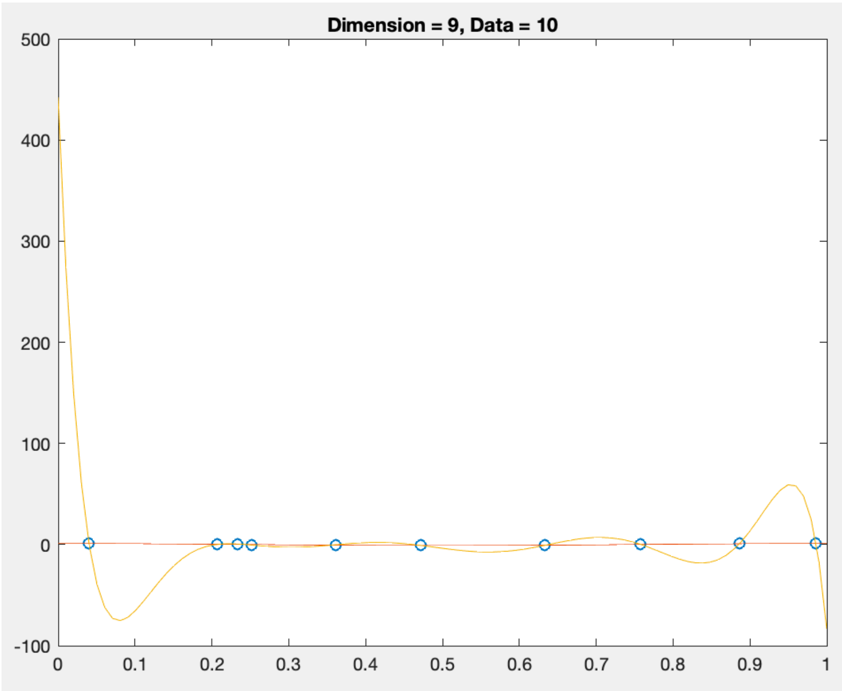

Figure 6

(2) Data Number 20
The number of data points in the first part is 10, and then I fit model through these data points. The result is shown in Figure 7.

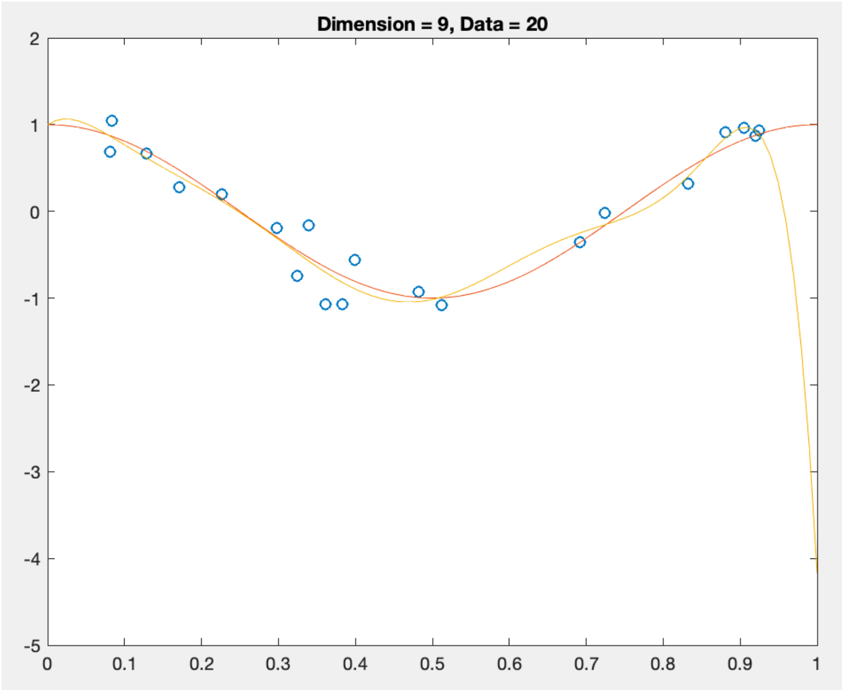

Figure 7

(3) Data Number 50
The number of data points in the first part is 50, and then I fit model through these data points. The result is shown in Figure 8.

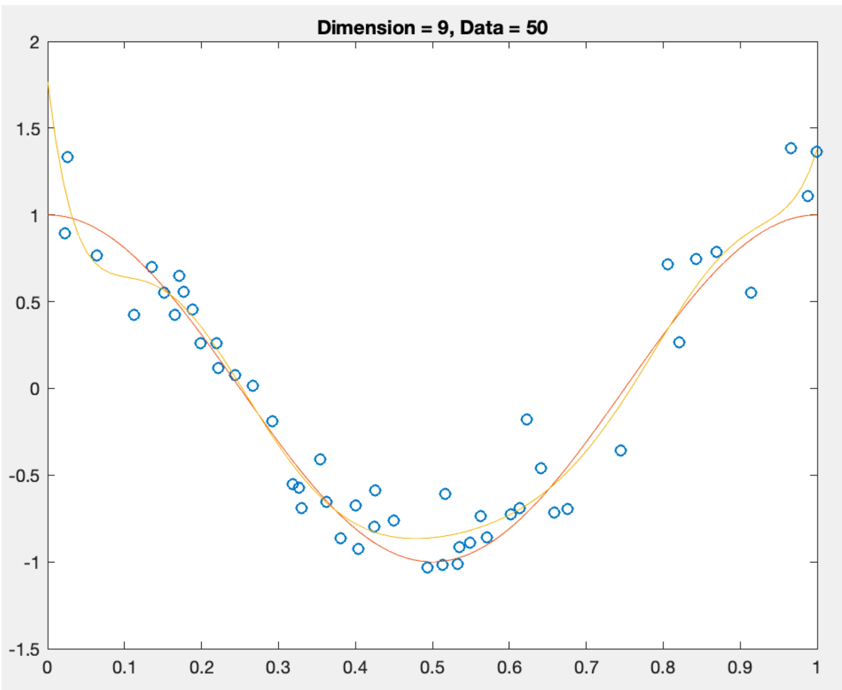

Figure 8

(4) Data Number 100

The number of data points in the first part is 100, and then I fit model through these data points. The result is shown in Figure 9.

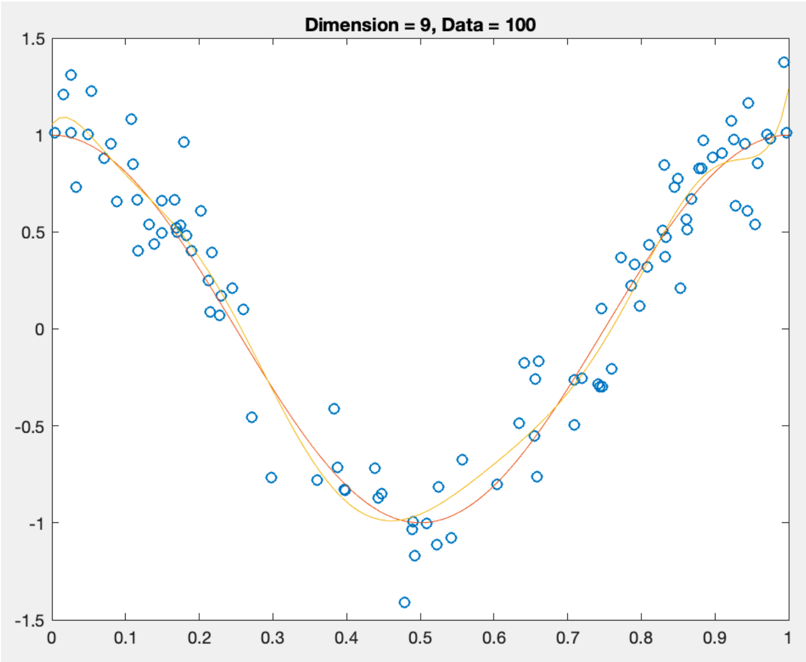

Figure 9

3.3 Regularizer Exploration

(1) No Regularizer

First, I try to fit the polynomial model without using any regularizer. The result is shown Figure 10.

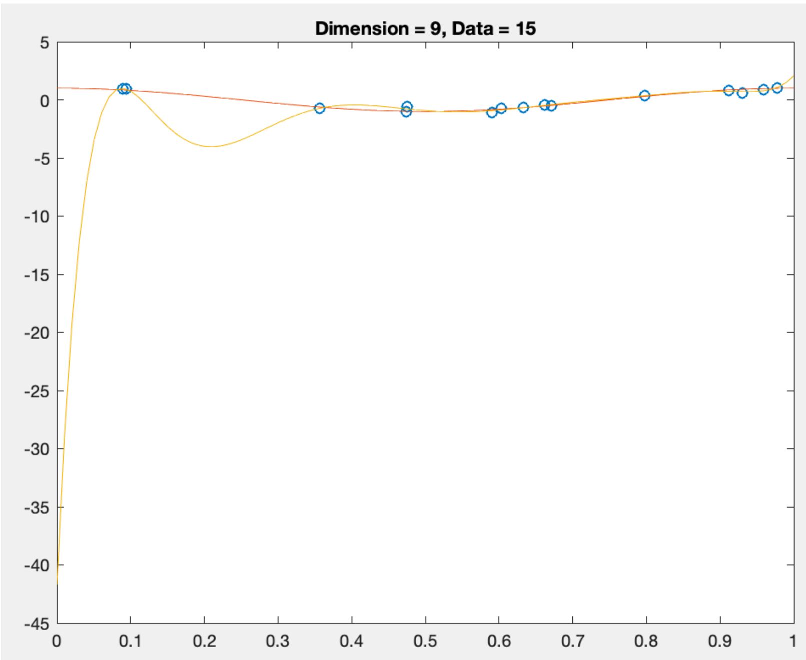

Figure 10

(2) Regularizer Coefficient lambda 0.001
Second, I try to fit polynomial model with the application of regularizer, and the coefficient that I set in the part is 0.001. The result is shown in Figure 11.

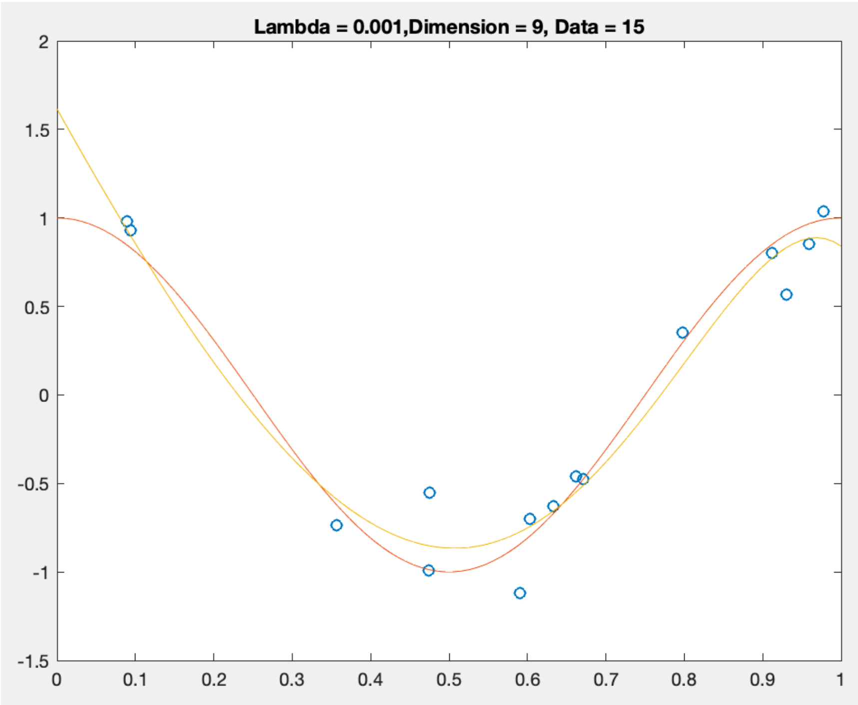

Figure 11

(3) Regularizer Coefficient lambda 0.1
Third, I try to fit polynomial model with the application of regularizer, and the coefficient that I set in the part is 0.1. The result is shown in Figure 12.

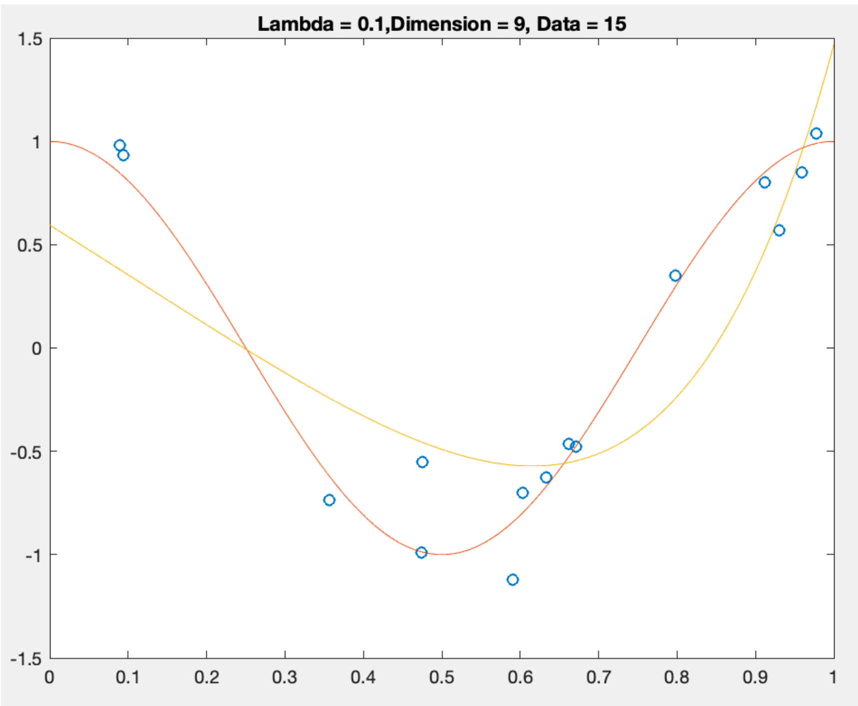

Figure 12

(4) Regularizer Coefficient lambda 1
Second, I try to fit polynomial model with the application of regularizer, and the coefficient that I set in the part is 0.001. The result is shown in Figure 13.

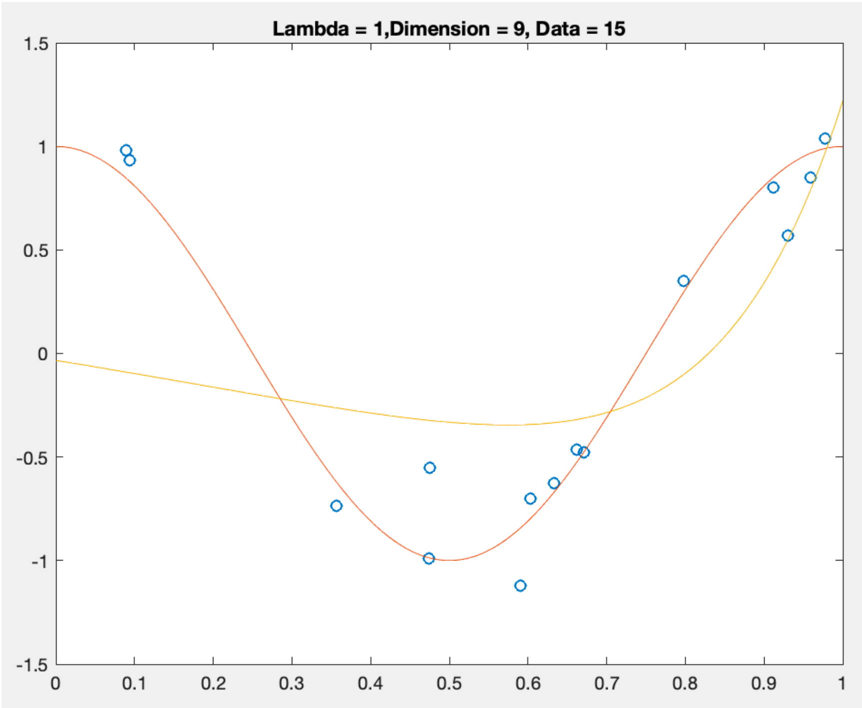

Figure 13

4. Reference

[1] ESE 524 Estimation and Detection Theory 2020 Spring, Lecture 13.pdf
[2] ESE 524 Estimation and Detection Theory 2020 Spring, Lecture Notes 09_linear_model.pdf
[3] https://www.mathworks.com/help/matlab/ref/polyval.html
[4] https://www.mathworks.com/help/matlab/math/polynomial-curve-fitting.html
[5] https://en.wikipedia.org/wiki/Vandermonde_matrix