PARUL UNIVERSITY - FACULTY OF ENGINEERING & TECHNOLOGY

Department of Applied Science & Humanities

1st Year B.Tech Programme

Mathematics – II (303191151)

Assignment 1-(HODE & LAPLACE TRANSFORM)

1. Solve the initial value problem $y'' - 9y = 0\,, y(0) = 2, y'(0) = -1$
2. Solve $(D^3 - 9D^2 + 8D - 4)\, y = 0$
3. Solve $(D^4 + 1)y = 0$
4. Solve $(D^2 - 2D + 3)\, y = x^2 + \sin x$
5. Solve $(D^2 - 3D + 2)\, y = x\, e^{2x} + \sin x$
6. Solve $(D^2 + 3D + 2)\, y = e^{e^x}$
7. Solve $(D^2 - 3D + 2)\, y = \frac{e^x}{1+e^x}$
8. Solve $(D^2 + 4)\, y = \cot 2x$
9. Solve $x^2y'' - 3xy' + 5y = x^2 \sin(logx)$
10. Solve $x^2y'' - 3xy' + 3y = 2 + 3\, logx$
11. Find the Laplace transform of $f(t) = \begin{cases} \cos t\,, 0 < t < 2\pi \\ 0, t > 2\pi \end{cases}$
12. Find the Laplace transform of $f(t) = t \quad 0 < t < \frac{1}{2}$
$= t - 1 \quad \frac{1}{2} < t < 1$
$= 0 \quad t > 1$
13. Find the Laplace transform of the following functions:
    1. $f(t) = \left(\sqrt{t} - 1\right)^2$
    2. $f(t) = sin2t\ sin3t$
    3. $f(t) = (\sin 2t - \cos 2t)^2$
    4. $f(t) = \cos(\omega t + b)$
    5. $f(t) = \cos t \cos 2t \cos 3t$
14. Find the Laplace transform of the following functions:
    1. $f(t) = \cosh^3 t$
    2. $f(t) = e^{\frac{5}{2}t} + 4t^3 - \sin 2t - \cos 3t$
    3. $f(t) = e^{\frac{-t}{2}} \left(1 + \sqrt{t}\right)^3$
    4. $f(t) = \cosh at \cos at$
    5. $f(t) = e^{-4t} \sin ht \sin t$
15. Find the Laplace transform of the following functions:
    1. $f(t) = t \cosh at$
    2. $f(t) = te^{2t}(\cos t - \sin t)$
    3. $f(t) = t \sin^3 t$
    4. $f(t) = t \sin 3t \cos 2t$
    5. $f(t) = t \cos^2 2t$

16. Find the Laplace transform of the following functions:
    1. $f(t) = \frac{\sin ht}{t}$
    2. $f(t) = \frac{e^{-at} - e^{-bt}}{t}$
    3. $f(t) = \frac{\sin^2 t}{t}$
    4. $f(t) = \frac{e^{2t} \sin t}{t}$
    5. $f(t) = \left(\frac{\sin 2t}{\sqrt{t}}\right)^2$

17. Find the Laplace transform of the following functions:
    1. $f(t) = \int_0^t e^{-2t} t^3 \, dt$
    2. $f(t) = \int_0^t \frac{e^t \sin t}{t} dt$
    3. $f(t) = e^{4t} \int_0^t t \sin 3t \, dt$
    4. $f(t) = t \int_0^t e^{-4t} \sin 3t \, dt$
    5. $f(t) = \int_0^t t cos^2 t \, dt$

18.
    1. Show that $\int_0^\infty e^{-5t} \sinh^3 t \, dt = \frac{1}{64}$
    2. Show that $\int_0^\infty e^{-3t} cos^2 t \, dt = \frac{11}{39}$
    3. Show that $\int_0^\infty e^{-2t} \, t \, sin^2 t \, dt = \frac{1}{8}$

19. Find the Laplace transform of the following periodic functions:
    1. $f(t) = e^t, \ 0 < t < 2\pi$ if $f(t) = f(t + 2\pi)$.
    2. $f(t) = t^2, \ 0 < t < 2$ if $f(t) = f(t + 2)$.
    3. $f(t) = t, \ 0 < t < 1$
       $= 0, \ 1 < t < 2$
       if $f(t) = f(t + 2)$.
    4. $f(t) = 1 \quad 0 < t < 1$
       $= 0 \quad 1 < t < 1$
       $= -1 \quad 2 < t < 3$
       if $f(t) = f(t + 2)$.
    5. $f(t) = \cos \omega t \quad 0 < t < \frac{\pi}{\omega}$
       $= 0 \quad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$