

PARUL UNIVERSITY
FACULTY OF ENGINEERING & TECHNOLOGY
B.Tech. Winter 2022-23 Examination

Semester: 2
Subject Code: 203191151
Subject Name: Mathematics-II

Date: 19/11/2022
Time: 10:30 am to 01:00 pm
Total Marks: 60

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 Objective Type Questions - (Fill in the blanks, one word answer, MCQ-not more than Five in case of MCQ) (All are compulsory) (Each of one mark)

1. Let $T: R^4 \rightarrow R^3$ be a linear transformation with $R(T)=3$, then what is the nullity of T ?
 a) 2 b) 1 c) 3 d) 4
2. Which of the following equation is linear?
 a) $\frac{dy}{dx} + xy^2 = \sin x$ b) $\frac{dy}{dx} + y = \sin x$ c) $\frac{dy}{dx} + xy = y^2$ d) $x \frac{dy}{dx} + xy^2 = e^x$
3. For the Differential Equation $(1 - x^2)y'' - 6xy' - 4y = 0$, $x=0$ is _____ point
 a) Ordinary b) Regular Singular c) Irregular Singular d) None of these
4. $\int_0^1 \int_0^x dy dx =$ _____
 a) 1 b) $\frac{1}{2}$ c) 2 d) 3
5. The Wronskian of the two functions $\sin 2x$ and $\cos 2x$ is _____
 a) 1 b) 3 c) -1 d) -2.
6. Define : Singular Point
7. Write the second order linear nonhomogeneous ordinary differential equation for LCR circuit.
8. Write the standard matrix for Shear in the x-direction on R^2
9. For the equation $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$, the integrating factor is _____
10. If $\phi = 3x^2y - y^2 + 3z$, then $\nabla \phi$ at $(1,0,2) =$ _____
11. Every set in V having more than n vectors is Linearly dependent. (True/False)
12. State Gauss Divergence Theorem.
13. The order and degree of the differential equation $\left[\frac{d^2y}{dx^2} + y \right]^{\frac{1}{2}} = \sin x$
14. What is the C.F. for $(D^2 + 1)y = 0$
15. A linear transformation $T: V \rightarrow W$ is one-one if and only if $\ker(T)=\{0\}$. (True/False)

Q.2 Answer the following questions. (Attempt any three)

(15)

A) Let $T: R^3 \rightarrow R^3$ be the linear transformation defined by

$$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$$

- (a) Find a basis and the dimension for the range of T .
- (b) Find a basis and dimension for the kernel of T .
- (c) Verify the dimension theorem.

B) Solve $y'' - 3y' + 2y = \cos(e^{-x})$

C) Evaluate $\iint y \, dx \, dy$ over the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$

D) Find work done in moving a particle in the force field $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the curve $x^2 = 4y$ and $3x^3 = 8z$ from $x = 0$ to $x = 2$.

- Q.3** A) (i) Solve $\frac{dy}{dx} + \frac{2y}{x} = y^2x^2$ (04)
(ii) Evaluate $(x^4 - 2xy^2 + y^4)dx - (2x^2y - 4xy^3 + \sin y)dy = 0$ (03)
B) Find the series solution of $(1 + x^2)y'' + xy' - 9y = 0$ (08)
- OR**
- B) Determine whether the set R^+ of all positive real numbers with operations.
 $x + y = xy$ and $kx = x^k$ is a vector space (08)
- Q.4** A) (i) Change the order of integration and Evaluate $\int_0^a \int_x^a (x^2 + y^2) dy dx$ (04)
(ii) Evaluate $\frac{d^3y}{dx^3} + 8y = 0$ (03)
- OR**
- A) Solve $(D^2 + 3D + 2)y = e^{x^x}$ (07)
B) (i) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dx dy dz$ (04)
(ii) Find unit vector normal to the surface $x^2y + 2xz^2 = 8$ at the point (1,0,2) (04)