Seat No: _____ **Enrollment No: _____**

PARUL UNIVERSITY

FACULTY OF ENGINEERING & TECHNOLOGY

B.Tech. Winter 2023 - 24 Examination

Semester: 2 **Date: 18/01/2024**
Subject Code: 303191151 **Time: 2:00 pm to 4:30 pm**
Subject Name: Mathematics-II **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1	**Objective Type Questions** - (Fill in the blanks, one word answer, MCQ-not more than Five in case of MCQ) (All are compulsory) (Each of one mark)	**(15)**	**CO**	**PO**	**Bloom's Taxonomy**
	1. If $\emptyset = x^2 - z^2$, then $\nabla\emptyset$ *at* (0,0, 0) = _____		**1**	**3**	**Apply**
	2. $L(\mathrm{Sec}^2 t - \tan^2 t) =$ _____		**2**	**1**	**Evaluate**
	3. If two roots m_1 and m_2 are same in second order homogeneous differential equation then general solution of differential equation is_____		**3**	**5**	**Evaluate**
	4.If $\overline{A} = x^2 z\,\hat{i} - 2y^3 z^2\,\hat{j} + xy^2 z\,\hat{k}$,find $\nabla \cdot \overline{A}$ at the point (1, 1, 0).		**1**	**3**	**Apply**
	5. Write down formula of the Fourier cosine integral of f(x).		**2**	**1**	**Remember**
	6. Write down formula of the Fourier sine integral of f(x).		**2**	**1**	**Remember**
	7. Find the value of $1 * t$ where $*$ denotes Convolution product.		**2**	**1**	
	8. Solve: $\int_0^3 \int_0^1 (x^2 + 3y^2)\, dy\, dx$		**5**	**4**	**Evaluate**
	9. A singular point $x_0$is called a regular singular point of equation $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)\, y = 0$ if $(x - x_0)\,P(x)$ and $(x - x_0)^2\,Q(x)$ both are analytic (i.e., differentiable) at x_0. **(True or False)**		**4**	**3**	**Understand**
	10. Let $\overline{F}$ be a scalar point function then $\lvert \mathrm{Curl}\ \overline{F} \rvert$ is vector quantity. **(True or False)**		**1**	**3**	**Understand**
	11. What is the assumed y_p of the differential equation $(D-2)^2 \cdot y = e^{2x}$. (a) $A\,e^{2x}$ (b) $A\,x\,e^{2x}$ (c) $A\,x^2 e^{2x}$ (d) $A\,x^3 e^{2x}$		**3**	**5**	**Analyze**
	12. The integrating factor for a linear equation $\frac{dx}{dy} + p(y)x = q(y)$ (a) $e^{-\int p(x)\,dx}$ (b) $e^{-\int p(y)\,dy}$ (c) $e^{-\int q(x)\,dx}$ (d) $e^{-\int q(y)\,dy}$		**3**	**5**	**Analyze**
	13. The region $\int_0^2 \int_0^4 dy\, dx$ represents (a) Square (b) Rectangle (c) Circle (d) Ellipse		**5**	**4**	**Analyze**
	14. The singular point of the differential equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ is (a) x = –1 (b) x = 2 (c) x = 1 (d) x = –2		**4**	**3**	**Evaluate**
	15. The value of $L\{e^{3t+3}\}$ is (a) $\frac{e^3}{s+3}$ (b) $\frac{e^3}{s-3}$ (c) $\frac{e^3}{s}$ (d) $\frac{e^3}{s^2+3}$		**2**	**1**	**Evaluate**
Q.2	**Answer the following questions**. (Attempt any three)	**(15)**			

	A) Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-3y=0.$		**3**	**5**	**Apply**
	B) Show that $\bar{F}=(y^2-z^2+3yz-2x)\,\hat{i}+(3xz+2xy)\,\hat{j}+(3xy-2xz+2z)\hat{k}$ is both solenoidal and irrotational.		**1**	**3**	**Analyze**
	C) Classify the singular points of the equation $x^3(x-2)\frac{d^2y}{dx^2}+x^3\frac{dy}{dx}+6y=0.$		**3**	**5**	**Understand**
	D) Change the order of integration and evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dy\,dx.$		**5**	**4**	**Evaluate**
Q.3	A) Solve the following initial value problem using Laplace transform. $y''-6y'+9y=t^2e^{3t},\ y(0)=2,\ y'(0)=6$	**(07)**	**2**	**1**	**Apply**
	B) 1) Using Convolution theorem Find the inverse Laplace transform of $\frac{1}{s(s+a)^3}$	**(04)**	**2**	**1**	**Apply**
	2) Find the Laplace transform of $\int_0^1 t\cosh t\,dt$	**(04)**	**2**	**1**	**Apply**
	OR				
	B) Find the power-series solution of $y''+xy=0.$	**(08)**	**4**	**3**	**Evaluate**
Q.4	A) Find the Fourier cosine and sine integral of $f(x)=e^{-kx}$ $(x>0,k>0)$	**(07)**	**3**	**5**	**Evaluate**
	OR				
	A) Verify Green's theorem for $\oint_C[(x-y)dx+3xy\,dy]$, where C is the boundary of the region bounded by the parabolas $x^2=4y$ and $y^2=4x.$	**(07)**	**5**	**4**	**Analyze**
	B) Using method of variation of parameters solve $\frac{d^3y}{dx^3}+\frac{dy}{dx}=\operatorname{cosec}x$	**(08)**	**3**	**5**	**Apply**