Tutorial - 3 Multivariable Calculus

Q:1 Evaluate the following integrals:

(i) $\int_0^1 \int_0^x e^x \, dy \, dx$

(ii) $\int_{-a}^{a} \int_0^x dy \, dx$

(iii) $\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^3 dz \, dy \, dx$

(iv) $\int_0^{\frac{\pi}{2}} \int_0^{\infty} e^{-r^2} r \, dr \, d\theta$

(v) $\int_0^{\pi} \int_0^{a(1+cos\theta)} r \, dr \, d\theta$

Q:2 Evaluate $\iint\limits_R f(x,y) dx \, dy$, where f(x, y) = 1-6x²y and R: $0 \le x \le 2$, $-1 \le y \le 1$.

Q:3 Evaluate $\iint\limits_R xy \, dy \, dx$, where R is the positive quadrant of the circle $x^2 + y^2 = a^2$.

Q:4 Change the order of integration in the integral $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy \, dx$ and evaluate it.

Q:5 By changing into polar co-ordinates, evaluate the integral $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dy \, dx$.

Q:6 By changing into polar co-ordinates, evaluate the integral $\int_0^1 \int_0^{\sqrt{1-y^2}} sin(x^2 + y^2) dx \, dy$.

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