PARUL UNIVERSITY
FACULTY OF ENGINEERING & TECHNOLOGY
B.Tech. Summer 2022 - 23 Examination

Semester: 2   Date: 15/05/2023
Subject Code: 303191151   Time: 02:00 pm to 04:30 pm
Subject Name: Mathematics-II   Total Marks: 60

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 Objective and short questions. (All are compulsory) (Each of one mark)   **(15)**

1. $y'' + 5y' + 6y = e^x$ is a _____ Linear differential equation.

   (a) Homogeneous   (b) Non homogeneous (c)Auxilliary   (d) none of above

2. In power series method second derivative of $\sum_{k=0}^{\infty} a_k x^k$ is _____

   (a) $\sum_{k=0}^{\infty} k(k-1)a_k x^k$ (b) $\sum_{k=1}^{\infty} ka_k x^{k-1}$

   (c) $\sum_{k=2}^{\infty} k(k-1)a_k x^{k-2}$ (d) $\sum_{k=0}^{\infty} ka_k x^{k-1}$

3. The line integral $\int_c \bar{F}(\hat{r}) \, d\hat{r}$ represents _____.

   (a)Work done   (b)Pressure   (c) Force   (d) velocity

4. If the linear differential equation have real and double root then solution is $y = $ _____.

   (a)$c_1 e^{m_1 x} + c_2 e^{m_2 x}$ (b) $(c_1 + c_2)e^{mx}$ (c)$(c_1 + c_2 x)e^{mx}$ (d) none of above

5. If R be the projections on S on the $xy - plane$, then _____ is the unit normal.

   (a)$\hat{i}$ (b)$\hat{k}$ (c) $\hat{j}$ (d) 0

6. For the given linear differential equation $y'' + 4y = 12$, particular integral value $y_p = $ _____

   (a) 3   (b) 4   (c) 12   (d) 0

7. If $\nabla \times F = 0$ then the vector function F is called _____

   (a)irrotational   (b) rotational   (c) Projection   (d) vector

8. Solution of $\int_0^1 \int_0^x (x^2 + y^2) dA$, where dA indicates small areas in $xy - plane$.

   (a)1   (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 2

9. $\int_0^{\frac{\pi}{2}} \int_1^2 y \sin xy \, dxdy = $ _____.

   (a) 1   (b)0   (c) 2   (d) -1

10. $f(x) = \int_0^{\infty} B(\lambda) \sin \lambda v \, d\lambda$ is a Fourier _____ integral.

   (a) cosine   (b) sine   (c) exponential   (d) periodic

11. Laplace transform of $\{cos^2 t\} = $ _____

12. $\int_1^2 \int_1^2 \frac{1}{xy} dxdy = $ _____.

13. $L^{-1}\left\{\frac{4}{s^2-121}\right\} = $ _____

14. $\frac{d^2 y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$ is not an analytic at _____ point.

15. $L\{e^t u(t-2)\} = $ _____

Q.2 Answer the following questions. (Attempt any three) (15)

1. Solve the linear differential equation $\frac{d^2y}{dx^2} + y = cosec\ x$ by variation parameter method.

2. Find Fourier cosine integral of $f(x) = e^{-kx}, (x > 0, k > 0)$.

3. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2}\ dxdy$ by changing it into polar coordinate.

4. Evaluate $\nabla r, \nabla \frac{1}{r}$ where $r = |r|$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

Q.3 (07)

(A) Inductance R in series and the charge Q at any time t satisfies the equation $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{c} = 0$, given that $L = 0.25, R = 250\ ohm, C = 2 \times 10^{-6}\ farad$ and that when $t = 0$, charge Q is 0.02 coulombs and the current $\frac{dQ}{dt} = 0$ obtain values of Q in terms of t.

(B) Using Fourier integral representation of the function $f(x) = \begin{cases} 1, & if\ |x| < 1 \\ 0, & if\ |x| > 1 \end{cases}$ (08)

hence evluate $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda}\ d\lambda$ and $\int_0^\infty \frac{\sin \lambda}{\lambda}\ d\lambda$

OR

(B) (i) Using convolution theorem evaluate $L^{-1}\left\{\frac{1}{S(S^2+4)}\right\}$, **(ii)** Evaluate $L^{-1}\left\{\log \frac{1}{S}\right\}$. (08)

Q.4

(A) Find the series solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$ using Power series method. (07)

OR

(A) Verify Green's theorem for $\oint(x^2 - 2xy)dx + (x^2y + 3)dy$ were C is the boundary of the region Bounded by parabola $y = x^2$ and $y = x$. (07)

(B) Changing the order of integration of $\int_0^a \int_{\frac{x^2}{a}}^{2x-a} xy\ dy\ dx$ hence Evaluate it. (08)