

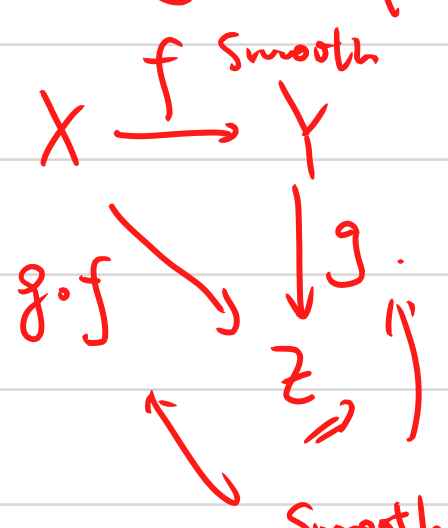
- variety: quasi-proj.
- affine: isom. to a Zar. closed subset of  $\mathbb{A}^n$
- noetherian top. space
- & noetherian induction.

- 2.1.4: every var. has smooth dense open subvar.
- 2.1.5: conn. in Zar. var.  $\Leftrightarrow$  conn. in ana. top.

2.1.9 next

Smooth morph.

analytically,  $\left(\frac{\partial g_i}{\partial x_j}\right)$  has rank  $n$ .



Generic Smoothness:

$f: X \rightarrow Y$  morph.

$X$  smooth  $\Rightarrow \exists U \subset Y$  open

$f^{-1}(U) \xrightarrow{f} U$  open.

étale: rel. dim. 0, smooth.

- local homeo.

surj., proper, étale  $\Rightarrow$  covering

Lemma.  $f: X \rightarrow Y$  fin.  $\exists U \subset Y$

$f|_{f^{-1}(U)}: f^{-1}(U) \rightarrow U$  fin. étale

- Divisor with simple normal crossing
  - $Z \subset X$  closed subvar. of dim.  $n-1$
  - $Z_1, \dots, Z_k$  irr.
  - $Z_i$  smooth
  - (1)  $\forall x \in X \setminus Z, \exists \{f_i\} \subset \mathcal{O}_{X,x}$  s.t.  $Z_i = \{f_i = 0\}$
  - &  $\{df_i|_x\}$  linearly indep.

Resol. Thm:  $X$  irr.  $Z \subset X$  closed, complement nonempty

$\exists f: \tilde{X} \rightarrow X$  smooth.

(1)  $\tilde{X}$  smooth.

(2)  $f$  res. to isom  $f^{-1}(X \setminus Z) \rightarrow X \setminus Z$

(3)  $f^{-1}(Z)$  div. with simple normal crossings.

- Loc. trivial fibrations:

$f: X \rightarrow Y$  diff. loc. triv. fib. (with fib.  $F$ )

$$\begin{array}{ccc} f^{-1}(U) & \xrightarrow{b} & F \times U \\ f|_U \downarrow & \nearrow \text{pr}_2 & \\ U & & \end{array} \quad \text{by } \exists U \ni y$$

Ehresmann's fib. thm:  $f: X \rightarrow Y$  smooth, surj. prop.  $\Rightarrow f$  diff. loc. triv. fibration.

$Z \subset X$  div. with —

$f: X \rightarrow Y$ , trans. to  $Z$  if  $f|_{Z_i}: Z_i \rightarrow Y$  smooth & surj.

$f$ : transv. locally triv. fibration.  $f|_{Z_i}: Z_i \rightarrow Y$  smooth & surj.?

$f$ : transv. loc. triv. fib.  $f$  surj.

if  $\forall y \in Y, \exists U$  diffeom.  $b: f^{-1}(U) \rightarrow f^{-1}(y) \times U$  that rest. to diffeo.

$$b|_{f^{-1}(U) \cap Z_i}: f^{-1}(U) \cap Z_i \rightarrow (f^{-1}(y) \cap Z_i) \times U$$

Thm:  $f: X \rightarrow Y$  smooth of smooth var. transv. to a div. with simple normal crossings  $Z \subset X$ .  $f$  proper  $\Rightarrow$  transv. loc. triv. fib.

Fundamental gps:

- $X$  smooth, conn.  $U \subset X$  Zar.  $\Rightarrow \pi_1(U, x_0) \rightarrow \pi_1(X, x_0)$  surj.

“横截相交”

- $f: X \rightarrow Y$  smooth, surj. w/ conn. fib.  $Y$  smooth & conn.  $\forall x_0 \in X, \pi_1(X, x_0) \rightarrow \pi_1(Y, f(x_0))$

- $X$  smooth

Nagata:

$$X \hookrightarrow X' \xrightarrow{\tilde{f}} Y$$

- assume  $X'$  smooth

2.1.13:  $\exists U \subset Y, \tilde{f}^{-1}(U) \rightarrow U$  smooth  $\Rightarrow \tilde{U}$  Zar. open.

$$U = X \cap \tilde{U}$$

$$\begin{array}{ccccc} U & \hookrightarrow & \tilde{U} & \longrightarrow & V \\ \downarrow & & \downarrow & & \downarrow \\ X & \hookrightarrow & X' & \xrightarrow{\tilde{f}} & Y \end{array}$$

$\tilde{f}$  prop.  $\Rightarrow \tilde{f}$  smooth, prop. surj. conn. fib.  $\Rightarrow$  conn.

$X$  conn.

$$\begin{array}{ccccc} \pi_1(U, x_0) & \xrightarrow{\pi_1(j)} & \pi_1(\tilde{U}, x_0) & \xrightarrow{\pi_1(\tilde{f})} & \pi_1(V, f(x_0)) \\ (\text{surj.}) \downarrow & \text{surj.} & \text{conn. fib.} \Rightarrow \text{surj.} & \downarrow \text{surj.} & \\ \pi_1(X, x_0) & \longrightarrow & \pi_1(Y, f(x_0)) & & \end{array}$$