- variety : quasi-proj.
- affine : isom. to a Zar. closed subset of $\mathbb{A}^n$
- noetherian top. space
  & noetherian induction.

- 2.1.4: every var. has smooth dense open subvar.
- 2.1.5: conn. in Zar. var. $\Leftrightarrow$ conn. in ana. top

2.1.9 next

Smooth morph:

analytically, $\left(\frac{\partial g_i}{\partial x_j}\right)_{ij}$ has rank n.

$$\begin{tikzcd} X \arrow[r, "f\ \text{smooth}"] \arrow[dr, "g\circ f"'] & Y \arrow[d, "g"] \\ & Z \end{tikzcd}$$

smooth

Generic Smoothness:

$f: X \to Y$ morph.

X smooth $\Rightarrow \exists U \subset Y$ open

$f^{-1}(U) \xrightarrow{f} U$ open.

étale : rel. dim. 0, smooth

- local homeo.

Surj. proper. étale $\Rightarrow$ covering

Lemma. $f: X \to Y$ fin. $\exists U \subset Y$

$f|_{f^{-1}(U)}: f^{-1}(U) \to U$ fin. étale

- Divisor with simple normal crossing
  - $Z \subset X$ closed. subvar. of dim. $n-1$
  - $Z_1 \cdots Z_k$ irr.

  (1) Z_i smooth

  (2) $\forall x$. $I = \{i \mid x \in Z_i\}$. $\exists U$

  $\exists \{f_i\}$ $Z_i \cap U = \{f_i = 0\}$

  & $\{df_i|_x\}$ linearly indep.

Resol: Thm: X irr. $Z \subset X$ closed. complement nonempty

$\exists f: \tilde{X} \to X$. smooth.

(1) $\tilde{X}$ smooth.

(2) f res. to isom $f^{-1}(X \setminus Z) \to X \setminus Z$

(3) $f^{-1}(Z)$ div. with simple normal crossings

- Loc. trivial fibrations:

$f: X \to Y$ diff. loc. triv. fib. (with fib. F)

$\forall y$. $\exists U \ni y$

$$\begin{tikzcd} f^{-1}(U) \arrow[r, "b", "\sim"'] \arrow[d, "f|"'] & F \times U \arrow[dl, "pr_2"] \\ U \end{tikzcd}$$

Ehresmann's fib. thm: $f: X \to Y$ smooth. surj. prop.

$\Rightarrow f$. diff. loc. triv. fibration.

$Z \subset X$ div. with —.

$f: X \to Y$, trans. to Z. if $f|_{Z_I}: Z_I \to Y$ smooth & surj.

f: transv. locally. triv. fibration.

$f|_{Z_I}: Z_I \to Y$ smooth. & surj. ?

f: transv. loc. triv. fib.

f surj

if $\forall y \in Y$. $\exists U$. diffeom.

$b: f^{-1}(U) \to f^{-1}(y) \times U$ that

rest. to diffeo.

$b|_{f^{-1}(U) \cap Z_I}: f^{-1}(U) \cap Z_I \to (f^{-1}(y) \cap Z_I) \times U$

Thm: $f: X \to Y$ smooth. morph. of smooth var.

transv. to a div. with simple normal crossings $Z \subset X$.

f proper $\Rightarrow$ transv. loc. triv. fib.

Fundamental gps:

① X smooth conn. $U \subset X$ Zar $\Rightarrow$

$\pi_1(U, x_0) \to \pi_1(X, x_0)$ surj.

"横截相交"

② $f: X \to Y$ smooth. surj. w/. conn. fib.

Y smooth & conn.

$\forall x_0 \in X$ $\pi_1(X, x_0) \to \pi_1(Y, f(x_0))$

- X smooth

Nagata:

$X \hookrightarrow X' \xrightarrow{\tilde{f}} Y$.

- assume X' smooth

2.1.13: $\exists V \subset Y$. $\tilde{f}^{-1}(V) \to V$ smooth

$\tilde{f}^{-1}(V) = \tilde{U}$ Zar open.

$U = X \cap \tilde{U}$

$$\begin{tikzcd} U \arrow[r, hook] \arrow[d, hook] & \tilde{U} \arrow[r] \arrow[d, hook] & V \arrow[d, hook] \\ X \arrow[r, hook] \arrow[rr, bend right] & X' \arrow[r, "\tilde{f}"] & Y \end{tikzcd}$$

$\tilde{f}$ prop. $\Rightarrow \tilde{f}'$ smooth. prop. surj.

conn. fib. $\Rightarrow$ conn.

X conn.

$$\begin{tikzcd} \pi_1(U, x_0) \arrow[r, "\pi_1(j)", "\text{surj.}"'] \arrow[d, "(\text{surj.})"'] & \pi_1(\tilde{U}, x_0) \arrow[r, "\pi_1(\tilde{f}')", "\substack{\text{conn. fib.} \\ \Rightarrow \text{surj.}}"'] & \pi_1(V, f(x_0)) \arrow[d, "\text{surj.}"] \\ \pi_1(X, x_0) \arrow[rr] & & \pi_1(Y, f(x_0)) \end{tikzcd}$$