

C)

For the first version:

Let  $T_n$  denotes the number of recursive calls by the ExponentialTetranacci(n) method.

$$T_0 = 1;$$

$$T_1 = 1;$$

$$T_2 = 1;$$

$$T_3 = 1;$$

$$T_4 = 1;$$

$$T_5 = T_1 + T_2 + T_3 + T_4 + 1 = 1 + 1 + 1 + 1 + 1 = 5;$$

$$T_6 = T_5 + T_4 + T_3 + T_2 + 1 = 5 + 1 + 1 + 1 + 1 = 9;$$

$$T_7 = T_6 + T_5 + T_4 + T_3 + 1 = 9 + 5 + 1 + 1 + 1 = 17;$$

$$T_8 = T_7 + T_6 + T_5 + T_4 + 1 = 17 + 9 + 5 + 1 + 1 = 33;$$

$$T_9 = T_8 + T_7 + T_6 + T_5 + 1 = 33 + 17 + 9 + 5 + 1 = 64;$$

$$T_{10} = T_9 + T_8 + T_7 + T_6 + 1 = 64 + 33 + 17 + 9 + 1 = 124;$$

$$T_{11} = T_{10} + T_9 + T_8 + T_7 + 1 = 124 + 64 + 33 + 17 + 1 = 239;$$

$$T_{12} = T_{11} + T_{10} + T_9 + T_8 + 1 = 239 + 124 + 64 + 33 + 1 = 461;$$

...

$$T_n = T_{n-1} + T_{n-2} + T_{n-3} + T_{n-4} + 1.$$

Since  $T_{n-1} > T_{n-2} > T_{n-3} > T_{n-4}$ ,

$$T_n < 4T_{n-1} < 4^2T_{n-2} < 4^3T_{n-3} < \dots < 4^{n-4}T_4 = 4^{n-4}.$$

Therefore, we can conclude that  $T_n$  is  $O(4^n)$ .

for the second version:

$$T_0 = 1;$$

$$T_1 = 1;$$

$$T_2 = 1;$$

$$T_3 = 1;$$

$$T_4 = 1;$$

$$T_5 = T_4 + 1 = 1 + 1 = 2;$$

$$T_6 = T_5 + 1 = 2 + 1 = 3;$$

$$T_7 = T_6 + 1 = 3 + 1 = 4;$$

$$T_8 = T_7 + 1 = 4 + 1 = 5;$$

$$T_9 = T_8 + 1 = 5 + 1 = 6;$$

$$T_{10} = T_9 + 1 = 6 + 1 = 7;$$

$$T_{11} = T_{10} + 1 = 7 + 1 = 8;$$

...

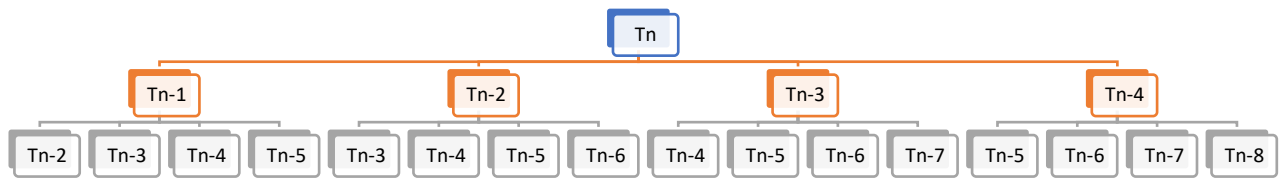
$$T_n = T_{n-1} + 1.$$

This means every recursive call decreases  $n$  by 1 when  $n > 4$ .

Therefore,  $T_n \begin{cases} 1 & n \leq 4 \\ n - 3 & n > 4 \end{cases}$

we can conclude that  $T_n$  is  $O(n)$ .

d)



When  $n > 4$ , each recursive call will invoke 4 recursive calls, each of which will call another 4 recursive calls respectively; then, the program will execute till  $n=4$ . Therefore, the first version is exponential.