No Butterfly Arbitrage Wing Model (Durrleman Condition)

1. Left Parabolic (dc < x <= 0)

In[]:=
```
w[x_] = vc + sc * x + pc * x^2
```

Out[]=

$$vc + sc\, x + pc\, x^2$$

In[]:=
```
g[x_] = (1 - x * w'[x] / (2 * w[x]))^2 - w'[x]^2 / 4 * (1 / w[x] + 1/4) + w''[x] / 2
```

Out[]=

$$pc - \frac{1}{4}(sc + 2\, pc\, x)^2\left(\frac{1}{4} + \frac{1}{vc + sc\, x + pc\, x^2}\right) + \left(1 - \frac{x\,(sc + 2\, pc\, x)}{2\left(vc + sc\, x + pc\, x^2\right)}\right)^2$$

In[]:=
```
Simplify[pc - 1/4 (sc + 2 pc x)^2 (1/4 + 1/(vc + sc x + pc x^2)) + (1 - x (sc + 2 pc x)/(2 (vc + sc x + pc x^2)))^2]
```
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Out[]=

$$pc + \frac{(2\, vc + sc\, x)^2}{4\,(vc + x\,(sc + pc\, x))^2} - \frac{1}{4}(sc + 2\, pc\, x)^2\left(\frac{1}{4} + \frac{1}{vc + x\,(sc + pc\, x)}\right)$$

2. Right Parabolic (0 < x <= uc)

In[]:=
```
w[x_] = vc + sc * x + cc * x^2
```

Out[]=

$$vc + sc\, x + cc\, x^2$$

In[]:=
```
g[x_] = (1 - x * w'[x] / (2 * w[x]))^2 - w'[x]^2 / 4 * (1 / w[x] + 1/4) + w''[x] / 2
```

Out[]=

$$cc - \frac{1}{4}(sc + 2\, cc\, x)^2\left(\frac{1}{4} + \frac{1}{vc + sc\, x + cc\, x^2}\right) + \left(1 - \frac{x\,(sc + 2\, cc\, x)}{2\left(vc + sc\, x + cc\, x^2\right)}\right)^2$$

In[]:=
```
Simplify[cc - 1/4 (sc + 2 cc x)^2 (1/4 + 1/(vc + sc x + cc x^2)) + (1 - x (sc + 2 cc x)/(2 (vc + sc x + cc x^2)))^2]
```
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Out[]=

$$cc + \frac{(2\, vc + sc\, x)^2}{4\,(vc + x\,(sc + cc\, x))^2} - \frac{1}{4}(sc + 2\, cc\, x)^2\left(\frac{1}{4} + \frac{1}{vc + x\,(sc + cc\, x)}\right)$$

3. Left Smoothing Range (dc(1 + dsm) < x <= dc)

In[•]:=
$$w[x\_] = vc - \left(1 + \frac{1}{dsm}\right) * pc * dc^2 - \frac{sc * dc}{2 * dsm} + \left(1 + \frac{1}{dsm}\right) * (2 * pc * dc + sc) * x - \left(\frac{pc}{dsm} + \frac{sc}{2 * dc * dsm}\right) * x^2$$

Out[•]=
$$-dc^2 \left(1 + \frac{1}{dsm}\right) pc - \frac{dc\ sc}{2\ dsm} + vc + \left(1 + \frac{1}{dsm}\right) (2\ dc\ pc + sc)\ x - \left(\frac{pc}{dsm} + \frac{sc}{2\ dc\ dsm}\right) x^2$$

In[•]:=
$$g[x\_] = \left(1 - \frac{x * w'[x]}{2 * w[x]}\right)^2 - \frac{w'[x]^2}{4} * \left(\frac{1}{w[x]} + \frac{1}{4}\right) + \frac{w''[x]}{2}$$

Out[•]=
$$-\frac{pc}{dsm} - \frac{sc}{2\ dc\ dsm} - \frac{1}{4}\left(\left(1 + \frac{1}{dsm}\right)(2\ dc\ pc + sc) - 2\left(\frac{pc}{dsm} + \frac{sc}{2\ dc\ dsm}\right) x\right)^2 \left(\frac{1}{4} + \frac{1}{-dc^2 \left(1 + \frac{1}{dsm}\right) pc - \frac{dc\ sc}{2\ dsm} + vc + \left(1 + \frac{1}{dsm}\right)(2\ dc\ pc + sc)\ x - \left(\frac{pc}{dsm} + \frac{sc}{2\ dc\ dsm}\right) x^2}\right) + \left(1 - \frac{x\left(\left(1 + \frac{1}{dsm}\right)(2\ dc\ pc + sc) - 2\left(\frac{pc}{dsm} + \frac{sc}{2\ dc\ dsm}\right) x\right)}{2\left(-dc^2 \left(1 + \frac{1}{dsm}\right) pc - \frac{dc\ sc}{2\ dsm} + vc + \left(1 + \frac{1}{dsm}\right)(2\ dc\ pc + sc)\ x - \left(\frac{pc}{dsm} + \frac{sc}{2\ dc\ dsm}\right) x^2\right)}\right)^2$$

In[•]:=
$$\text{Simplify}\left[-\frac{pc}{dsm} - \frac{sc}{2\ dc\ dsm} - \frac{1}{4}\left(\left(1 + \frac{1}{dsm}\right)(2\ dc\ pc + sc) - 2\left(\frac{pc}{dsm} + \frac{sc}{2\ dc\ dsm}\right) x\right)^2 \left(\frac{1}{4} + \frac{1}{-dc^2 \left(1 + \frac{1}{dsm}\right) pc - \frac{dc\ sc}{2\ dsm} + vc + \left(1 + \frac{1}{dsm}\right)(2\ dc\ pc + sc)\ x - \left(\frac{pc}{dsm} + \frac{sc}{2\ dc\ dsm}\right) x^2}\right) + \left(1 - \frac{x\left(\left(1 + \frac{1}{dsm}\right)(2\ dc\ pc + sc) - 2\left(\frac{pc}{dsm} + \frac{sc}{2\ dc\ dsm}\right) x\right)}{2\left(-dc^2 \left(1 + \frac{1}{dsm}\right) pc - \frac{dc\ sc}{2\ dsm} + vc + \left(1 + \frac{1}{dsm}\right)(2\ dc\ pc + sc)\ x - \left(\frac{pc}{dsm} + \frac{sc}{2\ dc\ dsm}\right) x^2\right)}\right)^2\right]$$
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Out[•]=
$$-\frac{pc}{dsm} - \frac{sc}{2\ dc\ dsm} + \frac{dc^2 \left(-2\ dc^2\ (1 + dsm)\ pc - dc\ sc + 2\ dsm\ vc + 2\ dc\ (1 + dsm)\ pc\ x + sc\ x + dsm\ sc\ x\right)^2}{\left(2\ dc^3\ (1 + dsm)\ pc + sc\ x^2 + dc^2\ (sc - 4\ (1 + dsm)\ pc\ x) - 2\ dc\ (x\ (sc - pc\ x) + dsm\ (vc + sc\ x))\right)^2} - \frac{1}{4\ dc^2\ dsm^2}(2\ dc\ pc + sc)^2\ (dc + dc\ dsm - x)^2 \left(\frac{1}{4} - \frac{2\ dc\ dsm}{2\ dc^3\ (1 + dsm)\ pc + sc\ x^2 + dc^2\ (sc - 4\ (1 + dsm)\ pc\ x) - 2\ dc\ (x\ (sc - pc\ x) + dsm\ (vc + sc\ x))}\right)$$

4. Right Smoothing Range (uc < x <= uc(1 + usm))

In[•]:=
$$w[x\_] = vc - \left(1 + \frac{1}{usm}\right) * cc * uc^2 - \frac{sc * uc}{2 * usm} + \left(1 + \frac{1}{usm}\right) * (2 * cc * uc + sc) * x - \left(\frac{cc}{usm} + \frac{sc}{2 * uc * usm}\right) * x^2$$

Out[•]=
$$-cc\,uc^2\left(1 + \frac{1}{usm}\right) - \frac{sc\,uc}{2\,usm} + vc + (sc + 2\,cc\,uc)\left(1 + \frac{1}{usm}\right)x - \left(\frac{cc}{usm} + \frac{sc}{2\,uc\,usm}\right)x^2$$

In[•]:=
$$g[x\_] = \left(1 - \frac{x * w'[x]}{2 * w[x]}\right)^2 - \frac{w'[x]^2}{4} * \left(\frac{1}{w[x]} + \frac{1}{4}\right) + \frac{w''[x]}{2}$$

Out[•]=
$$-\frac{cc}{usm} - \frac{sc}{2\,uc\,usm} - \frac{1}{4}\left((sc + 2\,cc\,uc)\left(1 + \frac{1}{usm}\right) - 2\left(\frac{cc}{usm} + \frac{sc}{2\,uc\,usm}\right)x\right)^2 \left(\frac{1}{4} + \frac{1}{-cc\,uc^2\left(1 + \frac{1}{usm}\right) - \frac{sc\,uc}{2\,usm} + vc + (sc + 2\,cc\,uc)\left(1 + \frac{1}{usm}\right)x - \left(\frac{cc}{usm} + \frac{sc}{2\,uc\,usm}\right)x^2}\right) + \left(1 - \frac{x\left((sc + 2\,cc\,uc)\left(1 + \frac{1}{usm}\right) - 2\left(\frac{cc}{usm} + \frac{sc}{2\,uc\,usm}\right)x\right)}{2\left(-cc\,uc^2\left(1 + \frac{1}{usm}\right) - \frac{sc\,uc}{2\,usm} + vc + (sc + 2\,cc\,uc)\left(1 + \frac{1}{usm}\right)x - \left(\frac{cc}{usm} + \frac{sc}{2\,uc\,usm}\right)x^2\right)}\right)^2$$

In[•]:=
$$\text{Simplify}\left[-\frac{cc}{usm} - \frac{sc}{2\,uc\,usm} - \frac{1}{4}\left((sc + 2\,cc\,uc)\left(1 + \frac{1}{usm}\right) - 2\left(\frac{cc}{usm} + \frac{sc}{2\,uc\,usm}\right)x\right)^2 \left(\frac{1}{4} + \frac{1}{-cc\,uc^2\left(1 + \frac{1}{usm}\right) - \frac{sc\,uc}{2\,usm} + vc + (sc + 2\,cc\,uc)\left(1 + \frac{1}{usm}\right)x - \left(\frac{cc}{usm} + \frac{sc}{2\,uc\,usm}\right)x^2}\right) + \left(1 - \frac{x\left((sc + 2\,cc\,uc)\left(1 + \frac{1}{usm}\right) - 2\left(\frac{cc}{usm} + \frac{sc}{2\,uc\,usm}\right)x\right)}{2\left(-cc\,uc^2\left(1 + \frac{1}{usm}\right) - \frac{sc\,uc}{2\,usm} + vc + (sc + 2\,cc\,uc)\left(1 + \frac{1}{usm}\right)x - \left(\frac{cc}{usm} + \frac{sc}{2\,uc\,usm}\right)x^2\right)}\right)^2\right]$$
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Out[•]=
$$-\frac{cc}{usm} - \frac{sc}{2\,uc\,usm} + \frac{uc^2\,(2\,usm\,vc - 2\,cc\,uc\,(1 + usm)\,(uc - x) + sc\,(-uc + x + usm\,x))^2}{\left(sc\left(uc^2 - 2\,uc\,(1 + usm)\,x + x^2\right) + 2\,uc\left(-usm\,vc + cc\left(uc^2\,(1 + usm) - 2\,uc\,(1 + usm)\,x + x^2\right)\right)\right)^2} - \frac{1}{4\,uc^2\,usm^2}(sc + 2\,cc\,uc)^2\,(uc + uc\,usm - x)^2\left(\frac{1}{4} - \frac{2\,uc\,usm}{sc\left(uc^2 - 2\,uc\,(1 + usm)\,x + x^2\right) + 2\,uc\left(-usm\,vc + cc\left(uc^2\,(1 + usm) - 2\,uc\,(1 + usm)\,x + x^2\right)\right)}\right)$$

5. Left Constant Level (x < dc(1 + dsm))

In[•]:=
```
w[x_] = vc + dc * (2 + dsm) * sc/2 + (1 + dsm) * pc * dc^2
```

Out[•]=

$$dc^2\ (1+dsm)\ pc + \frac{1}{2}\ dc\ (2+dsm)\ sc + vc$$

In[•]:=
```
g[x_] = (1 - x * w'[x]/(2 * w[x]))^2 - w'[x]^2/4 * (1/w[x] + 1/4) + w''[x]/2
```

Out[•]=

$$1$$

6. Right Constant Level (x > uc(1 + usm))

In[•]:=
```
w[x_] = vc + uc * (2 + usm) * sc/2 + (1 + usm) * cc * uc^2
```

Out[•]=

$$cc\ uc^2\ (1+usm) + \frac{1}{2}\ sc\ uc\ (2+usm) + vc$$

In[•]:=
```
g[x_] = (1 - x * w'[x]/(2 * w[x]))^2 - w'[x]^2/4 * (1/w[x] + 1/4) + w''[x]/2
```

Out[•]=

$$1$$

7. Dynamic Plotting (-1 <= x <= 1)

In[•]:=
```
Manipulate[Plot[If[x ≥ 0, vc + sc x + cc x^2, vc + sc * x + pc * x^2],
交互式操作  绘图  如果
  {x, -0.3, 0.3}, GridLines → {{{-sc / (2 * pc), Directive[Red, Thick]},
                  网格线                          指令  红色 粗
    {-sc / (2 * cc), Directive[Green, Thick]}}, {}}, PlotRange → {-0.05, 0.4}],
                     指令  绿色  粗              绘制范围
 {cc, 0.01, 100}, {pc, 0.01, 100}, {sc, -10, 10}, {vc, 0, 1}]
```

In[•]:=
```
Manipulate[
交互式操作
 Plot[If[x ≥ 0, cc + (2 vc + sc x)^2/(4 (vc + x (sc + cc x))^2) - 1/4 (sc + 2 cc x)^2 (1/4 + 1/(vc + x (sc + cc x))),
绘图  如果
   pc + (2 vc + sc x)^2/(4 (vc + x (sc + pc x))^2) - 1/4 (sc + 2 pc x)^2 (1/4 + 1/(vc + x (sc + pc x)))], {x, -0.3, 0.3}],
 {cc, -6.`, 6.`}, {pc, -6, 6}, {sc, -2, 2}, {vc, -2, 2}]
```