Irrational Borrowing and Financial Constraint

Abstract

Will households having hyperbolic discounting suffer welfare loss due to excessive debt? This paper builds a model in which rich people have higher desire to save. The resulting downward sloping long-run supply curve implies that the elevated debt demand tends to push the equilibrium interest rate lower. Therefore, hyperbolic discounting agents are, in fact, better off because of the cheaper debt, though they deviate from the optimal consumption path. As the credit limit increases, their welfare is improved in the long run. However, the economy is also more likely to be locked in an low-interest-rate steady state.

1 Introduction

US households had accumulated a remarkable amount of debt from 1980 to 2008. As an instrument of consumption smoothing, debt is welcomed by the households. However, if the households are irrational, they will deviate from the optimal debt level, which might cause them some welfare loss. For example, if the borrower has ”present bias” due to hyperbolic discounting (Laibson [1997]), then his lifetime consumption profile will be even steeper than the rational counterfactual, which means that the borrower will borrow more in his youth. The more irrational the agent is, the more excessive the debt he holds is. Debt is undesirable even though the households call for it.

In reality, people’s borrowing behavior is subject to the financial constraints. Can borrowers benefit from an increasing credit limit? For a financially constrained individual, debt is definitely welcomed as an instrument to keep herself on the consumption turnpike when income is temporarily low. The welfare of these hand-to-mouth people will be improved if more loans are offered. Nevertheless, it’s not necessarily true for the irrational households. If the credit limit exceed the amount that the rational borrowers would want to borrow, the hyperbolic households will still borrow every dollar he’s offered. Chances are that, the rational households and the hyperbolic discounting households leverage to the same amount decided by the credit limit before financial liberalization. But as the credit limit increases, the hyperbolic discounting agents get more indebted than rational agents. In this way, higher debt level might not be welfare improving for the hyperbolic discounting agents.

Nevertheless, the situation gets complicated in the equilibrium. In the equilibrium, the price of credit, namely the interest rate will respond to the changing demand. Whether the interest rate will rise or fall is model-specific. In a conventional lifetime-consumption model, the equilibrium interest rate is pinned down by the discount factor and doesn’t change as the borrowers leverage more. But the real economy might be non-homothetic. Mian et al. [2019] model an economy where the rich people have higher desire to save. In this model, a bouyant credit market leads to the savers holding more household debt as a financial asset. As the savers become richer, they are even more willing to offer credits and bid the interest rate even lower. As shown in fig. 1, the resulting supply curve is downward sloping, amplifying the welfare gain, or alleviating the welfare loss, from a liberalized credit market. An excessive debt demand might lead to lower interest rate in the long run. Therefore whether the hyperbolic discounting borrowers will be better or worse off is not as obvious as it seems.

This paper is intended to model irrational borrowing under loosing borrowing constraint. I'll focus on the welfare implication of the agents with hyperbolic discounting. Since the interest rate cut depend on the slope of the credit cupply curve, the welfare loss or gain might be dependent on the parameters of the economy. Thus this paper doesn't pursue a proof of welfare improvement nor the contrary. Instead, I'll quantitatively present simulated results with parameters calibrated from the reality and different extent of irrationality. The rest of the paper will be structured as the following. Section 2 will discuss related literature. Section 3 describes the model and solve the optimization problems of the savers and borrowers. In Section 4, I'll solve the steady state credit supply and demand curve and calculate the equilibrium and the welfare gain/loss. Section 5 discusses the implications and concludes.

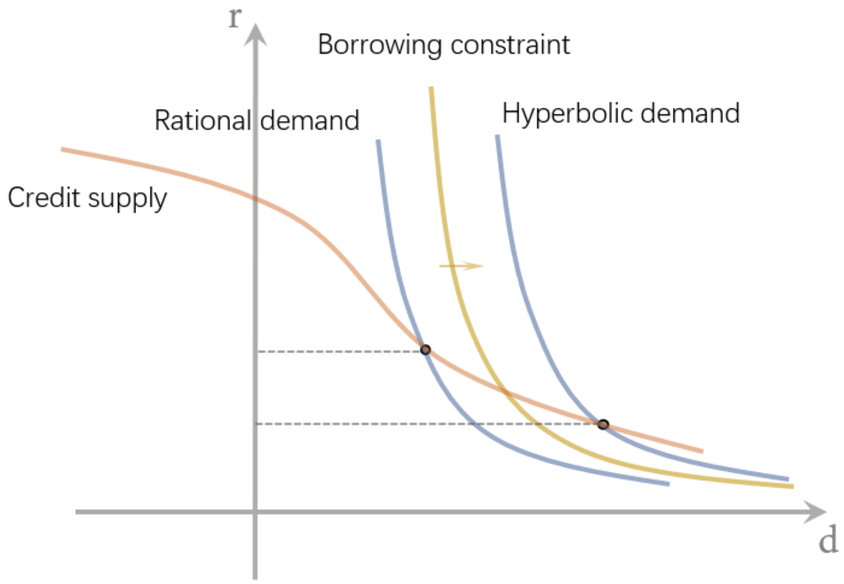


Figure 1: Credit Supply and Demand: rational savers with non-homothetic preference and irrational borrowers with huperbolic discounting.

2 Related literature

There are lists of literature that discuss the undesirability of high debt level. The first list focuses on its macro consequence. For example, excessive leverage leads to financial fragility (Minsky et al. [1982]) and lead to depressed demand (Mian et al. [2019]) and debt overhang (Myers [1977]) in the downturn. The second literature concerns the frictions in the financial market and people's perception biases also lead to suboptimal debt level. First, the financial institutions can take advantage of the households' "debt illiteracy" (Campbell et al. [2011]) and deceive them into paying more than expected. Second, the household themselves might have biased perceptions like exponential growth bias (Stango and Zinman [2009]) and neglected risk (Baron and Xiong [2017]), which makes them prone to overindebtedness.

But if we ignore these side-effects and frictions involved, would the current higher debt level be desirable? If the economy is free from uncertainty and the borrowers have the savvy of using debt, will the liberalized credit market benefit the households? The point of this paper is that even though the borrowers are not disturbed from the outside, they might still not be able to leverage to the optimal level due to irrationality or the ”present bias”.

The notion of ”hyperbolic discounting”(Laibson [1997]) means that the agent put more value to current consumption. While the agent is impatient between today and tomorrow, she always conceives the illusion of being patient in the future. As a result, in order to consume more today, the agent need to call for more debt. But tomorrow she'll again want to consume more instead of paying off the debt as planned. Therefore, such irrationality leads to a more steep life-time consumption and leverage profile which is suboptimal from the hindsight.

Few model has formally evaluated the excessiveness of debt due to hyperbolic discounting. There are many insightful irrational variations from fully rational models, such as Harris and Laibson [2003] and Cao and Werning [2018], which don't treat debt as a major concern. Most of the ”irrational” models concerning debt issues serve to provide empirical evidence for hyperbolic discounting in a structured way (Skiba and Tobacman [2008]). This paper is intended to investigate from a theoreticla perspective how hyperbolic discounting will change people's borrowing behavior and what's the welfare implications of such deviation.

3 Model

This section develops a model of irrational borrowing. The model is an endowment economy with normalized population and income, populated by two dynasties of agents, the savers and the borrowers. In line with the income inequality in reality, we let the top $\mu^s = 0.01$ fraction to be the savers and the rest $\mu^b = 0.99$ fraction to be the borrowers. The savers receive an income of $w^s = 0.14$, consistent with a 20% pre-tax income net of the 6% from the return of household and government debt. The borrowers own the rest $w^b = 0.86$ of the income, which is only 1/14 of the savers' income in per capita term. In this model, the wealthier are intrinsically the savers and the common workers are the natural borrowers.

The savers live forever, or equivalently, look into infinite horizons because of altruism. They have a ”warm-glow” bequeath motive and tend to accumulate and pass on their wealth. The borrowers, on the contrary, live only 60 years since adulthood, come and leave clean of any asset or debt, and don't internalize the wealfare of their prongeny nor value their own wealth directly. At the end of their life, they'll consume everything they have, pay off the debts, and then be replaced by the young cohort.

3.1 Saver

The savers have time-consistent discounting rate. Their utility comes from their own consumption and the discounted utility flow that their wealth, as a bequeath, can bring to their offspring if they pass away in the next period. The savers' current value function is given by

$$V^s(a_t^s) = \sum_{\tau=t}^{\infty} (\theta^s(1-\delta))^{\tau-t} \left\{ \log c_\tau^s + \frac{\delta}{\rho} v(a_\tau^s) \right\} \tag{1}$$

where $v(a)$ refers to the bequeath motive. δ is the mortality rate and $\theta^s = 1-\rho$ is the discount rate. To be consistent with the empirical fact that the rich people have a higher desire to save, I'll choose v to make sure that saving is a luxury. In other words, the marginal utility from their wealth must deminish less than that from consumption as the agent become richer. I'll construct an increasing function of asset $\eta(a)$ to be the extent that $v(a)$ deviate from logarithmic, or the degree of non-homotheticity, such that the marginal utility of wealth is

$$v'(a) \equiv \eta^i(a)/a \tag{2}$$

Here, $\eta(a)$ need to be almost constant when a is small, but becomes almost linear as the asset grows larger, so that wealth level doesn't affact the agent's MPC unless she's as rich as the top percentile. A desirable functional form is $\eta(a) = \tau \left(1 + \tilde{a}^{-1} \log\left(1 + e^{\kappa a - \tilde{a}}\right)\right)$. It also offers the flexibility for calibration. The parameter τ governs the relative importance that one attach to her wealth than consumption. $\tilde{a}$ provides a threshold beyong which the agents start to value wealth disproportionally. And κ is a parameter that determines how sensitive the agent is to the change of wealth.

The savers will maximize their current value function subject to the budget constraint

$$a^s_{t+1} = R_t a^s_t - c^s_t \tag{3}$$

where the asset $a^s_t = h^s_t - d^s_t$ is defined to be the value of the "real asset" net of debt (or plus financial assets). The value of the real asset is given by the discounted future income flow. The agent might want to extract the future resources for current consumptions or accumulate more by holding household debt as a financial asset.

$$h_t \equiv w^s/\mu^s \sum_{\tau=t}^{\infty} \left(\prod_{u=t}^{\tau} R_u\right)^{-1} \tag{4}$$

3.2 Borrower

The borrowers have hyperbolic discounting but are aware of their self-control problem. These sophisticated agents would make decisions on anticipation of the behaviors of future selves. On contrast to the savers, the borrowers only live for 60 years since adulthood and only get utility from consumption. The reason why we don't stick to the infinite-horizon model is that the hyperbolic discounting would be nothing but an extra source of impatience in an infinite-horizon setting. The steady state interest rate is still uniquely determined by (hyperbolic) discounting factors, and any other path would be explosive. But in an overlapping generation setting, though a higher interest rate would lead to a higher life-time saving, the economy won't explode because the agent would consume her savings as she comes to the end of her life, no matter how high the interest rate is. In this senario, the long run credit demand curve would be downward-sloping. Therefore, we divide the borrowers into 60 generations. In each period, the 80-year-old cohort are replaced by a 20-year-old cohort. The agent who enters the economy in period s would try to maximize the current value function $W(t-s, a^b_{s,t})$ given by

$$W(t-s, a^b_{s,t}) = \log c^b_{s,t} + \beta\theta^b V(t-s+1, a^b_{s,t+1}) \tag{5}$$

where θ^b is the discount rate, β is the extra discounting factor in the following period that generate bias toward current utility, and V is the countinuous value, or the value function had the discounting been time-consistent

$$V(t-s, a^b_{s,t}) = \log c^b_{s,t} + \theta^b V(t+1-s, a^b_{s,t+1}) \tag{6}$$

The borrowers are also subject to the budget constraint as eq. (3). But since they only live for a finite lifespan, the real resources will decline over their life cycle. The "real asset" is now

$$h_{s,t} \equiv w^b/\mu^b \sum_{\tau=t}^{s+59} \left(\prod_{u=t}^{\tau} R_u \right)^{-1} \tag{7}$$

For simplicity, I assume the elasticity of intertemporal substitution is one. However, IES can influence the borrowing pattern by affacting the agent's desire to smooth consumption. I'll extend to more realistic IES's and see whether a different IES will greatly influence the credit market equilibrium.

3.3 Equilibrium

Given initial debt $d^s_0, d^b_{-t,0}$, a competitive equilibrium is a set of variables $\{c^i_t, a^i_t, d^i_t, h^i_t, r_t\}$ such that both agents choose $\{c^i_t, a^i_t\}$ to maximize their current value function eq. (1) and eq. (5), subjecting to the budget constraint eq. (3) and the financial market clears $d^s + d^b = 0$.

A steady state is an equilibrium where all the variables are constant.

I'll use the following parametrization in later sections. Since the model is on an annual basis, the savers discount rate δ^s is set to 0.944 ($\rho = 0.056$). The borrower's current bias and discount rate are set to $\beta = 0.7$ and $\theta^b = 0.957$, taken from Angeletos 2001. However, there is radical disagreement on the value of β among different measures, I'll see how our result changes with different current biases. The δ is set to 0.05, not strictly aligned with mortality rate for two reasons. First, it also represents reasons for valuing wealth other than bequeath motive and the discount rate such as utility over status. Second, a discount rate calibrated based on homothetic model would seem insufficiently high for this model. Therefore, δ also serves to rise the "effective" discounting rate to what's consistent with a reasonable equilibrium interest rate. Since the interpretation and function of δ is multivariate, I calibrate it by targeting the equilibrium interest rate of 4%.

In calibrating the parameters in $\eta(a)$, I first target an equilibrium r of 4%. Second, I target an equilibrium debt level of 5 times of the borrower's annual income. Third, the slope of the credit supply curve around the steady state is aligned with the empirical value of $dr/d\log d = -0.032$. With the descipline, the relative importance of wealth to consumption is set to $\tau = 0.5$. The threshold $\tilde{a}$ and the sensitiveness κ are set to 1 and 0.004 respectively. [1]

[1] The asset threshold is around $\tilde{a}/\kappa = 250$. With an steady state interest rate of 4%, that means for nonhomotheticity to show up to the savers, their financial asset stock sould be as much as the value of real asset.

4 Excessive Debt and Welfare Cost

We first analyze the steady states. The saver's Euler equation is given by (after linearization)

$$\frac{\Delta c^s_{t+1}}{c^s_t} = r_t - \rho - \delta + \frac{\delta c^s_{t+1}}{\rho a^s_{t+1}} \eta(a^s_{t+1}) \tag{8}$$

In the steady state, asset is constant, thus the budget constraint is reduced to $c = ra$. Combined with $\Delta c^s_{t+1} = 0$, the long run credit supply is derived as

$$r = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta\left((w^s/r - d^s)/\mu^s\right)} \tag{9}$$

The borrower's Euler equation is given by

$$\frac{c^b_{s,t+1}}{c^b_{s,t}} = R_{t+1}\theta^b[1 - \frac{(1-\beta)}{R_{t+1}} \frac{dc^b_{s,t+1}}{da^b_{s,t+1}}] \tag{10}$$

The more biased the agent is to the current period, the lower the consumption growth will be. Therefore, the agent with hyperbolic discounting would call for more debt in early life stages and curtail consumption spending later to pay down the debt. Given the terminal condition $a^b_{s,s+T} = 0$, we can solve the agent's lifetime consumption profile backward. Since the borrowers' preference is homothetic, we can get the MPC for free along the way. Typical life cycle consumption and borrowing profiles for agents with time-consistent and hyperbolic discounting in the steady states are shown in fig. 3.

The aggregate debt demand from the borrowers is noramlized by taking the average of the credit demand from all cohorts

$$d^b_t = \frac{1}{T}\left(\sum_{s=0}^{T-1} d^b_{t-s,t}\right) \tag{11}$$

A unique aggregate credit demand corresponds to one steady state interest rate, generate long run the credit demand curve. Combined with the credti supply curve, the steady state is uniquely pinned down, as shown in fig. 2. As the agent becomes more impatient (in the sence of a lower β), the steady state debt level increases and the interest rate falls significantly because the savers are willing to hold household debt as an asset even at low interest rate as they become richer.

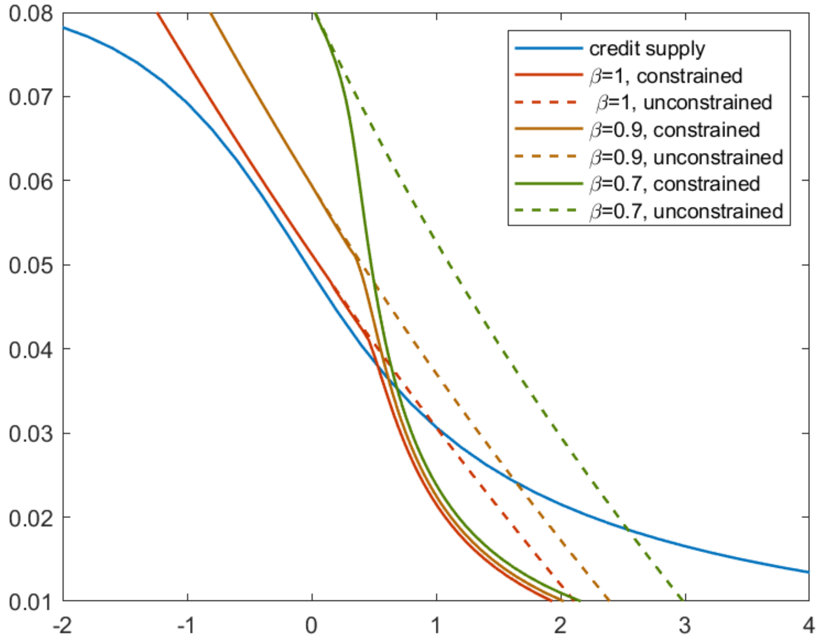
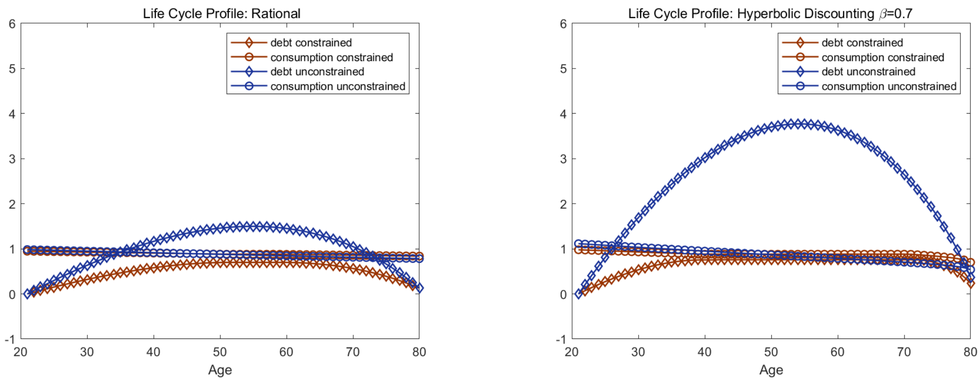


Figure 2: Credit Supply and Demand With Different β



(a) Rational

(b) Hyperbolic discounting $\beta = 0.7$

Figure 3: Life time consumption and debt profile

Despite the higher debt level, the interest rate also falls with hyperbolic discounting agents. Although the agents fail to optimize, they might be compensated by the lower debt service payment. Therefore, the change of their welfare is ambiguous. I compute the welfare cost/gain as the utility-equivalent consumption lost/gain in the rational case, denoted by $\Delta(\beta^*)$, using the following equation:

$$\sum_{t=0}^{T-1} u(c_{0,t}^b(1+\Delta(\beta^*)))\bigg|_{\beta=1} = \sum_{t=0}^{T-1} u(c_{0,t}^b)\bigg|_{\beta=\beta^*} \tag{12}$$

Surprisingly, medium hyperbolic discounting improves the agent's welfare. Although the agent is not acting optimally given the interest rate she faces, the benefit of a lower interest rate actually more than offsets the imperfection of the consumption plan. For example, the welfare gain of hyperbolic discounting with $\beta = 0.8$ is equivalent to increasing the consumption by 1.5% each period in the rational case, mainly benefited from an interest rate fall of 3%.

However, if the current bias is as high as $\beta = 0.7$, then the agent will suffer welfare cost because the interest rate won't decrease further while the higher debt payment later becomes more painful.

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