LSTMs

Learning to forget

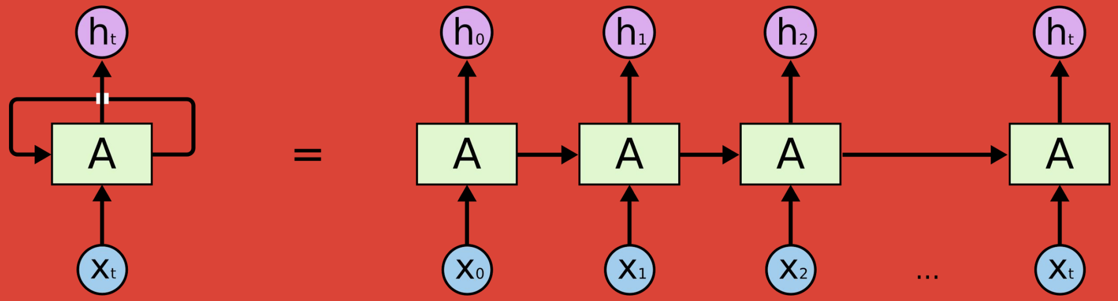
Devansh Kukreja and Eric Nie

Background/Motivation

- State-of-the-art in sequence learning
- Address the issue of remembering information
- Useful for analyzing sequential data
- Cells within RNNs and LSTMs maintain an internal state

Recurrent Neural Networks (A Brief Introduction)

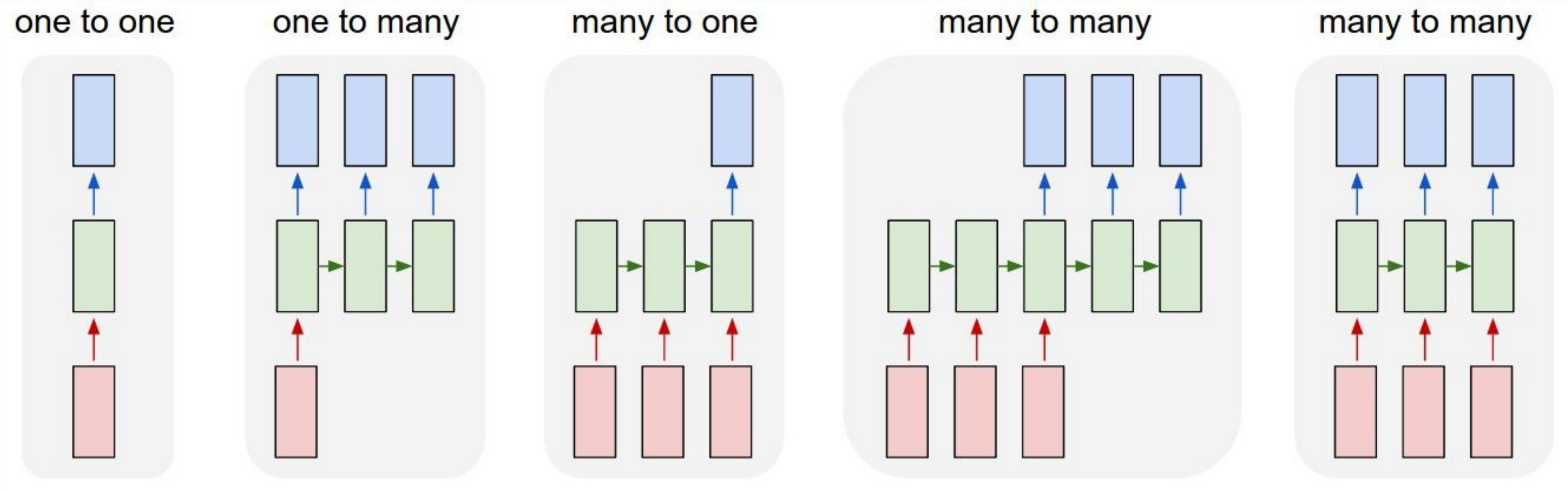
Intro



Recurrent Neural Networks

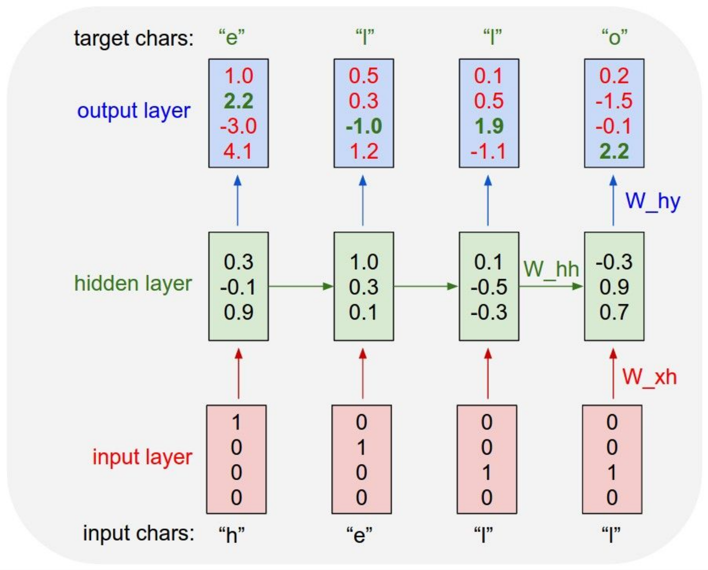
- Internal states
- Sequences of inputs and outputs
- Share weights across timesteps

More RNNs



https://karpathy.github.io/2015/05/21/rnn-effectiveness/

Character-level RNN



https://karpathy.github.io/2015/05/21/rnn-effectiveness/

- Predict the next character given input
- One-hot encoding

What can an RNN do?

Generate Text: Linux source code

```c
/*
 * Increment the size file of the new incorrect UI_FILTER group information
 * of the size generatively.
 */
static int indicate_policy(void)
{
  int error;
  if (fd == MARN_EPT) {
    /*
     * The kernel blank will coeld it to userspace.
     */
    if (ss->segment < mem_total)
      unblock_graph_and_set_blocked();
    else
      ret = 1;
    goto bail;
  }
  segaddr = in_SB(in.addr);
  selector = seg / 16;
  setup_works = true;
  for (i = 0; i < blocks; i++) {
    seq = buf[i++];
    bpf = bd->bd.next + i * search;
    if (fd) {
      current = blocked;
    }
  }
  rw->name = "Getjbbregs";
  bprm_self_clearl(&iv->version);
  regs->new = blocks[(BPF_STATS << info->historidac)] | PFMR_CLOBATHINC_SECONDS << 12;
  return segtable;
}
```

Produce Research Papers:

For $\bigoplus_{n=1,\ldots,m}$ where $\mathcal{L}_{m_\bullet} = 0$, hence we can find a closed subset $\mathcal{H}$ in $\mathcal{H}$ and any sets $\mathcal{F}$ on X, U is a closed immersion of S, then $U \to T$ is a separated algebraic space.

Proof. Proof of (1). It also start we get

$$S = \mathrm{Spec}(R) = U \times_X U \times_X U$$

and the comparicoly in the fibre product covering we have to prove the lemma generated by $\coprod Z \times_U U \to V$. Consider the maps M along the set of points Sch_{fppf} and $U \to U$ is the fibre category of S in U in Section, ?? and the fact that any U affine, see Morphisms, Lemma ??. Hence we obtain a scheme S and any open subset $W \subset U$ in $Sh(G)$ such that $\mathrm{Spec}(R') \to S$ is smooth or an

$$U = \bigcup U_i \times_{S_i} U_i$$

which has a nonzero morphism we may assume that f_i is of finite presentation over S. We claim that $\mathcal{O}_{X,x}$ is a scheme where $x, x', s'' \in S'$ such that $\mathcal{O}_{X,x'} \to \mathcal{O}'_{X',x'}$ is separated. By Algebra, Lemma ?? we can define a map of complexes $\mathrm{GL}_{S'}(x'/S'')$ and we win. □

To prove study we see that $\mathcal{F}|_U$ is a covering of $\mathcal{X}'$, and $\mathcal{T}_i$ is an object of $\mathcal{F}_{X/S}$ for $i > 0$ and $\mathcal{F}_p$ exists and let $\mathcal{F}_i$ be a presheaf of $\mathcal{O}_X$-modules on $\mathcal{C}$ as a $\mathcal{F}$-module. In particular $\mathcal{F} = U/\mathcal{F}$ we have to show that

$$\widetilde{M}^\bullet = \mathcal{I}^\bullet \otimes_{\mathrm{Spec}(k)} \mathcal{O}_{S,s} - i_X^{-1}\mathcal{F})$$

is a unique morphism of algebraic stacks. Note that

$$\mathrm{Arrows} = (Sch/S)^{opp}_{fppf}, (Sch/S)_{fppf}$$

and

$$V = \Gamma(S, \mathcal{O}) \longmapsto (U, \mathrm{Spec}(A))$$

is an open subset of X. Thus U is affine. This is a continuous map of X is the inverse, the groupoid scheme S.

Proof. See discussion of sheaves of sets. □

The result for prove any open covering follows from the less of Example ??. It may replace S by $X_{spaces,étale}$ which gives an open subspace of X and T equal to S_{Zar}, see Descent, Lemma ??. Namely, by Lemma ?? we see that R is geometrically regular over S.

Lemma 0.1. *Assume (3) and (3) by the construction in the description.*

Suppose $X = \lim |X|$ (by the formal open covering X and a single map $\underline{Proj}_X(\mathcal{A}) = \mathrm{Spec}(B)$ over U compatible with the complex

$$Set(\mathcal{A}) = \Gamma(X, \mathcal{O}_{X,\mathcal{O}_X}).$$

When in this case of to show that $\mathcal{Q} \to \mathcal{C}_{Z/X}$ is stable under the following result in the second conditions of (1), and (3). This finishes the proof. By Definition ?? (without element is when the closed subschemes are catenary. If T is surjective we may assume that T is connected with residue fields of S. Moreover there exists a closed subspace $Z \subset X$ of X where U in X' is proper (some defining as a closed subset of the uniqueness it suffices to check the fact that the following theorem

(1) *f is locally of finite type. Since $S = \mathrm{Spec}(R)$ and $Y = \mathrm{Spec}(R)$.*

Proof. This is form all sheaves of sheaves on X. But given a scheme U and a surjective étale morphism $U \to X$. Let $U \cap U = \coprod_{i=1,\ldots,n} U_i$ be the scheme X over S at the schemes $X_i \to X$ and $U = \lim_i X_i$. □

The following lemma surjective restrocomposes of this implies that $\mathcal{F}_{x_0} = \mathcal{F}_{x_0} = \mathcal{F}_{\mathcal{X},\ldots,0}$.

Lemma 0.2. *Let X be a locally Noetherian scheme over S, $E = \mathcal{F}_{X/S}$. Set $\mathcal{I} = \mathcal{J}_1 \subset \mathcal{I}'_n$. Since $\mathcal{I}^n \subset \mathcal{I}^n$ are nonzero over $i_0 \leq \mathfrak{p}$ is a subset of $\mathcal{J}_{n,0} \circ \overline{A}_2$ works.*

Lemma 0.3. *In Situation ??. Hence we may assume $\mathfrak{q}' = 0$.*

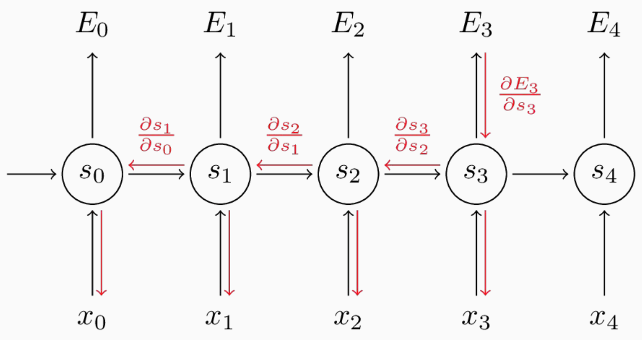
Proof. We will use the property we see that $\mathfrak{p}$ is the mext functor (??). On the other hand, by Lemma ?? we see that

$$D(\mathcal{O}_{X'}) = \mathcal{O}_X(D)$$

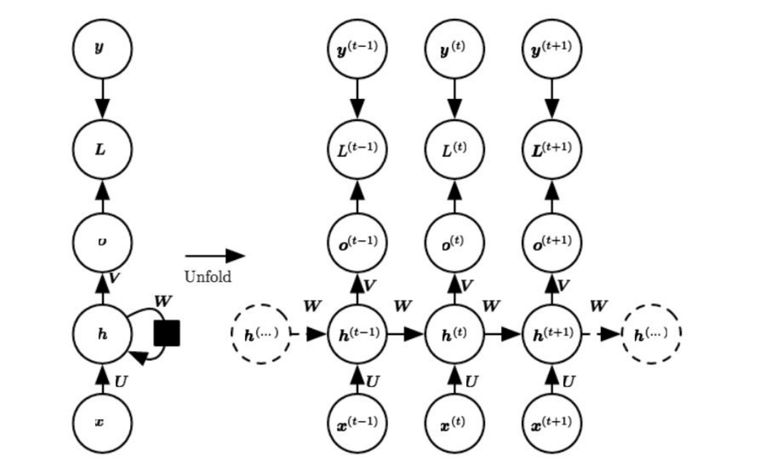
where K is an F-algebra where δ_{n+1} is a scheme over S. □

Training a RNN

Backpropagation through time:



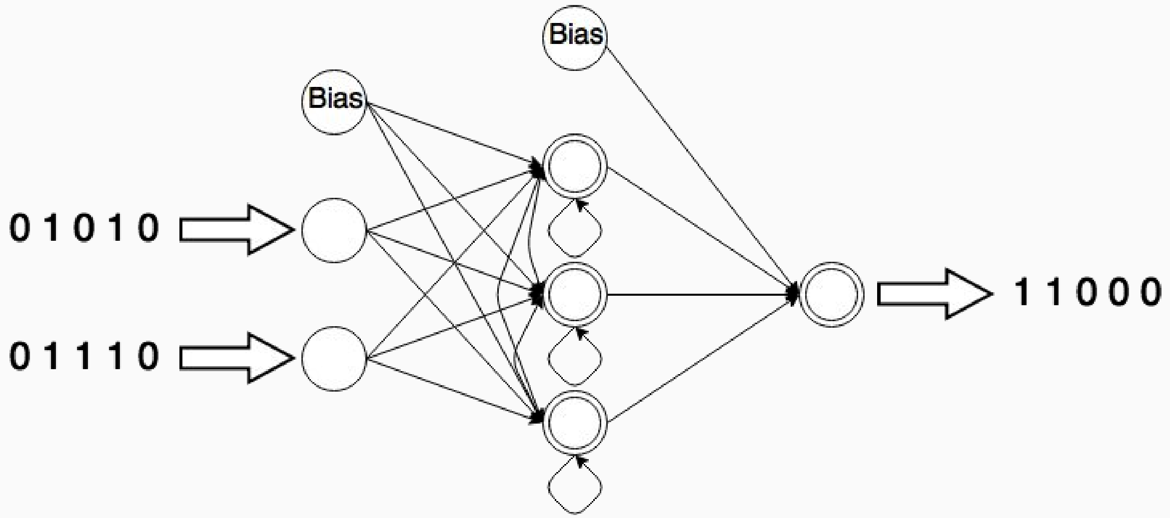
- Minibatches of subsequences
- Update weights after some number of inputs



$$\begin{aligned} \boldsymbol{a}^{(t)} &= \boldsymbol{b} + \boldsymbol{W}\boldsymbol{h}^{(t-1)} + \boldsymbol{U}\boldsymbol{x}^{(t)} & (10.8) \\ \boldsymbol{h}^{(t)} &= \tanh(\boldsymbol{a}^{(t)}) & (10.9) \\ \boldsymbol{o}^{(t)} &= \boldsymbol{c} + \boldsymbol{V}\boldsymbol{h}^{(t)} & (10.10) \\ \hat{\boldsymbol{y}}^{(t)} &= \operatorname{softmax}(\boldsymbol{o}^{(t)}) & (10.11) \end{aligned}$$

Equations for forward pass

Adding Binary Numbers

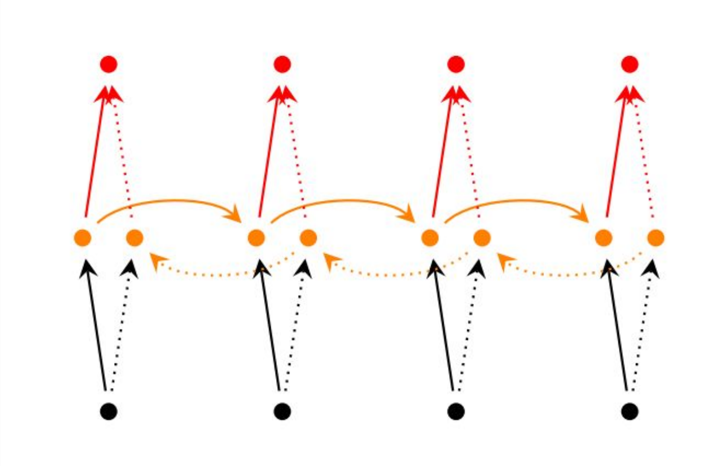


Issues with RNNs

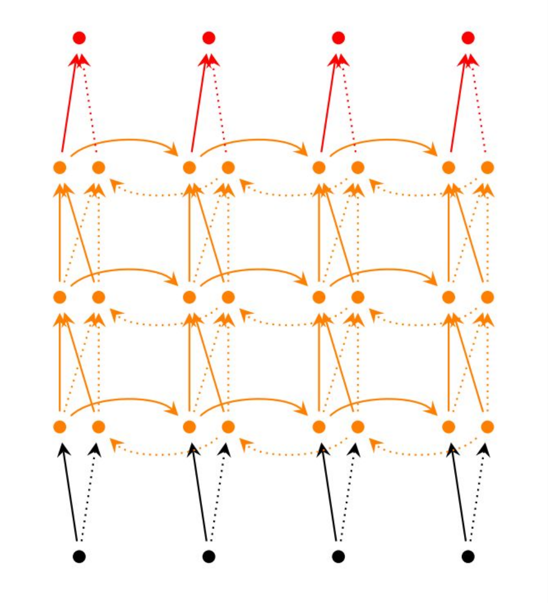
- Training RNNs uses backpropagation through time
- Exploding and vanishing gradient problem similar to that of deep neural networks

Extension to RNNs

Bidirectional RNNs

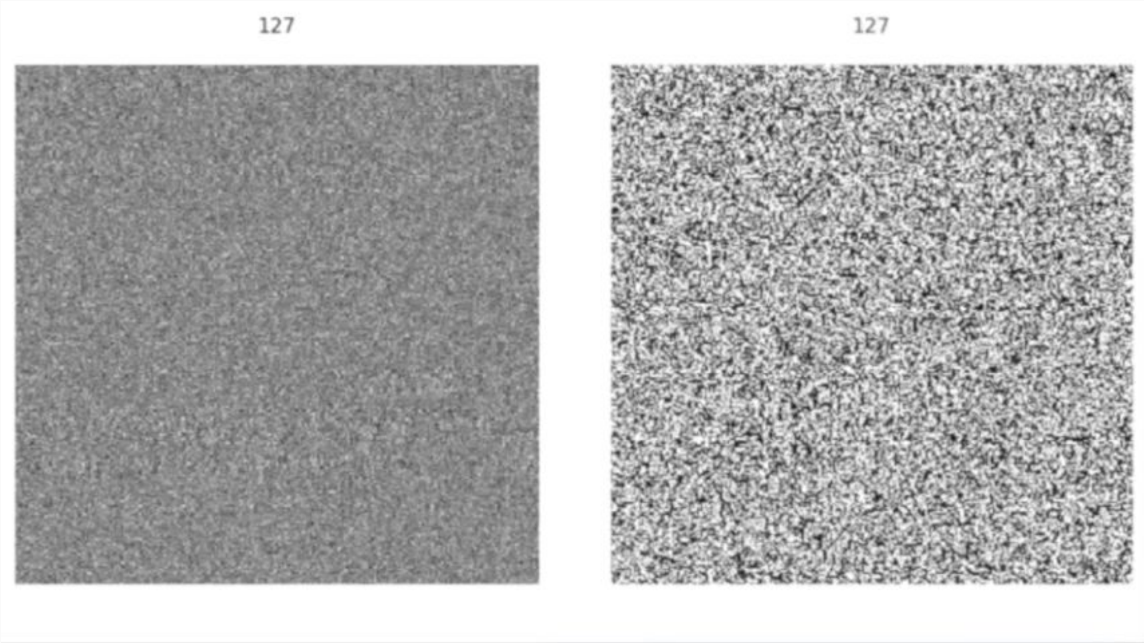


Deep Bidirectional RNNs



Final Word on RNNs

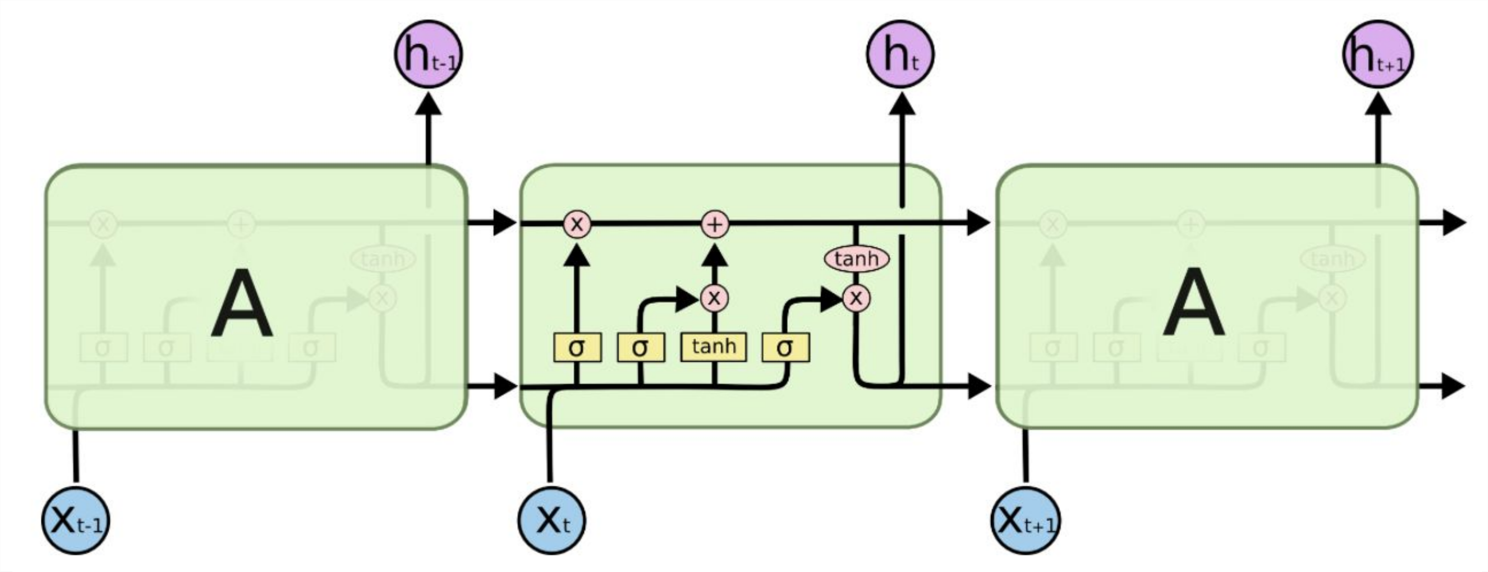
- Turing complete
- Useful for learning sequential data
- Trouble capturing long-term dependencies in practice



LSTMs are a variant RNN that uses “**gating units**” to control information flow adaptively to combat “**gradient vanishing**” *

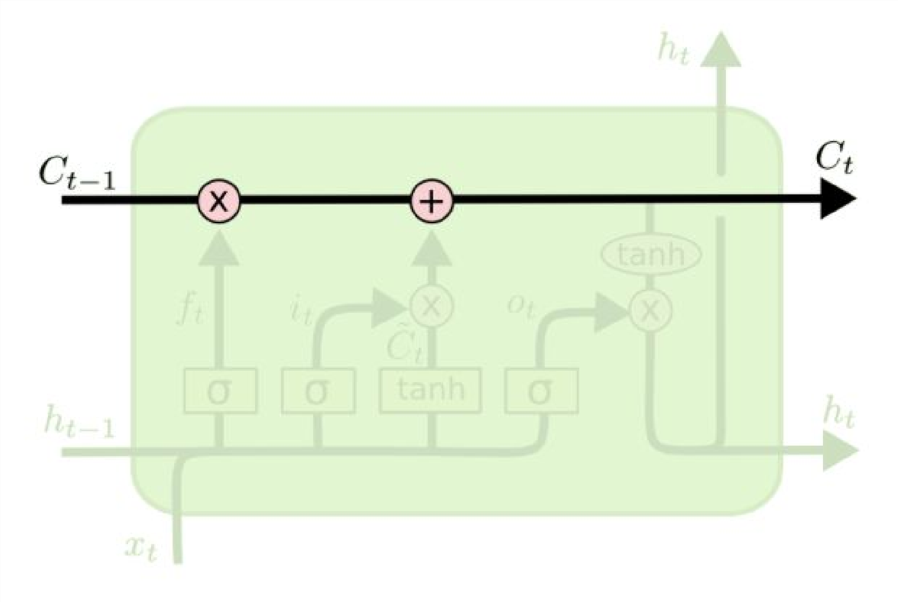
* Pulled from a whiteboard; likely written by a CMU Grad Student.

Long Short Term Memory Networks



http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Long Short Term Memory Networks



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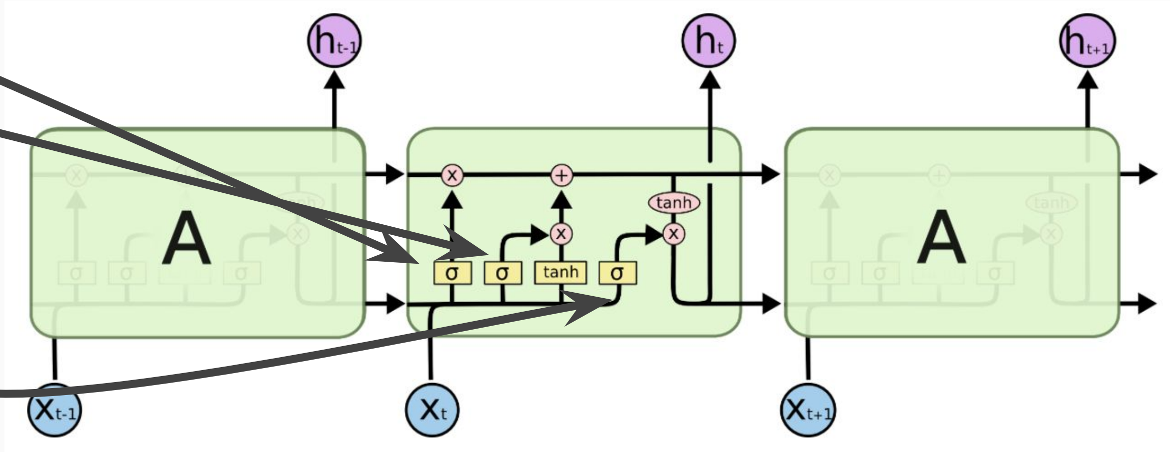
Long Short Term Memory Networks

1. **Forget** Gate
2. **Input** Gate
3. **Output** Gate



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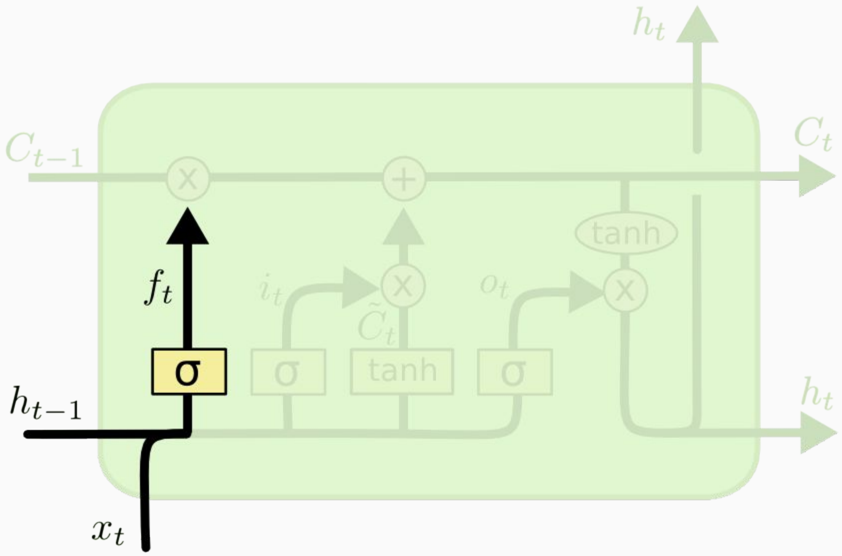
Forget Gate

1. Takes in h_{t-1} concat x_t and outputs a number between 0 and 1 for each number in the cell state.

2. That output filters the cell state, selectively choosing how much of each node to forget.

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$



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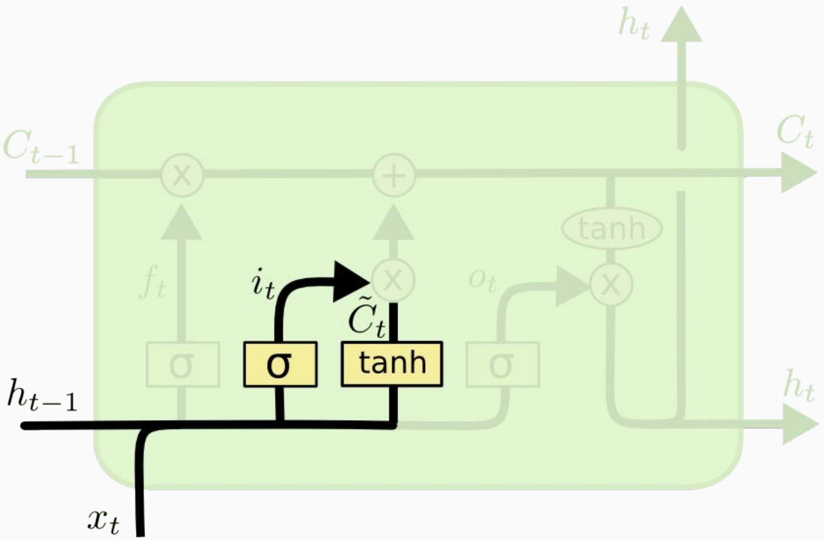
Input Gate

1. Sigmoid Input gate decides what new information can flow in.

2. Tanh layer creates a vector of new candidate values that could be added.

$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$



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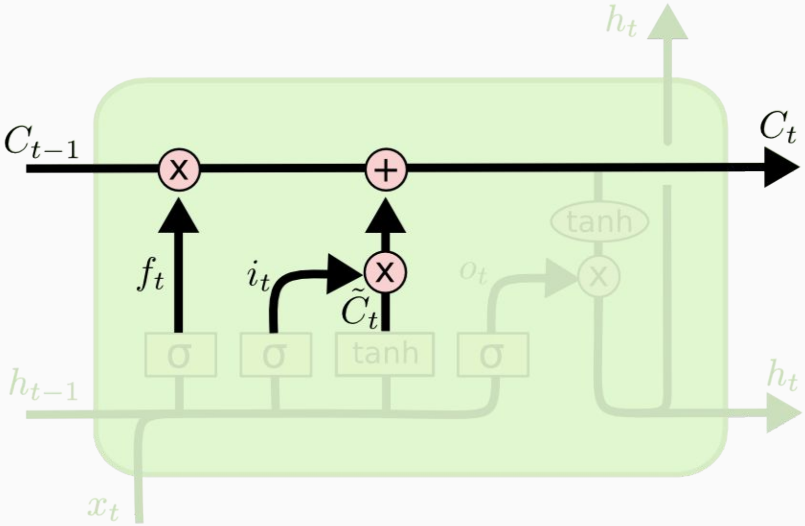
Putting it together

1. Apply forget gate
2. Compute new candidate cell state
3. Filter in new cell state



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

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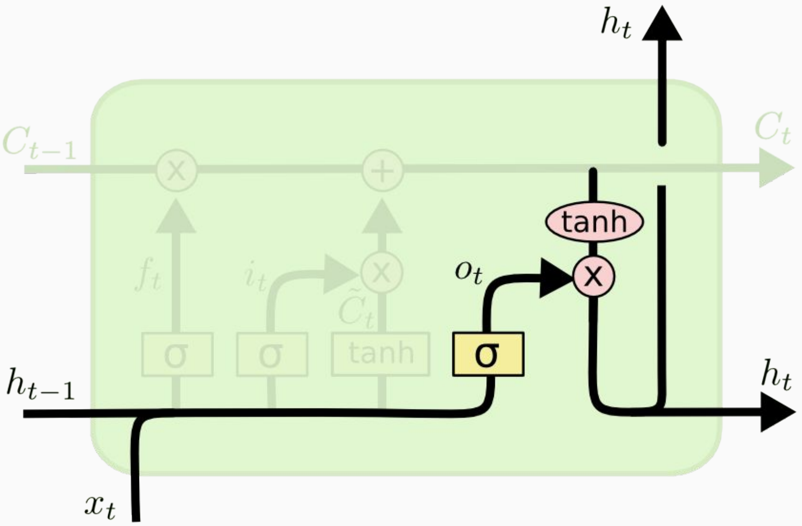
Output Gate

1. Run new cell state through tanh to put values between -1 and 1.

2. Multiply it by the output of the third sigmoid gate to decide what to output.

$$o_t = \sigma\left(W_o \left[h_{t-1}, x_t\right] + b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$



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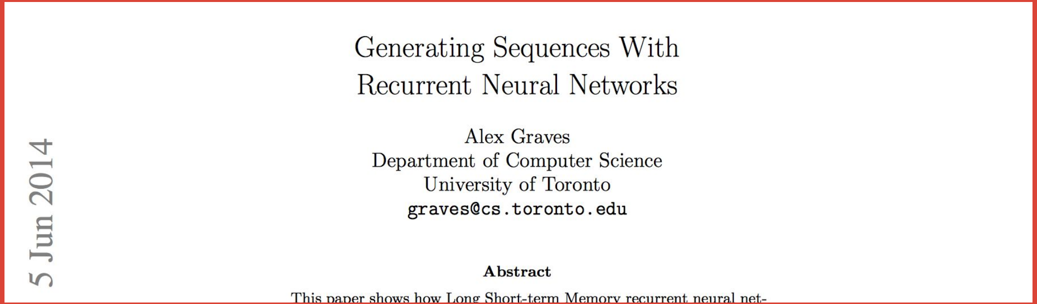
LSTM Demo


5 Jun 2014

Generating Sequences With
Recurrent Neural Networks

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Abstract

This paper shows how Long Short-term Memory recurrent neural net-

Row 1: Characters as they're recognized

Row 2: States of some memory cells

Row 3: Writing as it's analyzed

Row 4: Gradient backpropagated to inputs from the most active character

https://arxiv.org/pdf/1308.0850.pdf

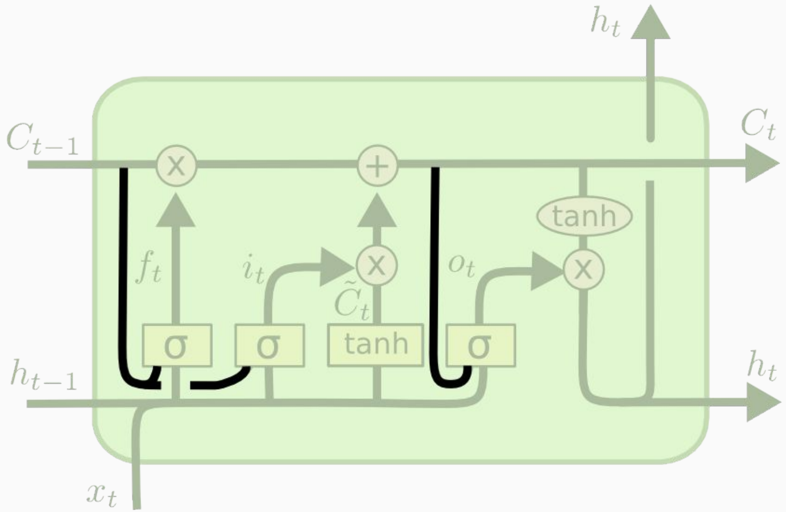
Variations on LSTM

Peephole Connections

1. Pass the gate layers the current cell state.

2. Helps the system learn from the size of the time lags.

$$f_t = \sigma\left(W_f \cdot [\boldsymbol{C_{t-1}}, h_{t-1}, x_t] \ + \ b_f\right)$$
$$i_t = \sigma\left(W_i \cdot [\boldsymbol{C_{t-1}}, h_{t-1}, x_t] \ + \ b_i\right)$$
$$o_t = \sigma\left(W_o \cdot [\boldsymbol{C_t}, h_{t-1}, x_t] \ + \ b_o\right)$$



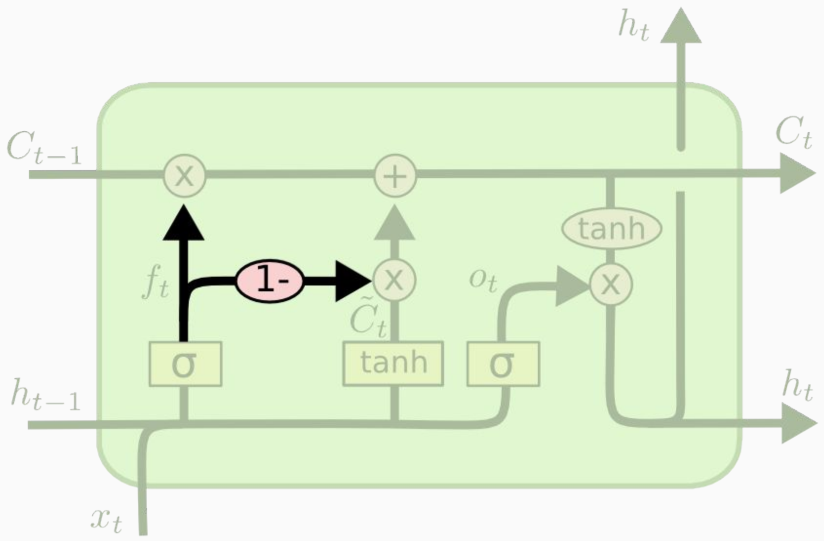
ftp://ftp.idsia.ch/pub/juergen/TimeCount-IJCNN2000.pdf

http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Couple Forget/Input Gates

1. Pass the gate layers the current cell state.

2. Simplifies the cell but reduced performance slightly.



$$C_t = f_t * C_{t-1} + (1 - f_t) * \tilde{C}_t$$

http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Gated Recurrent Unit (GRU)

1. Combine forget and input gates into an **"update gate"**

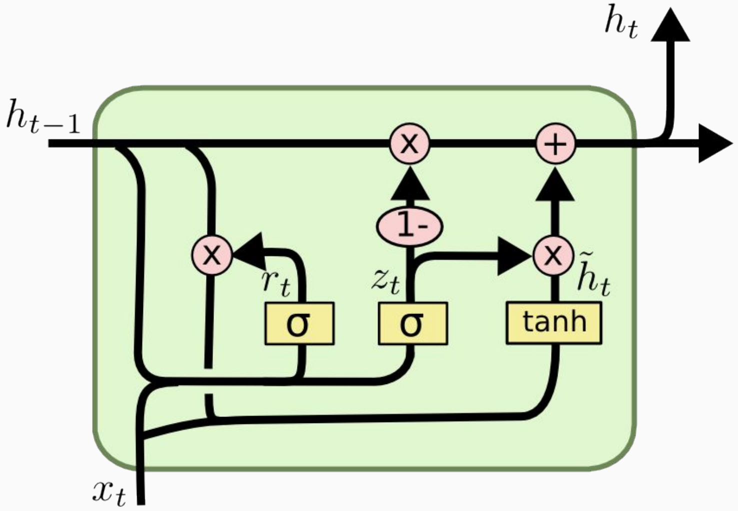
2. In general, equal to LSTMs in performance and faster in training

$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$



http://colah.github.io/posts/2015-08-Understanding-LSTMs/

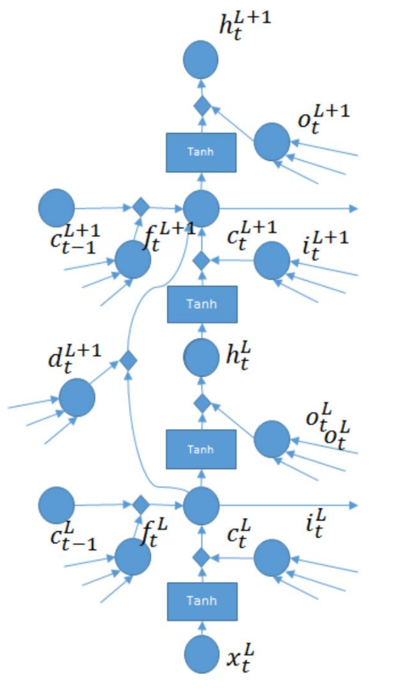
Depth Gated RNNs

1. Adds a fourth gate - the Depth Gate
2. Creates a gated linear connection between lower and upper layer memory cells
3. Relates to highway networks and Grid LSTM

Using the depth gate, a DGLSTM unit can be written as

$$
\begin{aligned}
i_t^{L+1} &= \sigma(W_{xi}^{L+1} x_t + W_{hi}^{L+1} h_{t-1}^{L+1} + W_{ci}^{L+1} c_{t-1}^{L+1}) && (13)\\
f_t^{L+1} &= \sigma(W_{xf}^{L+1} x_t + W_{hf}^{L+1} h_{t-1}^{L+1} + W_{cf} c_{t-1})^{L+1} && (14)\\
d_t^{L+1} &= \sigma(b_d^{L+1} + W_{xd}^{L+1} x_t^{L+1} + W_{cd}^{L+1} \odot c_{t-1}^{L+1} + W_{ld}^{L+1} \odot c_t^{L}) && (15)\\
c_t^{L+1} &= d_t^{L+1} c_t^{L} + f_t^{L+1} \odot c_{t-1} + i_t^{L+1} \odot tanh(W_{xc} x_t + W_{hc} h_{t-1}^{L+1}) && (16)\\
o_t^{L+1} &= \sigma(W_{xo}^{L+1} x_t + W_{ho}^{L+1} h_{t-1} + W_{co}^{L+1} c_t^{L+1}) && (17)\\
h_t^{L+1} &= o_t^{L+1} \odot tanh(c_t^{L+1}) && (18)
\end{aligned}
$$

where i_t^{L+1}, f_t^{L+1} o_t^{L+1}, and d_t^{L+1} are the input gate, forget gate, output gate and the depth gate.



Questions

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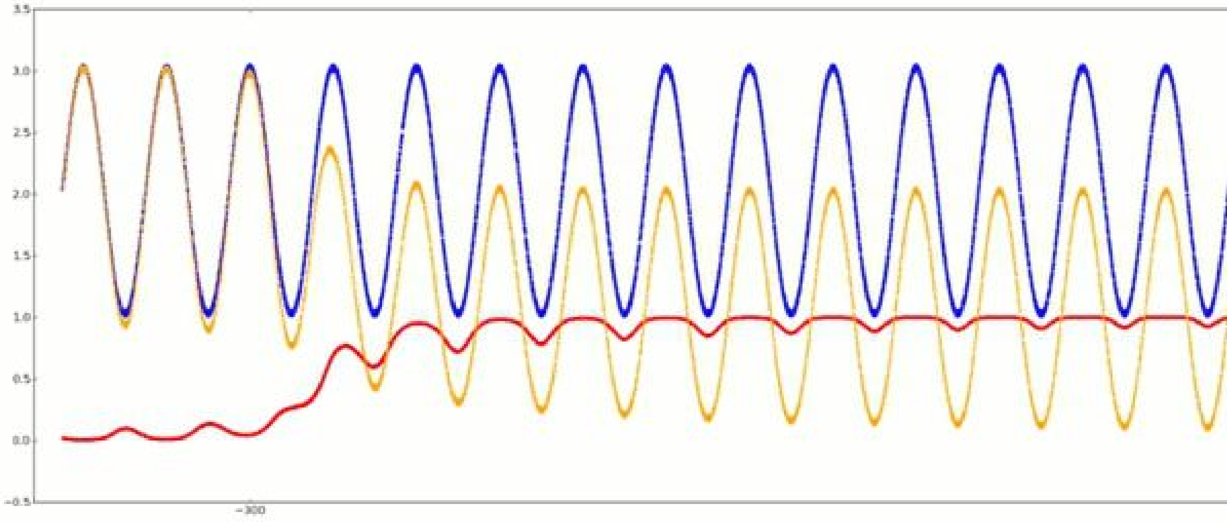
Devansh Kukreja

LSTMs



LSTM learning to predict next noised sinus value.

Training Signal
LSTM Prediction
Error