Solution to Thinking Recursively with JAVA.

— Eric Roberts.

Ex. 2-2.

(a) $1+2+4+8+\cdots+2^{N-1} = 2^N - 1.$

1° Let $N=1$,

$$2^{1-1} = 2^1 - 1.$$

2° If for $N=k$ we have

$$1+2+4+\cdots+2^{k-1} = 2^k - 1.$$

then for $N=k+1$, we have

$$1+2+4+\cdots+2^{k-1} + 2^{(k+1)-1} = 2^k + 2^k - 1$$

$$= 2^{k+1} - 1$$

(b). $1+3+9+27+\cdots+3^N = \dfrac{3^{N+1}-1}{2}$

1° Let $N=1$.

$$3^0 + 3^1 = \frac{3^2-1}{2} = 4.$$

2° let $N=k$, assume we have

$$1+3+\cdots+3^k = \frac{1}{2}(3^{k+1}-1)$$

then for $N=k+1$,

$$1+3+\cdots+3^k+3^{k+1} = \frac{1}{2}(3^{k+1}-1) + 3^{k+1}$$

$$= \frac{1}{2}(3^{(k+1)+1}-1)$$

(C). $1\times1 + 2\times2 + 3\times4 + \cdots + N\cdot2^{N-1} = (N-1)\,2^N + 1.$

1° let $N = 1$,

$$1\times2^{1-1} = 2^0 = (1-1)\cdot2^1 + 1 = 1.$$

2° let $N = k$, ~~for~~ we have,

$$1\times1 + 2\times2 + 3\times4 + \cdots + k\cdot2^{k-1} = (k-1)\,2^k + 1.$$

For $N = k+1$,

$$1\times1 + 2\times2 + 3\times4 + \cdots + k\cdot2^{k-1} + (k+1)\cdot2^k$$

$$= (k-1)\cdot2^k + 1 + (k+1)\,2^k$$

$$= 2k\cdot2^k + 1 = \big((k+1)-1\big)\cdot2^{k+1} + 1.$$

$\boxed{\text{Ex 2-3}}$

Same as Ex 2-2 (a).

$\boxed{\text{Ex 2-4}}$

(a) $O(1000)$

(b) $O(n^2)$

(c) $O(\log n)$

$1000 \le \log n$

$\Rightarrow n \geq 2^{1000}.$

Ex 2-5

From Ex. 2-2 (c).

$$1\times1 + 2\times2 + 3\times4 + 4\times8 + \cdots + G\times2^{G-1}$$

$$= (G-1)\,2^G + 1.$$

let $G = \log N$.

We have. $$\frac{(\log N - 1)N + 1}{N}$$

$$\doteq \log N - 1 + \frac{1}{N}.$$

So the average complexity is $\underline{\log N}$

As N doubles, the complexity increases a small factor.

Ex 2-6

The main issue here is the conjecture part.

namely, the base statement : Any set of horses is monochromatic", which is false according to our experience

$\boxed{\text{Ex 3-7}}$.

Key point:

the additions required for fib(n)

= the additions required for fib(n-1)

+

the additions required for fib(n-2)

+

1.

$\boxed{\text{Ex 3-8}}$

it computes fib(n).

$O(n)$

$\boxed{\text{Ex 3-9}}$.

$$\frac{\varphi^0 - \hat{\varphi}^0}{\sqrt{5}} = 0. = \text{fib}(0)$$

$$\frac{\varphi^1 - \hat{\varphi}^1}{\sqrt{5}} = \frac{\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}}{\sqrt{5}} = \frac{2\sqrt{5}}{2\sqrt{5}} = 1. = \text{fib}(1).$$

$$\cancel{\frac{\varphi^n - \hat{\varphi}^{n}}{\sqrt{5}}} \quad \frac{\varphi^{n-1} - \hat{\varphi}^{n-1}}{\sqrt{5}} + \frac{\varphi^{n-2} - \hat{\varphi}^{n-2}}{\sqrt{5}}$$

$$= \frac{\varphi^{n-2}(\varphi+1) - \hat{\varphi}^{n-2}(\hat{\varphi}+1)}{\sqrt{5}} = \underline{\frac{\varphi^n - \hat{\varphi}^n}{\sqrt{5}} = \text{fib}(n)}.$$

$\boxed{\text{EX. 5-4}}$

1° the number of moves to transfer a tower of Size 1, by the move Tower algorithm is $\underline{1}$.

2°. Suppose the number of moves to transfer a tower of Size n, by the move Tower ~~got~~ alg. is 2^n-1

3° then the steps to transfer a tower of size n+1. by move Tower is:

$$\underline{2^{n}-1+2^{n}-1+1} = \underline{2^{n+1}-1}.$$