HW 5- ANOVA

1. Implantable heart pacemakers contain small circuit boards called sub-strates. These substrates are assembled, cut to shape, and fired. Some of the substrates will separate, or delaminate, making them useless. The purpose of this experiment was to study the effects of two factors on the rate of delamination. The factors were A: furnace airflow, low versus high, with theory suggesting high is better; and B: laser, old versus new, with theory suggesting new cutting lasers are better. A large number of raw, assembled substrates are divided into groups. These groups are assigned at random to the four factor-level combinations of the two factors. The substrates are then processed, and the response is the fraction of substrates that delaminate. Data is

A	B	Y
Low	New	0.83
Low	New	0.78
Low	New	0.75
Low	New	0.83
Low	Old	0.86
Low	Old	0.67
Low	Old	0. 73
High	New	0.72
High	New	0.81
High	New	0.85
High	New	0.87
High	Old	0.68
High	Old	0.73
High	Old	0.74

(a) (2pt) Give the design matrix corresponding the the model

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

where

$$\begin{aligned} \alpha_1 + \alpha_2 &= 0, \beta_1 + \beta_2 = 0, (\alpha\beta)_{11} + (\alpha\beta)_{21} = 0, (\alpha\beta)_{12} + (\alpha\beta)_{22} = 0, \\ (\alpha\beta)_{11} + (\alpha\beta)_{12} &= 0, (\alpha\beta)_{21} + (\alpha\beta)_{22} = 0 \end{aligned}$$

(b) (2pt) Construct an interaction plot. Does it suggest that there is an interaction between fertilizer type and wheat type?

(c) (4pt) Test H_0 : No interaction against H_1 : there is an interaction, use $\alpha = 0.05$.

(d) (3pt) It is decided to fit a model without an interaction. Test H_0 : there is no A effect against H_a : there is an A effect. Use $\alpha = 0.05$.

(e) (3pt) It is decided to fit a model without an interaction. Test H_0 : there is no B effect against H_a : there is an B effect. Use $\alpha = 0.05$.

2. (6pt) (data in file mileage.csv) This problem is designed to review regression with categorical variables and the partial F test. International Oil Inc. is attempting to a develop a reasonably priced minimum unleaded gasoline that will deliver higher gasoline mileage than can be achieved by its current premium unleaded gasolines. As part of its development process, International Oil Inc. wishes to study the effect of one qualitative variable, x_1, premium gasoline unleaded type (A, B, C) and one quantitative variable x_2 amount of gasoline additive VST (0, 1, 2, 3 units) on the gasoline mileage y obtained by an automobile called Encore. For testing purposes a sample of 22 Encores is randomly selected and driven under normal driving conditions. The combination of x_1 and x_2 used in the experiment along with the corresponding values of y are in file mileage.csv.

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 x_2 + \epsilon_i$$

where $D_{1i} = 1$ gas type is B and 0 otherwise and $D_{2i} = 1$ is gas type is C and 0 otherwise.

(a) (2pt) Estimate the β_is and interpret your result (see note for how to fit this model)

(b) (2pt) Construct a 95% confidence interval for β_3 and interpret your result

(c) (2pt) Test $H_0 : \beta_1 = \beta_2 = 0$ against H_a : Not H_0 using $\alpha = 0.05$.

y	x1	x2
28.0	A	0
28.6	A	0
27.4	A	0
33.3	B	0
34.5	B	0
33.0	A	1
32.0	A	1
35.6	A	1
34.4	A	1
35.0	B	1
34.0	B	1
33.3	B	1
34.7	C	1
33.5	A	2
32.3	A	2
33.4	B	2
33.0	C	2
32.0	C	2
29.6	B	3
30.6	B	3
28.6	C	3
29.8	C	3