HW 7 solution

1. (12pt) Let π denote the probability that a randomly selected individual supports laws legalizing abortion predicted using gender ($G = 0$ if gender is male and $G = 1$ if gender is female), religion affiliation (protestant, catholic or jewish; $R_1 = 1$ if Protestant and 0 otherewise; $R_2 = 1$ if Catholic and 0 otherwise; $R_1 = R_2 = 0$ if Jewish) and political party (democrat, republican or independent ; $P_1 = 1$ if Democrat and 0 otherwise; $P_2 = 1$ if Republican and 0 otherwise; $P_1 = P_2 = 0$ if independent). The model used is

$$\text{logit}(\hat{\pi}) = \beta_0 + \beta_1 G + \beta_2 R_1 + \beta_3 R_2 + \beta_4 P_1 + \beta_5 P_2.$$

and the estimated logit model is

$$\text{logit}(\hat{\pi}) = 0.11 + 0.16G - 0.57R_1 - 0.66R_2 + 0.47P_1 - 1.67P_2.$$

(a) (2pt) Estimate the probability that a male, protestant and republican supports laws legalizing abortion. Estimate the probability that female, catholic and democrat supports laws legalizing abortion
Answer:

- The estimate of probability that a male, protestant and republican supports laws legalizing abortion is: $\hat{\pi} = \frac{e^{-2.13}}{1+e^{-2.13}} = 0.106$.
- The estimate of the probability that female, catholic and democrat supports laws legalizing abortion $\hat{\pi} = \frac{e^{0.08}}{1+e^{0.08}} = 0.520$.

(b) (2pt) Interpret $b_1 = 0.16$ and $b_2 = -0.57$.

- Holding religion affiliation and political party fixed, the odds that a female supports laws legalizing abortion is estimated to be $e^{0.16} = 1.174$ times the corresponding odds of a male. OR the odds that a female supports laws legalizing abortion is estimated to be 17.4% times higher than the corresponding odds of a male.
- Holding gender and political party fixed, the odds that a protestant supports laws legalizing abortion is estimated to be $e^{-0.57} = 0.566$ times the odds that a jewish person does OR holding gender and political party fixed, the odds that a protestant supports laws legalizing abortion is estimated to be 43.4% less than the odds that a jewish person does

(c) (2pt) If SE(b_1) $= 0.064$ construct a 95% confidence interval for β_1 and interpret your result.
The 95% confidence interval for β_1 is $b_1 \pm 1.96\ \text{SE}(\text{b}_1) = 0.16 \pm 1.96 * 0.064 = [0.035, 0.285]$. (Therefore a 95% confidence interval for e^{beta_1} is $[1.036, 1.330]$.)
Interpretation: We are 95% confident that, holding religion affiliation and political party fixed, the odds that a female supports laws legalizing abortion is a number between 1.036 and 1.330 times the odds that a male does.

(d) (2pt) Test $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 \neq 0$ Since the CI interval of β_1 does not contain zero, we reject H_0.

(e) (2pt) If SE(b_2) $= 0.38$ construct a 95% confidence interval for β_2 and interpret your result.
A 95% confidence interval for $beta_2$ is $b_2 \pm 1.96\ \text{SE}(\text{b}_2) = -0.57 \pm 1.96 * 0.38 = [-1.315, 0.175]$. (therefore a 95% confidence interval for e^{β_2} is $[0.268, 1.191]$)
Interpretation: We are 95% confident that if holding gender and political party fixed, the odds that a protestant supports laws legalizing abortion is a number between 0.268 and 1.191 times the odds that a Jewish person does,

(f) (2pt) Test $H_0 : \beta_2 = 0$ against $H_1 : \beta_2 \neq 0$
Since the confidence interval for β_2 includes 0 we fail to reject H_0.

2.

3. (8pt) The data in the file adolescent.csv appeared in a national study of 15 and 16 year-old adolescents. The event of interest is ever having sexual intercourse. The goal is to study the effect if any of race and gender on having sexual intercourse (Yes, No). Consider the following model

$$\text{logit}(\pi(\text{Intercourse=Yes}|\text{Gender, Race})) = \beta_0 + \beta_1\text{Gender} + \beta_2\text{Race}$$

(a) (4pt) Estimate β_1 and β_2 and interpret your result
Answer:
Here Gender = 1 if gender is male and Gender = 0 if gender is female. Race = 1 if race is white and Race = 0 if race is black. Then we get estimated coefficients: $b_1 = 0.6478,\ b_2 = -1.3135$.
Interpretations:

- Holding race fixed, the odds that a male adolescent has had intercourse is estimated to be $e^{0.6478} = 1.911$ times the odds that a female did. (the odds for a male are about 91.1% higher).
- Holding gender fixed, the odds that a white adolescent has had intercourse is estimated to be $e^{-1.3135} = 0.269$ times the odds that a black did.

(b) (2pt) Construct a 95% confidence interval to describe the effect of gender on the odds of Intercourse controlling for race (i.e. construct a 95% interval for e^{β_1}), Interpret your result
Answer:
From the output we have SE(b_1) = 0.2250 . Therefore a 95% confidence interval for $beta_1$ is: $b_1 \pm 1.96\ \text{SE}(\text{b}_1) = 0.6478 \pm 1.96 * 0.2250 = [0.207, 1.089]$.
Interpretation: We are 95% confident that holding race fixed, the odds that a male adolescent has had intercourse is a number between 1.230 and 2.971 times the odds that a female did. (this because a 95% interval for e^{β_1} is $[1.230, 2.971]$.

(c) (2pt) Test $H_0 : \beta_1 = 0$ against $H_a : \beta_1 \neq 0$. Use $\alpha = 0.05$.
Answer:
Since the 95% confidence interval for β_1 does not cover 0, we reject H_0.