HW5 GR5291

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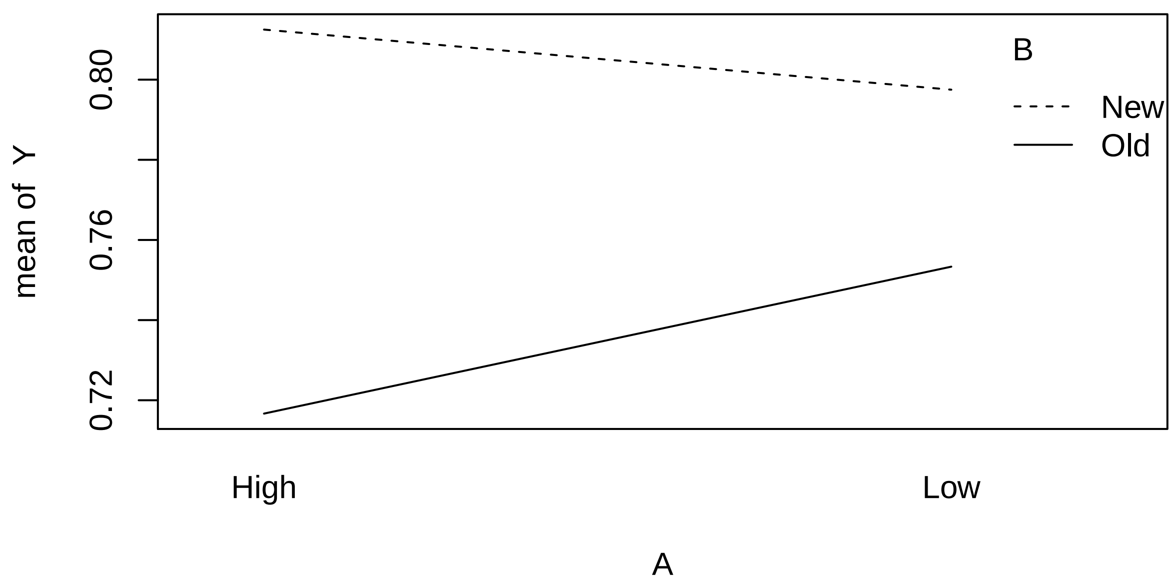
Problem 1

a. Give the design matrix corresponding to the model

$$\begin{bmatrix} y_{111} \\ y_{112} \\ y_{113} \\ y_{114} \\ y_{121} \\ y_{122} \\ y_{123} \\ y_{211} \\ y_{212} \\ y_{213} \\ y_{214} \\ y_{221} \\ y_{222} \\ y_{223} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} \mu \\ \alpha_1 \\ \beta_1 \\ (\alpha\beta)_{11} \end{bmatrix} + \begin{bmatrix} \epsilon_{111} \\ \epsilon_{112} \\ \epsilon_{113} \\ \epsilon_{114} \\ \epsilon_{121} \\ \epsilon_{122} \\ \epsilon_{123} \\ \epsilon_{211} \\ \epsilon_{212} \\ \epsilon_{213} \\ \epsilon_{214} \\ \epsilon_{221} \\ \epsilon_{222} \\ \epsilon_{223} \end{bmatrix}$$

b. Construct an interaction plot. Does it suggest that there is an interaction between furnace airflow and laser?

```r
A <- c(rep("Low",7), rep("High",7))
B <- c(rep("New",4), rep("Old",3), rep("New",4), rep("Old",3))
Y <- c(0.83,0.78,0.75,0.83,0.86,0.67,0.73,0.72,0.81,0.85,0.87,0.68,0.73,0.74)
interaction.plot(A,B,Y)
```



The interaction plot suggests that there is an interaction between furnace airflow and laser, since the lines are not parallel with each other. We need to do further test.

c. Test H_0 : No interaction against H_1 : there is an interaction, use $\alpha = 0.05$.

```
summary(aov(Y~A*B))
```

```
##             Df  Sum Sq  Mean Sq F value Pr(>F)
## A            1 0.00018 0.000179   0.046 0.8346
## B            1 0.01680 0.016800   4.321 0.0643 .
## A:B          1 0.00229 0.002288   0.588 0.4607
## Residuals   10 0.03888 0.003888
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the table above, we get the p-value of the test for interaction is 0.4607, which is larger than 0.05. This means we fail to reject H_0 and conclude that there is no interaction.

d. It is decided to fit a model without an interaction. Test H_0 : there is no A effect against H_a : there is an A effect. Use $\alpha = 0.05$.

```
summary(aov(Y~B+A))
```

```
##             Df  Sum Sq  Mean Sq F value Pr(>F)
## B            1 0.01680 0.016800   4.489 0.0577 .
## A            1 0.00018 0.000179   0.048 0.8311
## Residuals   11 0.04117 0.003743
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the table above, we get the p-value of the test for A effect is 0.8311, which is larger than 0.05. This means we fail to reject H_0 and conclude that there is no A effect.

e. It is decided to fit a model without an interaction. Test H_0 : there is no B effect against H_a : there is an B effect. Use $\alpha = 0.05$.

```
summary(aov(Y~A+B))
```

```
##             Df  Sum Sq  Mean Sq F value Pr(>F)
## A            1 0.00018 0.000179   0.048 0.8311
## B            1 0.01680 0.016800   4.489 0.0577 .
## Residuals   11 0.04117 0.003743
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the table above, we get the p-value of the test for B effect is 0.0577, which is larger than 0.05. This means we fail to reject H_0 and conclude that there is no B effect.

Problem 2

a. Estimate the $\beta_i s$ and interpret your result.

```
y <- c(28.0,28.6,27.4,33.3,34.5,33.0,32.0,35.6,34.4,35.0,34.0,33.3,34.7,33.5,32.3,33.4,33.0,32.0,29.6,3
x1 <- c("A","A","A","B","B","A","A","A","A","B","B","B","C","A","A","B","C","C","B","B","C","C")
x2 <- c(rep(0,5), rep(1,8), rep(2,5), rep(3,4))
D1 <- ifelse(x1=="B", 1, 0)
D2 <- ifelse(x1=="C", 1, 0)
fit <- lm(y ~ D1+D2+x2); fit
```

```
##
## Call:
```

```
## lm(formula = y ~ D1 + D2 + x2)
##
## Coefficients:
## (Intercept)           D1           D2           x2
##     32.0171       1.5218       0.5252      -0.4192
```

From above, we have $\beta_0 = 32.0171$, $\beta_1 = 1.5218$, $\beta_2 = 0.5252$, and $\beta_3 = -0.4192$.

For type A gasoline, we have $y = 32.0171 - 0.4192x_2 + \epsilon$.

For type B gasoline, we have $y = 32.0171 + 1.5218 - 0.4192x_2 + \epsilon = 33.5389 - 0.4192x_2 + \epsilon$.

For type C gasoline, we have $y = 32.0171 + 0.5252 - 0.4192x_2 + \epsilon = 32.5423 - 0.4192x_2 + \epsilon$.

The coefficients β_1 and β_2 indicate, repectively, how much higher or lower the reponse functions for gasoline B and gasoline C are than the one for gasoline A.

The coefficient β_3 means that if we increase x2 by one unit and hold other variables fixed, the average gasoline mileage y will decrease by 0.4192.

The coefficient β_0 indicates the intercept.

b. Construct a 95% confidence interval for β_3 and interpret your result.

```r
table <- summary(fit)$coefficients; table
```

```
##               Estimate Std. Error    t value     Pr(>|t|)
## (Intercept) 32.0170805  1.0004734 32.0019321 2.557742e-17
## D1           1.5218409  1.2650289  1.2030088 2.445581e-01
## D2           0.5251937  1.6194264  0.3243085 7.494429e-01
## x2          -0.4192156  0.6042243 -0.6938078 4.966597e-01
```

```r
table[4,1] + c(-1,1) * qt(0.975,22-4) * table[4,2]
```

```
## [1] -1.6886438  0.8502126
```

From the table, we can see the standard error for β_3 is 0.6042243, then we can get the 95% confidence interval is [-1.6886438, 0.8502126]. This means that we are 95% confidence that we increase x2 by 1 while holding other variables fixed, on average, y will change by an amount in this interval.

c. Test $H_0 : \beta_1 = \beta_2 = 0$ against H_a : Not H_0 using $\alpha = 0.05$.

```r
fit_red <- lm(y ~ x2)
anova(fit_red, fit)
```

```
## Analysis of Variance Table
##
## Model 1: y ~ x2
## Model 2: y ~ D1 + D2 + x2
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1     20 125.14
## 2     18 115.42  2    9.7138 0.7574 0.4832
```

Full model is $y = \beta_0 + \beta_1 D_1 + \beta_2 D_2 + \beta_3 x_2 + \epsilon$.

Reduced model is $y = \beta_0 + \beta_3 x_2 + \epsilon$.

From the table above, we can see the p-value of the test is 0.4832, which is greater than 0.05. This means we fail to reject H_0 and conclude that $\beta_1 = \beta_2 = 0$.