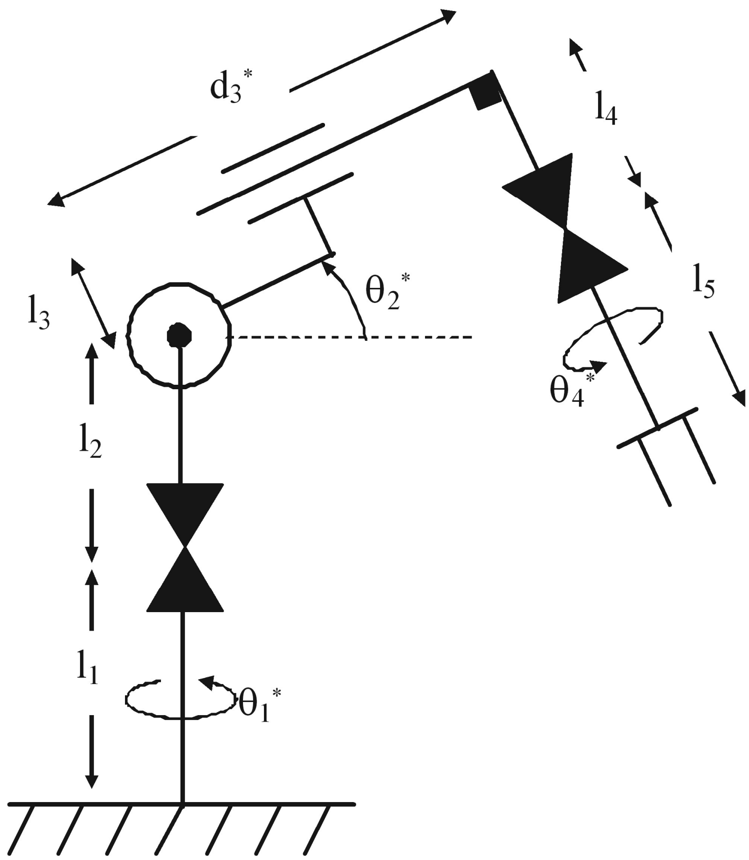
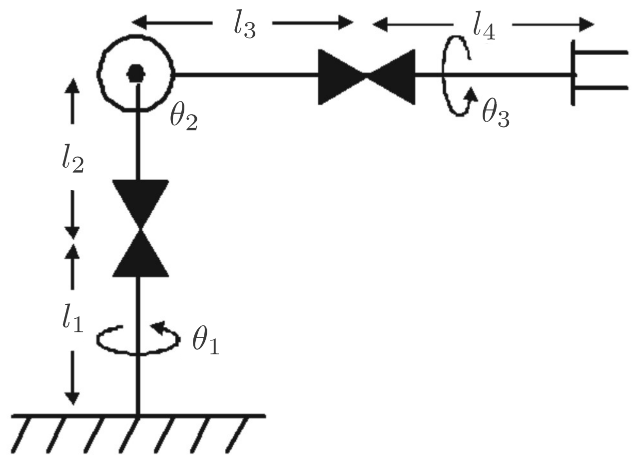
4. (21 points) Attach coordinate frames to the robot in the figure below using the DH convention taught in class, then fill in the DH parameter table.



i	a_i	α_i	d_i	θ_i
1				
2				
3				
4				
5				

4. (25 points) **Inverse Kinematics**

Consider the RRR manipulator below, which has no joint limits.



(a) (2 points) Suppose that $l_1 + l_2 = l_3 + l_4$ *for this part only*. Describe the set of points that the end effector can reach. Be specific about the geometry, and specify details such as the shape, location, and size of the workspace.

(b) (3 points) Given a desired position for the end effector, how many distinct solutions are there to the inverse kinematics problem? What if we are given a desired orientation as well?

(c) (10 points) The forward kinematics of the manipulator, with the base axes z_0 pointing up and x_0 pointing to the right, are as follows.

$$T_3^0 = \begin{bmatrix} s_1s_3 - c_1s_2c_3 & s_1c_3 + c_1s_2s_3 & c_1c_2 & (l_3+l_4)c_1c_2 \\ -c_1s_3 - s_1s_2c_3 & -c_1c_3 + s_1s_2s_3 & s_1c_2 & (l_3+l_4)s_1c_2 \\ c_2c_3 & -c_2s_3 & s_2 & l_1 + l_2 + (l_3+l_4)s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Given two desired coordinates x and z, what are all the possible y that the end effector can reach such that the point $p = (x, y, z)$ is in the end effector's workspace? If there are no solutions, explain why.

(d) (10 points) Find the 3×3 Jacobian matrix $J(\theta)$, such that $\dot{p} = J(\theta)\dot{\theta}$. What configurations make the robot singular, and why?

5. Consider a three-link planar RRR manipulator that has all three links 1 meter long. This robot has has the following forward kinematics:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 + c_{12} + c_{123} \\ s_1 + s_{12} + s_{123} \end{bmatrix}$$

   (a) (15 points) What is the Jacobian of this robot?

(b) (15 points) At the configuration

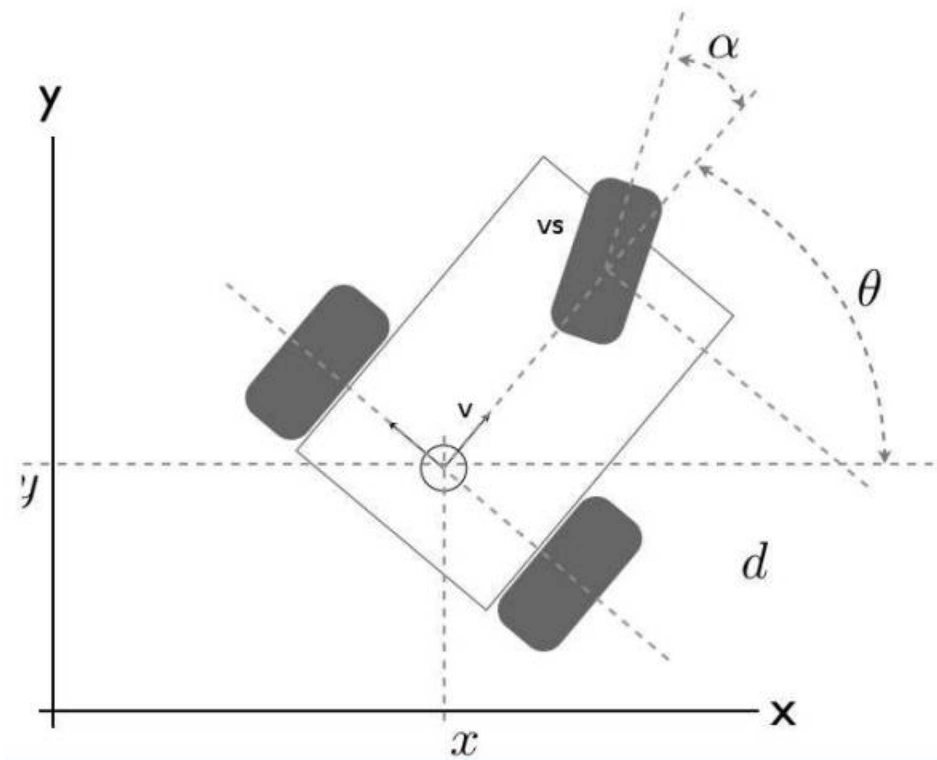
$$\Theta = \begin{bmatrix} 0 \\ \frac{\pi}{2} \\ \frac{\pi}{2} \end{bmatrix}$$

it is necessary to throw a ball with velocity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 2\mathrm{m/s} \\ 3\mathrm{m/s} \end{bmatrix}.$$

Compute the joint velocities that accomplish this. You do not need to crunch all of the numbers to get the final answer, just set up the equations, define all of the terms, and evaluate all of the sines and cosines. If no solution exists then say so and explain how you know. If multiple solutions exist, then find the "best" one and explain what "best" means.

Find the kinematicsof the tricycle robot below, which has a steerable input $\dot{\alpha}$ and a forward velocity input v_s.



3 Bug Algorithms

Provide short answers to the following questions on Bug algorithms. When asked to draw pictures, feel free to make drawings by hand and then take photos to put in your PDF.

Question 1

What is the difference between the Tangent Bug algorithm with zero range detector and Bug2? Draw examples.

Question 2

Consider the figure below. What are the differences between the path in this figure and the paths that Bug1 and Bug2 would have generated?

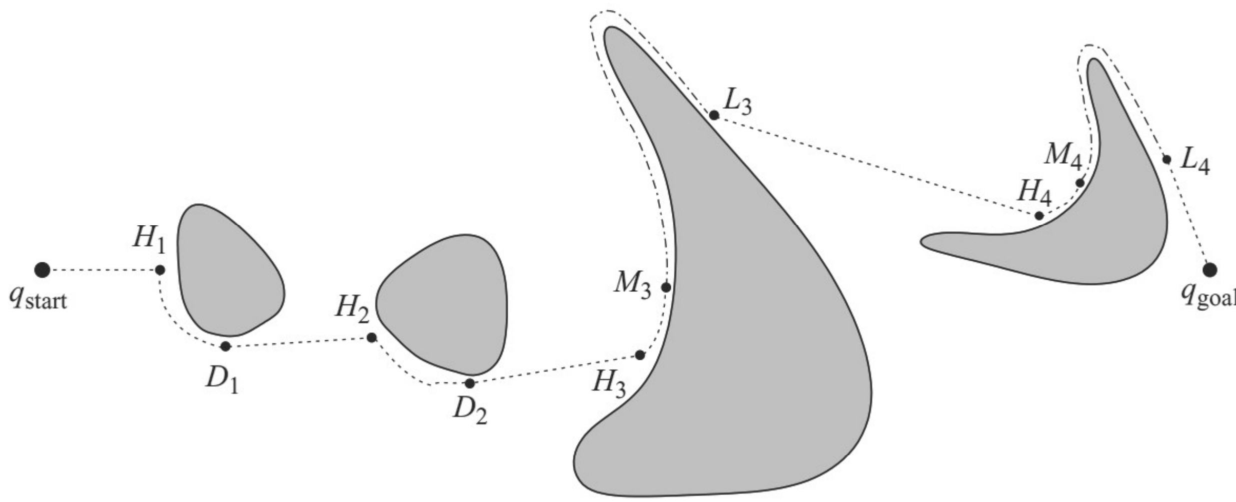


Figure 1: Question 2

Question 3

Draw an example obstacle-start-goal configuration for which Bug2 will fail to reach the goal.

Question 4

Consider an environment with two triangular objects. Draw an example of a start and goal position where the tangent bug will not take the shortest path.

Question 5

Consider an environment where there is only one circular obstacle, and the goal position is not visible from the start position. Prove that a tangent bug with infinite range radius will always take a shorter path than Bug1 or Bug2.