W4733 ROBOTICS HW5

Jing Qian (jq2282)

Q1. (a)

x_k is the state vector which describes the location and orientation of the robot at time instant k.

m is the set which includes all the time-invariant vector describing the location of landmarks.

$Z_{0:k}$ is the set of all landmark observations.

$U_{0:k}$ is the history of control inputs.

x_0 is the robot starting location and orientation.

$P(x_k|x_{k-1}, u_k)$ is the state transition model, which is the probability distribution of x_k conditioned on the previous state x_{k-1} and applied action u_k.

$P(z_k|x_k, m)$ is the observation model, which is the probability distribution of z_k conditioned on the state x_k and map m.

$P(x_{k-1}, m|Z_{0:k-1}, U_{0:k-1}, x_0)$ is our belief at time instant $k-1$, which is the probability distribution of x_{k-1} and m conditioned on the history set of all landmark ovservations at time $k-1$: $Z_{0:k-1}$, the history of control inputs at $k-1$: $U_{0:k-1}$ and initial state x_0.

$P(x_k, m|Z_{0:k-1}, U_{0:k}, x_0)$ is our belief of x_k and m distribution after we take action at time instant k but haven't observed yet, conditioned on $Z_{0:k-1}, U_{0:k}, x_0$.

$P(x_k, m|Z_{0:k}, U_{0:k}, x_0)$ is our belief of x_k and m distribution after we both take action and make observation at time instant k , conditioned on $Z_{0:k}, U_{0:k}, x_0$.

So Eq.(4) means that the posterior probability distribution of (x_k, m) conditioned on $(Z_{0:k-1}, U_{0:k}, x_0)$ is equal to the sum of transition probability of x_k conditioned on the previous state x_{k-1} and action u_k times the posterior probability distribution of (x_{k-1}, m) conditioned on $(Z_{0:k-1}, U_{0:k-1}, x_0)$.

Eq. (5) means that the posterior probability distribution of (x_k, m) conditioned on $(Z_{0:k}, U_{0:k}, x_0)$ is equal to the sum of observation probability of z_k conditioned on state x_k and map m times the posterior probability distribution of (x_k, m) conditioned on $(Z_{0:k-1}, U_{0:k}, x_0)$, devided by the posterior probability distribution of z_k conditioned on $(Z_{0:k-1}, U_{0:k})$.

Q1. (b)

Eq. (4)

$$P(x_k, m|Z_{0:k-1}, U_{0:k}, x_0) = \int P(x_k, m, x_{k-1}|Z_{0:k-1}, U_{0:k}, x_0)dx_{k-1} \text{ (Law of total probability)}$$
$$= \int P(x_k|m, x_{k-1}, Z_{0:k-1}, U_{0:k}, x_0) \times P(x_{k-1}, m|Z_{0:k-1}, U_{0:k}, x_0)dx_{k-1} \text{ (Chain rule)}$$
$$= \int P(x_k|x_{k-1}, u_k) \times P(x_{k-1}, m|Z_{0:k-1}, U_{0:k-1}, x_0)dx_{k-1} \text{ (Markov assumption)}$$

Eq. (5)

$$P(x_k, m|Z_{0:k}, U_{0:k}, x_0) = \frac{P(z_k|x_k, m, Z_{0:k-1}, U_{0:k}, x_0)P(x_k, m|Z_{0:k-1}, U_{0:k}, x_0)}{P(z_k|Z_{0:k-1}, U_{0:k}, x_0)} \text{ Bayes' theorem}$$
$$= \frac{P(z_k|x_k, m)P(x_k, m|Z_{0:k-1}, U_{0:k}, x_0)}{P(z_k|Z_{0:k-1}, U_{0:k})} \text{ Markov assumption}$$

Q2. (a)

Since it is a planar robot on map with n landmarks, we get the dimensionality of following quantities:

$\hat{x}_{k|k}$: 3 x 1.

$\hat{m}_k$: $2n$ x 1.

z_k: $2n$ x 1.

$P_{k|k}$: $(2n+3)$ x $(2n+3)$

Q_k: 3 x 3

R_k : $2n$ x $2n$

∇f: 3 x 3

∇h : $2n$ x $(2n+3)$

S_k: $2n$ x $2n$

W_k: $(2n+3)$ x $2n$

So the Eq. (10) should be:

$$\begin{bmatrix} \hat{x}_{k|k} \\ \hat{m}_k \end{bmatrix} = \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{m}_{k-1} \end{bmatrix} + W_k[z_k - h(\hat{x}_{k|k-1}, \hat{m}_{k-1})].$$

Q2. (b)

$$P_{k|k} = \begin{bmatrix} P_{xx} & P_{xm} \\ P_{xm}^T & P_{mm} \end{bmatrix}_{k|k}$$

$$P_{xx,k|k-1} = \nabla f P_{xx,k-1|k-1} \nabla f^T + Q_k$$

So we have:

$$P_{k|k-1} = \begin{bmatrix} P_{xx,k|k-1} & P_{xm,k|k-1} \\ P_{xm,k|k-1}^T & P_{mm,k|k-1} \end{bmatrix}$$
$$= \begin{bmatrix} P_{xx,k|k-1} & \nabla f P_{xm,k-1|k-1} \\ P_{xm,k-1|k-1}^T \nabla f^T & P_{mm,k-1|k-1} \end{bmatrix}$$

Q3. (a)

Figure 4 in the paper gives the graphical model of SLAM algorithm. We could see that through observations and robot states, the map landmarks are connected and hence dependent. However, if given robot trajectory, which means we block all the robot states, there is no path that could connect these map landmarks. So the landmarks are independent conditioned on the robot trajectory.

Since the landmarks are independent conditioned on the robot trajectory, the map accompanies each particle in sampling is composed of a set of independent Gaussian distributions multiplying continually. At each time step, particles are drawn from distributions and resampling step causes loss of historical particle information. Because of the conditional independence, the historical state information loss in particle resampling is exponential.

Q3. (b)

FastSLAM 1.0 use the motion model for its proposal distribution, which is:

$$x_k^{(i)} \sim P(x_k | x_{k-1}^{(i)}, u_k)$$

FastSLAM 2.0 includes the current observation for its proposal distribution, which is:

$$x_k^{(i)} \sim P(x_k | X_{0:k-1}^{(i)}, Z_{0:k}, u_k)$$

This means that the distribution that particles sampling from in FastSLAM1.0 only conditioned on previous state and action while the distribution that in FastSLAM2.0 also conditioned on the observations, which helps FastSLAM2.0 with the advantage of locally optimal proposal distribution. It means that for each particle, FastSLAM2.0 gives the smallest possible variance in importance weight $w_k^{(i)}$ conditioned upon the available information $X_{0:k-1}^{(i)}, Z_{0:k}, u_k$.

Q3. (c)

No, FastSLAM would not work properly if we never resample. Resample is used for deplete the distory, which helps exploration. If never resample, the FastSLAM will suffer from degeneration and will not work as expected.