W4733 ROBOTICS HW6

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Q1. (a)

From Eq.(1):

$$\dot{\mathbf{x}} = \begin{bmatrix} vR(\mathbf{r})\mathbf{e}_x \\ \omega M(\mathbf{r})\mathbf{e}_z \end{bmatrix} + \mathbf{u}_m$$

$$= \begin{bmatrix} v \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi & \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ \omega \begin{bmatrix} 1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi\sec\theta & \cos\varphi\sec\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} + \mathbf{u}_m$$

$$= \begin{bmatrix} v\cos\psi\cos\theta \\ v\sin\psi\cos\theta \\ -v\sin\theta \\ \omega\cos\varphi\tan\theta \\ -\omega\sin\varphi \\ \omega\cos\varphi\sec\theta \end{bmatrix} + \mathbf{u}_m$$

We could get $\mathbf{F}$ with differentiating Eq.(1) regarding to $(x, y, z, \varphi, \theta, \psi)$:

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & -v\cos\psi\sin\theta & -v\sin\psi\cos\theta \\ 0 & 0 & 0 & 0 & -v\sin\psi\sin\theta & v\cos\psi\cos\theta \\ 0 & 0 & 0 & 0 & -v\cos\theta & 0 \\ 0 & 0 & 0 & -\omega\sin\varphi\tan\theta & \omega\cos\varphi\sec^2\theta & 0 \\ 0 & 0 & 0 & -\omega\cos\varphi & 0 & 0 \\ 0 & 0 & 0 & -\omega\sin\varphi\sec\theta & \omega\cos\varphi\sec\theta\tan\theta & 0 \end{bmatrix}$$

Then:

$$\mathbf{\Phi}_k \approx \mathbf{I} + (t_{k+1} - t_k)\mathbf{F}$$

Q1. (b)

According to Eq.(6), we have:

$$\Delta\phi_i'(t_i') = \Delta\phi_i(t_i' - t_i')/(t_{i+1}' - t_i') = 0$$

Then,

$$\begin{aligned}
\begin{bmatrix}\Delta x_i'(t)\\ \Delta y_i'(t)\end{bmatrix} &= \Delta s_i \frac{t - t_i'}{t_{i+1}' - t_i'} \int_{t_i'}^{t} \begin{bmatrix}\cos(\Delta\phi_i'(\tau))\\ \sin(\Delta\phi_i'(\tau))\end{bmatrix} d\tau \\
&= \Delta s_i \frac{t - t_i'}{t_{i+1}' - t_i'} \begin{bmatrix}\sin(\Delta\phi_i'(t)) - \sin(\Delta\phi_i'(t_i'))\\ \cos(\Delta\phi_i'(t_i')) - \cos(\Delta\phi_i'(t))\end{bmatrix} \\
&= \Delta s_i \frac{t - t_i'}{t_{i+1}' - t_i'} \begin{bmatrix}\sin(\Delta\phi_i'(t))\\ 1 - \cos(\Delta\phi_i'(t))\end{bmatrix}
\end{aligned}$$

Since $\Delta\phi_1 = \pi/6$, $\Delta s_1 = 5$ meter, $t_1' = 0$ second and $t_2' = 2$ second, for sampling interval $[0, 2]$, we have:

$$\Delta\phi_i'(t) = \pi/6 \times t/2 = \frac{\pi}{12}t$$

$$\begin{bmatrix}\Delta x_i'(t)\\ \Delta y_i'(t)\end{bmatrix} = 5\frac{t}{2}\begin{bmatrix}\sin(\frac{\pi}{12}t)\\ 1 - \cos(\frac{\pi}{12}t)\end{bmatrix}$$

Then:

$$R(\tilde{\mathbf{r}}_i') = R\begin{bmatrix}0\\ 0\\ \pi/4\end{bmatrix} = \begin{bmatrix}\sqrt{2}/2 & -\sqrt{2}/2 & 0\\ \sqrt{2}/2 & \sqrt{2}/2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{aligned}
\tilde{\mathbf{p}}(t) &= \tilde{\mathbf{p}}_i' + R(\tilde{\mathbf{r}}_i')[\Delta x_i'(t), \Delta y_i'(t), 0]^T \\
&= \begin{bmatrix}0\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}\sqrt{2}/2 & -\sqrt{2}/2 & 0\\ \sqrt{2}/2 & \sqrt{2}/2 & 0\\ 0 & 0 & 1\end{bmatrix}\frac{5t}{2}\begin{bmatrix}\sin(\frac{\pi}{12}t)\\ 1 - \cos(\frac{\pi}{12}t)\\ 0\end{bmatrix} \\
&= \begin{bmatrix}\frac{5\sqrt{2}}{4}t[-1 + \sin(\frac{\pi}{12}t) + \cos(\frac{\pi}{12}t)]\\ \frac{5\sqrt{2}}{4}t[1 + \sin(\frac{\pi}{12}t) - \cos(\frac{\pi}{12}t)]\\ 0\end{bmatrix}
\end{aligned}$$

So:

$$\tilde{\mathbf{r}}(t) = R^{-1}(R(\tilde{\mathbf{r}}'_i)Rot_z(\Delta\phi'_i(t)))$$

$$= R^{-1}(\begin{bmatrix} \frac{5\sqrt{2}}{4}t[-1+\sin(\frac{\pi}{12}t)+\cos(\frac{\pi}{12}t)] \\ \frac{5\sqrt{2}}{4}t[1+\sin(\frac{\pi}{12}t)-\cos(\frac{\pi}{12}t)] \\ 0 \end{bmatrix} \begin{bmatrix} \cos(\frac{\pi}{12}t) & -\sin(\frac{\pi}{12}t) & 0 \\ \sin(\frac{\pi}{12}t) & \cos(\frac{\pi}{12}t) & 0 \\ 0 & 0 & 1 \end{bmatrix})$$

Q2

No, among all models that already pass a distance and viewing angle test, choosing a model in the middle of the range instead of the closest is not a good design choice. Since the models already pass the distance test, they are neither too close nor too far, which means they provide both enough and accurate visible features on the image. We don't need to worry that there might not be enough visible features on the image. Then we would want to maximize the features with the fixed camera resolution. So we choose the closest model to get the most accurate result.

Q3. (a)

There are both benefits and drawbacks using more than three pairs.

Drawbacks: it is more computational complex to use more than three pairs. More information to consider and more equations to solve.

Benefits: Using more than three pairs would provide more information, which could help us when facing multiple solutions. Often, we use more than three pairs when we have much measurement noise.

Q3. (b)

1) If we use the consensus set found only from a single threshold, we may fail to get an accurate matching. We initialize the consensus threshold with a large value to make sure to generate a roughly correct consensus set initially. Then through decreasing the threshold, we gradually eliminate the false positives and increase the accuracy. If we use the initial "large" value, the false positives may always be allowed, and we may not improve the accuracy.

2) It is better that we do not use the normalization procedure of dividing by the total projected length, which tends to underrate the correct poses when the model is slightly outside of the field of view. When evaluating the pose candidates, we accept the one with the highest metric scores after the loop as the correct pose. So we don't want to raise a high

standard and disqualify possible poses. On the other hand, if we overrate the poses, we could use criteria in the paper to eliminate unlikely ones. But it is hard to add eliminated poses after we underrate them.

Q4

In the figure above, I showed a building facade on campus where the matching may fail. I tried to replicate the robot's perspective as that in other figures in the paper. It is hard for the robot to match the image and the model due to the high degree of repetitiveness in the features. The windows in a row are identical. Specifically speaking, the robot may mistake point A for point B or C.