Mobile Robot Path Planning Based on Hierarchical Hexagonal Decomposition and Artificial Potential Fields

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In this article, a new algorithm based on an artificial potential field and hierarchical cell decomposition technique is developed to solve the find-path problem for a mobile robot. The complete map of the workspace including obstacle locations is assumed to be known a priori. The basic cell structure used for decomposition is a hexagon. The artificial potential field is based on an attractive force from the goal position and repelling forces from the obstacles. Computer simulations of the algorithm for various obstacle scenarios are also presented. © 1994 *John Wiley & Sons, Inc.*

この発表では、モービル・ロボットの経路検索問題を解決するために開発した、人工ポテンシャル・フィールドと、階層セルを分解する技法をベースにした新しいアルゴリズムについて説明する。障害物の位置を含むワークスペースの完全なマップは、既知のものであると仮定している。分解に使われるセルの基本的な形状は、6角形である。人工ポテンシャル・フィールドは、最終位置からの引力と障害物からの反発力で定義される。様々な障害物に関するアルゴリズムの働きは、コンピュータ・シミュレーションを使って説明する。

1. INTRODUCTION

Autonomous mobile robots have long been a dream of human beings. This dream is gradually becoming a reality due to the tremendous progress in robotics research. A major obstacle in achieving the goal of autonomous mobile robot is the path planning or find-path problem. This problem can be stated as: given an object with an initial configuration (position and orientation), a goal configuration, and a set of obstacles distributed in space, find a continuous path for the object from the initial configuration to the goal

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configuration without colliding with any obstacles along the path.

Although the problem of collision-free path planning has been extensively studied, finding an acceptable solution to it has not been easy. This is because the problem is inherently computationally intractable. In fact, it has been shown that the generalized path planning problem known as the "Piano mover's problem" is *P-space hard.* This means that it is at least as hard as a number of familiar problems for which the only known solution has a computational complexity that grows exponentially with the task size.[1]

Many path planning algorithms have been proposed. Sharir[2] provides an excellent overview on the path planning problem. These approaches can be divided into two categories: graph searching techniques and potential field methods. The graph searching techniques are so named because a chart or a graph is used to show free spaces where no collision will occur and forbidden spaces where a collision will occur. Based on this graph, a path is then selected by piecing together the free spaces or by tracing around the forbidden spaces. The potential field methods utilize artificial potential fields applied to the obstacles and the goal position, and use the resulting field to influence the path of the robot.

The representative algorithm of the graph searching techniques is the so-called Configuration-Space (C-space) method developed by Lozano-Perez.[3] The configuration of a rigid object is a set of independent parameters that characterize the position and orientation of every point in the object. The space of all possible configurations of an object is then its *configuration space.* Due to the presence of the immovable obstacles, some regions of the configuration space are not reachable. The find-path problem can then be formulated as finding a collision-free path in the configuration space. There are several difficulties in implementing this type of algorithm:

- Configuration space usually has six dimensions, whereas only three-dimensional or perhaps four-dimensional spaces seem to be tractable.[4] In practice, for a six degree-of-freedom (dof) arm, building and searching the six-dimensional C-space, although possible, is extremely cumbersome and slow.
- The graph describing the free space tends to have a very large number of nodes even at a minimal reasonable resolution.

As an alternative to graph searching, the hierarchical cell decomposition technique was introduced by Brooks and Lozano-Perez,[5] with subsequent contributions by other authors.[6,7] It consists of searching and decomposing the C-space of the mobile robot into cells at successive levels of approximations. A cell is classified as EMPTY if it does not intersect with any obstacles and FULL if it is completely inside an obstacle. If a cell is neither EMPTY nor FULL, it is classified as MIXED. At each level of decomposition, the algorithm searches for a sequence of adjoining EMPTY cells that connect the initial configuration of the mobile robot to the desired goal configuration. If a solution cannot be found, the algorithm will decompose some MIXED cells into smaller cells, and repeat the search process. The search terminates when a solution has been found, it is guaranteed that no solution exists, or MIXED cells cannot be decomposed further. The hierarchical cell decomposition approach is relatively easy to implement and quite efficient.[7]

Due to the difficulties with the configuration-space approach, another class of methods based on potential fields have been proposed. The idea of using "potential functions" for the specification of a robot was pioneered by Khatib[8] for obstacle avoidance, and further advanced by the work of Hogan[9] for force control, Warren[10] for global path planning, and Koditschek[11] for robot navigation. The methodology was developed independently by Arimoto[12] and Pavlov and Voronin.[13] The idea is to design an artificial potential field that would attract the robot to its goal configuration while repelling it when it is near obstacles. Although not as thorough as the graph searching techniques, the speed of the algorithms and the easy extension to higher dimensions made them an excellent alternative to the graph searching techniques. Unfortunately, it is acknowledged that there are obstacle arrangements that will result in potential fields with many spurious local minima that would trap the robot.

In this study, a new method for path planning is developed by combining the artificial potential field approach and the hierarchical cell decomposition technique. We also introduce a new structure for the cell used in the decomposition—a hexagon. A new technique of labeling a cell as PASSABLE or IMPASSABLE is also proposed.

The problem formulation and assumptions are described in section 2; the artificial potential field and the decomposition of hexagons are discussed in section 3; the path planning method will be described in section 4; and simulation results will be presented in section 5.

2. PROBLEM FORMULATION

The problem formulation and assumptions for mobile robot path planning are described in this section. We also present the model of the mobile robot and the free space.

In many manufacturing environments, mobile robots or automated guided vehicles are used to transport parts or raw materials between workstations. The movement of the mobile robot is constrained to the two-dimensional plane with two degrees of translation freedom and one degree of rotation freedom. Although the mobile robot and the obstacles are three-dimensional, we can project them to the ground and consider them as two-dimensional objects. Therefore, we consider our path planning to be a two-dimensional problem. We assume that if the two-dimensional projection of the mobile robot from its original three-dimensional entity will not collide with any obstacles projected in the same way, then the robot will not collide with any obstacles. Furthermore, without loss of generality, it is assumed that all objects are represented by polygons (can be convex or concave). The locations of the obstacles and workstations are assumed to be fixed and the complete map of the workspace including obstacle locations are assumed to be known *a priori*. The task of the mobile robot is to visit each workstation in a pre-defined order without colliding with any of the obstacles.

The mobile robot considered in this article can have both translation and rotation movements. The model of the mobile robot is illustrated in Figure 1. The mobile robot is represented by a rectangle with length l, width w ($l > w$) and center at (x_r, y_r). It can translate along X and Y axes and rotate about its center. We assume that the rectangle formed by l and w is bounded by a circumscribed circle and the diameter of this circle is defined as the minimal width, $t = \sqrt{l^2 + w^2}$, for a path. This approach is somewhat conservative as the minimal width can be reduced to $t = \min(l, w)$ if the mobile robot is able to rotate in the horizontal plane. The path planning problem can now be considered as finding a collision-free path with width t for a point (center of the mobile robot) among the obstacles. This model will suffice because if a point can pass through a path

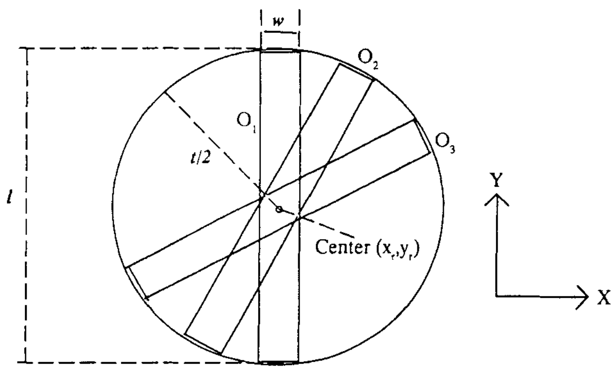

Figure 1. Model of the mobile robot.

with width t, then the mobile robot also can pass through it without any collisions with the obstacles.

A hierarchical decomposition method using hexagonal cells will be used to find the collision-free path. An artificial potential field is used as a criterion function for searching the hexagons created as the result of the decomposition. The basic idea behind the potential field approach is as follows. Each obstacle exerts a repulsive force on the moving object while the goal exerts an attractive force. With appropriate choice of the attractive force and the repulsive forces, the moving object will then be drawn toward the goal position while simultaneously being forced away from the obstacles. It can be shown that if the potential within the obstacles is set to infinity, collisions will never occur.[15] A hierarchical neighborhood-searching algorithm will be used to find a collision-free path.

3. POTENTIAL FIELD AND CELL DECOMPOSITION

In this section, we discuss the attractive force and the repulsive forces used to define the artificial potential field and also the procedure for decomposing a hexagon.

3.1. Potential Field and Region of Influence

There are various ways to define the artificial potential forces. One approach is to use the Newtonian inverse-squared law to define the artificial forces acting on a moving object by an obstacle. The potential acting on the moving object is then the maximum potential among all the obstacles plus a component decreasing linearly towards the goal configuration.[14]

We will adopt this approach with the following modifications:

1. Each obstacle is a source of a repelling force. The magnitude of the repelling force is proportional to $1/r^2$, where r is the distance between the center of the mobile robot and the center of the obstacle. The center of the obstacle is the center of gravity of the polygon representing the obstacle.
2. The target workstation is a source of an attractive force having a magnitude proportional to $(1 - k\alpha)$, where k is a scaling factor, and α is the angle between the line connecting the center of the workstation and the mobile robot and the direction of the motion of the mobile robot. The maximum attractive force is thus the straight line path connecting the center of the workstation and the mobile robot.
3. The potential within any obstacle is defined as infinity.
4. For the sake of simplicity, we assume that all sources of force (either attractive or repulsive) are point sources.
5. Therefore, the potential at any point will be the sum of the attractive force due to the target workstation and the repulsive force from each obstacle. It can be written as

$$\text{Potential}(P) = \sum_{i=1}^{n} k_1 \frac{1}{r_i^2} - C(1 - k_2\alpha) \quad (1)$$

where P is the point under consideration; r_i is the distance between the center of the ith obstacle and point P; k_1, k_2, and C are constants; and α is determined as shown in Figure 2. In Figure 2, S is the starting point of the mobile robot, T is the center of the immediate target workstation, R_c is the current position of the robot, and R_n is the next candidate point to which the robot will move.

According to Eq. (1), the potential of a point close to an obstacle is large and the attractive force provides a general direction for the robot to follow. Therefore, by choosing a path with minimal potential we can obtain a collision-free path for the robot. Eq. (1) will be used as the criteria function when we perform the neighborhood search.

To reduce the amount of computation, we introduce a region of influence for the mobile root (see Fig. 2). Only obstacles and workstations falling within this region will be considered when computing the potential. All obstacles and workstations outside this region will not be considered. All the workstations but the immediate target workstation within this region will be considered as obstacles. The region of influence is defined as a circle the diameter of which is the distance between the starting position of the moving robot and the center of the immediate target workstation. It is possible that a path to the target workstation does not exist within the region of influence. If this happens, the region of influence can be enlarged accordingly.

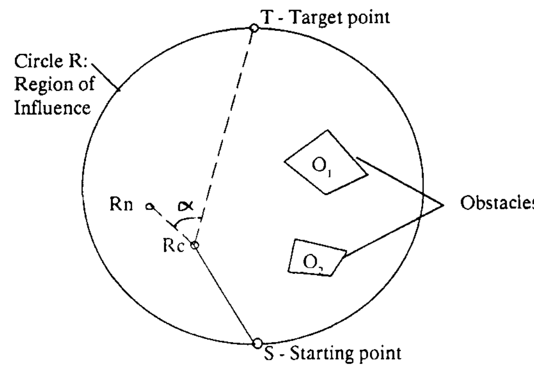

Figure 2. Determination of α.

3.2. Decomposition of Hexagons

The neighborhood-searching algorithm used to find the collision-free path will be based on a hierarchical decomposition approach. The basic cell structure for the decomposition is a hexagon. Some advantages of using hexagons are:

1. A hexagon can be recursively divided into smaller hexagons and half hexagons.
2. A hexagon has six edges or six neighbors (see Fig. 3). This simplifies the procedure of searching.

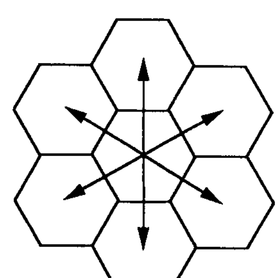

Figure 3. Neighbors of a hexagon.

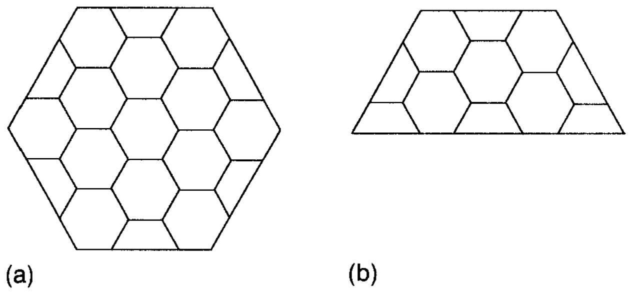

Figure 4. Decomposition of a hexagon: (a) decomposition of a full hexagon, (b) decomposition of a half hexagon.

3. The distances from the center of a hexagon to the centers of its neighbors are the same.

Although hexagons cannot be completely divided into smaller full hexagons, they can be decomposed into a combination of smaller full and half hexagons. Figure 4a illustrates the decomposition of a full hexagon. A full hexagon is divided into 13 smaller full hexagons and 6 smaller half hexagons. Notice that the half hexagons appear only on the edges of the parent hexagon. A half hexagon can be considered as a full hexagon except that half of it has been cut by the parent hexagon. Figure 4b illustrates the decomposition of a half hexagon. It is decomposed into 5 smaller full hexagons and 6 smaller half hexagons. The algorithm for performing the above decomposition consists of the following two steps:

1. The coordinates of the center point of the hexagons at the sub-level are computed.
2. Each smaller hexagon is reconstructed using the geometric properties of a hexagon.

From Figure 5, we can obtain the following characteristics of a hexagon:

1. Any hexagon can be characterized by the coordinates (H_x, H_y) of its center and its edge length l. Any other geometric feature of the hexagon can be derived from these two parameters. For example, the height (defined as the perpendicular distance from the center to one of the edges) is equal to $l\sqrt{3}/2$, and the coordinate (V_{1x}, V_{1y}) of the vertex V_1 can be determined by:

$$V_{1x} = H_x + \frac{1}{2}l \qquad V_{1y} = H_y + \frac{\sqrt{3}}{2}l$$

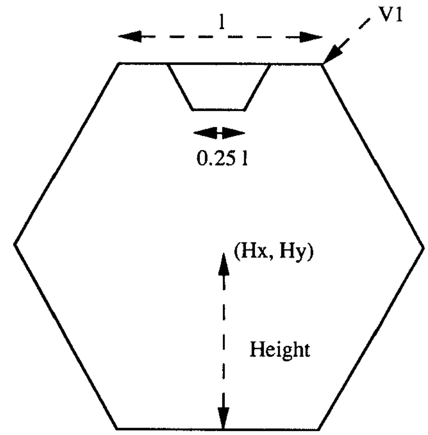

Figure 5. Geometry of a hexagon.

Similarly, we can determine all the coordinates of the vertices in a hexagon.

2. The edge length of a hexagon in the sub-level is 1/4 of the edge length of its parent hexagon.

With these useful features, the decomposition algorithm can be written as follows.

Decomposition Algorithm

Given the coordinate of the center point and the edge length of a hexagon.

1. Calculate the coordinates of the vertices of the parent hexagon.
2. Take the center of the parent hexagon as the center of the first sub-level hexagon.
3. Take the midpoint of each consecutive pair of vertices from the parent hexagon as the center of the next 6 sub-level hexagons. These are half hexagons.
4. Take the midpoint of each consecutive pair of centers obtained in step 3 as the center of another 6 sub-level hexagons.
5. Take the midpoint of each pair of points consisting of one point obtained in step 3 and the center of the parent hexagon as the center of the last 6 sub-level hexagons.
6. If any of the centers of the sub-level hexagons are outside the border of the parent hexagon, delete them from the list sub-level hexagons. This step is used for the decomposition of half hexagons.
7. The edge length of a sub-level hexagon is one quarter of the edge length of the parent hexagon.

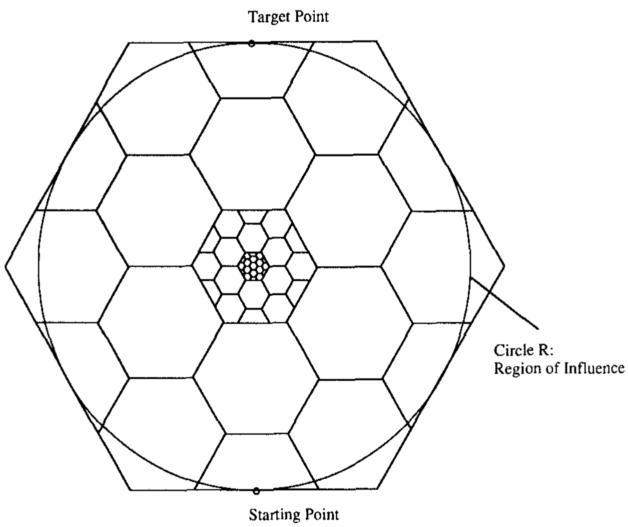

Figure 6. Three levels of decomposition of a hexagon.

Figure 6 illustrates a three-level decomposition of a hexagon using the algorithm given above.

4. NEIGHBORHOOD-SEARCHING ALGORITHM

In this section, we will describe in detail our collision-free path planning algorithm. Our method first forms a hexagon containing the starting and goal point. A neighborhood-searching algorithm guided by an artificial potential function is used to search for the collision-free path. The hexagons are decomposed heirarchically whenever it is necessary.

We assume that the work space has already been decomposed to a specific sub-level of hexagons depending on the sizes of the obstacles, and the mobile robot will travel from the center of a hexagon to the center of another hexagon.

4.1. Status of a Hexagon

Each hexagon is associated with two attributes, PASSABLE or IMPASSABLE, and EMPTY or MIXED. The second attribute is assigned only to hexagons that are PASSABLE. Such a classification will speed up the neighborhood-searching algorithm very effectively. The concept of PASSABLE and IMPASSABLE hexagons is illustrated in Figure 7. A hexagon B, is PASSABLE from its neighbor, A, if there is a collision-free path of width t from the center of A to the center of B; otherwise, it is IMPASSABLE. The potentials for the IMPASSABLE hexagons are assigned the value of infinity to ensure that they will not be selected. For

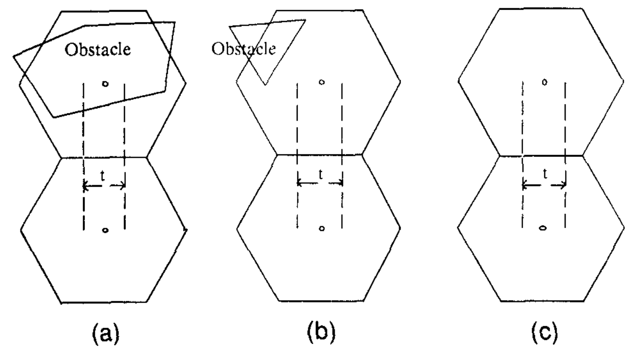

Figure 7. Status of a hexagon: (a) IMPASSABLE, (b) PASSABLE and MIXED, (c) PASSABLE and EMPTY.

the PASSABLE hexagons, if any part of an obstacle falls into the inscribed circle of the hexagon (see Fig. 5b), the hexagon is defined as MIXED; otherwise it is EMPTY. According to the above definitions, the hexagon in Figure 7a is IMPASSABLE, the one in Figure 7b is PASSABLE but MIXED, and the one in Figure 7c is PASSABLE and EMPTY.

To detect if a hexagon is PASSABLE or IMPASSABLE, we can use the following method. Suppose the mobile robot is going to move from hexagon A to hexagon B. We first form two parallel line segments of equal length that are equidistant ($t/2$) from the line connecting the centers of the hexagons (see Fig. 8a). The length of the line segment is the distance between the centers of hexagon A and hexagon B. If at least one of the line segments intersects with an obstacle, then hexagon B is labeled as IMPASSABLE; otherwise, a second test is carried out to see whether any vertices of the obstacles fall into the region

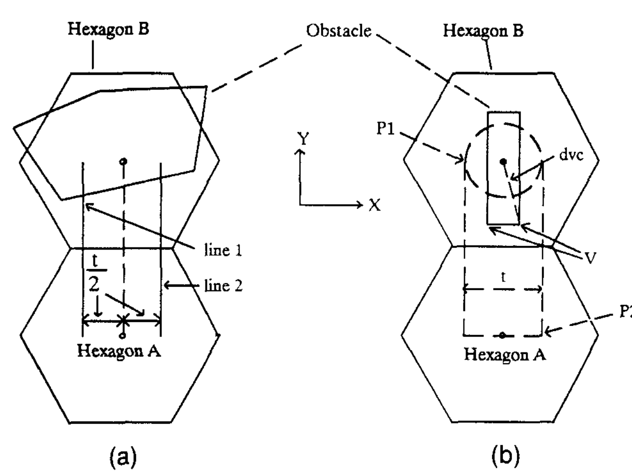

Figure 8. Determination of IMPASSABLE hexagons: (a) intersection, (b) coordinate and distance comparisons.

bounded by the dashed lines shown in Figure 8b. If there is such a vertex, the obstacle must be partly or fully inside the above region; then hexagon *B* is also IMPASSABLE; otherwise it is PASSABLE.

4.2. Selecting a Neighbor

As we perform the search, the mobile robot will be at the center of a hexagon. From there it will move a step towards the goal, or the target workstation. The mobile robot can choose from at most six different neighbors. Usually, the number of neighbors is less than six due to the presence of obstacles and the region of influence. For each neighboring hexagon, a test is made to see if it is PASSABLE. If it is, then the potential at the center of the candidate hexagon is computed according to Eq. (1). The hexagon with the minimum potential will be selected as the next path point and the robot will move to the center of that hexagon.

4.3. Searching with Hierarchical Decomposition

The searching algorithm guided by the potential function will attempt to search for a sequence of adjacent EMPTY cells connecting the initial configuration of the robot to the goal configuration. If no such sequence is found, it decomposes some mixed cells into smaller cells, labels them appropriately, and searches again for a sequence of EMPTY cells. The process ends when (1) a solution has been found, (2) it is guaranteed that no solution can be found, or (3) MIXED cells cannot be decomposed further.

The searching algorithm proceeds by examining the neighboring hexagons at its current position. It computes the potential function of the neighboring hexagons that are PASSABLE. It then selects the hexagon with the smallest potential value and take that hexagon as the next point on the path. If there are obstacles in the selected hexagon, i.e., it is MIXED, then the hexagon is decomposed and the search step size is decreased. If there are no obstacles, i.e., it is EMPTY, then the search step size will remain unchanged or become larger. This is because if the EMPTY hexagon lies on the edge of a larger hexagon at the previous level of decomposition, the search step size is reverted to the one for the larger hexagon. This search process continues until a solution is found, or a path cannot be found. Figure 9 illustrates this searching process. Suppose the current position of the robot is at the center of *H*1, and it selects *H*2 as the next point on the path. As *H*2 is a MIXED hexagon, it is decomposed into the next sub-level.

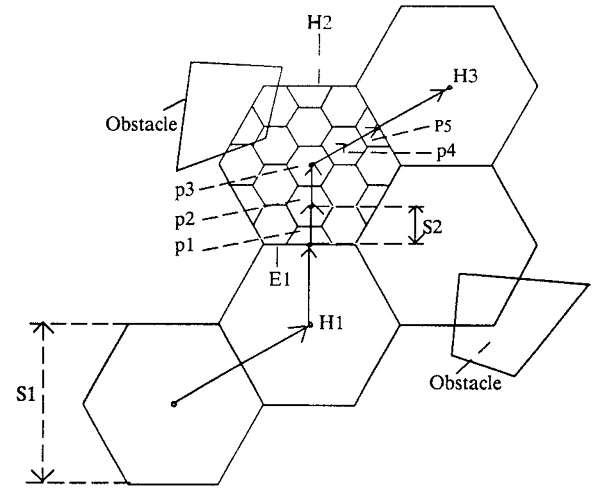

Figure 9. The searching process.

Because the robot is supposed to enter *H*2 from *H*1, it must pass through the edge *E*1. Along this edge, there are only three possible sub-hexagons; it will choose one of them according to their potentials. After that, the search step size will be changed from *S*1 to *S*2, which is 1/4 of *S*1. Then the robot will resume the search with step size *S*2 until it encounters *H*3, an EMPTY hexagon. Because the robot is now at a hexagon lying on the edge of a larger EMPTY hexagon, the search step size will revert to *S*1 and it will directly go to the center of *H*3.

4.4. Local Minima

Like any other method using artificial potential field, our path planner using the algorithm described above can be trapped into local minima. We have found that the following techniques are very effective for eliminating the local minima.

Consider the situation shown in Figure 10a. It is possible to place obstacles so that the robot will move to *P*2 when at *P*1 and move to *P*1 when at *P*2.

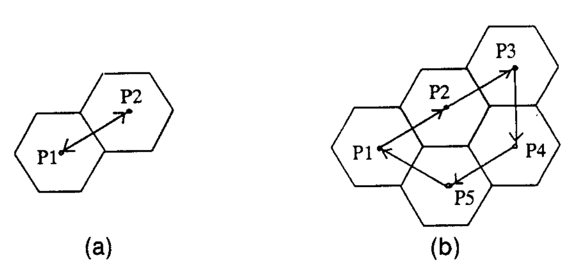

Figure 10. Local minima: (a) single point cycle, (b) multi-point cycle.

This will put the robot into an infinite cycle. To avoid this situation, our algorithm will remember the last point from which it came and its potential is set to infinity in the next round of selection. Therefore, this point will not be selected again.

Another class of trap is shown in Figure 10b. In this case, the robot is trapped into an endless loop consisting of several path points. Such a local minima can be eliminated by marking $P1$ as an IMPASSABLE hexagon, i.e., setting the potential of this hexagon to infinity. This can be done after this point has been visited twice.

4.5. Collision-Free Path Planning Algorithm

The complete algorithm for finding the collision-free path can be described as follows.

Find Collision-Free Path Algorithm

1. Obtain the obstacle information, such as the number of obstacles, vertices, and center of each obstacle; obtain the initial configuration of the mobile robot and the goal configuration.
2. Determine the region of influence according to the starting point and the target point.
3. Construct the initial hexagon, which is the smallest hexagon bounding the region of influence.
4. Decompose the initial hexagon to a specific sub-level, depending on the average length of the edges of all the obstacles; set the edge length of the smallest hexagon as the search step size.
5. From the current position, compute the potential function of each neighboring hexagon that is PASSABLE and select the hexagon with the smallest potential energy as the next path point. If the selected hexagon is the hexagon from which it came, set the potential to infinity for this hexagon.
6. If the selected hexagon is EMPTY and its edge length is larger than the current one, go to step 7; otherwise, decompose it to the next sub-level, decrease the search step size by 4, go to step 7.
7. Move to the center of the selected hexagon. If the new hexagon is at the boundary of a larger hexagon, then reset the search step size to the edge length of the larger hexagon. Repeat steps 5 and 6 until the robot reaches the target workstation.

5. SIMULATION RESULTS

We have implemented the collision-free path planning algorithm described in section 4 on a SUN SPARC-I workstation. The program allows the user to specify the obstacles and the starting and goal configurations, and will display the collision-free path graphically.

The algorithm has been extensively tested with many different scenarios;[15] we will only present several representative and interesting cases in Figures 11 to 16. In these figures, the largest hexagon corresponds to the region of influence under the given starting and goal configurations. The obstacles are shown as shaded polygons and the collision-free path is shown as line segments connecting the centers of the hexagons. In all the cases we have tested, a collision-free path is found in less than 0.5 seconds of CPU time.

Figure 11 illustrates a scenario with 12 obstacles. There are a total of 361 hexagons and half hexagons and they are at the second level of decomposition. The starting and goal configuration are, respectively, at the bottom and top of the largest hexagon. Initially, the step size of the mobile robot is at l (the edge length of the smaller hexagons) and the robot proceeds directly toward the goal due to the potential function. After the first two steps, the potential function for a direct path to the goal increases signifi-

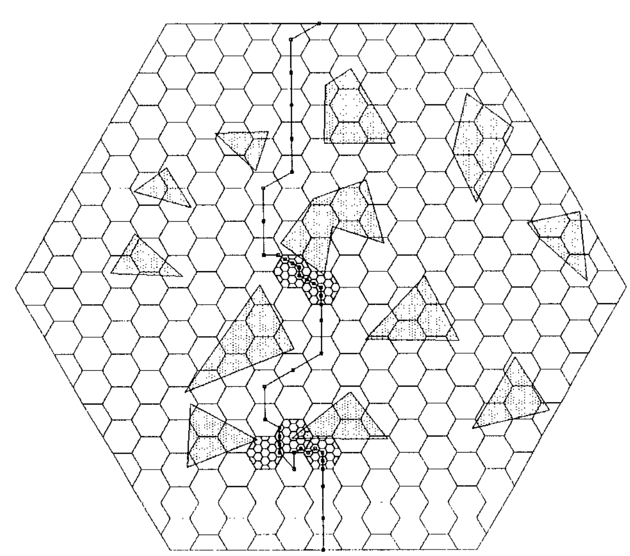

Figure 11. Scenario 1.

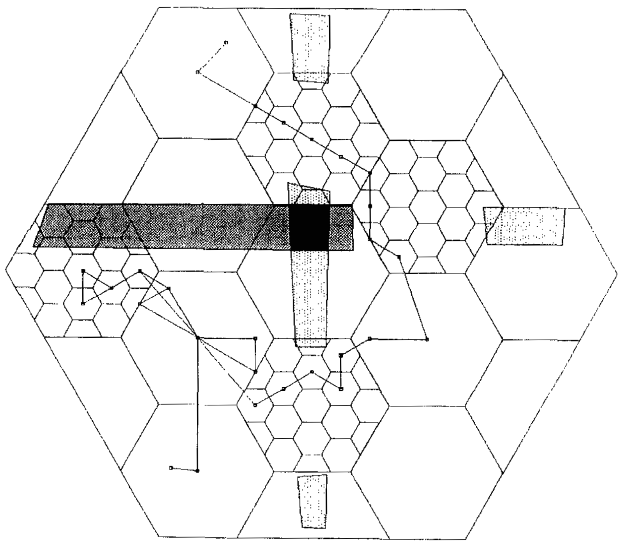

Figure 12. Scenario 2.

cantly due to the presence of the obstacle $O1$. The robot is steered away from $O1$ by the potential function and the robot selects the hexagon $H1$, which is PASSABLE and MIXED. $H1$ is then decomposed into smaller hexagons and the robot proceeds with a step size equal to $l/4$. As the robot navigates around the obstacle, it reaches the hexagon $H2$ which is PASSABLE and EMPTY, and the step size returns to the original value. The algorithm proceeds and finally reach the goal configuration.

Figure 12 illustrates a scenario where the robot is initially trapped in a local minima but eventually escapes and reaches the goal. The starting and goal configuration are, respectively, at the bottom and top of the largest hexagon. This simulation also demonstrates that the algorithm can escape from local minima. At the start configuration S, it is misled by the potential function and proceeds to its left side, which does not have a path to the goal configuration. After exploring several possible, futile paths, it reaches one of the local minima conditions discussed in Section 4 and proceed to find an opening which leads it to the goal configuration.

Figure 13 illustrates a scenario where there is only one possible path from the start to the goal configuration. The starting and goal configurations are, respectively, at the bottom and top of the largest hexagon. The path planning algorithm guided by the artificial potential field avoids the two obstacles and finds the opening to reach the goal configuration. As the space between the obstacles are wide, only one additional level of decomposition is required.

Figures 14 to 16 illustrate several different obstacle scenarios with various numbers of obstacles. In each case, the path planning algorithm was able to find a path to the target workstation.

6. CONCLUSION

A new collision-free path planning method for a mobile robot based on the hierarchical decomposition of hexagons and an artificial potential field is pre-

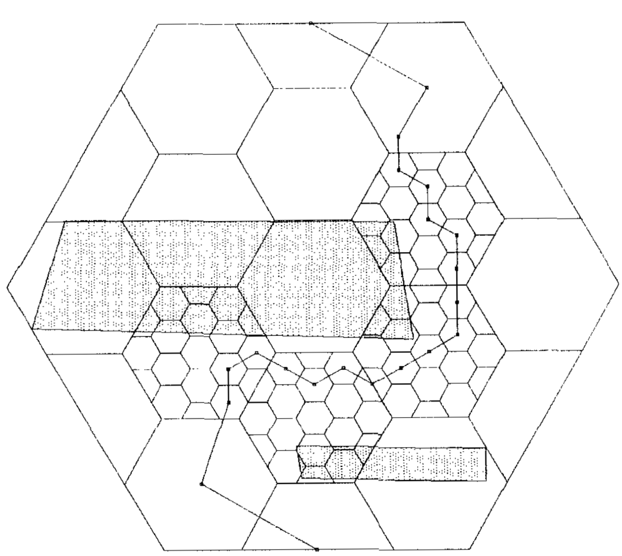

Figure 13. Scenario 3.

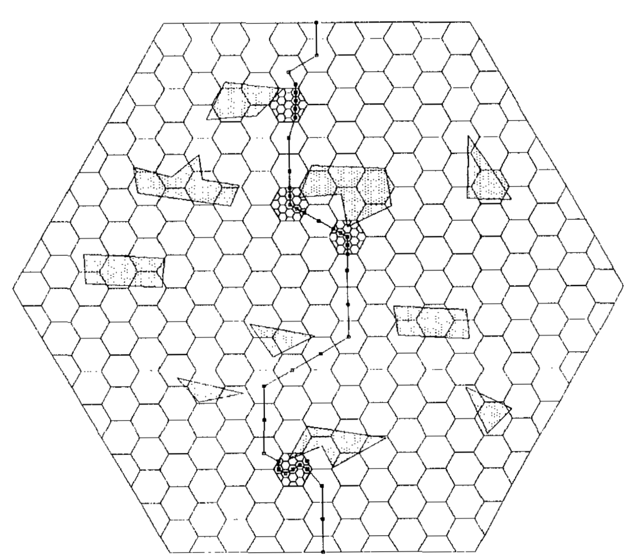

Figure 14. Scenario 4.

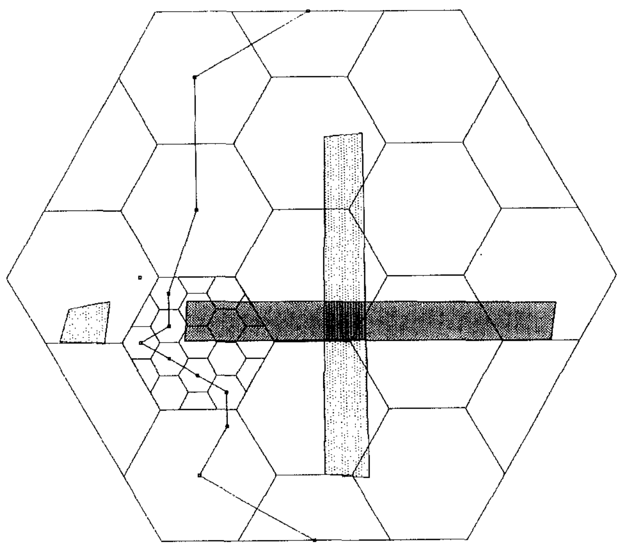

Figure 15. Scenario 5.

sented in this article. The workspace of the mobile robot is hierarchically decomposed into cells of full or half hexagons. Furthermore, two attributes, PASSABLE/IMPASSABLE and EMPTY/MIXED, are defined for the cells to facilitate the searching algorithm. A neighborhood-searching algorithm guided by an artificial potential field is used to search the hexagonal cells to find a collision-free path. One advantage of this approach is its fast response and robustness. The algorithm has been tested extensively and can find a collision-free path even for very complicated scenarios. Future research may be directed toward relaxing the limitation of the minimal path width required by the algorithm.

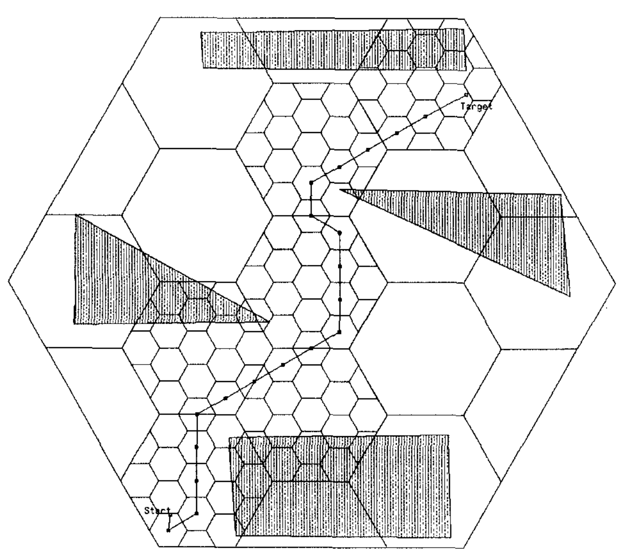

Figure 16. Scenario 6.

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