COMS W4733, HW2

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PROBLEM 1

(a) There are infinite solutions for this problem if we don't care about the angle. Because there is a laser in the end, like a prismatic joint with infinite range of length. Solving functions with three parameters θ_1, θ_2, d_3 with only two constraints p_x and p_y will lead to infinite solutions.

No, the number of solutions doesn't depend on l_1 or l_2.

(b) We could model the laser as a third prismatic joint with length d_3. So we try to solve $(\theta_1, \theta_2, d_3)$ with equations of (p_x, p_y, ϕ):

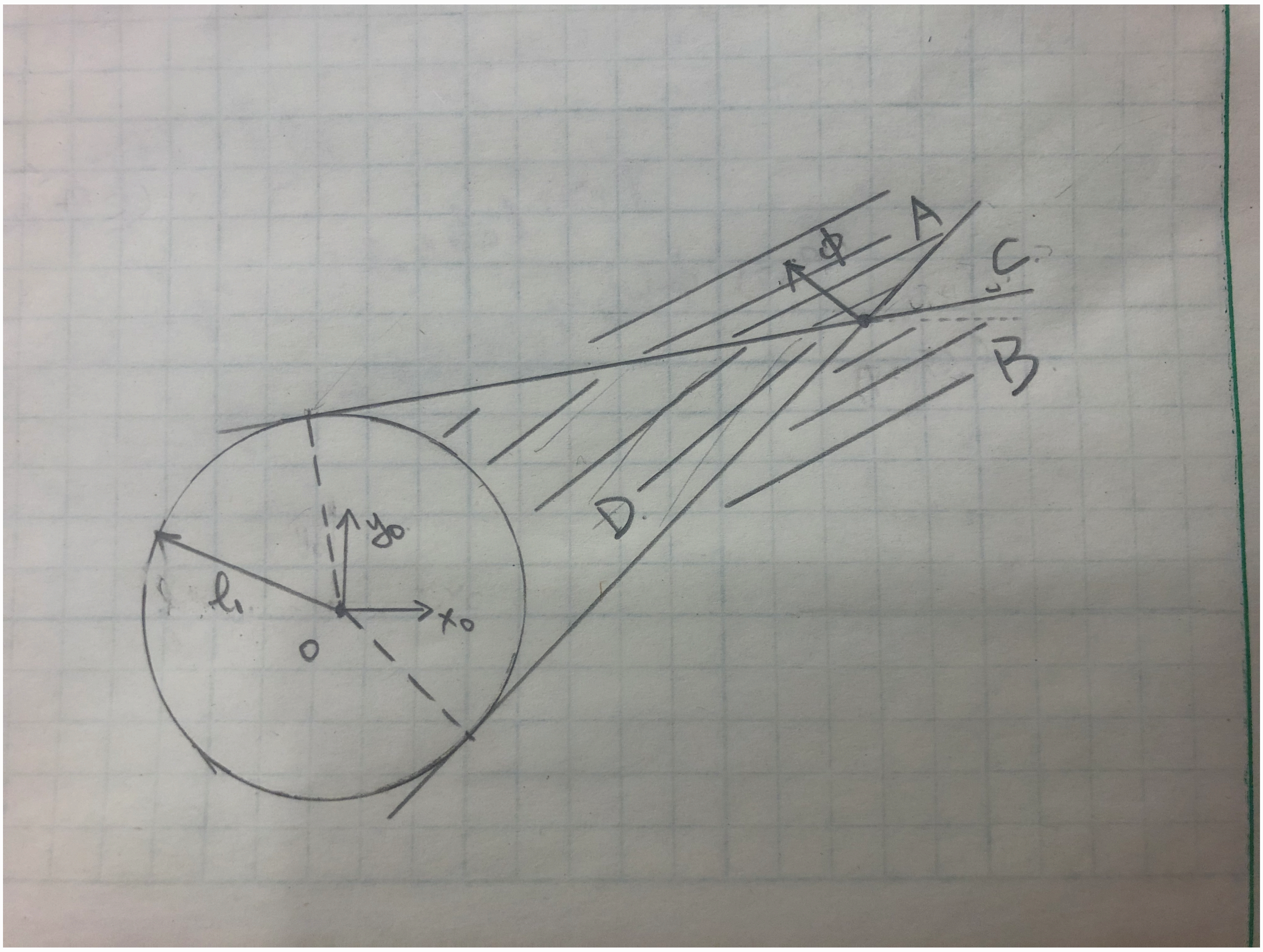
$$\begin{cases} \phi = \theta_1 + \theta_2 \\ p_x = l_1 c_1 + (l_2 + d_3) c_{12} \\ p_y = l_1 s_1 + (l_2 + d_3) s_{12} \end{cases}$$

We could get θ_1 and d_3 from:

$$\begin{cases} p_x = l_1 c_1 + (l_2 + d_3) c_\phi \\ p_y = l_1 s_1 + (l_2 + d_3) s_\phi \end{cases}$$

Then use $\theta_2 = \phi - \theta_1$.

(c) The robots' workspace is bound by the angle ϕ at (p_x, p_y). For point $p_x^2 + p_y^2 > l_1^2$, we do tangents from this point to the circle with raidus l_1. If ϕ points to a direction in the region C, there are solutions. But for regions, A, B and D, there is no solution.



PROBLEM 2

(a)

$$J_p(q) = \begin{bmatrix} \frac{\partial p_x}{\partial q_1} & \frac{\partial p_x}{\partial q_2} & \frac{\partial p_x}{\partial q_3} \\ \frac{\partial p_y}{\partial q_1} & \frac{\partial p_y}{\partial q_2} & \frac{\partial p_y}{\partial q_3} \\ \frac{\partial p_z}{\partial q_1} & \frac{\partial p_z}{\partial q_2} & \frac{\partial p_z}{\partial q_3} \end{bmatrix}$$
$$= \begin{bmatrix} -(L_1 + L_2 c_2 + L_3 c_{23}) s_1 & -L_2 c_1 s_2 - L_3 c_1 s_{23} & -L_3 c_1 s_{23} \\ (L_1 + L_2 c_2 + L_3 c_{23}) c_1 & -L_2 s_1 s_2 - L_3 s_1 s_{23} & -L_3 s_1 s_{23} \\ 0 & L_2 c_2 + L_3 c_{23} & L_3 c_{23} \end{bmatrix}$$

Link	a_i	α_i	d_i	θ_i
1	L_1	90	0	θ_1
2	L_2	0	0	θ_2

3	L_3	0	0	θ_3

From the DH parameter table, we could get homogeneous transformations between frame 0 and i. We know that z_i^0 is the top three elements of the third column from T_i^0. The angular velocity Jacobian is:

$$J_0 = \begin{bmatrix} z_0^0 & z_1^0 & z_2^0 \end{bmatrix} = \begin{bmatrix} 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

So the full Jacobian matrix is:

$$J = \begin{bmatrix} -(L_1 + L_2c_2 + L_3c_{23})s_1 & -L_2c_1s_2 - L_3c_1s_{23} & -L_3c_1s_{23} \\ (L_1 + L_2c_2 + L_3c_{23})c_1 & -L_2s_1s_2 - L_3s_1s_{23} & -L_3s_1s_{23} \\ 0 & L_2c_2 + L_3c_{23} & L_3c_{23} \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

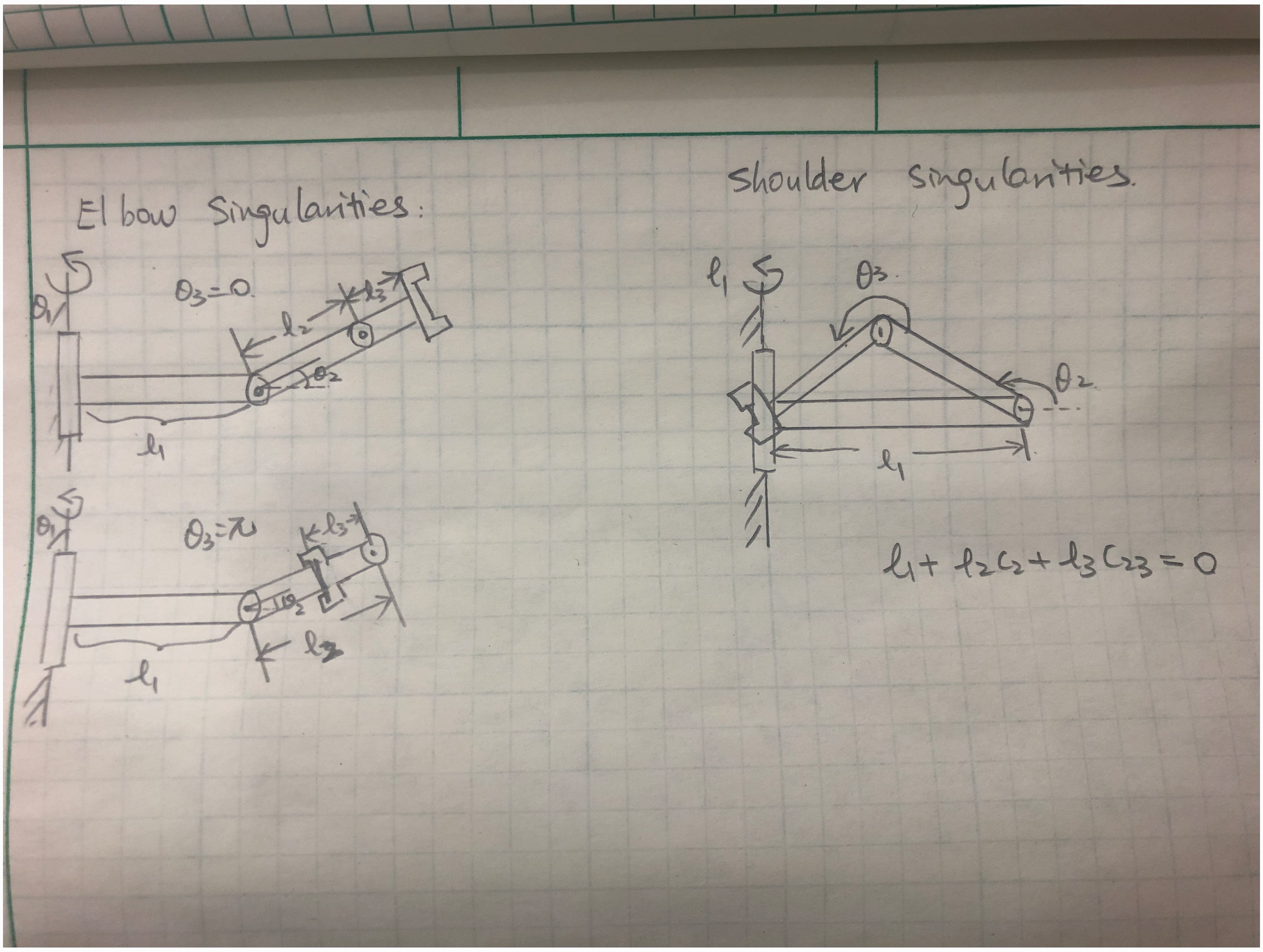
(b) The determination of J_P is:

$$\det(J_P) = -l_2l_3s_3(l_1 + l_2c_2 + l_3c_{23})$$

When $s_3 = 0$ or $l_1 + l_2c_2 + l_3c_{23} = 0$, $\det(J_p) = 0$ and singularities occur.

(c) When $\theta_3 = 0$ or π, $s_3 = 0$, elbow singularity occurs.

When $l_1 + l_2c_2 + l_3c_{23} = 0$, shoulder singularity occurs.



PROBLEM 3

(a) From the forward kinematics, we could get the linear velocity Jacobian:

$$
J_p(q) = \begin{bmatrix} \frac{\partial p_x}{\partial q_1} & \frac{\partial p_x}{\partial q_2} & \frac{\partial p_x}{\partial q_3} \\ \frac{\partial p_y}{\partial q_1} & \frac{\partial p_y}{\partial q_2} & \frac{\partial p_y}{\partial q_3} \\ \frac{\partial p_z}{\partial q_1} & \frac{\partial p_z}{\partial q_2} & \frac{\partial p_z}{\partial q_3} \end{bmatrix}
= \begin{bmatrix} -(d_2+2)s_1 - c_1 - 2s_{13} & c_1 & -2s_{23} \\ (d_2+2)c_1 - s_1 + 2c_{13} & s_1 & 2c_{13} \\ 0 & 0 & 0 \end{bmatrix}
$$

We could get the following DH parameter table:

Link	a_i	α_i	d_i	θ_i
1	1	90	0	θ_1+90
2	0	-90	d_2+2	0
3	2	0	0	θ_3-90

From the DH parameter table, we could get homogeneous transformations between frame 0 and i. We know that z_i^0 is the top three elements of the third column from T_i^0. Since the first and third joints are revolute and joint 2 is prismatic, the angular velocity Jacobian is:

$$\begin{aligned} J_0 &= \begin{bmatrix} z_0^0 & 0 & z_2^0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{aligned}$$

So the full 6×3 Jacobian matrix is:

$$J = \begin{bmatrix} -(d_2+2)s_1 - c_1 - 2s_{13} & c_1 & -2s_{23} \\ (d_2+2)c_1 - s_1 + 2c_{13} & s_1 & 2c_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) The determination of the three non-zero rows of J is:

$$\det \begin{bmatrix} -(d_2+2)s_1 - c_1 - 2s_{13} & c_1 & -2s_{23} \\ (d_2+2)c_1 - s_1 + 2c_{13} & s_1 & 2c_{13} \\ 1 & 0 & 1 \end{bmatrix} = -d_2 - 2$$

When $d_2 = -2$, the above determination turns to 0, singularities occurs. We couldn't differentiate the contribution of θ_1 and θ_3.

(c) The problem is underconstrained because there are only two desired velocities with three joints. The Jacobian is 2×3:

$$J = \begin{bmatrix} -(d_2+2)s_1 - c_1 - 2s_{13} & c_1 & -2s_{23} \\ (d_2+2)c_1 - s_1 + 2c_{13} & s_1 & 2c_{13} \end{bmatrix} = \begin{bmatrix} -2 - 1.5\sqrt{3} & 0.5\sqrt{3} & -\sqrt{3} \\ 0.5 + 2\sqrt{3} & 0.5 & 1 \end{bmatrix}$$

We could use the pseudoinverse to numerically find the "best" solution:

$$J_r^+ = \begin{bmatrix} -0.015 & 0.216 \\ 0.59 & 0.709 \\ -0.235 & -0.211 \end{bmatrix}$$

$$\dot{q}^* = J_r^+ v_d = \begin{bmatrix} -0.015 & 0.216 \\ 0.59 & 0.709 \\ -0.235 & -0.211 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.447 \\ 0.828 \\ -0.187 \end{bmatrix}$$

The right pseudo-inverse J_r^+ minimizes a cost function in joint velocities: $g(\dot{q}, v_d) = \frac{1}{2}(v_d - J\dot{q})^T(v_d - J\dot{q})$.

(d) All possible solutions to the above problem including the homogeneous solution is: $\dot{q} = J_r^+ v_d + (I - J_r^+ J)\dot{q}_0$. So:

$$\mathbf{P} = I - J_r^+ J = I - \begin{bmatrix} -0.015 & 0.216 \\ 0.59 & 0.709 \\ -0.235 & -0.211 \end{bmatrix} \begin{bmatrix} -2 - 1.5\sqrt{3} & 0.5\sqrt{3} & -\sqrt{3} \\ 0.5 + 2\sqrt{3} & 0.5 & 1 \end{bmatrix} = \begin{bmatrix} 0.075 & -0.095 & -0.242 \\ -0.098 & 0.123 & 0.313 \\ -0.244 & 0.314 & 0.804 \end{bmatrix}$$

$\dot{q}_0$ is arbitrary solutions to $v_d = J\dot{q}$.

(e) The problem is overconstrained because there are more specifications than DOFs. Jacobian has 6 rows and 3 columns.

$$J = \begin{bmatrix} -2 - 1.5\sqrt{3} & 0.5\sqrt{3} & -\sqrt{3} \\ 0.5 + 2\sqrt{3} & 0.5 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

We could use the pseudoinverse to numerically find the "best" solution:

$$J_l^+ = \begin{bmatrix} -0.123 & 0.218 & 0 & 0 & 0 & -0.432 \\ 0.730 & 0.706 & 0 & 0 & 0 & 0.559 \\ 0.123 & -0.218 & 0 & 0 & 0 & 1.432 \end{bmatrix}$$

$$\dot{q}^* = J_l^+ v_d = \begin{bmatrix} -0.123 & 0.218 & 0 & 0 & 0 & -0.432 \\ 0.730 & 0.706 & 0 & 0 & 0 & 0.559 \\ 0.123 & -0.218 & 0 & 0 & 0 & 1.432 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \\ -3 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1.423 \\ -0.436 \\ -3.423 \end{bmatrix}$$

The left pseudo-inverse tries to minimize the resulting error: $g(\dot{q}, v_d) = \frac{1}{2}(v_d - J\dot{q})^T(v_d - J\dot{q})$.

(f) Using the joint solution in the previous part, we could get the actual end effector velocities as:

$$v_e = J\dot{q}^* = \begin{bmatrix} -2-1.5\sqrt{3} & 0.5\sqrt{3} & -\sqrt{3} \\ 0.5+2\sqrt{3} & 0.5 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.423 \\ -0.436 \\ -3.423 \end{bmatrix} = \begin{bmatrix} -1.001 \\ 2.000 \\ 0 \\ 0 \\ 0 \\ -2.000 \end{bmatrix}$$

Comparing with the workspace velocities, we could find that $\dot{z}$, w_x could not achieve. It is worth mentioning that w_y could not achieve either although the w_y in v_e agrees with w_y given. Because these three components are related with the motion perpenticular to the plane, which could not be achieved by the planar manipulator.

PROBLEM 4

(a) Since the final velocity is 1 rad/s and the deceleration rate is 2 rad/s^2, if with another $t_m = \frac{1}{2}$s, the velocity at $t = t_f + t_m$ will be 0 rad/s. Then the modified position, velocity and acceleration profiles will be the same as the LSPD we learnt in class.

Using t_d to denote the time the joint begins to decelerate, we have:

$$2 = q(t_f) = 0.5 * \ddot{q}_c * t_c^2 * 2 + \ddot{q}_c * t_c * (t_d - t_c) - 0.5 * \ddot{q}_c * t_m^2 = 2t_c t_d - 0.25$$

$$\begin{cases} t_c * t_d = \frac{9}{8} \\ 2.5 - t_d = t_c \end{cases}$$

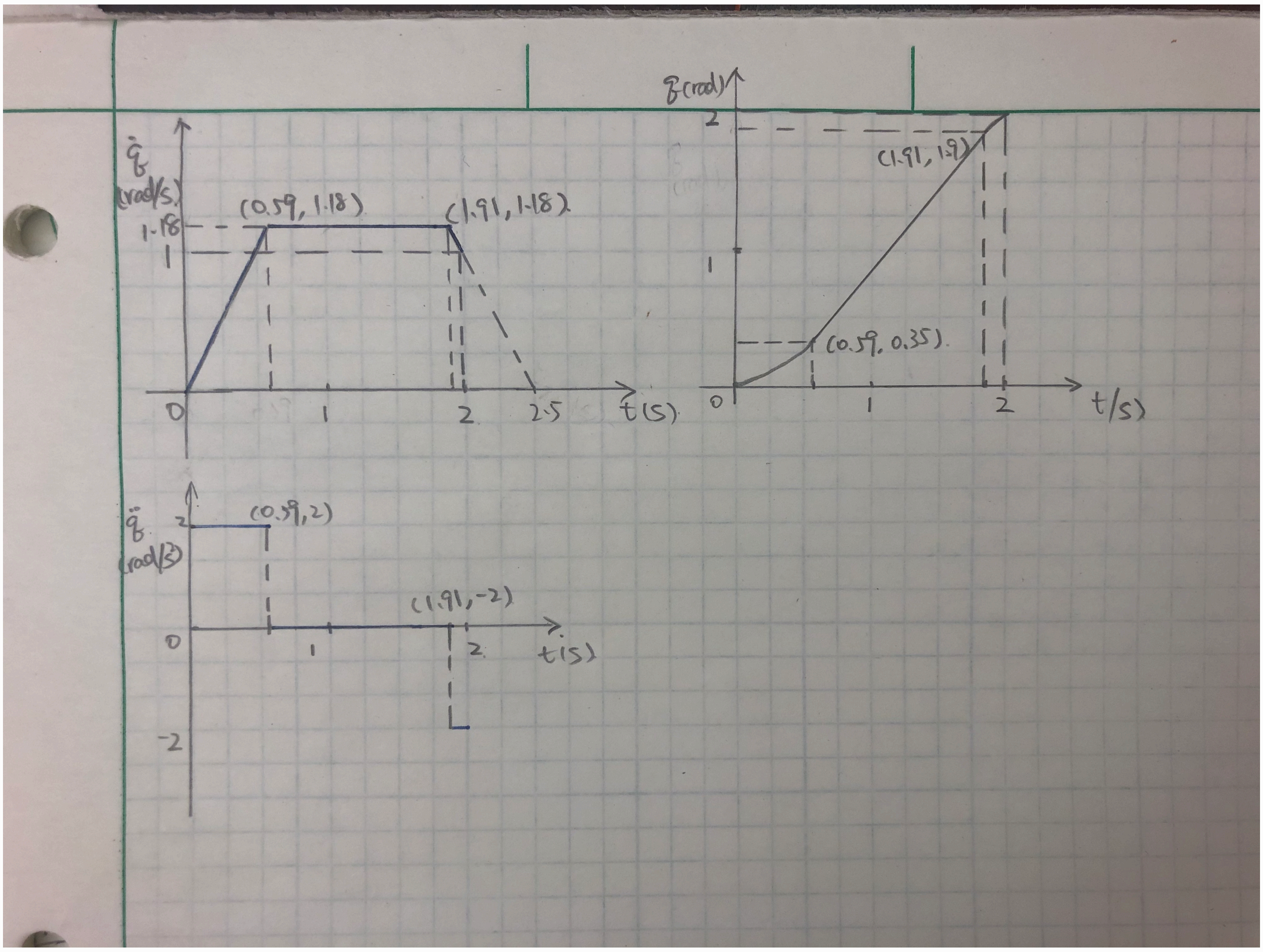
$$t_c = \frac{5-\sqrt{7}}{4},\ t_d = \frac{5+\sqrt{7}}{4}$$

We ignore the reverse solution because $t_c < t_d$.

So we have

$$\dot{q}(t_c) = \ddot{q}_c t_c \approx 1.18 \text{ rad/s},$$
$$\mathrm{q}(\mathrm{t_c}) = 0.5\ddot{\mathrm{q}}_\mathrm{c}\mathrm{t_c^2} \approx 0.35 \text{ rad},$$
$$\mathrm{q}(\mathrm{t_d}) = 2 + 0.5 * 2 * 0.5^2 - \mathrm{q}(\mathrm{t_c}) \approx 1.9 \text{ rad}.$$

The profiles are:



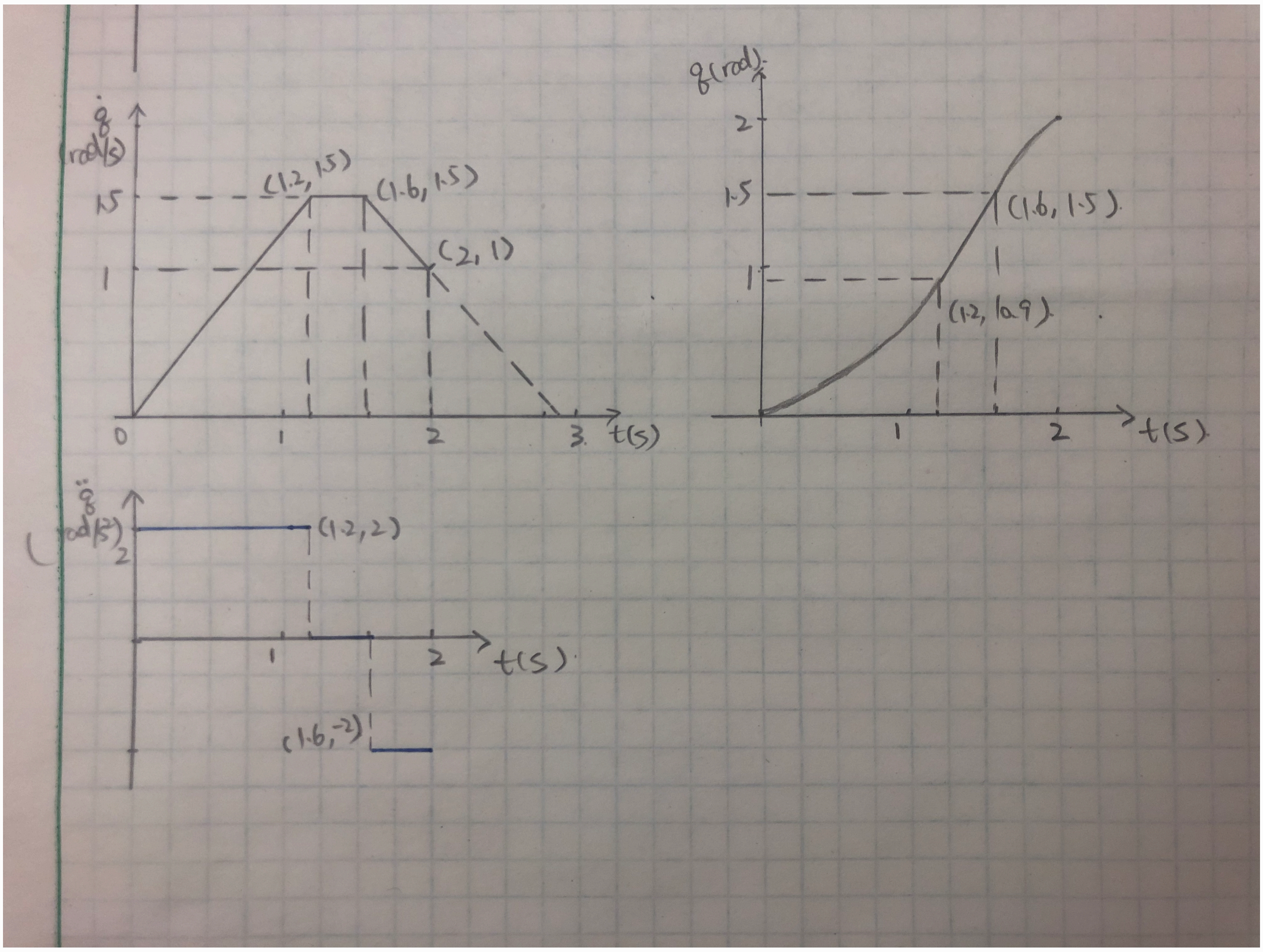
(b) Knowing that $\dot{q}_c = 1.5$ rad/s and final velocity is $\dot{q}_f = 1$ rad/s, similar to previous part, we could expolate the trajectory to the LSPD we learnt in class. Still use t_d to denote the deceleration time.

$$\begin{cases} \frac{2-t_d}{t_c} = \frac{1.5-1}{1.5} \\ 2 = 0.5\dot{q}_c t_c - (1-(\frac{2}{3})^2) * (0.5\dot{q}_c t_c) + \dot{q}_c(t_d - t_c) \end{cases}$$

And we could get the solution:$t_c = 1.2 \text{ s}, \ \mathrm{t_d} = 1.6 \text{ s}.$

So we have: $\ddot{q}_c = \dot{q}_c / t_c = 1.25 \ \mathrm{rad/s^2}$, $q_c = 0.5\dot{q}_c t_c$ = 0.9 rad, $q(t_d) = q_c + \dot{q}_c(t_d - t_c) =$1.5 rad.

The profiles are:



PROBLEM 5

(a) From the DH parameters, we could get the transformation matrix:

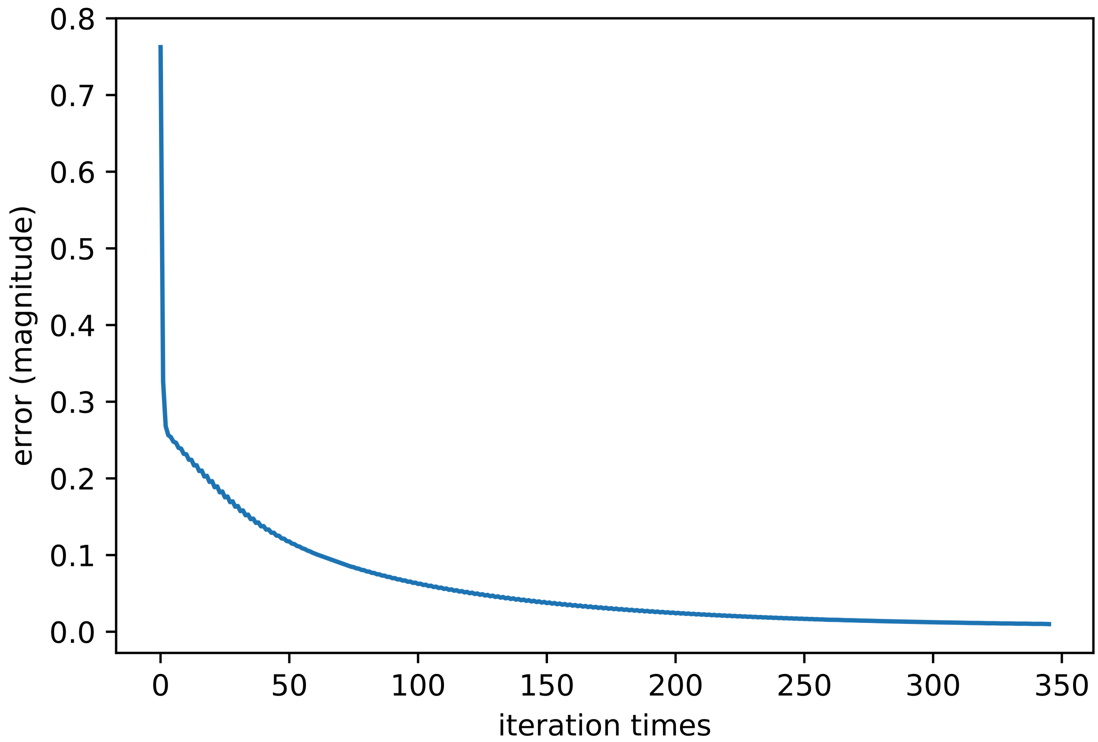
$$A_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0.333 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; A_2^1 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; A_3^2 = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & 0.316 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

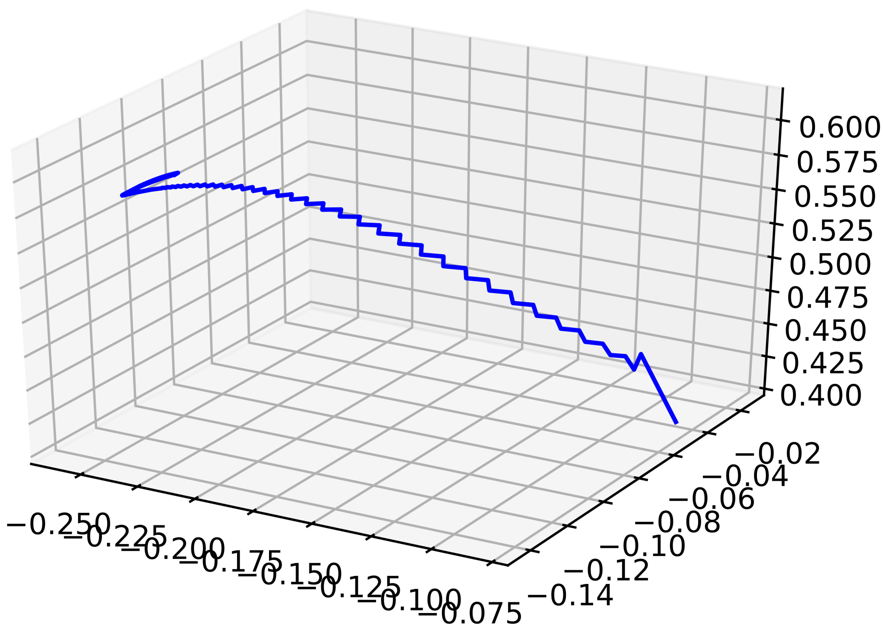
$$A_4^3 = \begin{bmatrix} c_4 & 0 & s_4 & 0.0825c_4 \\ s_4 & 0 & -c_4 & 0.0825s_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; A_5^4 = \begin{bmatrix} c_5 & 0 & -s_5 & -0.0825c_5 \\ s_5 & 0 & c_5 & -0.0825s_5 \\ 0 & -1 & 0 & 0.384 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; A_6^5 = \begin{bmatrix} c_6 & 0 & s_6 & 0 \\ s_6 & 0 & -c_6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_7^6 = \begin{bmatrix} c_7 & 0 & s_7 & 0.088c_7 \\ s_7 & 0 & -c_7 & 0.088s_7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Because of the computation complexity, we calculate the linear velocity Jacobian with $[J_{Pi}] = z_{i-1}^0 \times (p_e - p_{i-1})$ and angular velocity Jacobian with $[J_{Oi}] = z_{i-1}^0$. Corresponding code is provided.

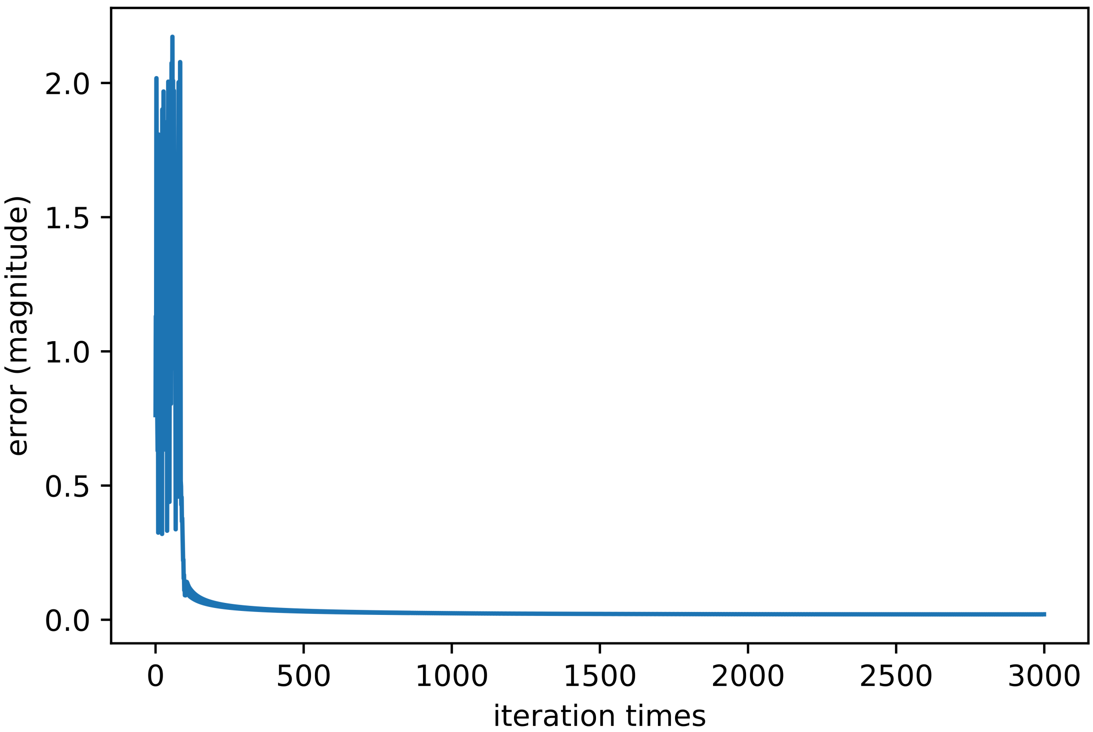
(b) The plot of error and 3D trajectory plot are given as following:



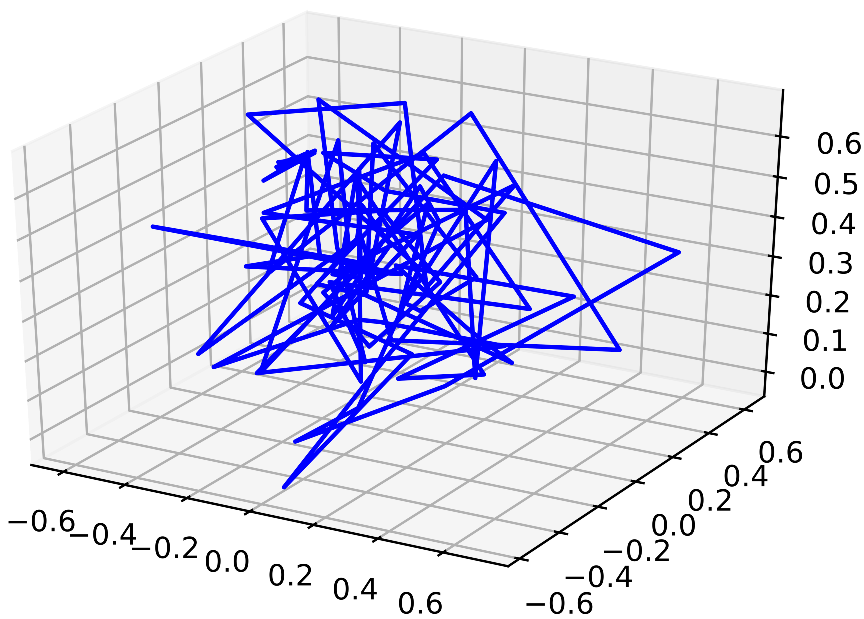


The algorithm runs fast when I choose error threshold =0.01 and converges fast. But the result is a little zigzag. I didn't use any weighting parameters. From the error plot, we could see that it converges around 300 times.The algorithm didn't run into singularities.

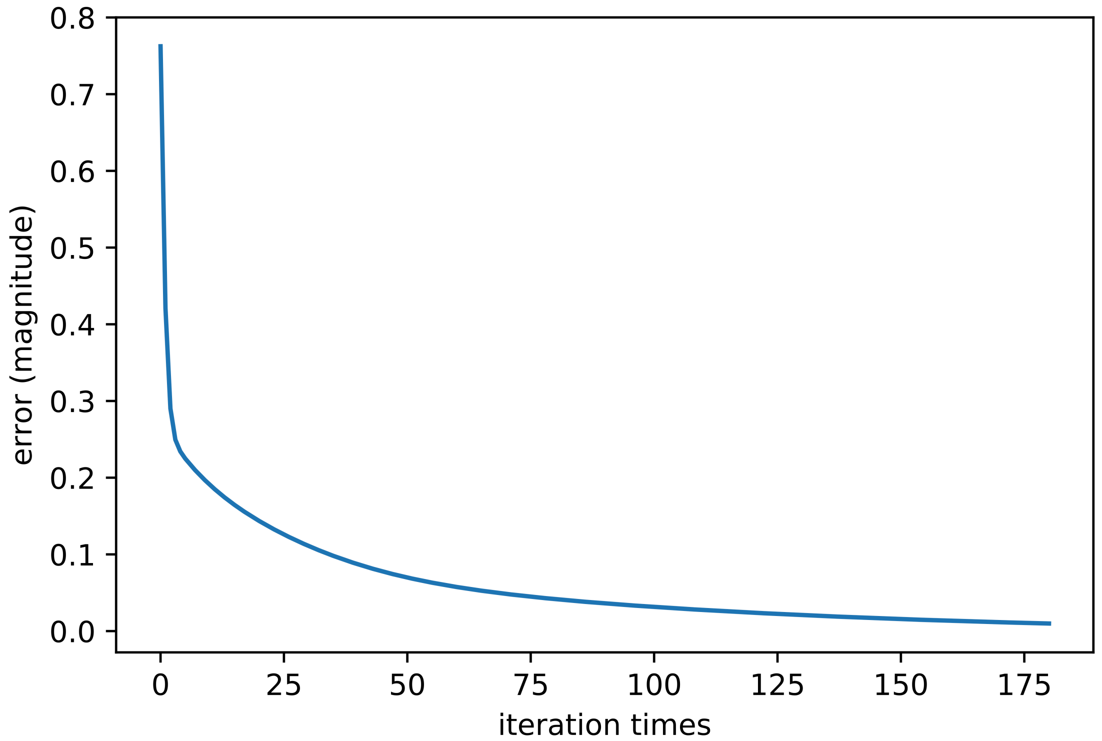
(b) The manipulator didn't run into singularities. The solution of pseudoinverse method is very unstable while the damped is quite smooth. Both methods take more time to run than simple Jacobian transpose. Figures see attached.



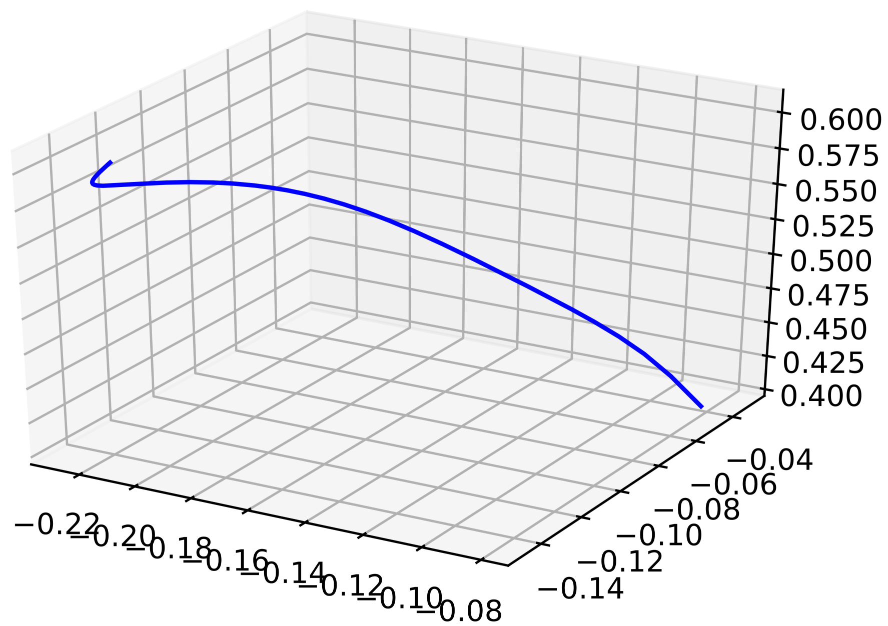
error (magnitude)
iteration times
0.0
0.5
1.0
1.5
2.0
0
500
1000
1500
2000
2500
3000



−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.0
0.1
0.2
0.3
0.4
0.5
0.6



0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
error (magnitude)
0
25
50
75
100
125
150
175
iteration times



0.600
0.575
0.550
0.525
0.500
0.475
0.450
0.425
0.400
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.22
−0.20
−0.18
−0.16
−0.14
−0.12
−0.10
−0.08