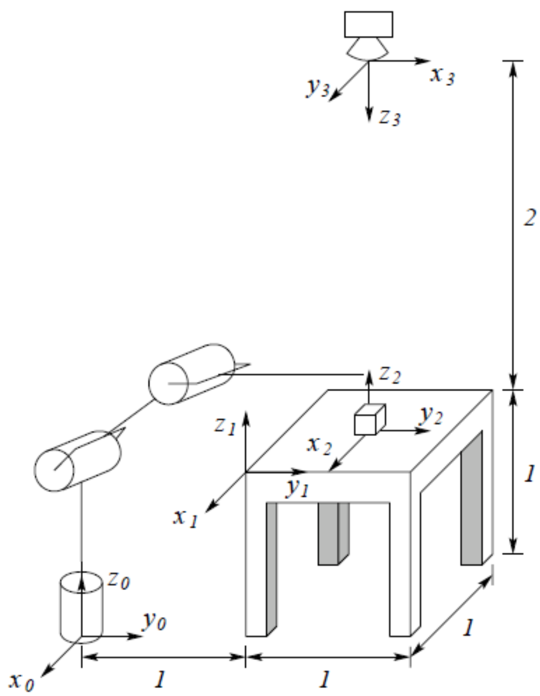
COMS W4733: Computational Aspects of Robotics

Homework 1

Due: February 11, 2019

Problem 1 (15 points)

Consider the diagram below. A robot with base frame O_0 is located 1 m away from a table, which is 1 m high and 1 m square. The table is described by a frame O_1 located at the corner closest to the robot as shown. A cube described by a frame O_2 is located at the center of the table, and a camera is described by a frame O_3 located 2 m directly above the table.



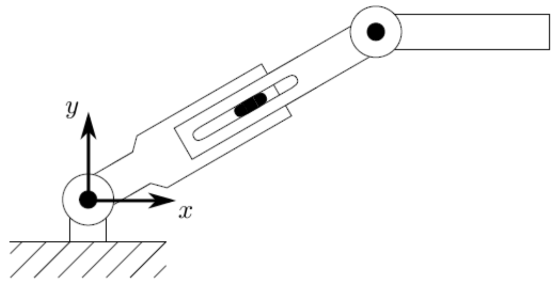
(a) Find the homogeneous transformations relating each frame to the base frame: A_1^0, A_2^0, A_3^0.

(b) Suppose that the camera is rotated 90° about z_3. Indicate which of the three transformations above change, and recompute those transformations, if any.

(c) After the camera is rotated, the block on the table is rotated 90° about z_2 and moved to the coordinates $(-0.2, 0.8, 0.2)^T$ m relative to frame O_1. Compute the two transformations relating the block frame to the base frame A_2^0 and relating the block frame to the camera frame A_2^3 (note the ordering of the last one!).

Problem 2 (20 points)

Consider the RPR manipulator below.



(a) Draw a simplified diagram of this manipulator and place coordinate frames, showing clearly where the origin of each is located (you may choose a more convenient reference configuration). Use the provided frame as frame O_0. Label the joints θ_1, d_2, θ_3, and the last link length a_3.

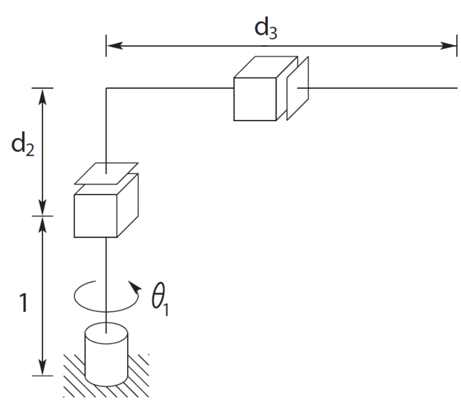
(b) Find the complete DH parameter table. There should be five columns: Link, a_i, α_i, d_i, θ_i.

(c) Write out the homogeneous transformations A_i^{i-1} corresponding to each row of the DH table, and then find the overall direct kinematics function T_n^0 (no need to expand the matrix product).

(d) Suppose that the prismatic joint is fixed to $d_2 = 2$ and the last link has length $a_3 = 1$. The rotational joints have no joint limits. Sketch the workspace of the arm.

Problem 3 (20 points)

Consider the cylindrical manipulator below.



(a) Place all coordinate frames on this manipulator, showing clearly where the origin of each is located. For frame O_0, draw the x_0 axis pointing to the right.

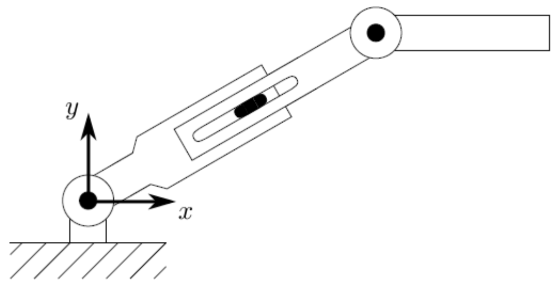
(b) Find the complete DH parameter table. There should be five columns: Link, a_i, α_i, d_i, θ_i.

(c) Write out the homogeneous transformations A_i^{i-1} corresponding to each row of the DH table, and then find the overall direct kinematics function T_n^0 (no need to expand the matrix product).

(d) Suppose that the manipulator has the following joint limits: $0 \leq \theta_1 \leq 90°$, $0 \leq d_2 \leq 2$, $0 \leq d_3 \leq 2$. Sketch the workspace of the arm.

Problem 4 (15 points)

Consider again the RPR manipulator from Problem 2.



(a) Given only an arbitrary desired position of the end effector, how many solutions are there to the inverse kinematics problem? Specify what assumptions have to be made about the desired position relative to the robot's workspace. What if the orientation is also specified?

(b) Find all inverse kinematics solutions given a desired position (p_x, p_y) and orientation ϕ with respect to frame O_0.

Problem 5 (30 points)

We will now look at the 7-dof Barrett WAM arm. Download the datasheet that is provided with this assignment. This arm has seven rotational joints labeled J_i: three at the base, one at the elbow, and three at the wrist. The corresponding coordinate frames has colored axes: x red, z blue, and y green. Note that the origins of frames O_0, O_1, and O_2 are coincident, as are those of O_5 and O_6. Frames O_3 and O_4 are slightly offset at the elbow joint J_4, as are frames O_6 and O_7 (wrist and end plate). Finally, a marker is attached vertically and centered on the robot's end plate. The marker is 120 mm long, providing the location of the origin of the frame O_{tool}.

(a) The DH table below corresponds to the parameters for the transformations between each of the frames, from frame O_0 all the way to frame O_7. Fill in the missing parameters, all of which are nonzero.

Link	a_i	α_i	d_i	θ_i
1	0		0	θ_1
2	0		0	θ_2
3				θ_3
4			0	θ_4
5	0			θ_5
6	0		0	θ_6
7	0	0		θ_7

(b) The robot has written some letters with the marker, with its joint trajectory provided in `qdata.txt`. Each line is a sequence of seven space-separated radian values for each joint J_1 to J_7. Create a program that implements the forward kinematics of the WAM arm and finds the trajectory of the marker tip with respect to the base frame O_0. Provide a short writeup describing your implementation. Submit your code file, the marker tip trajectory in a text file (same format as the joint trajectory), and the resultant drawing.