Hw2.3

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In Piazza @274.
said σ is a parameter should be estimated.

$y_{i,j} = N(u_i \cdot v_j, \sigma^2)$

r_{ij} → $n \times d$

$y_i = w \cdot x_i$

1) So # para = # u_i + # v_j = $(n+d)k$.

$w = u_i \cdot v_j$ $(nk+dk) - D$.

v_{ij} is only one set n.

$u_i \to k \times 1$

$v_j \to k \times 1$

② $\arg\max \prod_{i,j} P(i,j)$

$= \arg\max \sum_{i,j} \log P(i,j)$.

$= \cancel{\arg\max}$ $\arg\min \sum_{i,j} (u_i v_j - r_{ij})^2$.

$P(r_{ij})$. mean is $u_i \cdot v_j$.

? # para $= n \times d$.

$\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} =$

$r_{ij} = u_i v_j + \varepsilon_{ij}$.

$\underset{n\times d}{R} = \underset{n\times k}{U} \cdot \underset{k\times d}{V^T} + \underset{n\times d}{\Sigma}$

$P(U, \sigma^2)(x) = N(U \cdot V^T, \sigma^2)$

$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-u)^2}{2\sigma^2}}$

$\cancel{\arg\max}$ $\arg\min \sum_{i=1}^{n} \sum_{j=1}^{k} u_i^2 v_j^2 - 2 u_i v_j r_{ij}$ $\cancel{+ r_{ij}^2}$.

F.

$\frac{\partial F}{\partial \theta_e} = 0$.

∂F

← convex

G

eg: $\sum_{i=1}^{2} \sum_{j=1}^{3} -(u_i v_j - r_{ij})^2$

$= -(u_1 v_1 - r_{11})^2 - (u_1 v_2 - r_{12})^2 - (u_1 v_3 - r_{13})^2 - (u_2 v_1 - r_{21})^2 - (u_2 v_2 - r_{22})^2 - (u_2 v_3 - r_{23})^2$.

$\frac{\partial G}{\partial u_i} = -2(u_i v_j - r_{ij}) \cdot v_j$.

$\frac{\partial G}{\partial u_1} = -2(u_1 v_1 - r_{11}) v_1 - 2(u_1 v_2 - r_{12}) v_2 - 2(u_1 v_3 - r_{13}) v_3$

$= -2 \sum_{j=1} (u_1 v_j - r_{1j})$

$\arg\max \sum_{i=1}^{n} \sum_{j=1}^{k} -u_i^2 v_j^2 + 2 r_{ij} u_i v_j$.

See online: n movies
estimate how new client like movie.
θ.

p existing clients.
i-th client $x_i \in R^n$.

$y_i = N(w \cdot x_i, \sigma^2)$.
$w \to u_i$, $x_i \to v_j$
v u_i

(iv) prove convex, $f: \mathbb{R}^d \to \mathbb{R}$
for any two points x, x' & $\beta \in [0,1]$.
$f(\beta \vec{x} + (1-\beta)\vec{x}') \le \beta f(\vec{x}) + (1-\beta) f(\vec{x}')$

$\sigma = E[x^2] - E[x]^2$

① for u_i.

$f(\vec{x}) = \sum_i^n \sum_j^d -u_i^2 v_j^2 + 2 r_{ij} u_i v_j$

$f(\vec{x}') = \sum_i^n \sum_j^d -u_i'^2 v_j^2 + 2 r_{ij} u_i' v_j$

choose different u_i!

convex problem
→ convex function
→ convex constrain → no constrain?

$g(u) = u^2$
u_i, u_i'

u_i^2

$\therefore f(\beta\vec{x} + (1-\beta)\vec{x}') = \sum_i^n \sum_j^d -[(\beta u_i) + (1-\beta) u_i']^2 v_j^2 + 2 r_{ij} [\beta u_i + (1-\beta) u_i'] v_j$.

$g(\beta u_i + (1-\beta) u_i')$
$\ge \beta g(u_i) + (1-\beta) g(u_i')$

$\beta f(\vec{x}) + (1-\beta) f(\vec{x}') = \sum_i^n \sum_j^d -[\beta u_i^2 + (1-\beta) u_i'^2] v_j^2 + 2 r_{ij} [\beta u_i + (1-\beta) u_i'] v_j$.

? $f[\beta u_i + (1-\beta) u_i']^2 \not\le \ge f(\beta u_i^2 + (1-\beta) u_i'^2)$

$\beta^2 u_i^2 + (1-\beta)^2 u_i'^2 + 2\beta(1-\beta) u_i u_i' \ge \beta u_i^2 + (1-\beta) u_i'^2$

?

$g(u) = u^2$
convex fn

$g(\beta u_i + (1-\beta) u_i') \le \beta g(u_i) + (1-\beta) g(u_i')$

$[\beta u_i + (1-\beta) u_i']^2 \le$

② similarly for v_j.

(v). Similarly:

$f(\vec{x}) = \sum_i^n \sum_j^d - u_i^2 v_j^2 + 2 r_{ij} u_i v_j$.

$f(\vec{x}') = \sum_i^n \sum_j^d - u_i'^2 v_j'^2 + 2 r_{ij} u_i' v_j'$

$[\beta u_i + (1-\beta) u_i']^2$

$= \beta^2 u_i^2 + 2\beta(1-\beta) u_i u_i' + (1-\beta)^2 u_i'^2$

$= \beta^2 u_i^2 + \cancel{u_i'^2} - \cancel{2\beta u_i'^2} + \beta^2 u_i'^2 + 2\beta(1-\beta) u_i u_i'$

$-\beta u_i'^2$

L

$[\beta u_i^2 + (1-\beta) u_i'^2]$

$= \beta u_i^2 + \cancel{u_i'^2} - \cancel{\beta u_i'^2}$

R

$L - R = \beta(1-\beta) u_i^2 - \beta(1-\beta) u_i'^2 + 2\beta(1-\beta) u_i u_i'$

$0 < \beta < 1$

$= \beta(1-\beta) [u_i^2 - u_i'^2 + 2 u_i u_i']$.

$\sum_{i=1}^{n} (L - R) = \sum_{i=1}^{n} u_i^2 - u_i'^2 + 2 u_i u_i'$.

(vi). $\arg\max \sum_{i=1}^{n} \sum_{j=1}^{k} - u_i^2 v_j^2 + 2 r_{ij} u_i v_j$

$\frac{\partial f}{\partial u_i} = -2 \sum_{j=1}^{d} (u_i v_j - r_{ij}) = 0$

可以求二次求导.
f. ⇒ convex.

$\therefore u_i \sum_{j=1}^{d} v_j - \sum_{j=1}^{d} r_{ij} = 0$

$\therefore u_i = \sum_{j=1}^{d} \frac{r_{ij}}{v_j}$.

similarly, $v_j = \sum_{i=1}^{n} \frac{r_{ij}}{\cancel{\sum} u_i}$.

$\sum_{j=1}^{k} - \frac{\partial}{\partial u_i} (u_i^2 v_j^2) + \frac{\partial}{\partial u_i} (2 r_{ij} u_i v_j) = 0$

$\sum_{j=1}^{k} - f u_i v_j k + f r_{ij} v_j = 0$

$\sum_{j=1}^{k} r_{ij} - u_i \sum_{j=1}^{k} v_j = 0$

$\therefore u_i = \frac{\sum_{j=1}^{k} r_{ij}}{\sum_{j=1}^{k} v_j}$.

(vii)

$r_{ij} = N(u_i v_j, \sigma^2)$

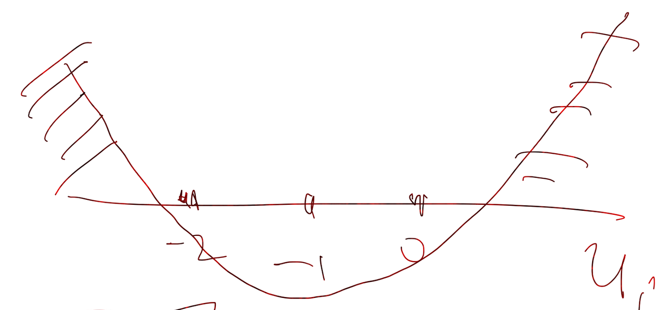
New client: $r_{ij} = N(\hat{u}_i v_j, \sigma^2)$

don't know.

or $\hat{v}_j$.

$[\beta u_i + (1-\beta) u_i']^2$

$\beta u_i^2 + (1-\beta) u_i'^2$



$f(x) = x^2$.

$f(x') = x'^2$.

$f(\beta x + (1-\beta) x') = [\beta x + (1-\beta) x']^2$

$= \beta^2 x^2 + (1-\beta)^2 x'^2$.

$\beta f(x) + (1-\beta) f(x')$
$\beta x^2 + (1-\beta) x'^2$

1 2 -1

$\frac{-2 \pm \sqrt{4+4}}{2} = \frac{-1 \pm \sqrt{2}}{1}$

$\arg\min \sum_{i=1}^{n} \sum_{j=1}^{k} u_i^2 v_j^2 - 2 r_{ij} u_i v_j$.

prove convex, $f: \mathbb{R}^d \to \mathbb{R}$
for any two points x, x' & $\beta \in [0,1]$.
$f(\beta \vec{x} + (1-\beta)\vec{x}') \le \beta f(\vec{x}) + (1-\beta) f(\vec{x}')$

(v) prove jointly convex.

$M_{i,j}$

$f(\vec{x}) = \sum_{i,j} u_i^2 v_j^2 - 2 r_{ij} u_i v_j = \sum_{i,j} (u_i v_j - r_{ij})^2$.

$f(\vec{x}') = \sum_{i,j} u_i'^2 v_j'^2 - 2 r_{ij} u_i' v_j' = \sum_{i,j} (u_i' v_j' - r_{ij})^2$

$M_{i,j}'$

$f(\beta\vec{x} + (1-\beta)\vec{x}') = \{[\beta u_i v_j + (1-\beta) u_i' v_j'] - r_{ij}\}^2$.