

# Mini-project 4

CMPSCI 670, Fall 2019, UMass Amherst  
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TAs: Aruni RoyChowdhury, Archan Ray

## Guidelines

**Submission.** Submit a *single pdf document* via moodle that includes your solutions, figures and code. The latex source file for the homework is provided which you can modify to produce your report. You are welcome to use other typesetting software as long as the final output is a pdf. For readability you may attach the code printouts at the end of the solutions within the same pdf. Note that we will not run your code. Similarly figures should be included in a manner which makes it easy to compare various approaches. Poorly written or formatted reports will make it harder for the TA to evaluate it and may lead to a partial deduction of credit.

**Late policy.** You could have 24 hours late submission with a 50% mark down. Late submission beyond 24 hours will not be given *any* credits.

**Plagiarism.** We might reuse problem set questions from previous years, covered by papers and webpages, we expect the students not to copy, refer to, or look at the solutions in preparing their answers. We expect students to want to learn and not google for answers.

**Collaboration.** The homework must be done individually, except where otherwise noted in the assignments. 'Individually' means each student must hand in their own answers, and each student must write their own code in the programming part of the assignment. It is acceptable, however, for students to collaborate in figuring out answers and helping each other solve the problems. We will be assuming that you will be taking the responsibility to make sure you personally understand the solution to any work arising from such a collaboration.

**Using other programming languages.** All of the starter code is in Matlab which is what we expect you to use. You are free to use other languages such as Octave or Python with the caveat that we may not be able to answer or debug non Matlab questions.

**Python requirements.** We will be using Python 2.7. The Python starter code requires [scipy](#), [numpy](#) (at least v1.12), [scikit-image](#) and [opencv-python](#). If you are not familiar with installing those libraries through some package manager (like [pip](#)), the easiest way of using them is installing [Anaconda](#). If you are using [pip](#), you probably want to install [opencv-python](#) and [opencv-python-contrib](#) at the same time:

```
pip install opencv-python==3.3.0.10 opencv-contrib-python==3.3.0.10.
```

# 1 Scale-space blob detection [45 points]



Figure 1: Scale-space blobs detection

In this homework you will implement a simple panoramic image stitching algorithm. The first part is to detect blobs as keypoints. The algorithm is outlined as follows:

1. Build a Laplacian scale space, starting with some initial scale and going for  $n$  iterations:
  - (a) Filter image with scale-normalized Laplacian at current scale.
  - (b) Save the square of Laplacian response for current level of scale space.
  - (c) Increase scale by a factor  $k$ .
2. Perform non-maximum suppression in scale space.
3. Display resulting circles at their characteristic scales for points above a threshold.

**Test images.** There are four images in the folder `data/blobs` to test your code. Fig. 1 provides the output of "butterfly" example as a reference to debug your code. Keep in mind that your output may look different depending on your threshold, range of scales, and other implementation details.

**Running the code.** Start from the entry code `evalBlobsDetection`. It calls the dummy implementation of `detectBlobs` and draws the blob with `drawBlobs` as Fig. 2. You need to implement the detail in `detectBlobs` which outputs a  $n \times 4$  matrix with a blob in each row as (x, y, radius, score).

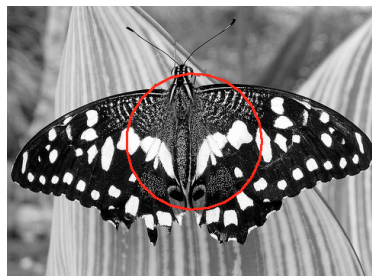


Figure 2: Output of running the default implementation of `evalBlobsDetection`

The following are the hints to implement the function:

- Don't forget to convert images to grayscale (`rgb2gray` command) and double (`im2double`).
- For creating the Laplacian filter, use the `fspecial` function (check the options). Pay careful attention to setting the right filter size.

- You have to choose the initial scale, the factor  $k$  by which the scale is multiplied each time, and the number of levels in the scale space. I typically set the initial scale to 2, and use 10 to 15 levels in the scale pyramid. The multiplication factor should depend on the largest scale at which you want regions to be detected.
- You may want to use a 3D array to represent your scale space. It would be declared as:

*scaleSpace = zeros(h,w,n); % [h,w] - dimensions of image, n - number of levels in scale space*

Then *scaleSpace(:, :, i)* would give you the  $i$ 'th level of the scale space. Alternatively, if you are storing different levels of the scale pyramid at different resolutions, you may want to use a cell array, where each "slot" can accommodate a different data type or a matrix of different dimensions. Here is how you would use it:

*scaleSpace = cell(n,1); %creates a cell array with n "slots"*  
*scaleSpace{i} = myMatrix; % store a matrix at level i*

- A point is the local maximum if it is higher than all its neighbors (each point has 26 neighbors in 3D). You may find functions `nlfilter`, `colfilt` or `ordfilt2` useful for doing non-maximum suppression in Matlab. In Python, you might want to use `scipy.ndimage.filters.generic_filter` in your implementation.
- You also have to set a threshold on the squared Laplacian response above which to report region detections. You should play around with different values and choose one you like best.
- To display the detected regions as circles, you can use the `drawBlobs` function. You should display the highest scoring 1000 detected blobs for each image. If your code returns fewer than 1000 blobs then you may have to lower your threshold. Don't forget that there is a multiplication factor that relates the scale at which a region is detected to the radius of the circle that most closely approximates the region.

## 1.a What to submit

To get full credit of this part, you have to

- Include your implementation of `detectBlobs` and include the results of given test images under `data/blobs`.

### My Answer:

Figure 3 shows the outputs of Scale-space blob detection.

### Implementation:

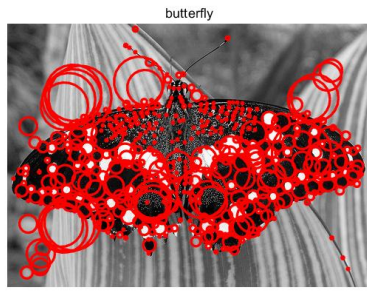
Step 1. Convert the images into grayscale and double [0,1].

Step 2. Setting parameters by using struct in MATLAB. Giving the default values to these parameters.

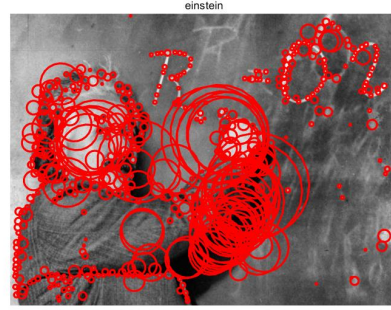
- sigma of the LoG filter: `param.sigma = 2;`
- number of intervals in an octave: `param.interval = 12;`
- threshold for blob detection: `param.threshold = 0.02;`

Step 3. Initialize a sigma of the smallest blob.

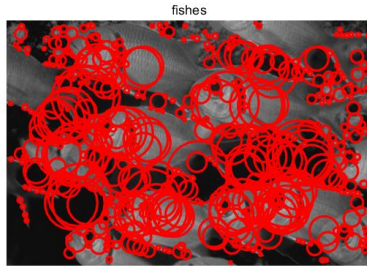
- The default value of sigma is 2.
- The radius is  $r = \text{ceil}(3 \cdot \text{sigma})$ .
- The LOG can be created by using `fspecial('log', 2*r+1, sigma)`.  $2 \cdot r + 1$  is the window size of the kernel.



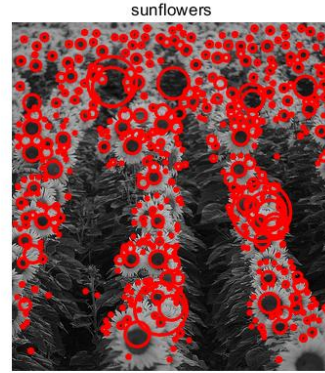
(a) butterfly



(b) einstein



(c) fishes



(d) sunflowers

Figure 3: Outputs of Scale-space blob detection

Step 4. Computing the scale space representation.

- Initialize the representation of pyramid of scale-space.
- The input of the pyramid is image, param.interval, and size(g,1).

Step 5. Computing the blob response pyra.scores.

- Initialize a zero array for pyra.scores which has the same size of width, length and depth.
- Each layer of scale needs to convolute with LOG.
- Pad 0 to the around of the outputs of last step. The radius of the padded 0 is r.
- Resize the results as the size of input images.
- For each layer of the pyramid,
  - (1) Score1 is the absolute value of each layer of scale-space convolute with LOG. After this step, the original image can be convoluted into different scales.
  - (2) (Because the mode of convolution used in here is “valid”. Therefore, there are area in the margin cannot be convoluted. The width of this area is r. Therefore, pad 0 to the surround the outputs of last step. The padded radius is r. In this way, the scale of each layer has the same size with the original image.)
  - (3) Resize the outputs of last step to the same size with the original images. This is each layer of scale-space. Until now, each layer of scale-space which be convoluted with LOG and resized can be return.

Step 6. Performing a spatial NMS (non-maximum suppression) in scale space.

- Create a matrix which all the values are 1. The size is 3\*3.
- For each layer of scale-space,

- (1) Replace each value by the maximum value of the 3\*3 domain. Using 'ordfilt2' in MATLAB can implemented quickly in this step.
- (2) The maxima is the absolute value of — local maximum value – original value— which less than eps. Return the array which has the same size with original image, but there is only 0 or 1 inside the array. The pixel which is 1, is the local maxima of the local domain.
- (3) Multiply the results of the previous step with each layer of the scale-space. The result is reassigned to each layer of the scale-space. In the current matrix, only the local maximum in the full matrix range is retained, and the other values are zero.

Step 7. Perform nms on the across scale.

- Getting the location (width, length, depth) of maxima by using `max(pyra.score, [], 3)`
- Perform nms across space again to collapse values to a point. In Step 6. The nms only performed on the scale. Now it needs to be performed in the space.
- Calculating the radius of window of nms, called 'nms\_r'.
- Building the kernel window of nms by using nms\_r called 'nms\_w'
- Finding the local maxima value by using `ordfilt2(blobScore, numel(nms_w), nms_w)`. Replacing value of each pixel by using local maxima value.
- Setting a threshold. When the results of last step are bigger than this threshold, the results are the final maxima of the domain. Called "blob\_find".
- Getting the index [y,x] of the maxima by using `find(blob_find)`.
- The responses of LOG is biggest when  $r = \sigma * \sqrt{2}$ , the radius id blob is  $(1./pyra.scale(blobScale(blob\_find)))) * \sigma * s$

Step 8. Creating a function to compute scale-space representation of the image. (This step is mainly calling the functions or methods which created above and input parameter into the function.)

- Call the method `scaleSpace(im, interval, minSize)` to create scale-space.
- Record the size of the image
- Precompute how many scales and steps you need
- (1) First enlarge the length and width of the image to twice by using `im = imresize(im, 2)`. Doubling the length and width of the original image before creating the scale space to preserve the original image information and increase the number of feature points.
- (2) Compare the length and width of im in which dimension is smaller.
- (3) Determine the number of Octaves by `ceil(log(smallDim/minSize))+1`, named `Octaves_num`.
- Create scale-space for the images
- (1) Create an empty pyramid named "pyra.im" by using `cell(1, numOctaves*interval)`.
- (2) Create an empty `pyra.scale` by using `zeros(1, numOctaves*interval)`
- Loop over scales and compute pyramid representation Initialize `stepSize` which is  $2^{1/\text{interval}}$
- Initialize `scale = 2` and `offset = 0`;

```

for each layer of Octaves s, the scale of s is (1:numOctaves):
    for each octave i, the scale of i is (1:interval):
        Scaling each layer into  $1/\text{stepSize}^{(i-1)}$  by using
        pyra.im{offset+i} = imresize(im, 1/stepSize^(i-1))
        pyra.scale(offset+i) = currScale/stepSize^(i-1)
    end
    Resize the outputs into half size of the original image.
    The current scale also scaling to the half of previous one by multiplying 0.5.
    (Because the images were enlarged twice at the beginning.)
    offset = offset + interval
end

```



## 2 Image stitching [35 points]

In this part you will implement the RANSAC algorithm to stitch two images. The input to the algorithm are two images which are related by an unknown transformation. You will use your blobs detector implemented in the first part to extract keypoints and extract feature descriptors on them. Your goal is to estimate an affine transformation using feature matching and RANSAC to produce a combined image.

Recall that the overall steps in the alignment algorithm are:

1. Detect keypoints in each image (part 1).
2. Extract SIFT features at each detected keypoints.
3. Match features based on pairwise distance.
4. Use RANSAC to estimate the best affine transformation.
5. Stitch the two images using the estimated transformation.

The provided codebase contains an implementation of Step 2 and Step 5. You will implement the remaining steps. The entry code for this homework is in [evalStitching](#). The code loads two images which contain overlapping areas. Figure 4 shows the “hill” example included in the homework. More test examples are given in the [data/stitching](#) folder compressed in [p4\\_data.zip](#).



Figure 4: Input images to the image stitching algorithm.

Below is an outline of the steps you need to implement the algorithm.

1. **[0 points] Detect keypoints** The first step is to detect blobs in each image independently. You already implemented this in the first part. Remember to set the initial scale and threshold accordingly to get enough keypoints.
2. **[0 points] Feature extraction.** The next step is to extract feature descriptors on detected keypoints. The code to compute SIFT is included in the code base. The file [compute\\_sift](#) takes an image and (x, y, radius) of  $N$  blobs as input and return a matrix of size  $N \times 128$  with 128 dimensional SIFT descriptor in each row. If you are using Matlab, the function uses different external libraries which are included in [mex](#) folder depending on the operating system. The first few lines in [evalStitching.m](#) sets up the environment variables in Matlab based on your operating system. The code has been tested to work on MATLAB 2019a version and earlier, and should work out of the box on Windows, MAC, and Linux platforms. Talk to the TA in case the mex files are not compatible with your operating system. If you are using Python, just install [opencv](#) using your favorite package manager.
3. **[10 points] Computing matches.** The next step is to compute matches between the two sets of features. Implement the function `matches = computeMatches(f1, f2)` that returns the best match

of each feature `f1` to `f2` using the smallest sum-of-squared-differences. The input are two matrices `f1=d×N` and `f2=d×M` and the output is a array of size `N×1` where each entry `matches(i) ∈ [0, 1, ..., M]` is the closest feature in `f2` to the  $i^{th}$  feature `f1(:, i)`, with `matches(i) = 0` indicating no matching (using MATLAB's 1-starting indexing; for Python a no-match can be indicated by -1).

Considering only nearest neighbors returns many false matches. To refine the matches, compute the ratio of distance from the closest feature to the distance from the second closest feature and reject the matches with the ratio greater than a threshold. David Lowe's paper suggest a threshold 0.8. Set `matches(i)=0` if  $i^{th}$  feature `f1(:, i)` fails on the ratio test.

You can visualize the matches using the `showMatches(im1, im2, c1, c2, matches)` function provided in the codebase. Figure 5 shows the output of my implementation for reference.

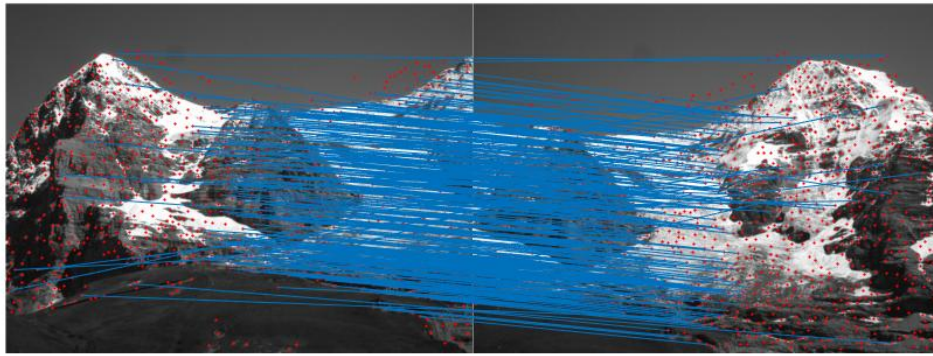


Figure 5: Matches between features.

4. [25 points] **Estimating affine transformation using RANSAC.** The matching in the previous step is noisy and contains many outliers. Using RANSAC, estimate the affine translation that agrees with most matches (inliers). Implement the function `[inliers, transf] = ransac(matches, c1, c2)`. Here `inliers` is the indices of the inlier matches, `transf` is the estimated affine translation of  $2 \times 3$  matrix  $[m_1, m_2, t_1; m_3, m_4, t_2]$  between `c1` and `c2` (See the notation in Lecture slides). You need at least three pairs of points to estimate the this. See lecture slides for details.

You can visualize the inlier matches using the `showMatches` function. Figure 6 shows the output of my implementation for reference. The estimated affine transformation for this example is given as follows for your reference:

$$\begin{bmatrix} 0.9827 & 0.1855 & 124.1568 \\ -0.1782 & 0.9804 & 0.9023 \end{bmatrix}$$

5. [0 points] **Visualizing the stitched image.** Merge the images using `mergeImages(im1, im2, transf)` function provided in the codebase. The result is shown as Figure 7.

## 2.a What to submit

To get full credit for this part you have to

- Include your implementation of `computeMatches.m` and `ransac.m`.
- Visualize the intermediate results and the output of each of the provided test image pairs. You can do this by setting the `exampleIndex` variable in the `evalStitching.m` file.
- Report the estimated affine transformation for given test images.

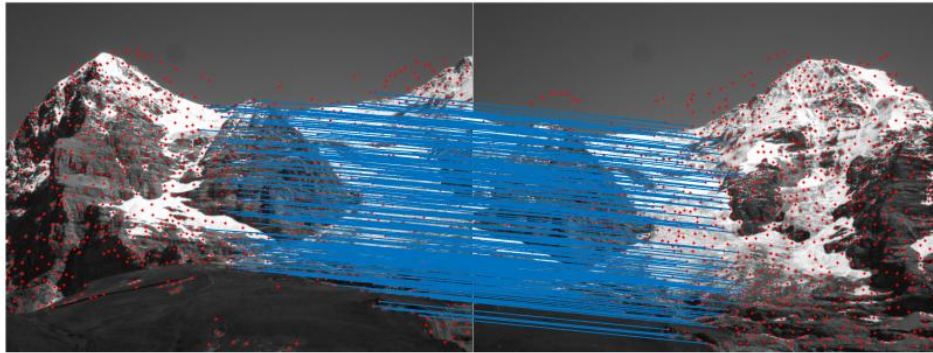


Figure 6: Inliers obtained from RANSAC.

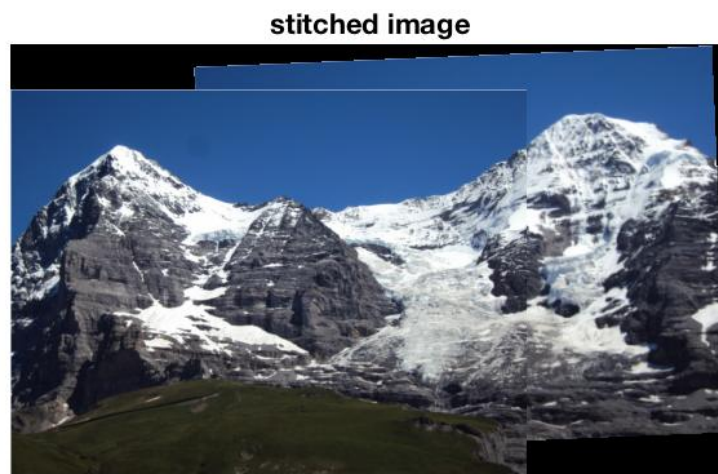


Figure 7: Visualization of output.

### My Answer:

#### computeMatches.m:

- Create a function named 'computeMatches', the inputs of computeMatches are two sets of features (f1, f2).
- Call the function dist2(f1, f2) which is built below, get the output pairwiseDist.
- Get the size of pairwiseDist. [width, length]
- Create an empty array m which size is (1, width)

```
For i in (1, width),
    Get the sorted values and related index by using [value,ord] = sort(pairwiseDist(i,:))
    Get the ratio of value (1)/ value (2);
    if the ratio is less than threshold (assume is 0.8),
        m(i) = ord(1);
    end
```

- Create a function to calculates the squared Euclidean distance between two sets of points, length = dist2(data, centre).



- Get the size of data and centre respectively,  
 $[ndata, dim\_data] = size(data)$   
 $[ncentres, dim\_cen] = size(centre);$
- The length equals to  $(ones(ncentres, 1) * sum((data.^2)', 1))' + ones(ndata, 1) * sum((centre.^2)', 1) - 2. * (data * (centre'))$
- Let the negative entries in length =0.

**ransac.m:**

- Create a function [inliers, transf] = ransac(matches, blobs1, blobs2)  
 Input:  
 blobs1 = blobs1'; Convert blobs1 to N1 \* 2 matrix, each row is a point.  
 blobs2 = blobs2'; Convert blobs2 to N2 \* 2 matrix, each row is a point  
 matches: M \* 2 matrix, each row represents a match [index of p1, index of p2]
- Solve for the line between the points of blobs1 and blobs2
- Count the number of inliers to the line L
- Create a function to estimate affine transformation. The input in here is (src, dst). The purpose of here is if L has the highest number of inliers so far, save it  
 Calculate the different size of source and target.  
 Get the minimum number of 4 matches required for estimating affine transformation.
- Repeat for N rounds, return the best L.
- Output:  
 A robust estimation of affine transformation from blobs1 to blobs2.

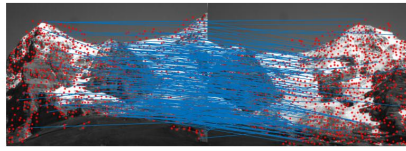
The estimated affine transformation for given test images:

image	inliers	affine transformation
hill	194	[0.99, 0.19, 122.85; -0.18, 0.99, -0.44]
field	75	[0.99, 0.22, 231.91; -0.22, 0.99, 25.16]
ledge	207	[0.98, 0.07, 117.24; -0.05, 0.98, -116.20]
pier	180	[1.00, 0.09, 275.44; -0.08, 1.00, 34.99]
river	61	[0.98, -0.23, 299.51; 0.23, 0.98, -61.01]
roofs	69	[0.98, 0.21, -189.18; -0.14, 0.98, -0.17]
building	98	[0.98, 0.03, 108.20; -0.02, 1.02, 9.17]
uttower	141	[1.01, -0.00, -210.20; -0.04, 1.00, -0.20]

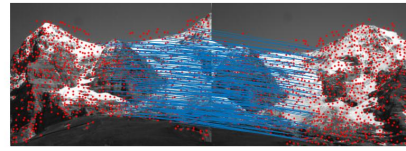
Figure 8, 9, 10, 11, 12, 13, 14, and 15 visualizes the intermediate results and the output of each of the provided test image pairs.

### 3 Report Writing and Presentation [10 points]

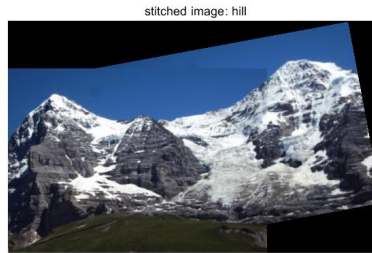
Please follow the guidelines for writing a good report. Graders will penalize reports that are poorly written and fail to present the results in a reasonable manner



(a) Matches between features

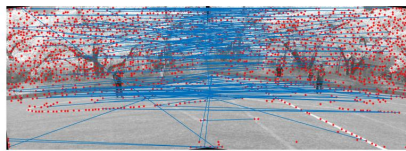


(b) Inliers obtained from RANSAC

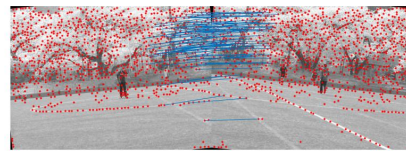


(c) Image stitching

Figure 8: Outputs of 'hill' stitching



(a) Matches between features



(b) Inliers obtained from RANSAC

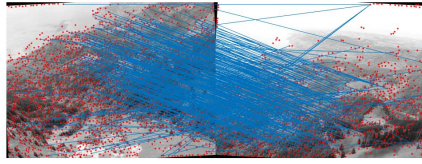


(c) Image stitching

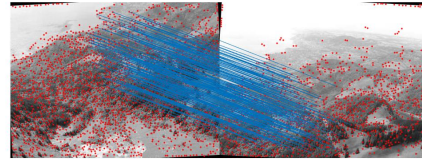
Figure 9: Outputs of 'field' stitching

## 4 Extensions for extra credit

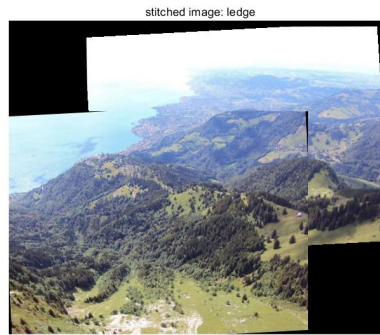
- To detect blobs at multiple scales, there are two strategies to complete the task: (1) iteratively filter the image with kernel of increasing size. (2) iteratively downsample the image and filter it with the kernel of fixed size. You will have to upsample the result or do some interpolation in order to find maxima in scale space for the second approach. Which approach is more efficient? Run the both implementations and compare the performance between them.
- Implement the difference-of-Gaussian pyramid as mentioned in class and described in [David Lowe's](#)



(a) Matches between features

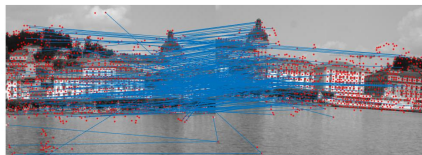


(b) Inliers obtained from RANSAC

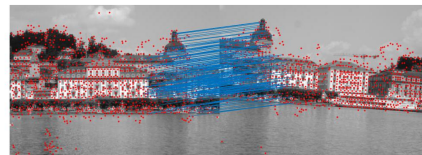


(c) Image stitching

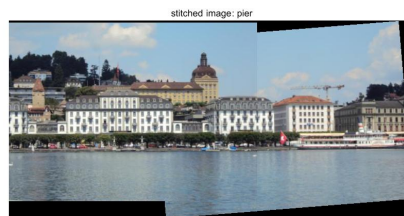
Figure 10: Outputs of 'ledge' stitching



(a) Matches between features



(b) Inliers obtained from RANSAC

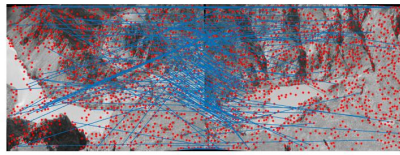


(c) Image stitching

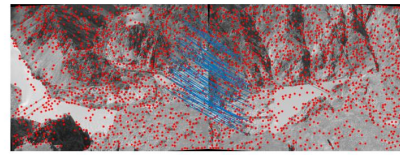
Figure 11: Outputs of 'pier' stitching

[paper](#). Compare the results and the running time to the direct Laplacian implementation.

- Implement the affine adaptation step to turn circular blobs into ellipses as shown in the lecture. The selection of the correct window function is essential here. You should use a Gaussian window that is a factor of 1.5 or 2 larger than the characteristic scale of the blob. Note that the lecture slides show how to find the relative shape of the second moment ellipse, but not the absolute scale (i.e., the axis lengths are defined up to some arbitrary constant multiplier). A good choice for the absolute scale is



(a) Matches between features

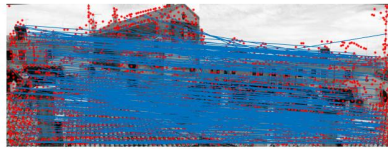


(b) Inliers obtained from RANSAC

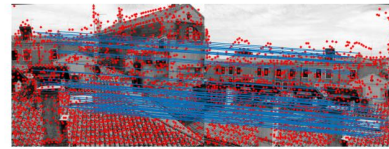


(c) Image stitching

Figure 12: Outputs of 'river' stitching



(a) Matches between features



(b) Inliers obtained from RANSAC

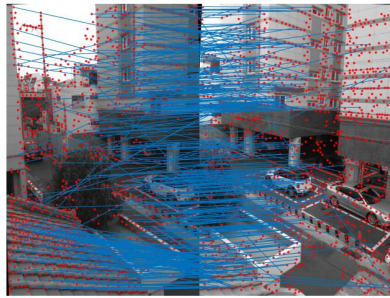


(c) Image stitching

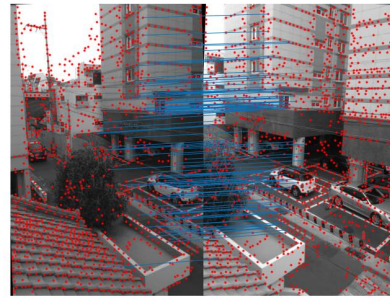
Figure 13: Outputs of 'roofs' stitching

to set the sum of the major and minor axis half-lengths to the diameter of the corresponding Laplacian circle. To display the resulting ellipses you should modify the circle drawing function or look for a better function in the MATLAB or on the Internet.

- The Laplacian has a strong response not only at blobs, but also along edges. However, recall from the class lecture that edge points are not "repeatable". So, implement an additional thresholding step that computes the Harris response at each detected Laplacian region and rejects the regions that have only one dominant gradient orientation (i.e., regions along edges). If you have implemented the affine adaptation step, these would be the regions whose characteristic ellipses are close to being degenerate



(a) Matches between features



(b) Inliers obtained from RANSAC



(c) Image stitching

Figure 14: Outputs of 'building' stitching

(i.e., one of the eigenvalues is close to zero). Show both "before" and "after" detection results.

- There are few difficult test images where affine transformation will not produce perfect results since some 2D transformations cannot be modeled as affine (See section 2.1 in [Richard Szeliski's book](#)). For example, you could to align two images with a homography.

If you complete any work for extra credit, be sure to clearly mark that work in your report, explain it and include the code.

## 5 Code (MatLab)

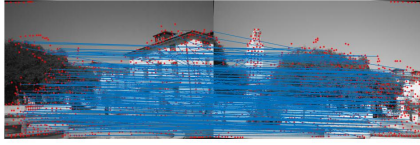
### 1. Scale-space blob detection

---

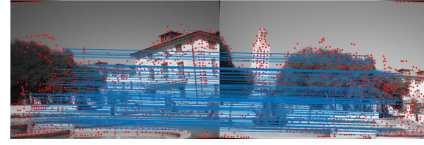
```
% Convert the images into grayscale and double [0,1].
im=rgb2gray(im);
if ~isfloat(im)
    im = im2double(im);
end

% Setting parameters by using struct
```





(a) Matches between features



(b) Inliers obtained from RANSAC



(c) Image stitching

Figure 15: Outputs of 'uttower' stitching

```

if nargin < 2
    param.sigma = 2;
    param.interval = 12;
    param.threshold = 1e-2;
end

% Initialize a sigma of the smallest blob.
sigma = param.sigma;
r = ceil(3*sigma);
g = fspecial('log', 2*r+1, sigma);

% Computing the scale space representation
pyra = scaleSpace(im, param.interval, size(g,1));

% Computing the blob response pyra.scores.
pyra.scores = zeros(size(im,1), size(im,2), length(pyra.scale));
tic;
for i = 1:length(pyra.scale),
    score = abs(conv2(pyra.im{i}, g, 'valid'));
    score = padarray(score,[r r], 0, 'both');
    pyra.score(:,:,i) = imresize(score, size(im));
    if param.display,
        figure(1); clf;
        subplot(1,2,1);
        imagesc(pyra.im{i}); axis image; colormap gray;
        subplot(1,2,2);
        imagesc(pyra.score(:,:,i)); axis image; colormap gray;
        pause(0.01);
    end
end

end

% Performing a spatial NMS (non-maximum suppression) in scale space.
nbhd = ones(3);
for i = 1:length(pyra.scale),

```

```

ordscore = ordfilt2(pyra.score(:,:,i), numel(nbhd), nbhd);
ismax = abs(ordscore - pyra.score(:,:,i)) < eps;
pyra.score(:,:,i) = ismax.*pyra.score(:,:,i);
if param.display,
    figure(1); clf;
    subplot(1,2,1);
    imagesc(pyra.im{i}); axis image; colormap gray;
    subplot(1,2,2);
    imagesc(pyra.score(:,:,i)); axis image; colormap gray;
    pause(0.01);
end
end

% Performing nms on across scales
[blobScore, blobScale] = max(pyra.score, [], 3);
nmsRadius = max(1, ceil(0.005*sqrt(size(im,1)*size(im,1) +
    size(im,2)*size(im,2))));
nbhd = ones(2*nmsRadius+1);
ordscore = ordfilt2(blobScore, numel(nbhd), nbhd);
ismax = abs(ordscore - blobScore) < eps;
blobScore = blobScore.*ismax;
inds = blobScore > param.threshold;
[y,x] = find(inds);
blobs = [x y (1./pyra.scale(blobScale(inds))) "' "*sigma*sqrt(2) blobScore(inds)];

% Creating a function to compute scale-space representation of the image
function pyra = scaleSpace(im, interval, minSize)
pyra.size = size(im);

im = imresize(im, 2);
smallDim = min(size(im, 1), size(im,2));
numOctaves = ceil(log(smallDim/minSize))+1;

pyra.im = cell(1, numOctaves*interval);
pyra.scale = zeros(1, numOctaves*interval);

stepSize = 2^(1/interval);
currScale = 2;
offset = 0;
for s = 1:numOctaves,
    for i = 1:interval,
        pyra.im{offset+i} = imresize(im, 1/stepSize^(i-1));
        pyra.scale(offset+i) = currScale/stepSize^(i-1);
    end
    im = imresize(im, 0.5);
    currScale = currScale*0.5;
    offset = offset + interval;
end
end

```

---

## 2. Image stitching (computeMatches.m)

---

```

function m = computeMatches(f1,f2)
pairwiseDist = dist2(f1, f2);
[n1, n2] = size(pairwiseDist);
m = zeros(1, n1);
for i = 1:n1,

```

```

    [d,ord] = sort(pairwiseDist(i,:));
    ratio = d(1)/d(2);
    if ratio < 0.8,
        m(i) = ord(1);
    end
end

function n2 = dist2(x, c)
[ndata, dimx] = size(x);
[ncentres, dimc] = size(c);
if dimx ~= dimc
    error('Data dimension does not match dimension of centres')
end

n2 = (ones(ncentres, 1) * sum((x.^2)', 1))' + ...
    ones(ndata, 1) * sum((c.^2)', 1) - ...
    2.*(x*(c'))';

if any(any(n2<0))
    n2(n2<0) = 0;
end

```

---

## Image stitching (ransac.m)

---

```

function [inliers, transf] = ransac(matches, blobs1, blobs2)

blobs1 = blobs1'; blobs2 = blobs2';

maxIter = 500;
valid = find(matches > 0);
src = blobs1(1:2, valid);
dst = blobs2(1:2, matches(valid));
numValid = length(valid);
maxInliers = 3;
for iter = 1:maxIter,
    sel = randperm(numValid);

    sel = sel(1:3);
    src1 = src(:, sel);
    dst1 = dst(:, sel);
    A = estimateAffine(src1, dst1);
    tdst = A*[dst; ones(1, size(dst, 2))];
    fn = @estimateAffine;

    err = sum((tdst - src).^2);

    isInlier = err < 10;
    currInliers = sum(isInlier);
    if currInliers > maxInliers
        maxInliers = currInliers;
        transf = fn(src(:,isInlier), dst(:,isInlier));
        inliers = valid(isInlier);
    end
end

fprintf(['RANSAC finished with %i iters [%i inliers]\n' ...

```

```

    'affine transformation = [%.2f, %.2f, %.2f; %.2f, %.2f, %.2f]\n'], ...
    iter, maxInliers,transf(1),transf(3),transf(5),transf(2),transf(4),transf(6));

function transf = estimateAffine(src, dst)

assert(size(src, 2) == size(dst, 2), 'different size of source and target')
assert(size(dst, 2) >= 3, 'minimum number of 4 matches required for estimating affine
    transformation')

A = kron(eye(2), dst');
A = cat(2, A, kron(eye(2), ones(size(dst, 2), 1)));

b = [src(1,:), src(2,:)]';

M = A\b;
transf = [M(1), M(2), M(5); M(3), M(4), M(6)];

```

---