# Fluid Mechanics Homework #4

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**1、已知不可压缩液体平面流动的流速场为$v_x = xt + 2y,\ v_y = xt^2 - yt$，试求在 1s 时点 *A*(1, 2)处液体质点的加速度.**

解：不可压缩平面流体质点的加速度为

$$\begin{cases} a_x = \dfrac{\mathrm{d}v_x}{\mathrm{d}t} = \dfrac{\partial v_x}{\partial t} + v_x \dfrac{\partial v_x}{\partial x} + v_y \dfrac{\partial v_x}{\partial y} \\ a_y = \dfrac{\mathrm{d}v_y}{\mathrm{d}t} = \dfrac{\partial v_y}{\partial t} + v_x \dfrac{\partial v_y}{\partial x} + v_y \dfrac{\partial v_y}{\partial y} \end{cases} \tag{1.1}$$

这里 $x = 1\mathrm{m},\ y = 2\mathrm{m},\ t = 1\mathrm{s}$ , 故

$$\begin{cases} a_x = x + \left(xt + 2y\right)t + \left(xt^2 - yt\right)\cdot 2 = 1 + \left(1+4\right)\times 1 + \left(1-2\right)\times 2 = 4\ \mathrm{m/s^2} \\ a_y = 2xt - y + \left(xt + 2y\right)t^2 + \left(xt^2 - yt\right)\left(-t\right) = 2 - 2 + \left(1+4\right)\times 1^2 + \left(1-2\right)\times\left(-1\right) = 6\ \mathrm{m/s^2} \end{cases}$$

所以质点的加速度为

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{4^2 + 6^2} = 2\sqrt{13}\ \mathrm{m/s^2} \approx 7.2\ \mathrm{m/s^2}$$

**2、已知不可压缩空间流动中的两个流速分量分别为：① $v_x = 7x,\ v_y = -5y$ ，② $v_x = xyzt,\ v_y = -xyzt^2$，试求第三个速度分量$v_z$，假设$z = 0$时，$v_z = 0$.**

解：不可压缩流体满足

$$\nabla\cdot\vec{V} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{2.1}$$

对于①，

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 7 - 5 + \frac{\partial v_z}{\partial z} = 0 \tag{2.2}$$

解此偏微分方程得

$$v_z = -2z + C\left(x, y\right) \tag{2.3}$$

代入定解条件可知$C\left(x, y\right) = 0$，所以

$$v_z = -2z. \tag{2.4}$$

对于②，

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=yzt-xzt^2+\frac{\partial v_z}{\partial z}=0 \tag{2.5}$$

解此偏微分方程得

$$v_z=\frac{1}{2}z^2\left(xt^2-yt\right)+C\left(x,y\right) \tag{2.6}$$

代入定解条件可知$C\left(x,y\right)=0$，所以

$$v_z=\frac{1}{2}z^2\left(xt^2-yt\right). \tag{2.7}$$

**3、已知不可压缩粘性流体平面流动的流速分量为：$v_x=Ax,\ v_y=-Ay$，其中 $A$ 为常数. 试求：**

**（1）应力 $p_{xx},\ p_{yy},\ \tau_{xy},\ \tau_{yx}$ ；**

**（2）假设忽略外力作用，且 $x=y=0$ 处压强为 $p_0$，写出压强分布表达式.**

解：（1）平面不可压缩粘性牛顿流体的本构方程满足

$$\begin{cases} p_{xx}=-p+2\mu\dfrac{\partial v_x}{\partial x} \\ p_{yy}=-p+2\mu\dfrac{\partial v_y}{\partial y} \\ \tau_{xy}=\tau_{yx}=\mu\left(\dfrac{\partial v_x}{\partial y}+\dfrac{\partial v_y}{\partial x}\right) \end{cases} \tag{3.1}$$

所以

$$\begin{cases} p_{xx}=-p+2\mu A \\ p_{yy}=-p-2\mu A \\ \tau_{xy}=\tau_{yx}=0 \end{cases} \tag{3.2}$$

（2）忽略外力作用，平面不可压缩粘性牛顿流体的运动方程为

$$\begin{cases} \rho\left(\dfrac{\partial v_x}{\partial t}+v_x\dfrac{\partial v_x}{\partial x}+v_y\dfrac{\partial v_x}{\partial y}\right)=-\dfrac{\partial p}{\partial x}+\mu\left(\dfrac{\partial^2 v_x}{\partial x^2}+\dfrac{\partial^2 v_x}{\partial y^2}\right) \\ \rho\left(\dfrac{\partial v_y}{\partial t}+v_x\dfrac{\partial v_y}{\partial x}+v_y\dfrac{\partial v_y}{\partial y}\right)=-\dfrac{\partial p}{\partial y}+\mu\left(\dfrac{\partial^2 v_y}{\partial x^2}+\dfrac{\partial^2 v_y}{\partial y^2}\right) \end{cases} \tag{3.3}$$

所以

$$\begin{cases} \rho A^2 x = -\dfrac{\partial p}{\partial x} \\ \rho A^2 y = -\dfrac{\partial p}{\partial y} \end{cases} \tag{3.4}$$

解得

$$p = C - \frac{1}{2}\rho A^2\left(x^2 + y^2\right) \tag{3.5}$$

代入初始条件：$x = y = 0$ 时 $p = p_0$，可知常数 $C = p_0$，所以压强分布表达式为

$$p = p_0 - \frac{1}{2}\rho A^2\left(x^2 + y^2\right). \tag{3.6}$$