Fluid Mechanics Homework #2

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1、写出在直角坐标系中，连续性方程和运动方程的形式。

解：连续性方程的直角坐标形式为

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \tag{1.1}$$

运动方程的直角坐标形式为

$$\begin{cases}\dfrac{\partial(\rho u_x)}{\partial t}+\dfrac{\partial(\rho u_x u_x)}{\partial x}+\dfrac{\partial(\rho u_x u_y)}{\partial y}+\dfrac{\partial(\rho u_x u_z)}{\partial z}=\rho g_x+\dfrac{\partial \tau_{xx}}{\partial x}+\dfrac{\partial \tau_{xy}}{\partial y}+\dfrac{\partial \tau_{xz}}{\partial z}\\ \dfrac{\partial(\rho u_y)}{\partial t}+\dfrac{\partial(\rho u_y u_x)}{\partial x}+\dfrac{\partial(\rho u_y u_y)}{\partial y}+\dfrac{\partial(\rho u_y u_z)}{\partial z}=\rho g_y+\dfrac{\partial \tau_{yx}}{\partial x}+\dfrac{\partial \tau_{yy}}{\partial y}+\dfrac{\partial \tau_{yz}}{\partial z}\\ \dfrac{\partial(\rho u_z)}{\partial t}+\dfrac{\partial(\rho u_z u_x)}{\partial x}+\dfrac{\partial(\rho u_z u_y)}{\partial y}+\dfrac{\partial(\rho u_z u_z)}{\partial z}=\rho g_z+\dfrac{\partial \tau_{zx}}{\partial x}+\dfrac{\partial \tau_{zy}}{\partial y}+\dfrac{\partial \tau_{zz}}{\partial z}\end{cases} \tag{1.2}$$

2、一不可压缩流体的流动，x 方向的速度分量是 $u=ax^2+by$，z 方向的速度分量为零，求 y 方向的速度分量 v，其中 a 和 b 为常数. 已知 $y=0$ 时，$v=0$.

解：不可压缩流体的连续性方程为

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 . \tag{2.1}$$

所以，

$$\frac{\partial(ax^2+by)}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial 0}{\partial z}=2ax+\frac{\partial v}{\partial y}=0 .$$

解此偏微分方程可得

$$v=-2axy+C(x,z),$$

代入初值条件可知 $C(x,z)=0$.

所以 y 方向的速度分量

$$v=-2axy .$$

3、二维、定常不可压缩流动，x 方向的速度分量为$u = e^{-x}\cos(hy)+1$，求 y 方向的速度分量 v. 设 $y=0$ 时，$v=0$.

解：由式(2.1)可知

$$\frac{\partial\left[e^{-x}\cos(hy)+1\right]}{\partial x}+\frac{\partial v}{\partial y}=-e^{-x}\cos(hy)+\frac{\partial v}{\partial y}=0.$$

解此偏微分方程可得

$$v=\frac{e^{-x}\sin(hy)}{h}+C(x),$$

代入初值条件可知$C(x)=0$.

所以 y 方向的速度分量

$$v=\frac{e^{-x}\sin(hy)}{h}.$$

4、试证下述不可压缩流体的运动不可能存在：$u=x, v=y, w=z$.

证明：由题设可得

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\frac{\partial x}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial z}{\partial z}=1+1+1=3\neq 0,$$

故此不可压缩流体的运动不可能存在.

证毕.