Harbin Institute of Technology, ShenZhen
Operations Research
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Homework 1

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1.1. The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is \$5 per unit on special risk insurance and \$2 per unit on mortgages. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

	Work-Hours per Unit		
Department	**Special Risk**	**Mortgage**	**Work-Hours Available**
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

(a) Formulate a linear programming model for this problem.

i. Decision variables:

$$x_1 = \text{number of special risk insurances}$$
$$x_2 = \text{number of mortgages}$$

ii. Objective function:

$$\max \quad Z = 5x_1 + 2x_2$$

iii. Constraints:

$$\begin{aligned} 3x_1 + 2x_2 &\leqslant 2400 \\ x_2 &\leqslant 800 \\ 2x_1 &\leqslant 1200 \\ x_1,\ x_2 &\geqslant 0 \end{aligned}$$

(b) Use the graphical method to solve this model.

First draw the feasible region, which is the shaded area shown in Fig.1. Then use the slope-intercept form of the objective function to find the optimal solution, which passes through the CPF $(600, 300)$.

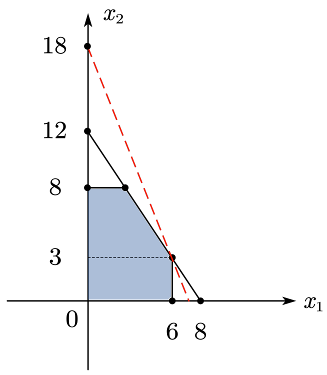

Figure 1: graphical solution

Therefore, the optimal solution is $x_1 = 600,\ x_2 = 300$, with $Z = 3600$.

1.2. A cargo plane has three compartments for storing cargo: front, center, and back. These compartments have capacity limits on both weight and space, as summarized below:

Compartment	Weight Capacity (Tons)	Space Capacity (Cubic Feet)
Front	12	7,000
Center	18	9,000
Back	10	5,000

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the airplane.

The following four cargoes have been offered for shipment on an upcoming flight as space is available:

Cargo	Weight (Tons)	Volume (Cubic Feet/Ton)	Profit ($/Ton)
1	20	500	320
2	16	700	400
3	25	600	360
4	13	400	290

Any portion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo should be accepted and how to distribute each among the compartments to maximize the total profit for the flight.

(a) Formulate a linear programming model for this problem.

i. Decision variables:
x_{ij} := tons of cargo i in compartment j, $i = 1, 2, 3, 4,\ j = 1, 2, 3$, listed in Tab.1.

Table 1: Decision variables

Cargo	Front	Center	Back
1	x_{11}	x_{12}	x_{13}
2	x_{21}	x_{22}	x_{23}
3	x_{31}	x_{32}	x_{33}
4	x_{41}	x_{42}	x_{43}

ii. Objective function:

$$\max \quad Z = 320\sum_{k=1}^{3} x_{1k} + 400\sum_{k=1}^{3} x_{2k} + 360\sum_{k=1}^{3} x_{3k} + 290\sum_{k=1}^{3} x_{4k}$$

iii. Constraints:

1. Usable weight in each compartment:

$$\sum_{j=1}^{4} x_{j1} \leqslant 12,\ \sum_{j=1}^{4} x_{j2} \leqslant 18,\ \sum_{j=1}^{4} x_{j3} \leqslant 10$$

2. Available space in each compartment:

$$\begin{aligned} 500x_{11} + 700x_{21} + 600x_{31} + 400x_{41} &\leqslant 7000 \\ 500x_{12} + 700x_{22} + 600x_{32} + 400x_{42} &\leqslant 9000 \\ 500x_{13} + 700x_{23} + 600x_{33} + 400x_{43} &\leqslant 5000 \end{aligned}$$

3. Total cargoes of each kind:

$$\sum_{k=1}^{3} x_{1k} \leqslant 20,\ \sum_{k=1}^{3} x_{2k} \leqslant 16,\ \sum_{k=1}^{3} x_{3k} \leqslant 25,\ \sum_{k=1}^{3} x_{4k} \leqslant 13$$

4. Equal weight proportion of cargoes in each compartment:

$$\frac{\sum_{j=1}^{4} x_{j1}}{12} = \frac{\sum_{j=1}^{4} x_{j2}}{18} = \frac{\sum_{j=1}^{4} x_{j3}}{10}$$

5. Nonnegativity:

$$x_{ij} \geqslant 0,\ i = 1,2,3,4,\ j = 1,2,3.$$