Operations Research

Homework 2

Notes:
1. Please submit a hardcopy of the solution, with clear writing.
2. Homework 2 is due on Nov. **18**. Each homework will be collected at the beginning of the class it is due.

Q1（Textbook 4.1-8）
Label each of the following statements about linear programming problems as true or false.
(a) For minimization problems, if the objective function evaluated at a CPF solution is no larger than its value at every adjacent CPF solution, then that solution is optimal.
(b) Only CPF solutions can be optimal, so the number of optimal solutions cannot exceed the number of CPF solutions.
(c) If multiple optimal solutions exist, then an optimal CPF solution may have an adjacent CPF solution that also is optimal (the same value of Z).

Q2（Textbook 4.4-7）
Solve the following problem by the simplex method in tabular form.

$$\text{Maximize} \quad Z = 2x_1 - x_2 + x_3,$$

subject to

$$\begin{aligned} 3x_1 + x_2 + x_3 &\le 6 \\ x_1 - x_2 + 2x_3 &\le 1 \\ x_1 + x_2 - x_3 &\le 2 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

Q3（Textbook 4.6-3）
Consider the following problem.

$$\text{Minimize} \quad Z = 2x_1 + 3x_2 + x_3,$$

subject to

$$\begin{aligned} x_1 + 4x_2 + 2x_3 &\ge 8 \\ 3x_1 + 2x_2 &\ge 6 \end{aligned}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0.$$

Using the Big M method, work through the simplex method step by step to solve the

problem.

Q4（Textbook 4.6-7）

Consider the following problem.

Maximize $Z = 2x_1 + 5x_2 + 3x_3,$

subject to

$$x_1 - 2x_2 + x_3 \geq 20$$
$$2x_1 + 4x_2 + x_3 = 50$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

Using the two-phase method, work through the simplex method step by step to solve the problem.

Q5（Textbook 5.2-4）

Work through the matrix form of the simplex method step by step to solve the following model.

Maximize $Z = x_1 + 2x_2,$

subject to

$$x_1 + 3x_2 \leq 8$$
$$x_1 + x_2 \leq 4$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

Q6 (Textbook 5.3-2)

Consider the following problem.

Maximize $Z = 4x_1 + 3x_2 + x_3 + 2x_4,$

subject to

$$4x_1 + 2x_2 + x_3 + x_4 \leq 5$$
$$3x_1 + x_2 + 2x_3 + x_4 \leq 4$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0.$$

Let x_5 and x_6 denote the slack variables for the respective constraints. After you apply the simplex method, a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of:							Right Side
		Z	x_1	x_2	x_3	x_4	x_5	x_6	
Z	(0)	1					1	1	
x_2	(1)	0					1	−1	
x_4	(2)	0					−1	2	

(a) Use the fundamental insight presented in Topic 3 to identify the missing numbers in the final simplex tableau. Show your calculations.
(Hint: You can refer to Table 5.8 on Slide 101.)
(b) Identify the defining equations of the CPF solution corresponding to the optimal CPF solution in the final simplex tableau.