CS1101S - Programming Methodology I

Studio 3

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Tutorial Group 8D

Admin Stuff

Attendance Taking

Make sure you have taken your temperature.

We will take photo when everyone is present.

Regarding SourceAcademy tasks

Paths

- By now, you should have done 4 paths:
  1. Elements of Programming (ungraded)
  2. Runes (140XP)
  3. Substitution model and recursion (140XP)
  4. Orders of Growth (160XP)
- These will be autograded at a later time.
- Paths are meant to assess your basic understanding of the topics taught in lectures and briefs. Please try to do them on time.

Mission

- Most, if not all of you, have completed the Rune Reading Mission.
- Some things to note:
  - Read the comments
  - Please make your code readable! This is for your future projects as well as my sanity in marking.
  - Read the Source styleguide. Some common mistakes include:

Common mistakes

- Put your comments for a function outside the function, not inside the function body. Example
- For things other than functions, put the comments either on the line or after the line. Example
- Indentation style: try to stick to one tab (not two) - this one is more forgivable, but you might get some trouble when it got to longer code, because
- You need to keep to 80 characters per line. Example
- Also check the styleguide for indentation, as there are quite a bit on it.

Recap

Substitution Model

- Reason about programs
- Replace eligible sub-expression with result until you can't anymore (fully simplified/reduced) - some expressions are *irreducible*.
- By performing repeated reductions, we can simplify and find the result of any given statement.

Applicative Order Reduction

Works like your normal arithmetic: evaluate from the left, and the deepest expression.

- Evaluate arguments first.
- Then substitute function call(s) with body.

Example:

```
12345 % math_pow(10, math_floor(math_log10(12345)));
```

This will be evaluated as:

```
12345 % math_pow(10, math_floor(math_log10(12345)));
12345 % math_pow(10, math_floor(4.09...));
12345 % math_pow(10, 4);
12345 % 10000;
2345;
```

Normal Order Reduction

Do not evaluate arguments unless absolutely needed.

- Substitute function call(s) with body.
- Then evaluate the arguments.

Normal Order Reduction (Example)

Example:

```
function sq(x) {
    return x * x
};
function dist(x, y) {
    return math_sqrt(sq(x) + sq(y))
};
dist(1 + 5, 2 * 10);
```

This program will be executed as follows:

```
dist(1 + 5, 2 * 10);
math_sqrt(sq(1 + 5) + sq(2 * 10))
math_sqrt((1 + 5) * (1 + 5) + (2 * 10) * (2 * 10))
math_sqrt((6) * (1 + 5) + (2 * 10) * (2 * 10))
math_sqrt((6) * (6) + (2 * 10) * (2 * 10))
...
```

Applicative vs Normal Order Reduction

```
function p() {
    return p();
}

function test(x, y) {
    return x === 0 ? 0 : y;
}

test(0, p());
```

Recursive Problem-solving

1. Base Case - the simplest problem you can think of
2. Think of one slightly simpler function call. Can you build the desired function call from this one?

Example: Tower of Hanoi

The thing with recursion is belief!
You need to assume that the smaller problem is correct.
Do not try to trace the recursion.

Recursive vs Iterative Processes

- Both involve calling the function inside the function body (i.e. recursion)
- A recursive process has **deferred operations**
- An iterative process does not

Example: https://share.sourceacademy.nus.edu.sg/itervsrec
(Use Stepper for visualisation)

Time Complexity

Why do we care?

- Need an abstract way to talk about resources consumed
- The same for every languages, architecture, CPU
- Some abstract measure of time taken for the program to run.
- How do we characterize it?
    - Number of operations performed.
    - Number of “simple” operations performed for some input size.
    - Simple operations:
        - All arithmetic e.g. 4 * 5
        - Memory read and write e.g. const a = 4;
        - Conditionals e.g if (a === 4)

Space Complexity

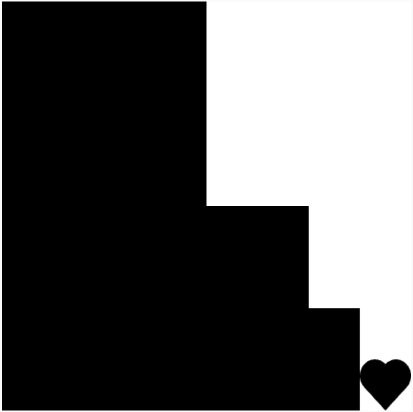
Why do we care?

- We don't want to run out of space :(
- Some abstract measure of space taken for the program to run is needed
- Characterized by maximum number of symbols created at one point in time (usually deferred operator)

Studio Sheet (and Photo Taking)

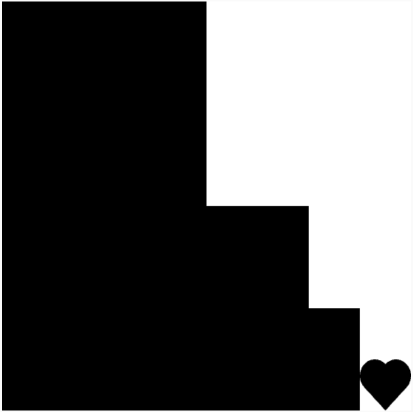
Question 1

Someone try to answer :D



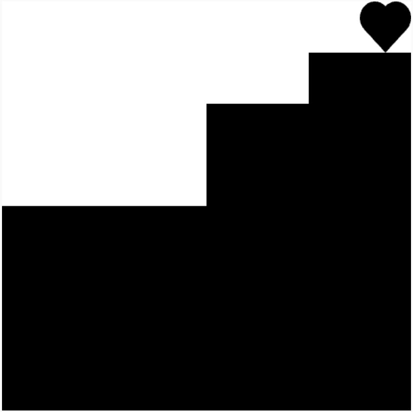
Question 1

Intuitive solution:



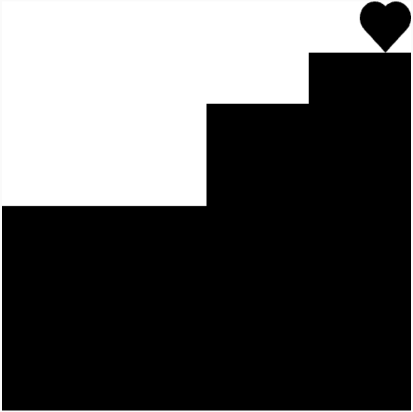
Question 2

The proper solution: here



Question 2

The intuitive solution: (on the board)



Question 3

Take some time to think about the question, and someone try to explain.

Question 4

```
function power(b, n) {
    return n === 0 ? 1 : b * power(b, n - 1);
}
```

- Iterative or recursive process? Can you write it in the other manner?
- Use the Θ notation to characterize the running time and space consumption of *power* as the argument n grows.

Question 4

Consider the following example: `power(2, 3)`

```
power(2, 3)
3 === 0 ? 1 : 2 * power(2, 2) // false
2 * power(2, 2)
2 * (2 === 0 ? 1 : 2 * power(2, 1)) // false
2 * (2 * power(2, 1))
2 * (2 * (1 === 0 ? 1 : 2 * power(2, 0))) // false
2 * (2 * (2 * (power(2, 0))))
2 * (2 * (2 * (0 === 0 ? 1 : power(2, -1)))) // true
2 * (2 * (2 * (1)))
2 * (2 * (2))
2 * (4)
8
```

Question 5

Given that

$$b^n = \begin{cases} b^{n/2} b^{n/2} & \text{if } n \text{ is even} \\ b^{(n-1)/2} b^{(n-1)/2} & \text{if } n \text{ is even} \end{cases}$$

- Implement a function *fast_power*(b, n) which computes b^n in $O(log(n))$ time, where n is a natural number. [answer]
- Can you extend this to integer powers?
- Iterative or recursive process? Can you write it in the other manner?
- Use the Θ notation to characterize the running time and space consumption of *fast_power* as the argument n grows.

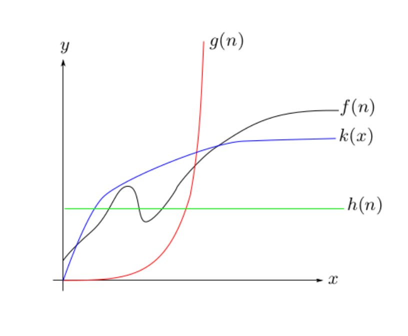
Additional Material

Big-O, Big-Ω, Big-Θ

The intuitive definition of Big-O, Big-Ω, Big-Θ are:
When the input is big enough,

$f(n) = O(g(n))$	f is bounded above by g
$f(n) = \Omega(g(n))$	f is bounded below by g
$f(n) = \Theta(g(n))$	f is bounded by g

Big-O, Big-Ω, Big-Θ



In this example, what is the relation between

- f and g
- f and h
- f and k

Big-O, Big-Ω, Big-Θ

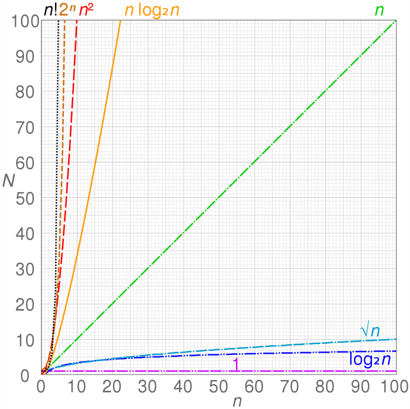


Figure 1: Graphs of commonly encountered time complexities

Big-O, Big-Ω, Big-Θ

Optional content - try if you like math, at this juncture.
What are the complexities of:

- $4n^2 - n$
- $5n^2 + n$
- $\sqrt{n} + n$
- $3^n n^2$

Verify the following:

- $log_5 n = \Theta(\log n)$
- $10n \log n = O(n^2)$
- $n^3 = O(2^n)$

Optional content - try if you want some extra practice, at this juncture.

What is the (space and time) complexity of:

```
function factorial(n) {
    return n === 1
        ? 1
        : n * factorial(n - 1);
}
```

Optional content - try if you want some extra practice, at this juncture.

What is the (space and time) complexity of:

```
function helper(n, res) {
    return n === 1
        ? res
        : helper(n - 1, n * res);
}

function factorial(n) {
    return helper(n, 1);
}
```

Optional content - try if you want some extra practice, at this juncture.

What is the (space and time) complexity of:

```
function fizz(n) {
    if (n === 0) {
      return "done";
    } else {
        n % 3 === 0
            ? display("fizz")
            : n % 5 === 0
                ? display("buzz")
                : display(n);
        return fizz(n - 1);
    }
}
```

Big-O, Big-Ω, Big-Θ

Optional content - try if you want some extra practice, at this juncture.
Implement the following using both recursive and iterative processes:

- Factorial
- Fibonacci
- Power
- GCD
- LCM
- Coin-change problem (covered this week in lecture!)
- ...(continued next slide)

Big-O, Big-Ω, Big-Θ

Optional content - try if you want some extra practice, at this juncture.

Implement the following using both recursive and iterative processes:

- Pascal triangle (covered this week in lecture!)
- Tower of Hanoi
- Permutations
- Combinations
- $sqrt(x)$ by Newton-Raphson method (or Newton's method)
- sine approximation (see the book)

Some cannot be implemented using iterative.
Analyze their space and time complexity.
Are there any closed form for these questions?