Project on CPU, GPU, and TPU

Lixian Chen, Zhanhao Zhang, Jingbin Cao

4/14/2021

Preparing Required Packages

Read Data

```
data <- read.csv("../data/Runtime.csv")
head(data)
```

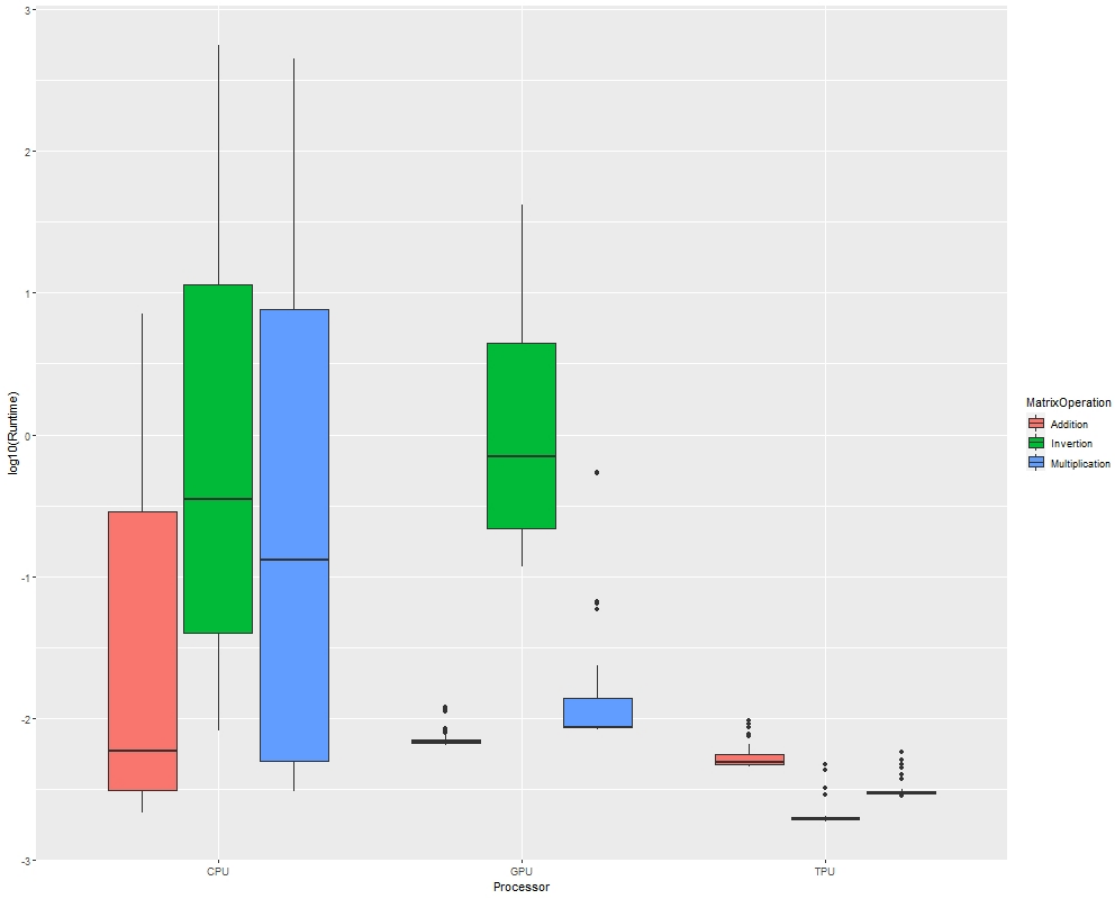
```
##       Runtime Processor MatrixSize MatrixOperation Trial
## 1 0.003411770       CPU         10        Addition     1
## 2 0.006412983       CPU         10        Addition     2
## 3 0.003450394       CPU         10        Addition     3
## 4 0.003098965       CPU         10        Addition     4
## 5 0.002490997       CPU         10        Addition     5
## 6 0.002594948       CPU         20        Addition     1
```

We tested three types of processors CPU, GPU, and TPU for three kinds of matrix operation, addition, multiplication, and inversion, with the matrix from size 10 to size 2160. We repeat each test for five times. **We measured log10(run-time) for each trial, and we use that as the evaluation of the performances.**

Simple Plots

Here is the general visualization for the performances of each processor under three matrix operations:

```
jpeg(filename = "../figs/overview.jpeg", width = 1000, height = 800,quality = 10000)
ggplot(data = data, aes(x = Processor, y = log10(Runtime))) +
geom_boxplot(aes(fill = MatrixOperation))
while (!is.null(dev.list()))  dev.off()
```



Jingbin Cao Part I

One Way Anova for different matrix sized for each pair of processor and matrix operation: $\mu_1 = Matrix_{Size} = 320$ $\mu_2 = Matrix_{Size} = 640$ $\mu_3 = Matrix_{Size} = 1280$ $\mu_4 = Matrix_{Size} = 2160$

Getting Data

```
cpu_add <- data[data$Processor == "CPU" & data$MatrixOperation=="Addition" & data$MatrixSize >= 320,]
cpu_mult <- data[data$Processor == "CPU" & data$MatrixOperation=="Multiplication" & data$MatrixSize >= 3
cpu_inv <- data[data$Processor == "CPU" & data$MatrixOperation=="Invertion" & data$MatrixSize >= 320,]
gpu_add <- data[data$Processor == "GPU" & data$MatrixOperation=="Addition" & data$MatrixSize >= 320,]
gpu_mult <- data[data$Processor == "GPU" & data$MatrixOperation=="Multiplication" & data$MatrixSize >= 3
gpu_inv <- data[data$Processor == "GPU" & data$MatrixOperation=="Invertion" & data$MatrixSize >= 320,]
tpu_add <- data[data$Processor == "TPU" & data$MatrixOperation=="Addition" & data$MatrixSize >= 320,]
tpu_mult <- data[data$Processor == "TPU" & data$MatrixOperation=="Multiplication" & data$MatrixSize >= 3
tpu_inv <- data[data$Processor == "TPU" & data$MatrixOperation=="Invertion" & data$MatrixSize >= 320,]
```

Anovas

```
summary(aov(Runtime ~ as.factor(MatrixSize), data=cpu_add))
```

```
##                       Df Sum Sq Mean Sq F value Pr(>F)
## as.factor(MatrixSize)  3 160.47   53.49  105384 <2e-16 ***
## Residuals             16   0.01    0.00
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aov(Runtime ~ as.factor(MatrixSize), data=cpu_mult))
```

```
##                       Df Sum Sq Mean Sq F value Pr(>F)
## as.factor(MatrixSize)  3 678487  226162 1792321 <2e-16 ***
## Residuals             16      2       0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aov(Runtime ~ as.factor(MatrixSize), data=cpu_inv))
```

```
##                       Df  Sum Sq Mean Sq F value Pr(>F)
## as.factor(MatrixSize)  3 1071784  357261 1400421 <2e-16 ***
## Residuals             16       4       0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aov(Runtime ~ as.factor(MatrixSize), data=gpu_add))
```

```
##                       Df    Sum Sq   Mean Sq F value  Pr(>F)
## as.factor(MatrixSize)  3 3.297e-05 1.099e-05   6.868 0.00348 **
## Residuals             16 2.560e-05 1.600e-06
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aov(Runtime ~ as.factor(MatrixSize), data=gpu_mult))
```

```
##                       Df Sum Sq Mean Sq F value Pr(>F)
## as.factor(MatrixSize)  3 0.9873  0.3291   20672 <2e-16 ***
## Residuals             16 0.0003  0.0000
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aov(Runtime ~ as.factor(MatrixSize), data=gpu_inv))
```

```
##                       Df Sum Sq Mean Sq F value Pr(>F)
## as.factor(MatrixSize)  3   5080    1694 1803746 <2e-16 ***
## Residuals             16      0       0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aov(Runtime ~ as.factor(MatrixSize), data=tpu_add))
```

```
##                       Df    Sum Sq   Mean Sq F value Pr(>F)
## as.factor(MatrixSize)  3 1.195e-05 3.982e-06   3.541 0.0387 *
## Residuals             16 1.799e-05 1.125e-06
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aov(Runtime ~ as.factor(MatrixSize), data=tpu_mult))
```

```
##                       Df    Sum Sq   Mean Sq F value Pr(>F)
## as.factor(MatrixSize)  3 1.413e-06 4.710e-07   1.188  0.346
## Residuals             16 6.344e-06 3.965e-07
```

```
summary(aov(Runtime ~ as.factor(MatrixSize), data=tpu_inv))
```

```
##                       Df   Sum Sq   Mean Sq F value Pr(>F)
## as.factor(MatrixSize)  3 6.87e-07 2.291e-07   0.748  0.539
## Residuals             16 4.90e-06 3.063e-07
```

Zhanhao Zhang Part I

Pros & Cons of Each Processor

When matrix size is the same, is there any processor or operation effects? Or is there any interactive effect?

```
df_cpu <- data[data$Processor == "CPU",]
lm(Runtime ~ MatrixSize + as.factor(MatrixOperation), data = df_cpu) %>%
  summary()
```

```
##
## Call:
## lm(formula = Runtime ~ MatrixSize + as.factor(MatrixOperation),
##     data = df_cpu)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -234.810  -41.042   -6.483   48.307  248.752
##
## Coefficients:
##                                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                             -67.641331  13.699933  -4.937 2.37e-06
## MatrixSize                                0.120877   0.009097  13.287  < 2e-16
## as.factor(MatrixOperation)Invertion      71.726373  17.944905   3.997 0.000107
## as.factor(MatrixOperation)Multiplication 55.821041  17.944905   3.111 0.002291
##
## (Intercept)                             ***
## MatrixSize                              ***
## as.factor(MatrixOperation)Invertion     ***
```

```
## as.factor(MatrixOperation)Multiplication **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 85.12 on 131 degrees of freedom
## Multiple R-squared:  0.5971, Adjusted R-squared:  0.5879
## F-statistic: 64.73 on 3 and 131 DF,  p-value: < 2.2e-16
```

```
df_gpu <- data[data$Processor == "GPU",]
lm(Runtime ~ MatrixSize + as.factor(MatrixOperation), data = df_gpu) %>%
  summary()
```

```
##
## Call:
## lm(formula = Runtime ~ MatrixSize + as.factor(MatrixOperation),
##     data = df_gpu)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.4178  -3.7532   0.9807   2.6766  24.7084
##
## Coefficients:
##                                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                             -2.9430365  1.0021808  -2.937  0.00392
## MatrixSize                               0.0051962  0.0006655   7.808 1.64e-12
## as.factor(MatrixOperation)Invertion      6.7906886  1.3127100   5.173 8.42e-07
## as.factor(MatrixOperation)Multiplication 0.0669200  1.3127100   0.051  0.95942
##
## (Intercept)                             **
## MatrixSize                              ***
## as.factor(MatrixOperation)Invertion     ***
## as.factor(MatrixOperation)Multiplication
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.227 on 131 degrees of freedom
## Multiple R-squared:  0.4237, Adjusted R-squared:  0.4105
## F-statistic:  32.1 on 3 and 131 DF,  p-value: 1.273e-15
```

```
df_tpu <- data[data$Processor == "TPU",]
lm(Runtime ~ MatrixSize + as.factor(MatrixOperation), data = df_tpu) %>%
  summary()
```

```
##
## Call:
## lm(formula = Runtime ~ MatrixSize + as.factor(MatrixOperation),
##     data = df_tpu)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0008794 -0.0003316 -0.0002244 -0.0000993  0.0041166
##
## Coefficients:
```

```
##                                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                              5.460e-03  1.389e-04  39.296   <2e-16
## MatrixSize                              -2.635e-08  9.226e-08  -0.286    0.776
## as.factor(MatrixOperation)Invertion     -3.320e-03  1.820e-04 -18.243   <2e-16
## as.factor(MatrixOperation)Multiplication -2.238e-03  1.820e-04 -12.296   <2e-16
##
## (Intercept)                              ***
## MatrixSize
## as.factor(MatrixOperation)Invertion      ***
## as.factor(MatrixOperation)Multiplication ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0008633 on 131 degrees of freedom
## Multiple R-squared:  0.7256, Adjusted R-squared:  0.7193
## F-statistic: 115.4 on 3 and 131 DF,  p-value: < 2.2e-16
```

```
# Reduced Model for TPU
df_tpu <- data[data$Processor == "TPU",]
lm(Runtime ~ as.factor(MatrixOperation), data = df_tpu) %>%
  summary()
```

```
##
## Call:
## lm(formula = Runtime ~ as.factor(MatrixOperation), data = df_tpu)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0008647 -0.0003214 -0.0002179 -0.0001272  0.0041294
##
## Coefficients:
##                                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                              0.0054450  0.0001282   42.46   <2e-16
## as.factor(MatrixOperation)Invertion     -0.0033202  0.0001814  -18.31   <2e-16
## as.factor(MatrixOperation)Multiplication -0.0022379  0.0001814  -12.34   <2e-16
##
## (Intercept)                              ***
## as.factor(MatrixOperation)Invertion      ***
## as.factor(MatrixOperation)Multiplication ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0008603 on 132 degrees of freedom
## Multiple R-squared:  0.7254, Adjusted R-squared:  0.7212
## F-statistic: 174.3 on 2 and 132 DF,  p-value: < 2.2e-16
```

```
# Reduced Model for GPU
df_gpu <- data[data$Processor == "GPU" & data$MatrixOperation != "Multiplication",]
lm(Runtime ~ MatrixSize + as.factor(MatrixOperation), data = df_gpu) %>%
  summary()
```

```
##
## Call:
```

```
## lm(formula = Runtime ~ MatrixSize + as.factor(MatrixOperation),
##     data = df_gpu)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.464  -2.908  -1.598   3.785  19.597
##
## Coefficients:
##                                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)                        -4.3996659  1.1188890  -3.932 0.000169 ***
## MatrixSize                          0.0077617  0.0008792   8.828 1.01e-13 ***
## as.factor(MatrixOperation)Invertion  6.7906886  1.4161219   4.795 6.66e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.717 on 87 degrees of freedom
## Multiple R-squared:  0.537,  Adjusted R-squared:  0.5264
## F-statistic: 50.46 on 2 and 87 DF,  p-value: 2.824e-15
```

Lixian Chen Part I

When the processor is the same, is there any operation and matrix size effect? We want to answer the following question: in each scenario, which processor should we use?

Getting Data Ready

```
# Read Data
df <- fread("../data/Runtime.csv", header=TRUE)
attach(df)

MatrixOperation<-factor(MatrixOperation)
Processor<-factor(Processor)



CPUdata <- df %>% filter(Processor=="CPU")
GPUdata <- df %>% filter(Processor=="GPU")
TPUdata <- df %>% filter(Processor=="TPU")



describeBy(df$Runtime,group = df$MatrixSize, mat = TRUE) %>%  #create dataframe
  select(MatrixSize=group1, N=n, Mean=mean, SD=sd, Median=median, Min=min, Max=max,
         Skew=skew, Kurtosis=kurtosis, SEM=se)
```

```
##     MatrixSize  N         Mean           SD      Median         Min         Max
## X11         10 45   0.02117285   0.04190229 0.005617142 0.001893044   0.1475253
## X12         20 45   0.02285517   0.04503445 0.004884481 0.001916409   0.1613362
## X13         40 45   0.03386522   0.07211264 0.005309582 0.001902819   0.2506576
## X14         80 45   0.06023479   0.12131721 0.006777525 0.001910210   0.4039948
```

```
## X15        160 45   0.13455488   0.22988237 0.006753206 0.001870394   0.7055206
## X16        320 45   0.54802946   0.77266887 0.008684158 0.001933098   1.8301501
## X17        640 45   2.65963363   4.07807665 0.064900875 0.001965523  11.5108950
## X18       1280 45  16.76243327  28.75264487 0.538181782 0.001962900  80.1931298
## X19       2560 45 117.16431474 210.69591026 0.008469582 0.001975775 562.2822752
##          Skew    Kurtosis          SEM
## X11 2.2815817  3.52258988  0.006246425
## X12 2.2598210  3.47569251  0.006713340
## X13 2.3149204  3.63635579  0.010749918
## X14 2.1514343  3.03908564  0.018084903
## X15 1.6153802  1.18296414  0.034268841
## X16 0.7633051 -1.35170085  0.115182675
## X17 1.1600905 -0.21560402  0.607923773
## X18 1.3579285  0.07252682  4.286191230
## X19 1.3393614 -0.10016629 31.408691862
```

```
#boxplot(Runtime~Processor*MatrixOperation)
#tapply(Runtime,list(Processor, MatrixOperation),mean)
#tapply(Runtime,MatrixOperation,mean)

group_by(df, Processor) %>%
  summarise(
    count = n(),
    mean = mean(Runtime, na.rm = TRUE),
    sd = sd(Runtime, na.rm = TRUE)
  )
```

```
## # A tibble: 3 x 4
##   Processor count     mean        sd
## * <chr>     <int>    <dbl>     <dbl>
## 1 CPU         135 43.5     133.
## 2 GPU         135  2.29      8.11
## 3 TPU         135  0.00359   0.00163
```

```
group_by(df, MatrixOperation) %>%
  summarise(
    count = n(),
    mean = mean(Runtime, na.rm = TRUE),
    sd = sd(Runtime, na.rm = TRUE)
  )
```

```
## # A tibble: 3 x 4
##   MatrixOperation count   mean     sd
## * <chr>           <int>  <dbl>  <dbl>
## 1 Addition          135  0.334   1.35
## 2 Invertion         135 26.5   107.
## 3 Multiplication    135 19.0    84.5
```

```
#detach(data)

size320data <- df %>% filter(MatrixSize==320)
size640data <- df %>% filter(MatrixSize==640)
size1280data <- df %>% filter(MatrixSize==1280)
size2560data <- df %>% filter(MatrixSize==2560)
```

Analysis

Anova

```
############ 320
fit2<-lm(Runtime~Processor+MatrixOperation, data = size320data)
summary(fit2)
```

```
##
## Call:
## lm(formula = Runtime ~ Processor + MatrixOperation, data = size320data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.63252 -0.44075  0.05662  0.39766  0.58682
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     0.5072     0.1496   3.389  0.00159 **
## ProcessorGPU                   -0.4163     0.1639  -2.540  0.01508 *
## ProcessorTPU                   -1.0252     0.1639  -6.254 2.08e-07 ***
## MatrixOperationInvertion        1.1525     0.1639   7.031 1.70e-08 ***
## MatrixOperationMultiplication   0.4116     0.1639   2.511  0.01619 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4489 on 40 degrees of freedom
## Multiple R-squared:  0.6931, Adjusted R-squared:  0.6624
## F-statistic: 22.59 on 4 and 40 DF,  p-value: 8.149e-10
```

```
fit1<-lm(Runtime~Processor*MatrixOperation, data = size320data)
summary(fit1)
```

```
##
## Call:
## lm(formula = Runtime ~ Processor * MatrixOperation, data = size320data)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0259142 -0.0004408 -0.0000341  0.0006995  0.0137136
##
## Coefficients:
##                                             Estimate Std. Error t value
## (Intercept)                                 0.067873   0.003147   21.57
## ProcessorGPU                               -0.060596   0.004450  -13.62
## ProcessorTPU                               -0.062968   0.004450  -14.15
## MatrixOperationInvertion                    1.647123   0.004450  370.15
## MatrixOperationMultiplication               1.234854   0.004450  277.50
## ProcessorGPU:MatrixOperationInvertion       0.165856   0.006293   26.36
## ProcessorTPU:MatrixOperationInvertion      -1.649847   0.006293 -262.17
## ProcessorGPU:MatrixOperationMultiplication -1.233006   0.006293 -195.93
## ProcessorTPU:MatrixOperationMultiplication -1.236834   0.006293 -196.54
```

```
##                                             Pr(>|t|)
## (Intercept)                                  < 2e-16 ***
## ProcessorGPU                                9.04e-16 ***
## ProcessorTPU                                2.80e-16 ***
## MatrixOperationInvertion                     < 2e-16 ***
## MatrixOperationMultiplication                < 2e-16 ***
## ProcessorGPU:MatrixOperationInvertion        < 2e-16 ***
## ProcessorTPU:MatrixOperationInvertion        < 2e-16 ***
## ProcessorGPU:MatrixOperationMultiplication   < 2e-16 ***
## ProcessorTPU:MatrixOperationMultiplication   < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.007036 on 36 degrees of freedom
## Multiple R-squared:  0.9999, Adjusted R-squared:  0.9999
## F-statistic: 6.633e+04 on 8 and 36 DF,  p-value: < 2.2e-16
```

```
anova(fit2, fit1)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ Processor + MatrixOperation
## Model 2: Runtime ~ Processor * MatrixOperation
##   Res.Df    RSS Df Sum of Sq     F    Pr(>F)
## 1     40 8.0610
## 2     36 0.0018  4    8.0592 40700 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
mod320<-aov(Runtime~Processor*MatrixOperation, data = size320data)
#summary(mod320)
Anova(mod320,type="III")
```

```
## Anova Table (Type III tests)
##
## Response: Runtime
##                            Sum Sq Df  F value    Pr(>F)
## (Intercept)                0.0230  1   465.30 < 2.2e-16 ***
## Processor                  0.0127  2   128.65 < 2.2e-16 ***
## MatrixOperation            7.3464  2 74200.69 < 2.2e-16 ***
## Processor:MatrixOperation 8.0592  4 40700.27 < 2.2e-16 ***
## Residuals                  0.0018 36
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
modF<-lm(Runtime~Processor+MatrixOperation, data = size320data)
modA<-lm(Runtime~Processor, data = size320data)
modB<-lm(Runtime~MatrixOperation, data = size320data)

anova(modA, modF)
```

```
## Analysis of Variance Table
```

```
##
## Model 1: Runtime ~ Processor
## Model 2: Runtime ~ Processor + MatrixOperation
##   Res.Df    RSS Df Sum of Sq      F   Pr(>F)
## 1     42 18.293
## 2     40  8.061  2    10.232 25.387 7.62e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(modB, modF)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ MatrixOperation
## Model 2: Runtime ~ Processor + MatrixOperation
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1     42 16.036
## 2     40  8.061  2    7.9754 19.788 1.061e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
################## 640
fit2_640<-lm(Runtime~Processor+MatrixOperation, data = size640data)
summary(fit2)
```

```
##
## Call:
## lm(formula = Runtime ~ Processor + MatrixOperation, data = size320data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.63252 -0.44075  0.05662  0.39766  0.58682
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     0.5072     0.1496   3.389  0.00159 **
## ProcessorGPU                   -0.4163     0.1639  -2.540  0.01508 *
## ProcessorTPU                   -1.0252     0.1639  -6.254 2.08e-07 ***
## MatrixOperationInvertion        1.1525     0.1639   7.031 1.70e-08 ***
## MatrixOperationMultiplication   0.4116     0.1639   2.511  0.01619 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4489 on 40 degrees of freedom
## Multiple R-squared:  0.6931, Adjusted R-squared:  0.6624
## F-statistic: 22.59 on 4 and 40 DF,  p-value: 8.149e-10
```

```
fit1_640<-lm(Runtime~Processor*MatrixOperation, data = size640data)
summary(fit1_640)
```

```
##
## Call:
```

```
## lm(formula = Runtime ~ Processor * MatrixOperation, data = size640data)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.070744 -0.001994 -0.000012  0.000669  0.055012
##
## Coefficients:
##                                            Estimate Std. Error t value
## (Intercept)                                 0.282343   0.008379   33.70
## ProcessorGPU                               -0.275710   0.011849  -23.27
## ProcessorTPU                               -0.277395   0.011849  -23.41
## MatrixOperationInvertion                   11.186470   0.011849  944.06
## MatrixOperationMultiplication               7.371087   0.011849  622.07
## ProcessorGPU:MatrixOperationInvertion      -6.741354   0.016758 -402.29
## ProcessorTPU:MatrixOperationInvertion     -11.189419   0.016758 -667.73
## ProcessorGPU:MatrixOperationMultiplication  -7.313893   0.016758 -436.45
## ProcessorTPU:MatrixOperationMultiplication  -7.373071   0.016758 -439.99
##                                            Pr(>|t|)
## (Intercept)                                  <2e-16 ***
## ProcessorGPU                                 <2e-16 ***
## ProcessorTPU                                 <2e-16 ***
## MatrixOperationInvertion                     <2e-16 ***
## MatrixOperationMultiplication                <2e-16 ***
## ProcessorGPU:MatrixOperationInvertion        <2e-16 ***
## ProcessorTPU:MatrixOperationInvertion        <2e-16 ***
## ProcessorGPU:MatrixOperationMultiplication   <2e-16 ***
## ProcessorTPU:MatrixOperationMultiplication   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01874 on 36 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 2.606e+05 on 8 and 36 DF,  p-value: < 2.2e-16
```

```
anova(fit2_640, fit1_640)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ Processor + MatrixOperation
## Model 2: Runtime ~ Processor * MatrixOperation
##   Res.Df     RSS Df Sum of Sq      F    Pr(>F)
## 1     40 184.706
## 2     36   0.013  4    184.69 131541 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
mod640<-aov(Runtime~Processor*MatrixOperation, data = size640data)
Anova(mod640,type="III")
```

```
## Anova Table (Type III tests)
##
## Response: Runtime
##                           Sum Sq Df   F value    Pr(>F)
```

```
## (Intercept)                 0.40  1   1135.52 < 2.2e-16 ***
## Processor                   0.25  2    363.15 < 2.2e-16 ***
## MatrixOperation           323.38  2 460630.58 < 2.2e-16 ***
## Processor:MatrixOperation 184.69  4 131541.31 < 2.2e-16 ***
## Residuals                   0.01 36
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
modF<-lm(Runtime~Processor+MatrixOperation, data = size640data)
modA<-lm(Runtime~Processor, data = size640data)
modB<-lm(Runtime~MatrixOperation, data = size640data)

anova(modA, modF)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ Processor
## Model 2: Runtime ~ Processor + MatrixOperation
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1     42 388.42
## 2     40 184.71  2    203.71 22.058 3.496e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(modB, modF)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ MatrixOperation
## Model 2: Runtime ~ Processor + MatrixOperation
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1     42 528.04
## 2     40 184.71  2    343.33 37.176 7.521e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
###################### 1280
fit2_1280<-lm(Runtime~Processor+MatrixOperation, data = size1280data)
summary(fit2)
```

```
##
## Call:
## lm(formula = Runtime ~ Processor + MatrixOperation, data = size320data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.63252 -0.44075  0.05662  0.39766  0.58682
##
## Coefficients:
##                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)                    0.5072     0.1496   3.389  0.00159 **
## ProcessorGPU                  -0.4163     0.1639  -2.540  0.01508 *
```

```
## ProcessorTPU                    -1.0252     0.1639  -6.254 2.08e-07 ***
## MatrixOperationInvertion         1.1525     0.1639   7.031 1.70e-08 ***
## MatrixOperationMultiplication    0.4116     0.1639   2.511  0.01619 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4489 on 40 degrees of freedom
## Multiple R-squared:  0.6931, Adjusted R-squared:  0.6624
## F-statistic: 22.59 on 4 and 40 DF,  p-value: 8.149e-10
```

```
fit1_1280<-lm(Runtime~Processor*MatrixOperation, data = size1280data)
summary(fit1_1280)
```

```
##
## Call:
## lm(formula = Runtime ~ Processor * MatrixOperation, data = size1280data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.32048 -0.00256 -0.00001  0.00333  0.51330
##
## Coefficients:
##                                             Estimate Std. Error t value
## (Intercept)                                  1.50458    0.05293   28.43
## ProcessorGPU                                -1.49482    0.07485  -19.97
## ProcessorTPU                                -1.49793    0.07485  -20.01
## MatrixOperationInvertion                    78.17525    0.07485 1044.41
## MatrixOperationMultiplication               56.11665    0.07485  749.71
## ProcessorGPU:MatrixOperationInvertion      -66.68873    0.10586 -630.00
## ProcessorTPU:MatrixOperationInvertion      -78.17992    0.10586 -738.56
## ProcessorGPU:MatrixOperationMultiplication -55.58840    0.10586 -525.14
## ProcessorTPU:MatrixOperationMultiplication -56.11973    0.10586 -530.16
##                                            Pr(>|t|)
## (Intercept)                                  <2e-16 ***
## ProcessorGPU                                 <2e-16 ***
## ProcessorTPU                                 <2e-16 ***
## MatrixOperationInvertion                     <2e-16 ***
## MatrixOperationMultiplication                <2e-16 ***
## ProcessorGPU:MatrixOperationInvertion        <2e-16 ***
## ProcessorTPU:MatrixOperationInvertion        <2e-16 ***
## ProcessorGPU:MatrixOperationMultiplication   <2e-16 ***
## ProcessorTPU:MatrixOperationMultiplication   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1183 on 36 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 3.246e+05 on 8 and 36 DF,  p-value: < 2.2e-16
```

```
anova(fit2_1280, fit1_1280)
```

```
## Analysis of Variance Table
##
```

```
## Model 1: Runtime ~ Processor + MatrixOperation
## Model 2: Runtime ~ Processor * MatrixOperation
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1     40 9812.3
## 2     36    0.5  4    9811.8 175129 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
modF<-lm(Runtime~Processor+MatrixOperation, data = size1280data)
modA<-lm(Runtime~Processor, data = size1280data)
modB<-lm(Runtime~MatrixOperation, data = size1280data)

anova(modA, modF)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ Processor
## Model 2: Runtime ~ Processor + MatrixOperation
##   Res.Df     RSS Df Sum of Sq     F    Pr(>F)
## 1     42 16666.1
## 2     40  9812.3  2    6853.7 13.97 2.505e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(modB, modF)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ MatrixOperation
## Model 2: Runtime ~ Processor + MatrixOperation
##   Res.Df     RSS Df Sum of Sq      F    Pr(>F)
## 1     42 29521.7
## 2     40  9812.3  2     19709 40.173 2.708e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
mod1280<-aov(Runtime~Processor*MatrixOperation, data = size1280data)
Anova(mod1280,type="III")
```

```
## Anova Table (Type III tests)
##
## Response: Runtime
##                             Sum Sq Df   F value    Pr(>F)
## (Intercept)                   11.3  1    808.11 < 2.2e-16 ***
## Processor                      7.5  2    266.44 < 2.2e-16 ***
## MatrixOperation            16245.1  2 579906.51 < 2.2e-16 ***
## Processor:MatrixOperation   9811.8  4 175128.60 < 2.2e-16 ***
## Residuals                      0.5 36
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
################## 2560
fit2_2560<-lm(Runtime~Processor+MatrixOperation, data = size2560data)
summary(fit2)
```

```
##
## Call:
## lm(formula = Runtime ~ Processor + MatrixOperation, data = size320data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.63252 -0.44075  0.05662  0.39766  0.58682
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     0.5072     0.1496   3.389  0.00159 **
## ProcessorGPU                   -0.4163     0.1639  -2.540  0.01508 *
## ProcessorTPU                   -1.0252     0.1639  -6.254 2.08e-07 ***
## MatrixOperationInvertion        1.1525     0.1639   7.031 1.70e-08 ***
## MatrixOperationMultiplication   0.4116     0.1639   2.511  0.01619 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4489 on 40 degrees of freedom
## Multiple R-squared:  0.6931, Adjusted R-squared:  0.6624
## F-statistic: 22.59 on 4 and 40 DF,  p-value: 8.149e-10
```

```r
fit1_2560<-lm(Runtime~Processor*MatrixOperation, data = size2560data)
summary(fit1_2560)
```

```
##
## Call:
## lm(formula = Runtime ~ Processor * MatrixOperation, data = size2560data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.85301 -0.00118 -0.00007  0.00026  1.23586
##
## Coefficients:
##                                             Estimate Std. Error  t value
## (Intercept)                                   7.0363     0.1765    39.87
## ProcessorGPU                                 -7.0296     0.2496   -28.17
## ProcessorTPU                                 -7.0315     0.2496   -28.18
## MatrixOperationInvertion                    554.0101     0.2496  2219.89
## MatrixOperationMultiplication               437.5216     0.2496  1753.13
## ProcessorGPU:MatrixOperationInvertion      -512.2048     0.3529 -1451.25
## ProcessorTPU:MatrixOperationInvertion      -554.0125     0.3529 -1569.71
## ProcessorGPU:MatrixOperationMultiplication -437.5188     0.3529 -1239.64
## ProcessorTPU:MatrixOperationMultiplication -437.5234     0.3529 -1239.65
##                                            Pr(>|t|)
## (Intercept)                                  <2e-16 ***
## ProcessorGPU                                 <2e-16 ***
## ProcessorTPU                                 <2e-16 ***
```

```
## MatrixOperationInvertion                     <2e-16 ***
## MatrixOperationMultiplication                <2e-16 ***
## ProcessorGPU:MatrixOperationInvertion        <2e-16 ***
## ProcessorTPU:MatrixOperationInvertion        <2e-16 ***
## ProcessorGPU:MatrixOperationMultiplication   <2e-16 ***
## ProcessorTPU:MatrixOperationMultiplication   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3946 on 36 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 1.568e+06 on 8 and 36 DF,  p-value: < 2.2e-16
```

```
anova(fit2_2560, fit1_2560)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ Processor + MatrixOperation
## Model 2: Runtime ~ Processor * MatrixOperation
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1     40 541548
## 2     36      6  4    541542 869482 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
mod2560<-aov(Runtime~Processor*MatrixOperation, data = size2560data)
Anova(mod2560,type="III")
```

```
## Anova Table (Type III tests)
##
## Response: Runtime
##                            Sum Sq Df    F value    Pr(>F)
## (Intercept)                   248  1    1589.81 < 2.2e-16 ***
## Processor                     165  2     529.08 < 2.2e-16 ***
## MatrixOperation            853203  2 2739748.40 < 2.2e-16 ***
## Processor:MatrixOperation 541542  4  869481.88 < 2.2e-16 ***
## Residuals                      6 36
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
modF<-lm(Runtime~Processor+MatrixOperation, data = size2560data)
modA<-lm(Runtime~Processor, data = size2560data)
modB<-lm(Runtime~MatrixOperation, data = size2560data)

anova(modA, modF)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ Processor
## Model 2: Runtime ~ Processor + MatrixOperation
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1     42 859034
```

```
## 2     40 541548  2    317486 11.725 9.829e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(modB, modF)
```

```
## Analysis of Variance Table
##
## Model 1: Runtime ~ MatrixOperation
## Model 2: Runtime ~ Processor + MatrixOperation
##   Res.Df     RSS Df Sum of Sq      F    Pr(>F)
## 1     42 1635796
## 2     40  541548  2   1094248 40.412 2.501e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Mean of Runtime

```
tapply(Runtime,list(Processor, MatrixOperation),mean, data = size320data)
```

```
##        Addition    Invertion Multiplication
## CPU 0.989863337 72.716236062   56.810904413
## GPU 0.007234912  6.797923491    0.074154954
## TPU 0.005444972  0.002124808    0.003207064
```

```
tapply(Runtime,list(Processor, MatrixOperation),mean, data = size640data)
```

```
##        Addition    Invertion Multiplication
## CPU 0.989863337 72.716236062   56.810904413
## GPU 0.007234912  6.797923491    0.074154954
## TPU 0.005444972  0.002124808    0.003207064
```

```
tapply(Runtime,list(Processor, MatrixOperation),mean, data = size1280data)
```

```
##        Addition    Invertion Multiplication
## CPU 0.989863337 72.716236062   56.810904413
## GPU 0.007234912  6.797923491    0.074154954
## TPU 0.005444972  0.002124808    0.003207064
```

```
tapply(Runtime,list(Processor, MatrixOperation),mean, data = size2560data)
```

```
##        Addition    Invertion Multiplication
## CPU 0.989863337 72.716236062   56.810904413
## GPU 0.007234912  6.797923491    0.074154954
## TPU 0.005444972  0.002124808    0.003207064
```

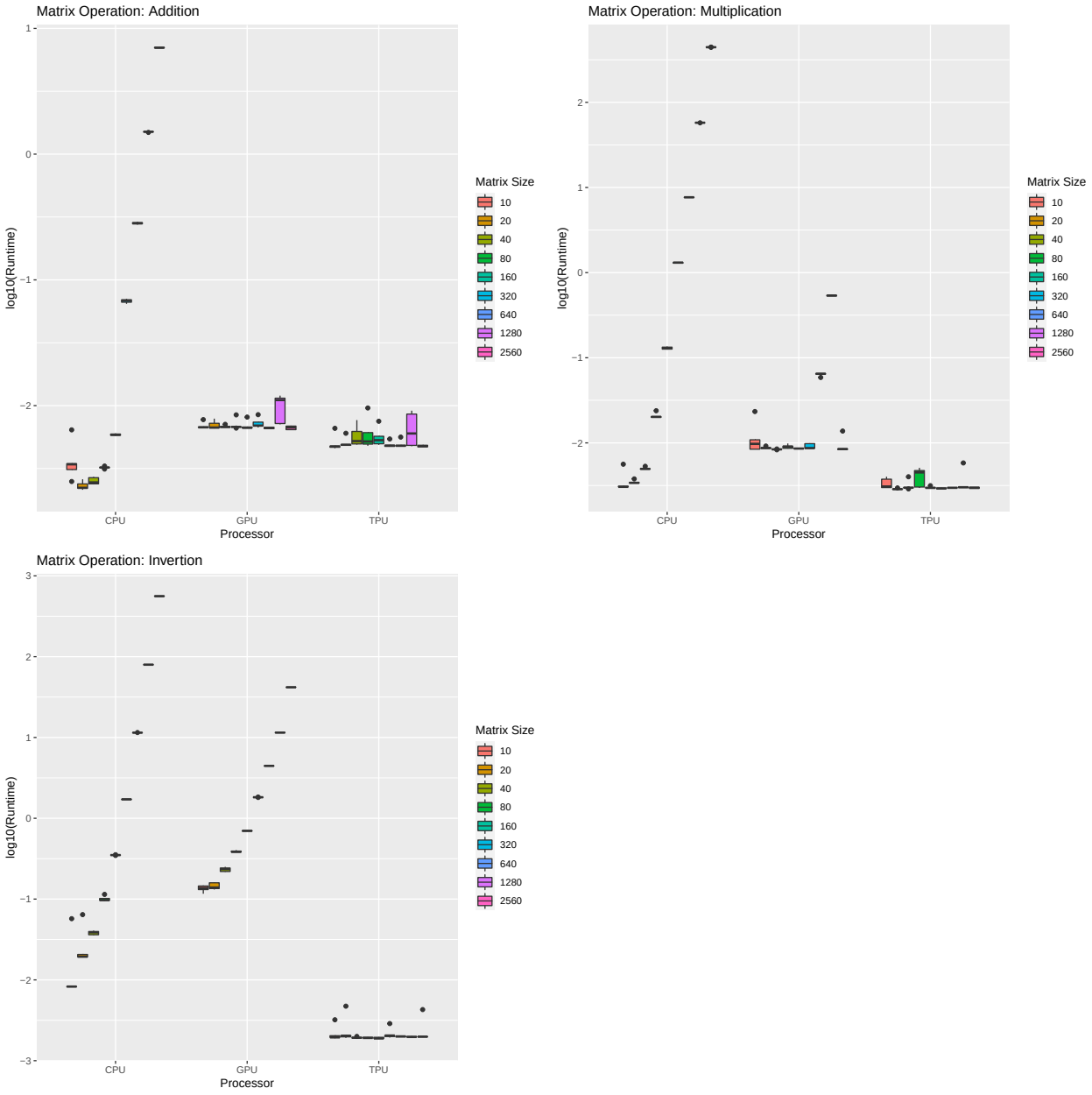


Figure 1: Operation v.s. Processors for Each Matrix Size

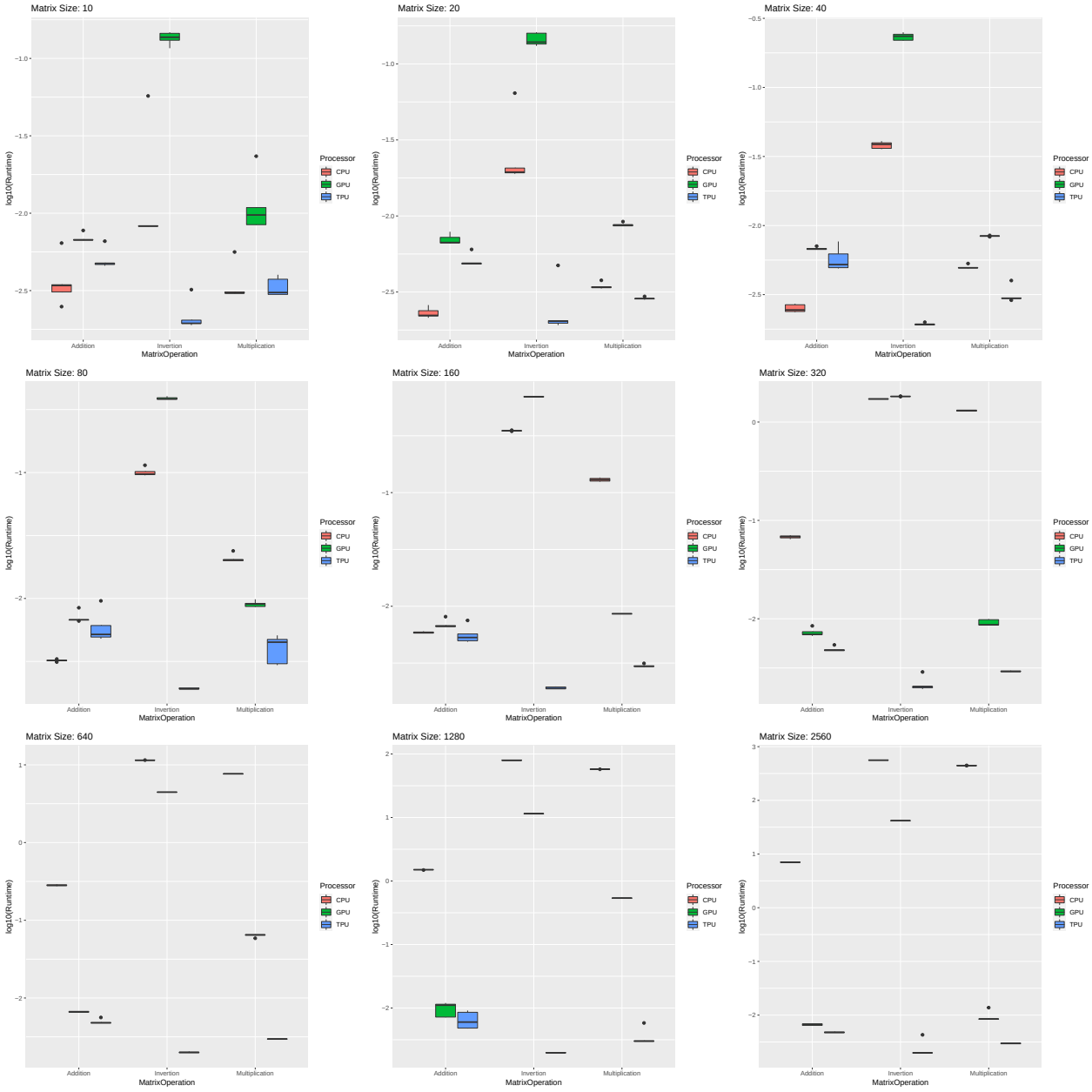


Figure 2: Matrix Size v.s. Operations for Each Processor.

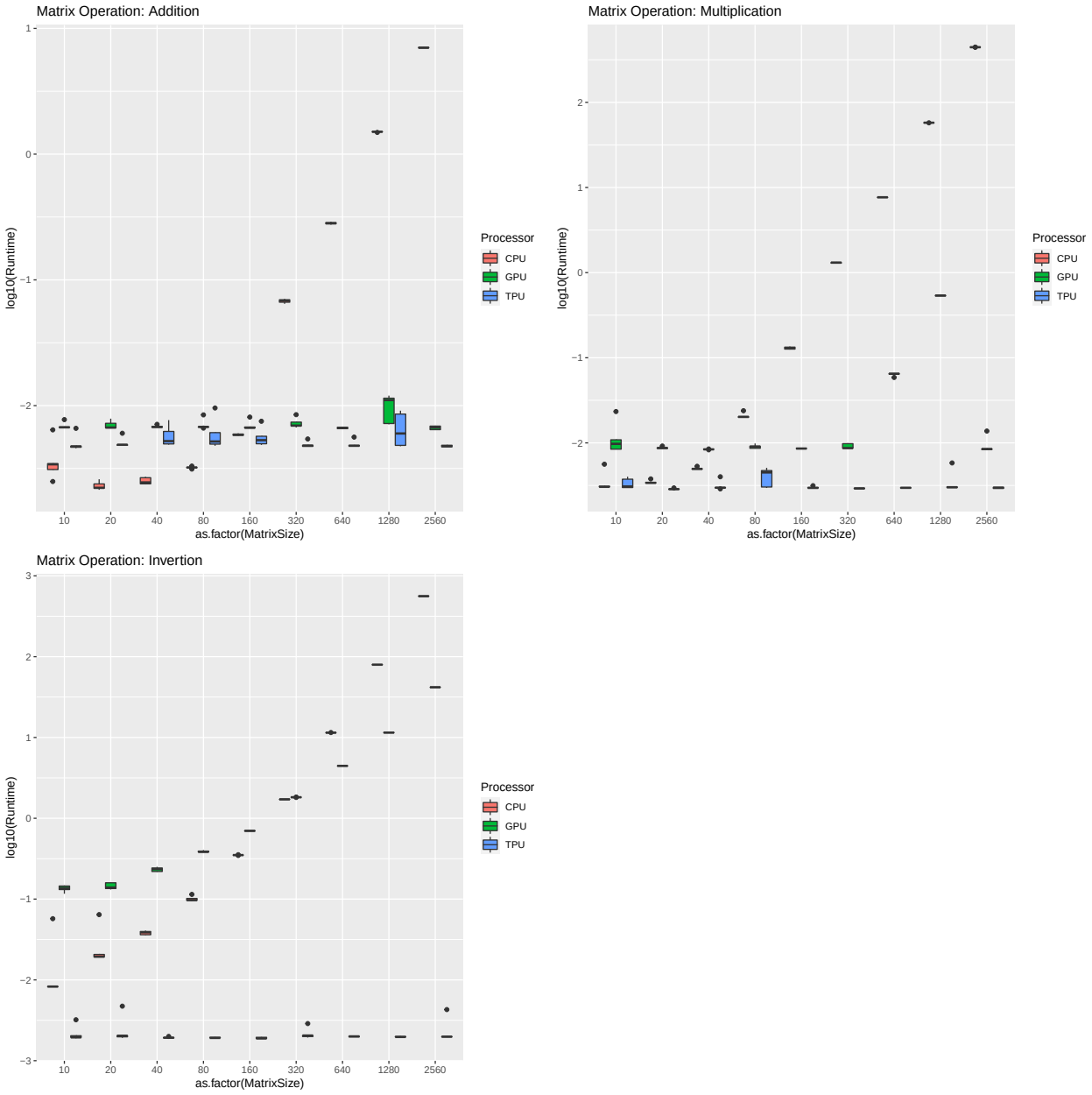


Figure 3: Operation v.s. Matrix Size for Each Processor

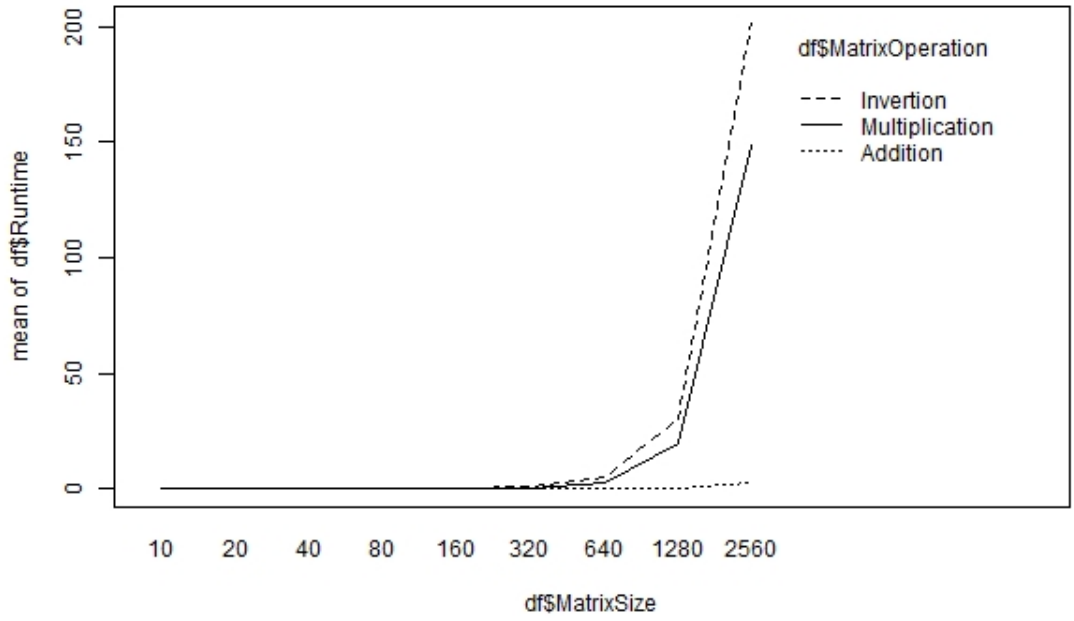
Zhanhao Zhang Part II

General Visualization

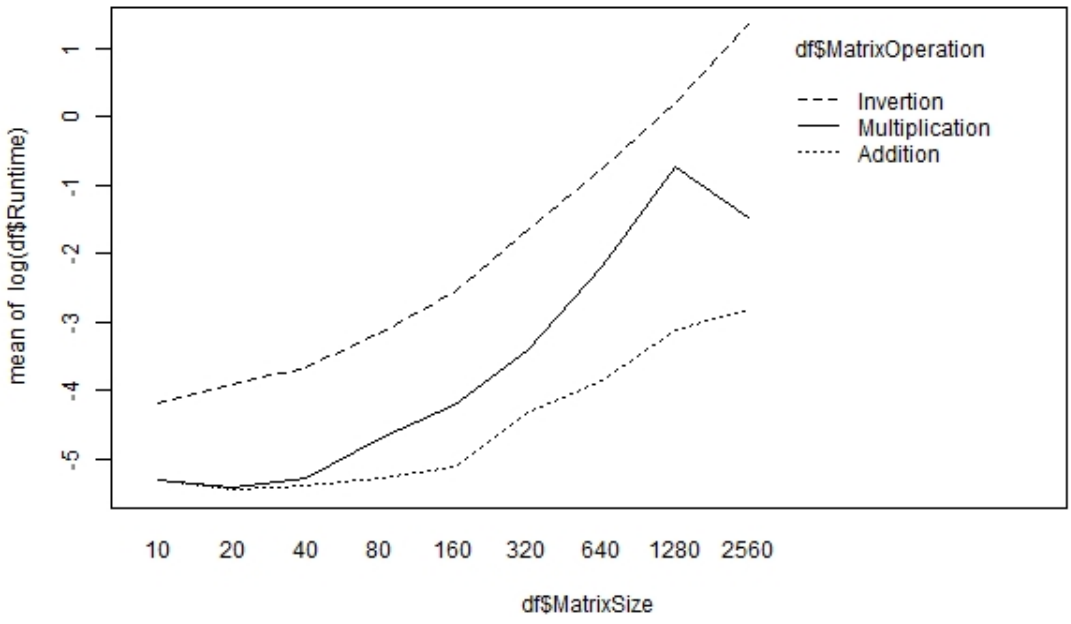
Lixian Chen Part II: Plots

Interaction Plots

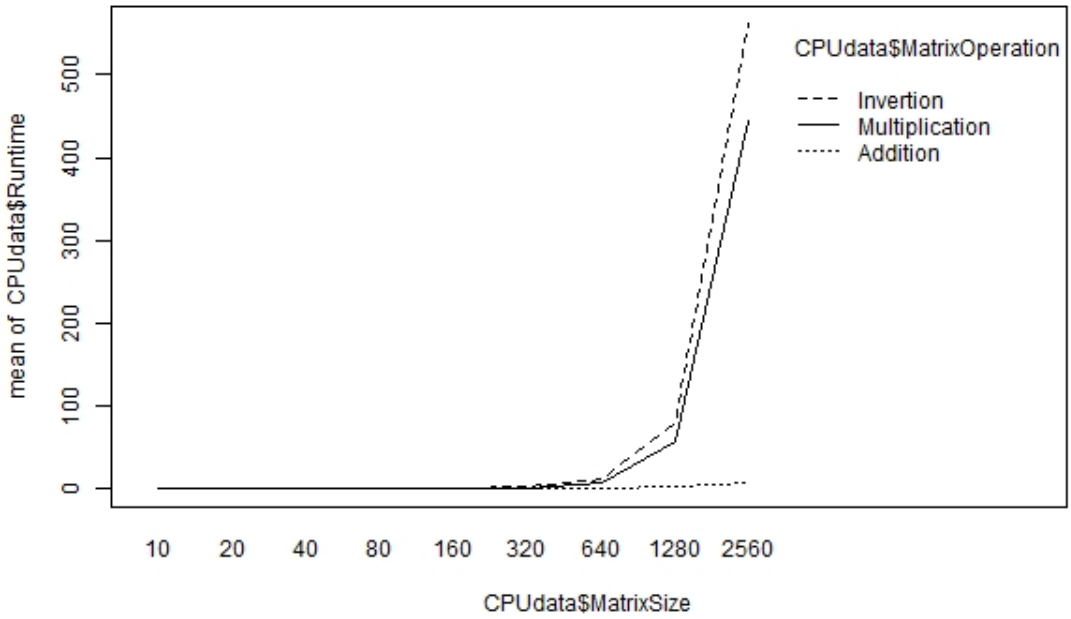
```
jpeg(filename = "../figs/interaction_size_time.jpeg", width = 600, height = 400,quality = 10000)
interaction.plot(df$MatrixSize, df$MatrixOperation, df$Runtime)
while (!is.null(dev.list()))  dev.off()
```



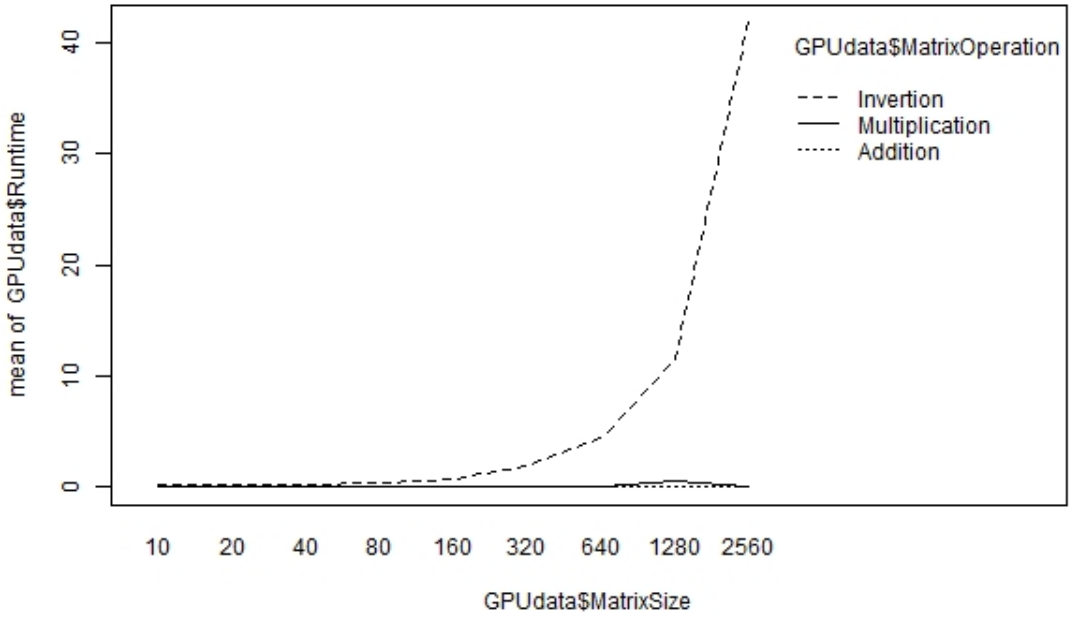

```
jpeg(filename = "../figs/interaction_size_log_time.jpeg", width = 600, height = 400,quality = 10000)
interaction.plot(df$MatrixSize, df$MatrixOperation, log(df$Runtime))
while (!is.null(dev.list()))  dev.off()
```



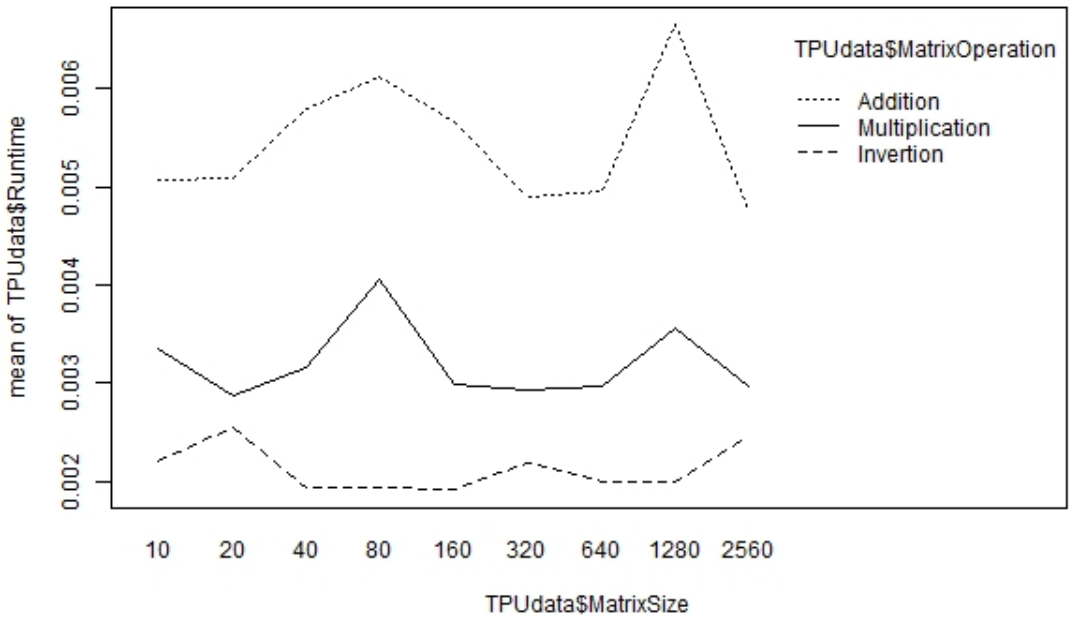

```
jpeg(filename = "../figs/interaction_CPU_size_time.jpeg", width = 600, height = 400,quality = 10000)
interaction.plot(CPUdata$MatrixSize, CPUdata$MatrixOperation, CPUdata$Runtime)
while (!is.null(dev.list()))  dev.off()
```



```
jpeg(filename = "../figs/interaction_GPU_size_time.jpeg", width = 600, height = 400,quality = 10000)
interaction.plot(GPUdata$MatrixSize, GPUdata$MatrixOperation, GPUdata$Runtime)
while (!is.null(dev.list()))  dev.off()
```

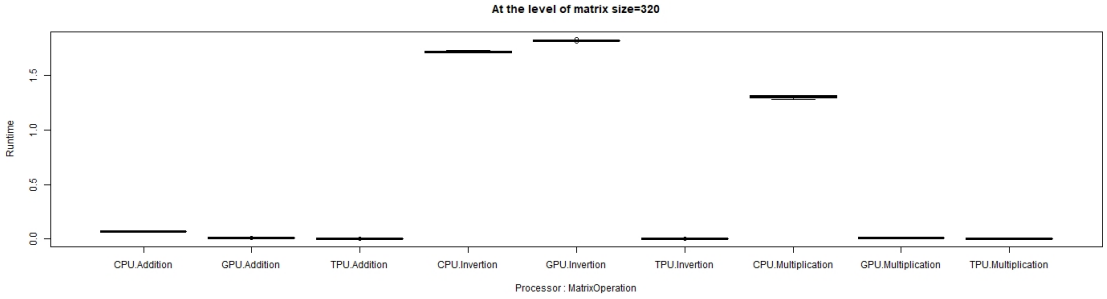



```
jpeg(filename = "../figs/interaction_TPU_size_time.jpeg", width = 600, height = 400,quality = 10000)
interaction.plot(TPUdata$MatrixSize, TPUdata$MatrixOperation, TPUdata$Runtime)
while (!is.null(dev.list()))  dev.off()
```

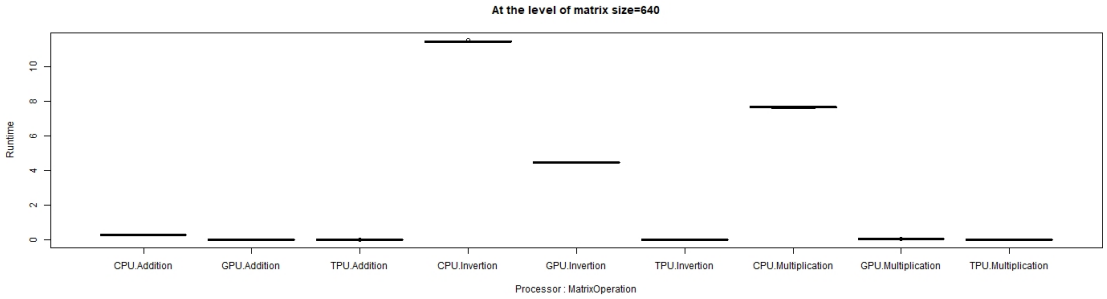


Boxplots

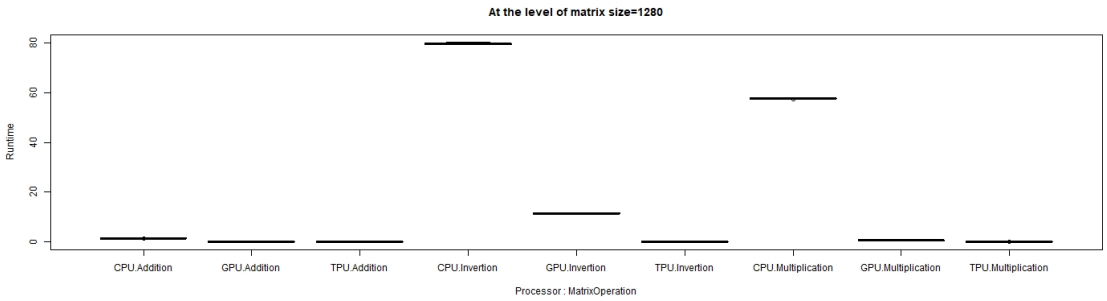
```
jpeg(filename = "../figs/Operation_vs_runtime_size320.jpeg", width = 1400, height = 400,quality = 10000)
boxplot(Runtime~Processor*MatrixOperation, data = size320data, main="At the level of matrix size=320")
while (!is.null(dev.list()))  dev.off()
```



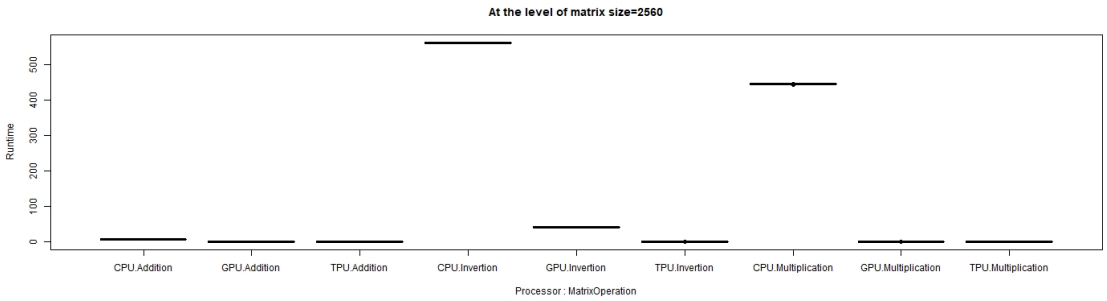

```
jpeg(filename = "../figs/Operation_vs_runtime_size640.jpeg", width = 1400, height = 400,quality = 10000)
boxplot(Runtime~Processor*MatrixOperation, data = size640data, main="At the level of matrix size=640")
while (!is.null(dev.list()))  dev.off()
```



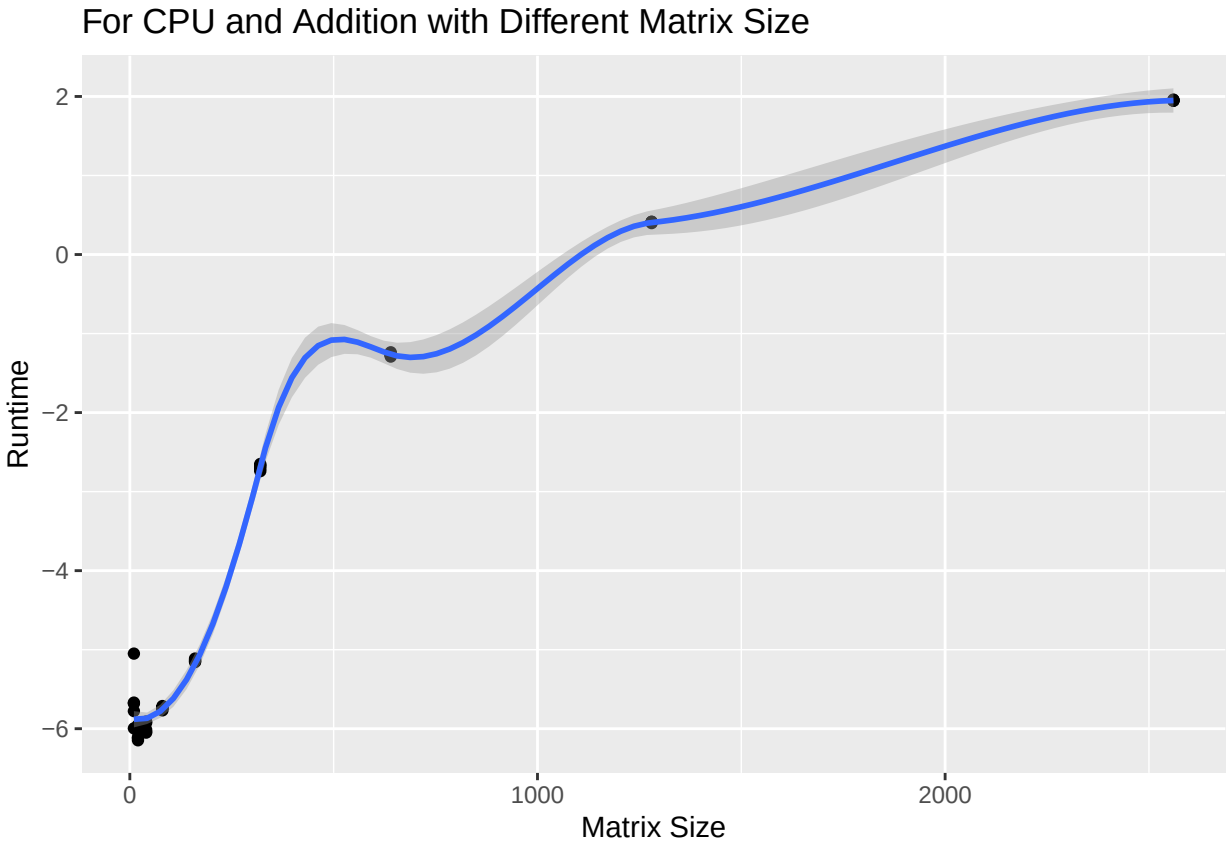

```
jpeg(filename = "../figs/Operation_vs_runtime_size1280.jpeg", width = 1400, height = 400,quality = 10000
boxplot(Runtime~Processor*MatrixOperation, data = size1280data, main="At the level of matrix size=1280")
while (!is.null(dev.list()))  dev.off()
```

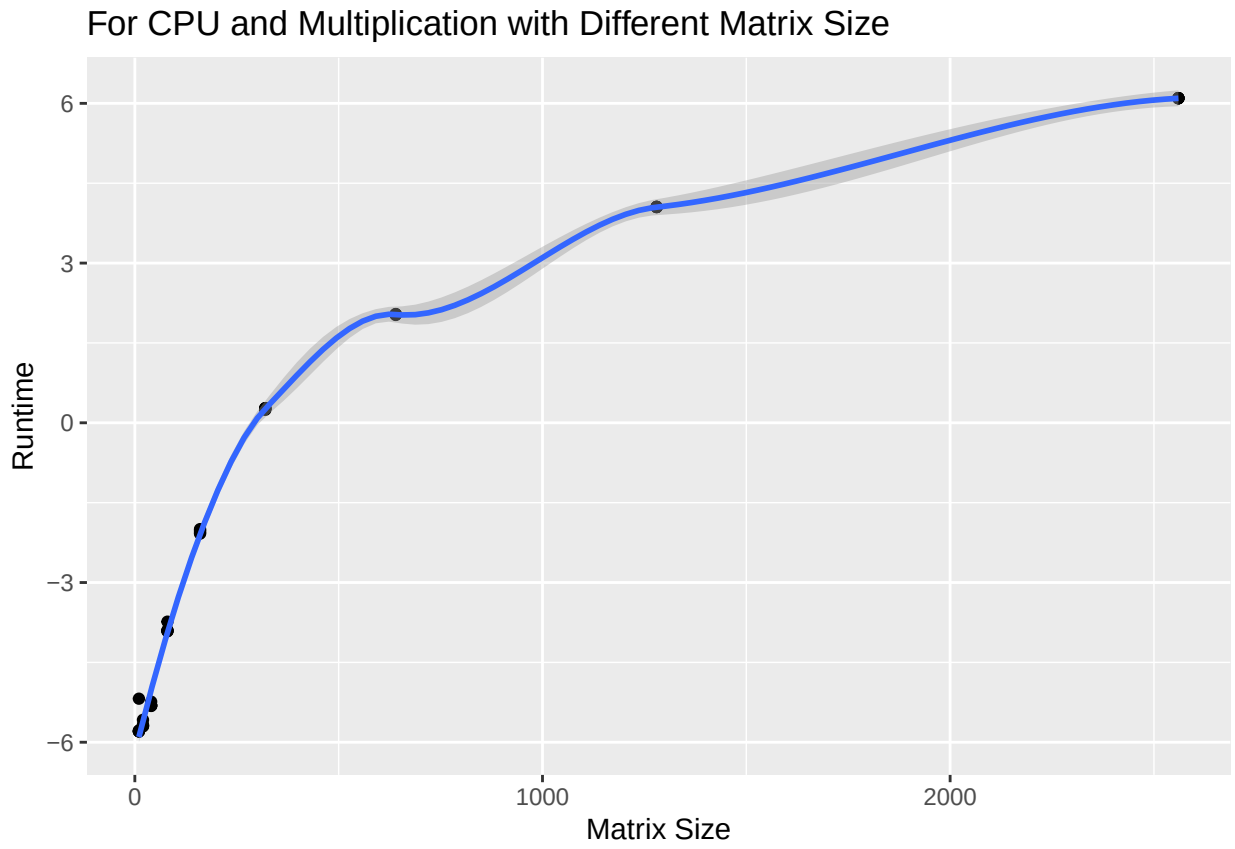


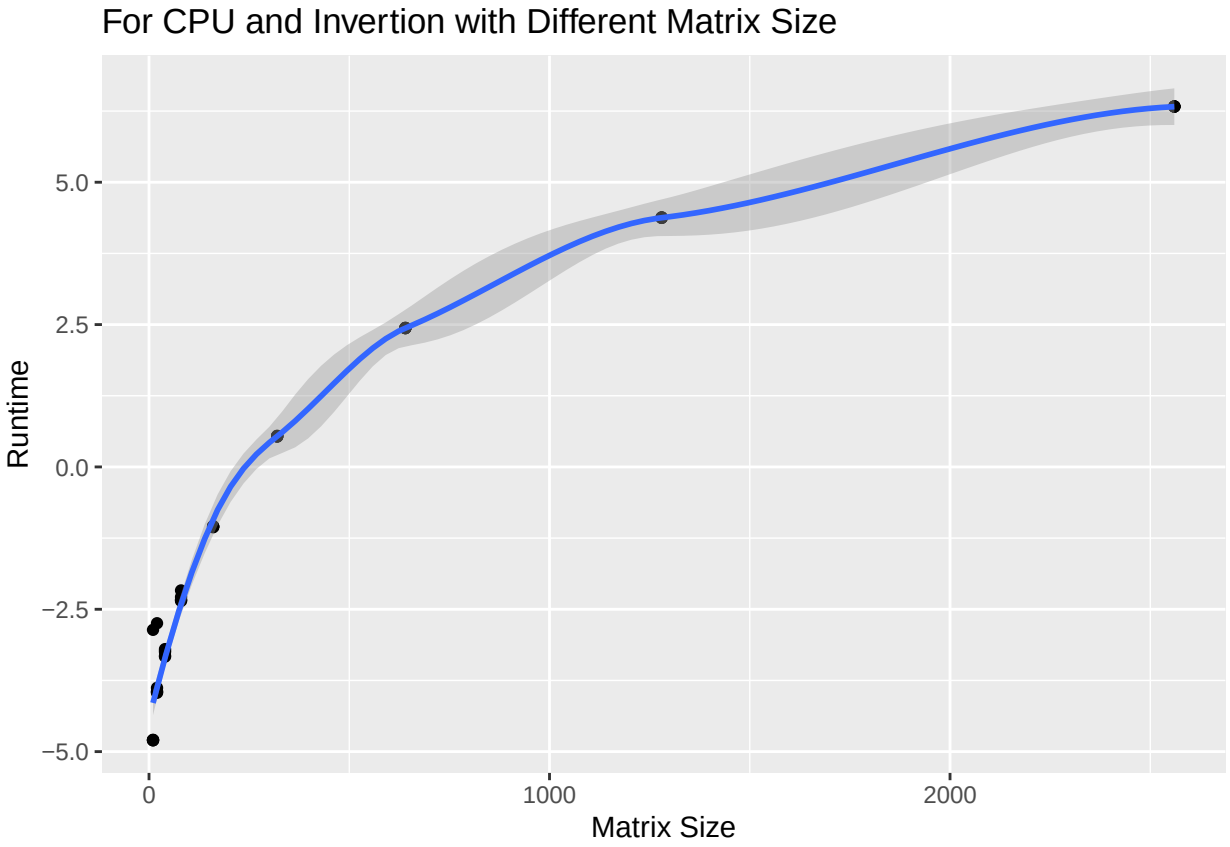

```
jpeg(filename = "../figs/Operation_vs_runtime_size2560.jpeg", width = 1400, height = 400,quality = 10000
boxplot(Runtime~Processor*MatrixOperation, data = size2560data, main="At the level of matrix size=2560")
while (!is.null(dev.list()))  dev.off()
```

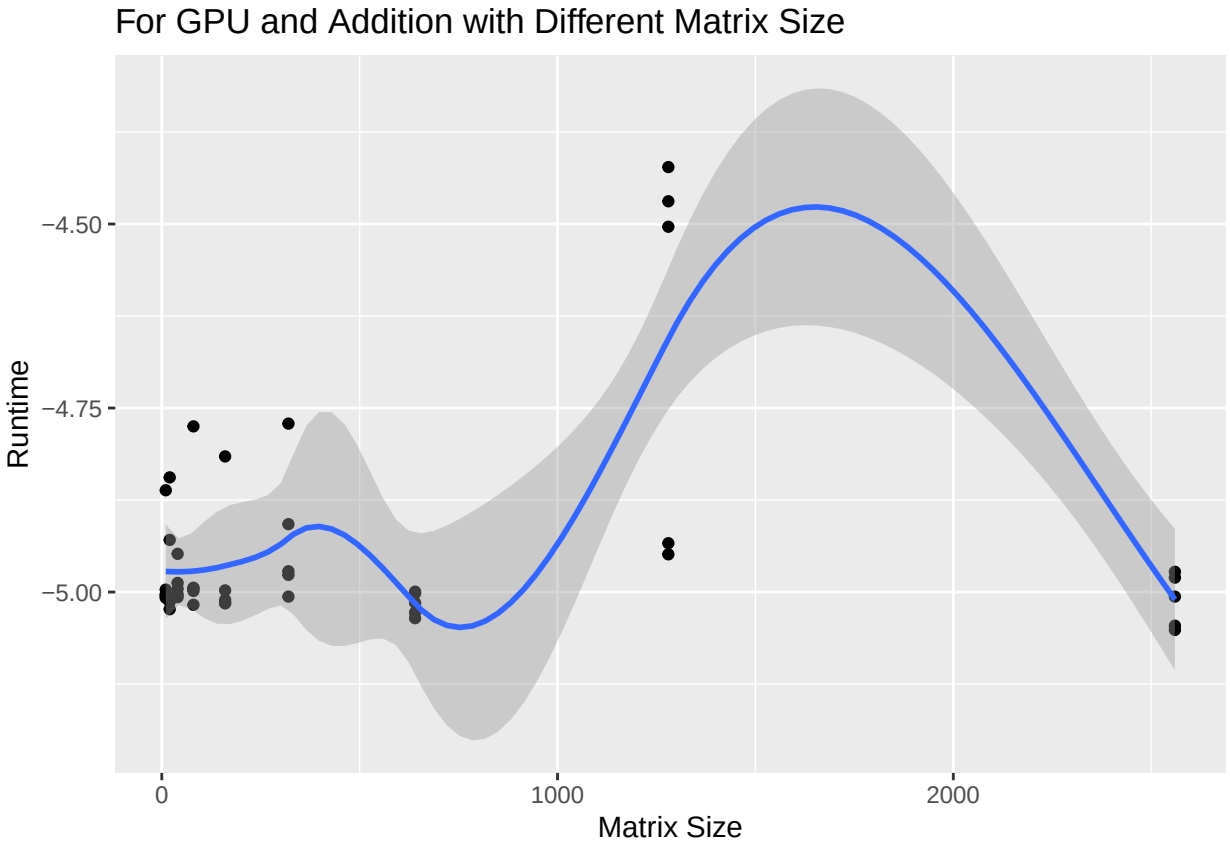


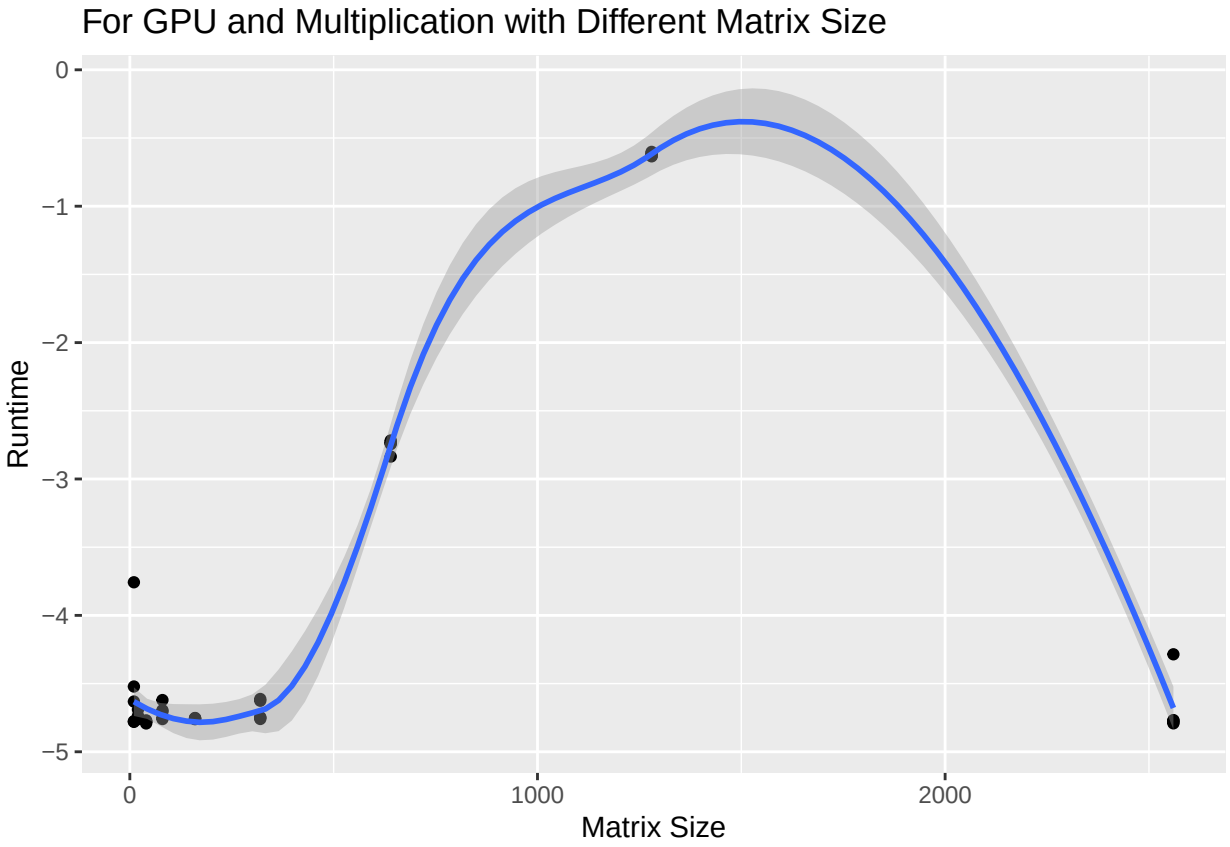
At the level of matrix size=2560, avoid using CPU for inversion and multiplication because its run-times are much bigger.

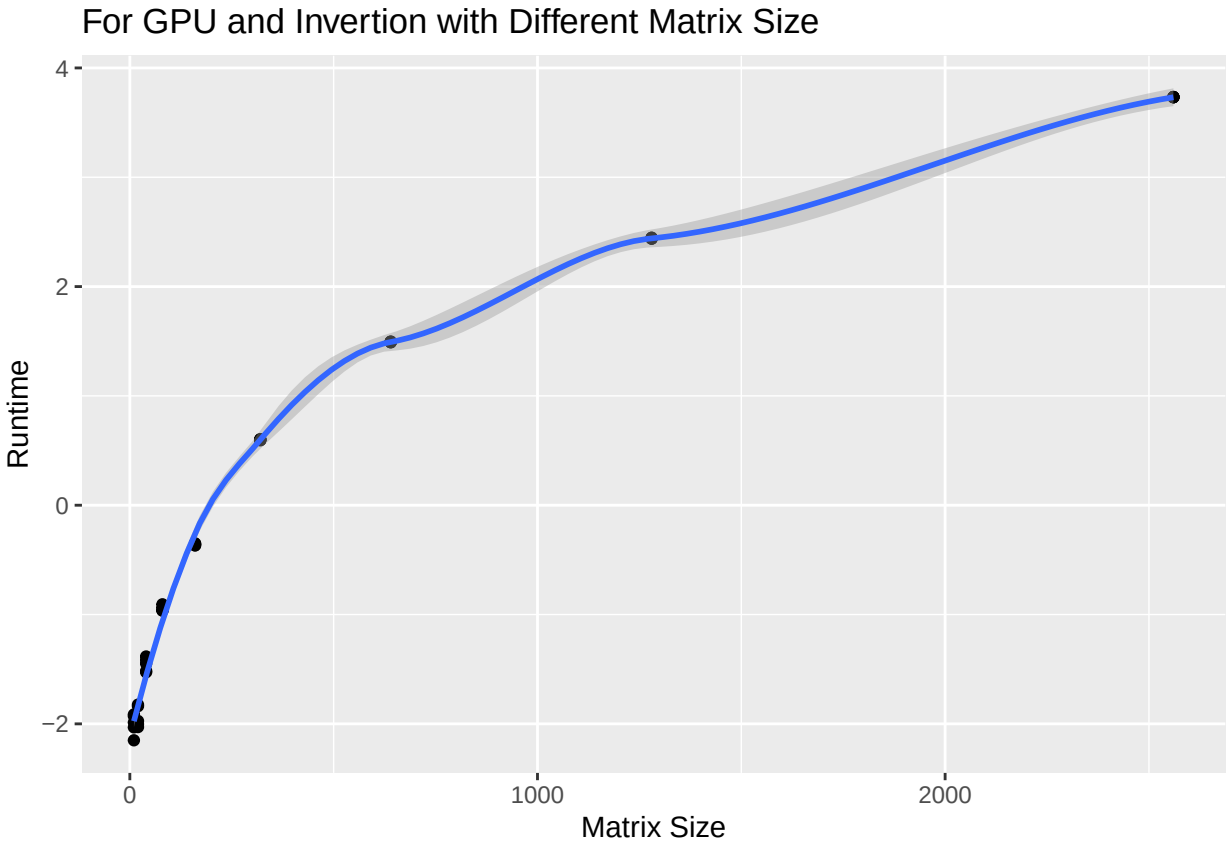

For CPU and Addition with Different Matrix Size
Runtime
2
0
−2
−4
−6
0
1000
2000
Matrix Size

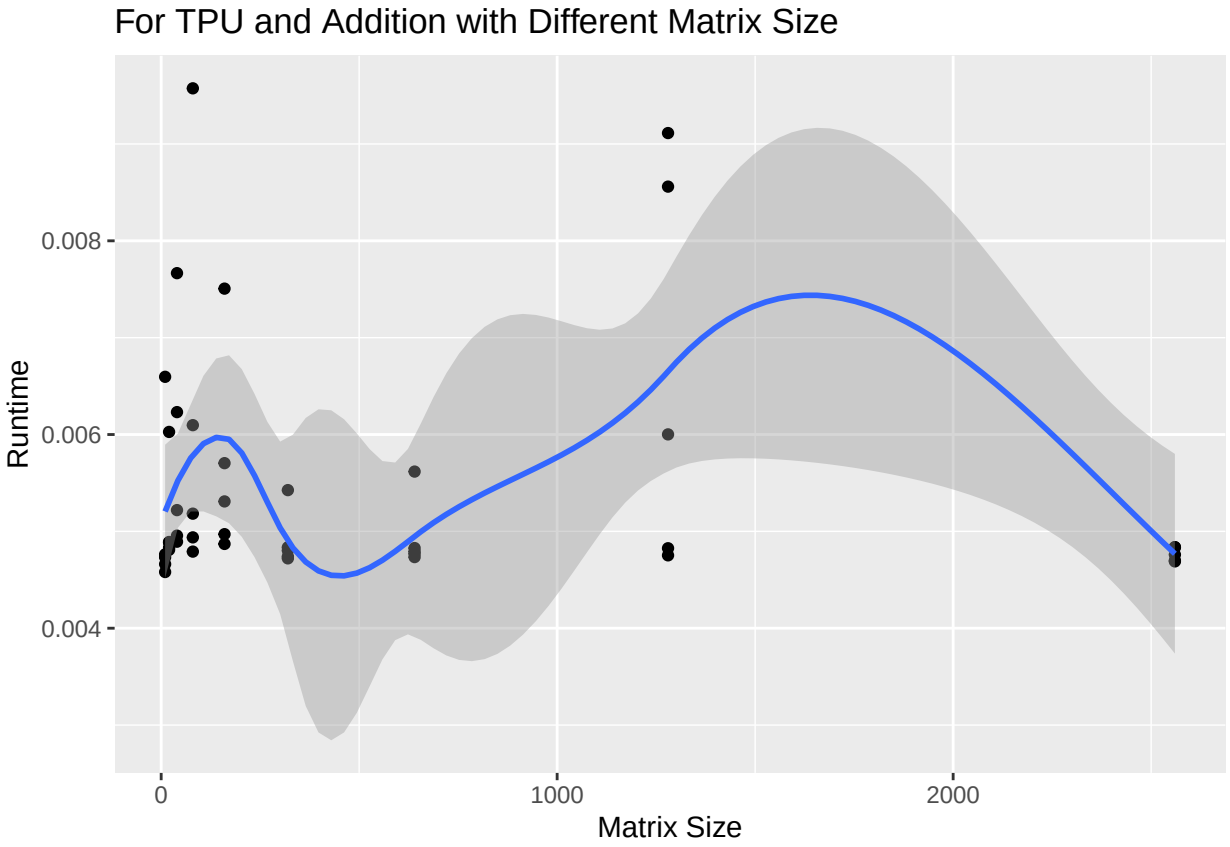

For CPU and Multiplication with Different Matrix Size
Runtime
6
3
0
−3
−6
0
1000
2000
Matrix Size

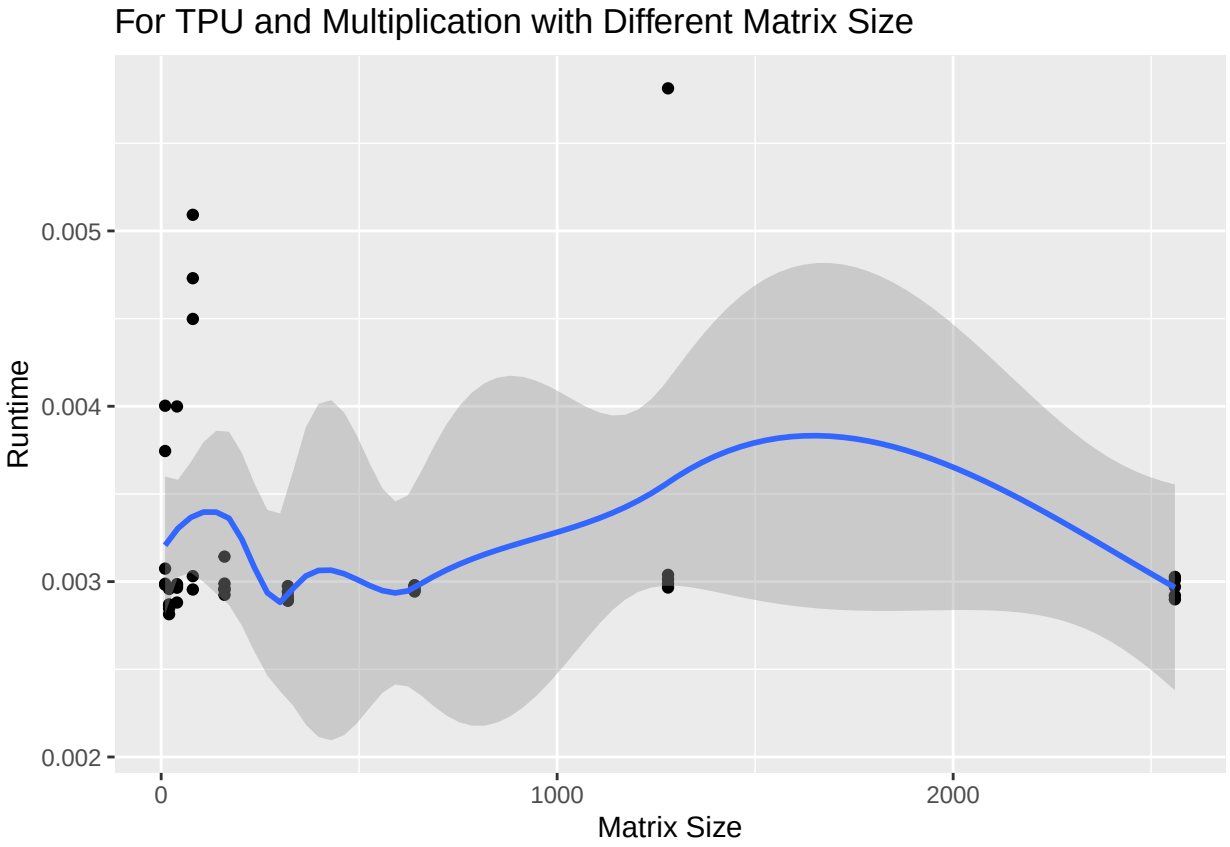

For CPU and Invertion with Different Matrix Size
Runtime
5.0
2.5
0.0
−2.5
−5.0
0
1000
2000
Matrix Size

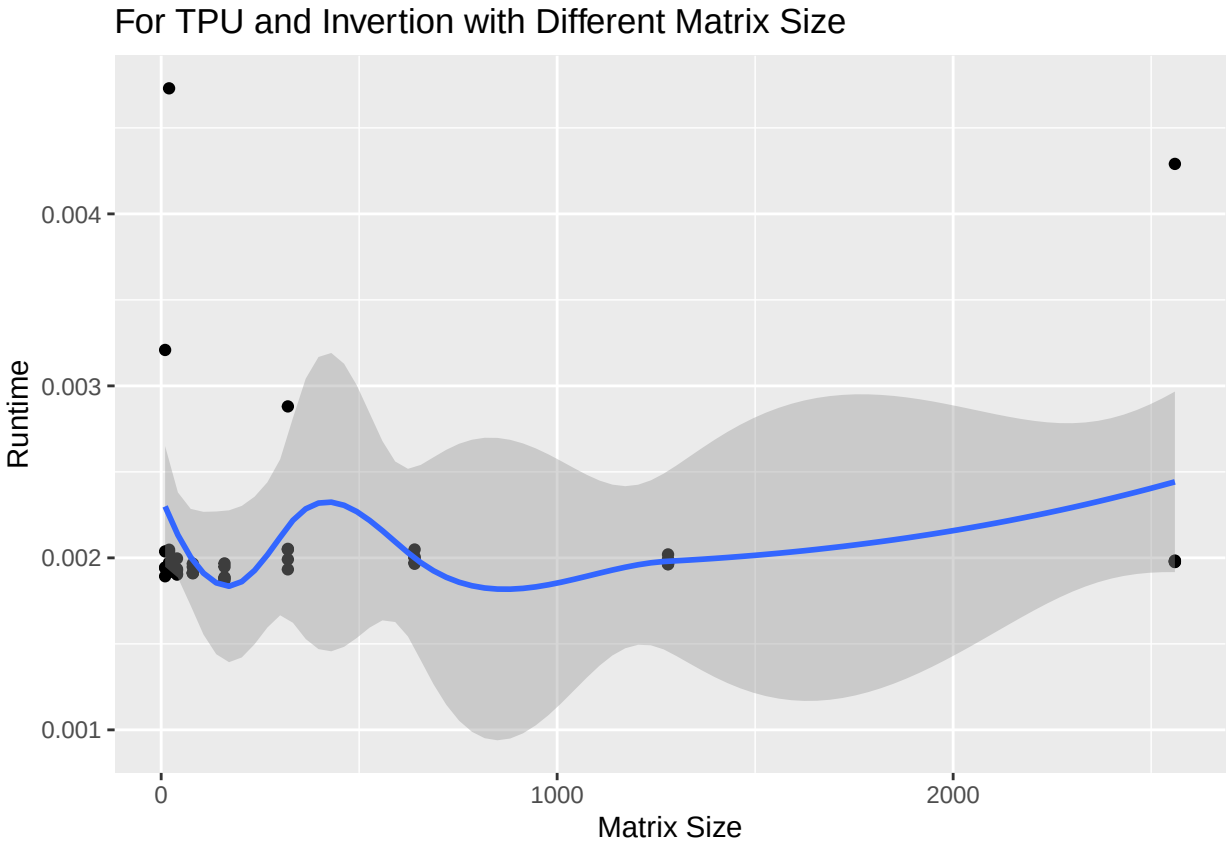

For GPU and Addition with Different Matrix Size
Runtime
−4.50
−4.75
−5.00
0
1000
2000
Matrix Size


For GPU and Multiplication with Different Matrix Size
Runtime
0
−1
−2
−3
−4
−5
0
1000
2000
Matrix Size


For GPU and Invertion with Different Matrix Size
Runtime
4
2
0
−2
0
1000
2000
Matrix Size


For TPU and Addition with Different Matrix Size
Runtime
0.008
0.006
0.004
0
1000
2000
Matrix Size


For TPU and Multiplication with Different Matrix Size
Runtime
0.005
0.004
0.003
0.002
0
1000
2000
Matrix Size


For TPU and Invertion with Different Matrix Size
Runtime
0.004
0.003
0.002
0.001
0
1000
2000
Matrix Size