MATHEMATICS

2INT HIGHER LEVEL

PAPER 2

December 2016

ÉCOLE Jeannine Manuel

International understanding through a bilingual education

1 hour - Calculator Allowed

5 mn reading time + 60 mn

S. Coursaget

CANDIDATE NAME : Jingjie YANG

Grade : 59/ **60 marks**

19,7/ **20**

Excellent!

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INSTRUCTIONS TO CANDIDATES

- Write your name above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is <u>required</u> for this paper.
- Answer all questions in the spaces provided
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to **three significant figures**.
- The maximum mark for this examination paper is ***[60 marks]***.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, ***if graphs are used to find a solution, you should sketch these as part of your answer.*** *Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.*

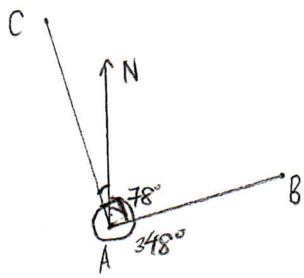
Exercise 1 : (8 marks)

Romeo and Juliet met for lunch. At 1 p.m., they parted ways.

Romeo drove on a true bearing of 078° at 80 km·h^{-1} and Juliet drove on a true bearing of 348° at 120 km·h^{-1}.

1. Draw a sketch and find the angle between the paths of Romeo and Juliet. Show your working.

2. How far will they be from each other at 3:30 p.m. ?



① $\hat{CAN} = 360° - 348° = 12°$

$\hat{CAB} = \hat{CAN} + \hat{NAB} = 12° + 78° = 90°$

∴ The angle between the paths of Romeo and Juliet is 90°.

② $t = 3{:}30\text{pm} - 1\text{p.m.} = 2\text{h }30\text{ min} = \frac{5}{2}\text{ h}$

$AB = V_{Romeo} \cdot t = 80 \times \frac{5}{2} = 200\text{ km}$

$AC = V_{Juliet} \cdot t = 120 \times \frac{5}{2} = 300\text{ km}$

Using Pythagoras

$BC^2 = AB^2 + AC^2$

$BC^2 = 200^2 + 300^2$

$BC \simeq 361\text{ km}$ $(BC \geq 0)$

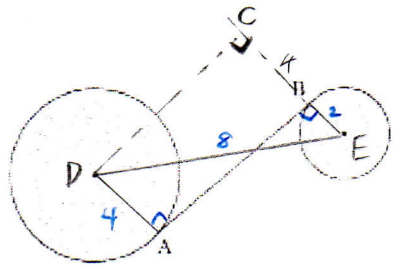
∴ They will be 361 km away from each other at 3:30 p.m.

Exercise 2 : (5 marks)

The illustration shows two circles of radii $4\ cm$ and $2\ cm$ respectively.

The distance between the two centers is $8\ cm$.

Find the length of the common tangent $[AB]$, to three significant figures.



We construct a rectangle ABCD

hence $BC = AD = 4$ and $AB = CD$, $CD \perp CE$

Using Pythagoras,

$$CD^2 + CE^2 = DE^2$$

$$AB^2 + (CB+BE)^2 = 8^2$$

$$AB^2 + 6^2 = 8^2$$

$$AB^2 = 28$$

$$AB = 2\sqrt{7}$$

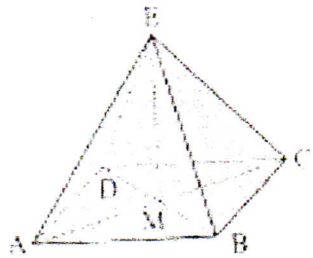
$$AB \simeq 5.29\ cm$$

$\therefore$ The length of $[AB]$ is $5.29\ cm$.

Exercise 3 : (6 marks)

A pyramid of height $40\ m$ has a square base with edges $50\ m$.

Determine the length of the slant edges .



Using Pythagoras,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 50^2 + 50^2$$

$$AC = \sqrt{5000} = 50\sqrt{2}$$

$$AM = \frac{1}{2}AC = \frac{50\sqrt{2}}{2} = 25\sqrt{2}$$

Using Pythagoras,

$$EA^2 = EM^2 + AM^2$$

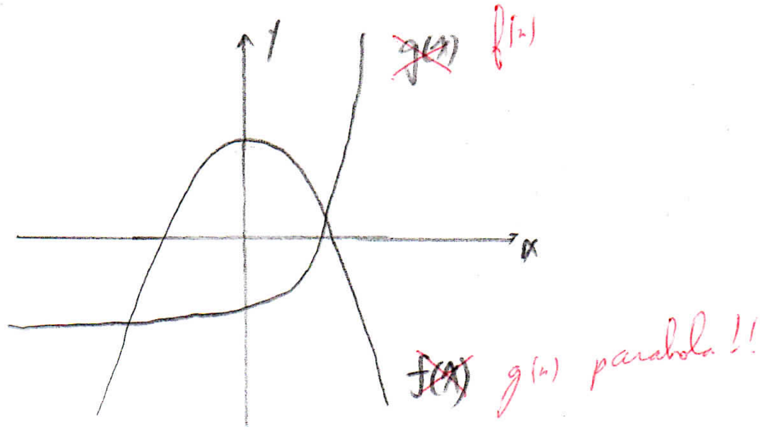
$$EA^2 = 40^2 + (25\sqrt{2})^2$$

$$EA = 5\sqrt{114} \simeq 53.4\ m$$ $\therefore$ The length of the slant edges is $53.4\ m$

Exercise 4 : ~~(7 marks)~~ 6

Let $f(x) = 2^x - 3$ and $g(x) = -x^2 + 6$.

a) Using your graphics display calculator, make a sketch of the graphs of these two functions :



b) Solve, giving your answer(s) correct to three significant figures :

]-2.98 ; 2.14 [

$f(x) = g(x)$: {-2.98 ; 2.14} ✓ $f(x) < g(x)$:]-∞ ; -2.98[∪]2.14 ; +∞[

c) Determine the range of f :]-∞ ; 6] and the range of g :]-3 ; +∞[

Exercise 5 : (10 marks) 6

1. Find the constant term in the expansion of $(3x - \frac{2}{x})^6$:

$$20 \times (3x)^3 \times \left(-\frac{2}{x}\right)^3 \quad ✓$$

$$= -20 \times 27 \times 8 \times \frac{x^3}{x^3}$$

$$= -4320 \quad ✓$$

∴ The constant term is −4320.

2. Find the value of the real number a given that the coefficient of x^3 in the expansion of $(ax+5)^5$ is 2000 :

$$10 \cdot (ax)^3 \times 5^2 = 2000x^3 \quad ✓$$

$$250a^3 = 2000$$

$$a^3 = 8$$

$$a = 2 \quad ✓$$

∴ The value of the real number a is 2.

Exercise 6 : (8 marks)

A rectangular swimming pool is $12\ m$ long by $6\ m$ wide. It is surrounded by a pavement of uniform width, the area of the pavement being $\frac{7}{8}$ of the area of the pool.

1. If the pavement is $x\ m$ wide, show that the area of the pavement is $4x^2 + 36x \quad m^2$.

2. Hence, by solving an equation, determine the width of the pavement.

① $A_{pavement} = (12+2x)(6+2x) - 72$
$= 4x^2 + 12x + 24x + 72 - 72$
$= (4x^2 + 36x)\ m^2$

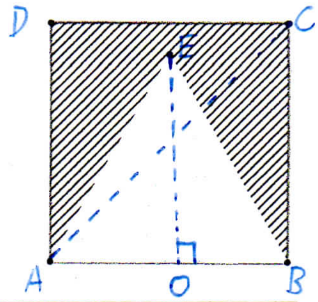
② $4x^2 + 36x = \frac{7}{8} \times 12 \times 6$
$4x^2 + 36x = 63$
$4x^2 + 36x - 63 = 0$
$(2x+21)(2x-3) = 0$
$x = -\frac{21}{2}$ or $x = \frac{3}{2}$
$\because x > 0 \quad \therefore x = \frac{3}{2} = 1.5m$ $\quad \therefore$ The with of the pavement is $1.5m$

Exercise 7 : (6 marks)

An equilateral triangle is inscribed inside a square.

Given that the diagonal of the square is $10\sqrt{2}\ cm$, find the area of the shaded area.

Use exact values and show all your working.



Using Pythagoras,
$AB^2 + BC^2 = AC^2$
$2AB^2 = (10\sqrt{2})^2$
$AB^2 = 100$
$AB = 10 \quad (AB > 0)$
In the equilateral triangle ABE,
$BE = AB = AE = 10, \quad EO \perp AB$

hence $OB = \frac{1}{2}AB = \frac{10}{2} = 5$
Using Pythagoras,
$EO^2 + OB^2 = BE^2$
$EO^2 = 10^2 - 5^2$
$EO = \sqrt{75} = 5\sqrt{3}$
$A_{\triangle ABE} = \frac{1}{2} \cdot AB \cdot EO$
$= \frac{1}{2} \cdot 10 \cdot 5\sqrt{3}$
$= 25\sqrt{3}$

$A_{shaded} = A_{ABCD} - A_{\triangle ABE}$
$= AB^2 - A_{\triangle ABE}$
$= 10^2 - 25\sqrt{3}$
$= 100 - 25\sqrt{3}\ cm^2$
$\simeq 56.7\ cm^2$
$\therefore$ The area of the shaded area is $100 - 25\sqrt{3}\ cm^2$ or $56.7\ cm^2$.

Exercise 8 : (10 marks)

Solve for x and y given that they are rational :

1. $(x + y\sqrt{3})(2\sqrt{3}) = 5 - 6\sqrt{3}$

$2\sqrt{3}x + 6y = 5 - 6\sqrt{3}$

$\Leftrightarrow \begin{cases} 2\sqrt{3}x = -6\sqrt{3} \\ 6y = 5 \end{cases}$

$\begin{cases} x = -3 \\ y = \frac{5}{6} \end{cases}$

$\therefore S = (-3; \frac{5}{6})$

2. $(x + y\sqrt{2})(3 - \sqrt{2}) = -2\sqrt{2}$

$3x - \sqrt{2}x + 3\sqrt{2}y - 2y = -2\sqrt{2}$

$\Leftrightarrow \begin{cases} 3x - 2y = 0 \\ -\sqrt{2}x + 3\sqrt{2}y = -2\sqrt{2} \end{cases}$

$\begin{cases} x = \frac{2}{3}y \\ -\frac{2}{3}y + 3y = -2 \end{cases}$

$\begin{cases} x = \frac{2}{3}y \\ \frac{7}{3}y = -2 \end{cases}$

$\begin{cases} x = \frac{2}{3}y \\ y = -\frac{6}{7} \end{cases}$

$\begin{cases} x = -\frac{4}{7} \\ y = -\frac{6}{7} \end{cases}$

$\therefore S = (-\frac{4}{7}; -\frac{6}{7})$

3. $(x + y\sqrt{2})^2 = 66 - 40\sqrt{2}$

$x^2 + 2\sqrt{2}xy + 2y^2 = 66 - 40\sqrt{2}$

$\Leftrightarrow \begin{cases} x^2 + 2y^2 = 66 \\ 2\sqrt{2}xy = -40\sqrt{2} \end{cases}$

$\begin{cases} x^2 + 2y^2 = 66 \\ xy = -20 \end{cases}$

$\begin{cases} x^2 + 2y^2 = 66 \\ y = -\frac{20}{x} \end{cases}$

$\begin{cases} x^2 + 2 \times \frac{400}{x^2} = 66 \\ y = -\frac{20}{x} \end{cases}$

$\begin{cases} x^4 - 66x^2 + 800 = 0 \\ y = -\frac{20}{x} \end{cases}$

$(x^2 - 50)(x^2 - 16) = 0$

$(x - 5\sqrt{2})(x + 5\sqrt{2})(x - 4)(x + 4) = 0$

$x = 5\sqrt{2}$ or $x = -5\sqrt{2}$ or $x = 4$ or $x = -4$

$\because x \in \mathbb{Q}$

$\therefore \begin{cases} x = 4 \\ y = -5 \end{cases}$ or $\begin{cases} x = -4 \\ y = 5 \end{cases}$

$\therefore S = (-4; 5) \cup (4; -5)$