MATHEMATICS
2INT HIGHER LEVEL
PAPER 1

December 2016

ÉCOLE Jeannine Manuel
International understanding through a bilingual education

1 hour – No Calculator

5 mn reading time + 60 mn

S. Coursaget

CANDIDATE NAME : Jingjie YANG

Grade : 59,5/60 **marks**

19,8 **/20**   Excellent

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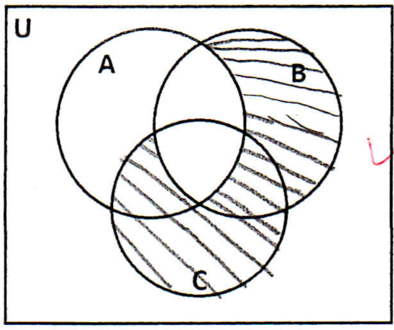
INSTRUCTIONS TO CANDIDATES

- Write your name above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[60 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

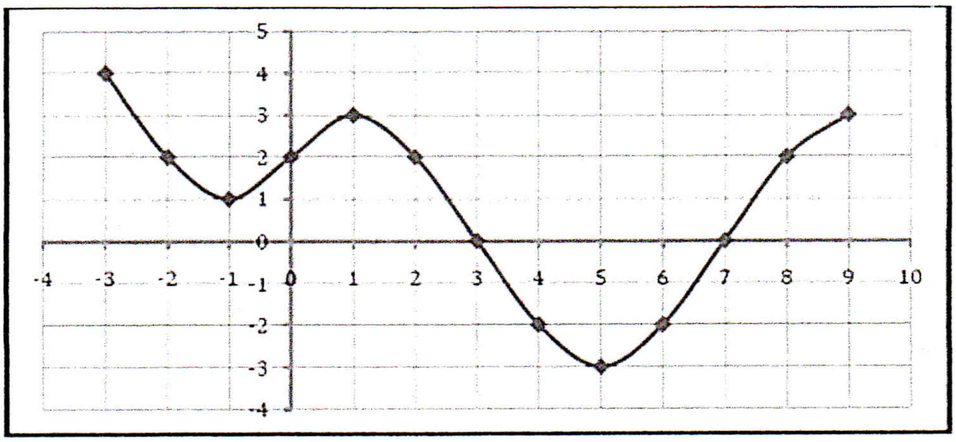
Exercise 1 : (2 marks)

Shade the region representing $(A \cap B)' \cap (C \cup B)$ in the diagram below :



Exercise 2 : (8 marks)

The following curve is the graph of a function f :



1. Fill in the blanks : $f(2) = $ ..2.. and $f($..5..$) = -3$
2. Write down the image of -1 : 1 And the pre-image(s) of 0 : 3 and 7
3. Find the domain of f : $[-3; 9]$ And the range of f : $[-3; 4]$
4. Solve the following equations and inequations :

   a) $f(x) = -2$: $S = \{4; 6\}$ b) $f(x) = -4$: $S = \emptyset$

   c) $f(x) \leq 2$: $S = [-2; 0] \cup [2; 8]$ d) $f(x) > -3$: $S = [-3; 9] \setminus \{5\}$
5. Find the set of values for k such that k has exactly four pre-images by function f :

   $k =]1; 3[$ yes!
6. Find the domain of $g(x) = \frac{1}{f(x)}$: $[-3; 9] \setminus \{3; 7\}$

   Working:

   $f(x) \neq 0$

   $\because f(3) = f(7) = 0$

   $\therefore$ domain of $g(x) = [-3; 9] \setminus \{3; 7\}$

Exercise 3 : (4 marks)

Show that the number $2.\overline{27}$ which is 2.27272727 … is a rational number.

Let x be the number $2.\overline{27}$

$x = 2.\overline{27}$

$100x = 227.\overline{27}$

$99x = 225$

$x = \frac{225}{99}$

$x = \frac{25}{11}$

Hence we can write $2.\overline{27}$ under the form of a fraction as $\frac{25}{11}$

where $11, 25 \in \mathbb{Z}$.

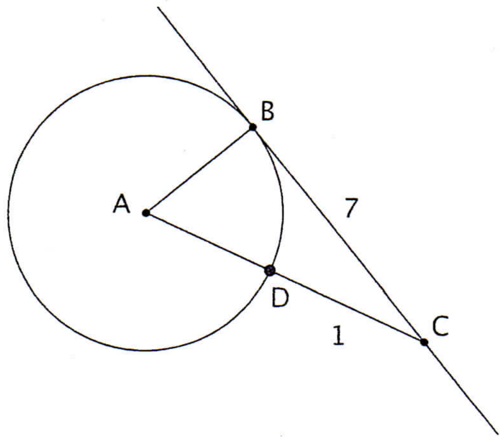
$\therefore 2.\overline{27} \in \mathbb{Q}$ hence it is a rational number.

Exercise 4 : (5 marks)

Find the radius of the circle with centre A.

(BC) is tangent to the circle, BC=7 and DC=1.

The diagram is not to scale.



Let x be the radius of the circle with centre A, thus $AB = AD = x$.

Using Pythagoras,

$AB^2 + BC^2 = AC^2$

$x^2 + 7^2 = (x+1)^2$

$x^2 + 49 = x^2 + 2x + 1$

$2x = 48$

$x = 24$

$\therefore$ The radius of the circle with centre A is 24.

7.5

Exercise 5 : (8 marks)

1. Let $f(x) = 2x^2 + x - 15$ Find the pre-image(s) of 0 :

$f(x) = 0$
$2x^2 + x - 15 = 0$
$2x^2 - 5x + 6x - 15 = 0$
$x(2x-5) + 3(2x-5) = 0$
$(2x-5)(x+3) = 0$
$2x - 5 = 0$ or $x + 3 = 0$
$x = \frac{5}{2}$ or $x = -3$
$\therefore$ $\frac{5}{2}$ and -3 are the pre-images of 0.

2. Let $g(x) = \frac{x+1}{1-x}$:

a) Find the image of $\frac{\sqrt{3}}{2}$, giving your answer with integer denominator :

$$g\left(\frac{\sqrt{3}}{2}\right) = \frac{\frac{\sqrt{3}}{2} + 1}{1 - \frac{\sqrt{3}}{2}}$$
$$= \frac{\frac{\sqrt{3}+2}{2}}{\frac{2-\sqrt{3}}{2}}$$
$$= \frac{\sqrt{3}+2}{2-\sqrt{3}} \times \frac{\cancel{2}}{\cancel{2}}$$
$$= \frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})}$$
$$= \frac{4+4\sqrt{3}+3}{1}$$
$$= 7 + 4\sqrt{3}$$

b) Write down the domain of g : $\mathbb{R} \setminus \{1\}$

3. Let $h(x) = -(x+2)^2 + 5$:

a) Write down the range of h : $]-\infty; 5]$

b) Solve $h(x) = 6$:

$-(x+2)^2 + 5 = 6$
$-(x+2)^2 = 1$
$(x+2)^2 = -1$
$\because (x+2)^2 \geq 0$
$\therefore S = \phi$

$h(x) = -4$:

$-(x+2)^2 + 5 = -4$
$-(x+2)^2 = -9$
$(x+2)^2 = 9$
$x + 2 = 3$ or $x + 2 = -3$
$x = 1$ or $x = -5$
$\therefore S = \{-5; 1\}$

Exercise 6 : (5 marks)

Given x and y such that $2 < x < 7$ and $1 < y < 3$, find the range of values of :

$xy - 1$:
$2 < xy < 21$
$\therefore 1 < xy - 1 < 20$

$2x - y$:
$4 < 2x < 14$
$-3 < -y < -1$
$\therefore 1 < 2x - y < 13$

$\sqrt{x+2}$:
$4 < x + 2 < 9$
$\therefore 2 < \sqrt{x+2} < 3$

$x + \frac{1}{y}$:
$\frac{1}{3} < \frac{1}{y} < 1$
$\therefore \frac{7}{3} < x + \frac{1}{y} < 8$

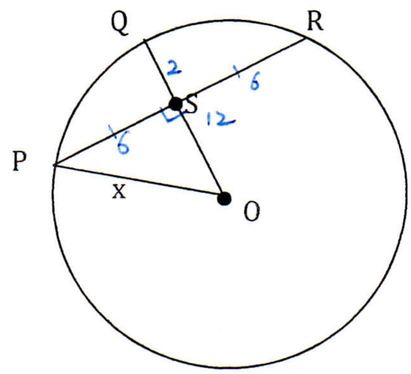
$(x-5)^2$:
$-3 < x - 5 < 2$
$(x-5)^2 \geq 0$
$(x-5)^2 < 9$
$\therefore 0 \leq (x-5)^2 < 9$

Exercise 7 : (6 marks)

In the circle with center O , (OQ) is perpendicular to (PR) .

$PR = 12$ units and $SQ = 2$ units.

Let x be the length of $[OP]$. Determine x .



In the circle with centre O, PR is a chord perpendicular to OQ, the radius.
OQ bisects PR and
$\therefore PS = \frac{1}{2} PR = \frac{1}{2} \times 12 = 6$

In the circle with centre O, OP, OQ are all radii of the circle.
$\therefore OP = OQ = x$

Using Pythagoras,
$OS^2 + PS^2 = OP^2$
$(OQ - QS)^2 + PS^2 = OP^2$
$(x-2)^2 + 6^2 = x^2$
$x^2 - 4x + 4 + 36 = x^2$
$x = 10$ $\quad \therefore$ The length of $[OP]$ is 10 units.

Exercise 8 : (6 marks)

Solve the following simultaneous inequalities over $\mathbb{Z}$:

$$\begin{cases} -4 \leq \frac{5-2x}{3} \leq 5 \\ \frac{1-3x}{4} > -2 \end{cases}$$

$-4 \leq \frac{5-2x}{3} \leq 5$ $\qquad$ $\frac{1-3x}{4} > -2$

$-12 \leq 5-2x \leq 15$ $\qquad$ $1-3x > -8$

$-17 \leq -2x \leq 10$ $\qquad$ $-3x > -9$

$-5 \leq x \leq \frac{17}{2}$ $\qquad$ $x < 3$

$\therefore$ Over $\mathbb{R}$, $S = [-5; \frac{17}{2}] \cap]-\infty; 3[= [-5; 3[$

$\therefore$ Over $\mathbb{Z}$, $S = \{-5; -4; -3; -2; -1; 0; 1; 2\}$

Exercise 9 : (13 marks)

1. Simplify the following rational expression : $\frac{3x^2-5x-2}{4x^2-7x-2}$

$$\frac{3x^2-5x-2}{4x^2-7x-2}$$

$$= \frac{3x^2+x-6x-2}{4x^2+x-8x-2}$$

$$= \frac{(3x+1)(x-2)}{(4x+1)(x-2)}$$

$$= \frac{3x+1}{4x+1}$$

2. Find x when $\frac{3x}{x-4} - \frac{60}{(x+1)(x-4)}$ is :

a) undefined : when denominator is 0:

$x-4=0$ or $(x+1)(x-4)=0$

$x=4$ or $x=-1$ or $x=4$

$\therefore$ ~~S~~ = $\{-1;4\}$ ✗

no "S" here

$x \in \{-1;4\}$

b) equal to 0 :

$$\frac{3x}{x-4} - \frac{60}{(x+1)(x-4)} = 0$$

$3x(x+1) - 60 = 0 \quad (x \neq -1 \text{ and } x \neq 4)$

$3x^2+3x-60 = 0$

$x^2+x-20 = 0$

$(x+5)(x-4) = 0$

$x+5=0$ or $x-4=0$

$x=-5$ or $x=4$

$\because x \neq 4$

$\therefore S = \{-5\}$

3. Simplify : $\frac{2a+4}{1-\frac{4}{a^2}}$

$$= \frac{2a+4}{\frac{a^2-4}{a^2}}$$

$$= 2(a+2) \times \frac{a^2}{(a+2)(a-2)}$$

$$= \frac{2a^2}{a-2}$$

Exercise 10 : (3 marks)

Make x the subject of $y = \frac{3x+2}{x-1}$.

$(x-1)y = 3x+2$

$xy - 3x = y+2$

$(y-3)x = y+2$

$$\therefore x = \frac{y+2}{y-3}$$