杨[illegible] 121090699 hw1

1. (a) Decision variables: quantity x of the first type product, quantity y of the second type product.

Objective: maximize daily profit: $(9x-1.2x)+(8y-0.9y) = 7.8x+7.1y$

Constraints: ① assembly labor: $\frac{1}{8}x+\frac{1}{2}y \le 90$

② testing: $\frac{1}{4}x+\frac{1}{6}y \le 80$

The optimization problem can be written as:

$$\begin{aligned}\text{maximize}_{x,y}\quad & 7.8x+7.1y\\ \text{subject to}\quad & \begin{cases}\frac{1}{8}x+\frac{1}{2}y\le 90\\ \frac{1}{4}x+\frac{1}{6}y\le 80\\ x,y\ge 0\end{cases}\end{aligned}$$

(b) standard form:

$$\begin{aligned}\text{minimize}_{x}\quad & -7.8x_1-7.1x_2\\ \text{subject to}\quad & \frac{1}{8}x_1+\frac{1}{2}x_2+s_1=90\\ & \frac{1}{4}x_1+\frac{1}{6}x_2+s_2=80\\ & x_1,x_2,s_1,s_2\ge 0\end{aligned}$$

(c) Decision variables: additional z assembly labor.

The optimization problem can be written as:

$$\begin{aligned}\max\quad & 7.8x+7.1y-8z\\ \text{subject to}\quad & \frac{1}{8}x+\frac{1}{2}y-z\le 90\\ & \frac{1}{4}x+\frac{1}{6}y\le 80\\ & z\le 40\\ & x,y,z\ge 0\end{aligned}$$

see code: q1.m

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Status: Solved
Optimal value (cvx_optval): +2724

x1 =

  240.0000

x2 =

  120.0000
```

(d) Daily profit maximization: 2724

240 first type products, 120 second type products.

2. Define $G=(V,E)$, where $V=\{1,2,3,4,5\}$ is the set of locations, and E is the set of pairs of regions.

$W=\{w_{ij}\}$ is the set of cost of moving a car between each pair of regions.

Decision variables: use x_{ij} to denote moving x_{ij} cars from i to j.

Optimization formulation:

$$\text{minimize}\quad \sum_{(i,j)\in E} w_{ij}x_{ij}$$

$$\text{subject to}\quad \sum_{i=1}^{5} x_{i1} - \sum_{j=1}^{5} x_{1j} \ge 40$$

$$\sum_{j=1}^{5} X_{2j} - \sum_{i=1}^{5} X_{i2} \le 135$$

$$\sum_{i=1}^{5} X_{i3} - \sum_{j=1}^{5} X_{3j} \geq 200$$

$$\sum_{j=1}^{5} X_{4j} - \sum_{i=1}^{5} X_{i4} \le 220$$

$$\sum_{j=1}^{5} X_{5j} - \sum_{i=1}^{5} X_{i5} \le 220$$

$$\sum_{j=1}^{5} X_{1j} \le 110$$

$$\sum_{j=1}^{5} X_{2j} \le 335$$

$$\sum_{j=1}^{5} X_{3j} \le 400$$

$$\sum_{j=1}^{5} X_{4j} \le 420$$

$$\sum_{j=1}^{5} X_{5j} \le 610$$

$$X_{ij} \geq 0, \quad \forall (i,j) \in E$$

```
-----------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +2400


X =

    0.0000    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000   20.0000    0.0000
    0.0000    0.0000    0.0000    0.0000    0.0000
   40.0000    0.0000  200.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000    0.0000
```

see code: q2.m.

Hence, least cost 2400.

move 20 cars from 2 to 4.

move 40 cars from 4 to 1.

move 200 cars from 4 to 3.

3. See attached code $\boxed{(q3.m)}$

(If there was no edge for (i,j), set $W(i,j) = 100$.)

Optimal path: $S \to 3 \to 5 \to 7 \to T$

Optimal length: 8

```
>> q3
Input an n*n matrix W(n>2): [100,5,4,100,100,100,100,100;
        5,100,100,3,100,7,100,100;
        4,100,100,100,1,2,100,100;
        100,3,100,100,2,100,100,100;
        100,100,1,2,100,100,2,5;
        100,7,2,100,100,100,100,3;
        100,100,100,100,2,100,100,1;
        100,100,100,100,5,2,1,100]
```

$\rightarrow$

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Status: Solved
Optimal value (cvx_optval): +8


X =

   -0.0000    0.0000    1.0000   -0.0000   -0.0000   -0.0000   -0.0000   -0.0000
   -0.0000   -0.0000   -0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000
   -0.0000   -0.0000   -0.0000   -0.0000    1.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000   -0.0000   -0.0000    0.0000   -0.0000   -0.0000   -0.0000
   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    1.0000    0.0000
   -0.0000   -0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000    0.0000
   -0.0000   -0.0000   -0.0000   -0.0000    0.0000   -0.0000   -0.0000    1.0000
   -0.0000   -0.0000   -0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000
```

4. (a) $\min \; 2x + 3|y - x|$

s.t. $|x+2| + |y| \le 5$, where $x, y \in \mathbb{R}$.

This can be equivalently written as:

Define $Z = |y-x|$, $Q = |x+2|$, $M = |y|$.

$\min \; 2x + 3Z$

s.t. $Z \geq y - x$

$Z \geq x - y$

$Q + M \le 5$

$Q \geq x + 2$

$Q \geq -x - 2$

$M \geq y$

$M \geq -y$

(b) $\min \; c^T x + f(d^T x)$

s.t. $Ax \geq b$

where $x, c, d \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $f(\alpha) = \max\{\alpha, 0, 2\alpha - 4\}$ for $\alpha \in \mathbb{R}$.

This can be equivalenetly written as:

Define $y = f(d^T x) = \max\{d^T x, 0, 2d^T x - 4\}$.

$$\begin{aligned} \text{minimize}_{x, y} \quad & c^T x + y \\ \text{s.t.} \quad & y \geq d^T x \\ & y \geq 0 \\ & y \geq 2d^T x - 4 \\ & Ax \geq b \end{aligned}$$

(c) $\min \; c^T x$

$$\begin{aligned} \text{s.t.} \quad & \|Ax - b\|_\infty \leq \delta \\ & x \geq 0 \end{aligned}$$

where $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$.

This can be equivalenetly written as:

$$\begin{aligned} \text{minimize}_{x} \quad & c^T x \\ \text{s.t.} \quad & A_i x - b_i \leq S, \quad \forall i \in [n] \\ & b_i - A_i x \leq S, \quad \forall i \in [n] \\ & x \geq 0 \end{aligned}$$