HW3: MATH/CSCI-4800-02 Numerical Computing

1. Text problem on p.43: problem 8.
   (*if you use 2nd edition of the book, it is problem 8 on p. 41*).

2. Find each fixed point of $g(x) = x^2 - \frac{3}{2}x + \frac{3}{2}$ and decide whether fixed point iteration is locally convergent to it.

3. Express $2x^3 - x + e^x = 0$ as a fixed point problem $x = g(x)$ in three different ways.

4. Text problem on p.44: problem 14 (refer to Definition 1.5 in the textbook for the linear convergence rate S, as well as Theorem 1.6).
   (*if you use 2nd edition of the book, it is problem 14 on p.41*).

5. Text problem on p.121: problem 2(c).
   (*if you use 2nd edition of the book, it is problem 2(c) on p.116.*).

6. This is to revisit some examples and theories discussed in class. To solve $Ax = b$, where

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}. \tag{1}$$

   Note that this A is strictly diagonally dominant. It is known that both Jacobi and Gauss-Seidel methods lead to iterative methods in the following form

$$x^{k+1} = Bx^k + d, \tag{2}$$

   for some $B \in \mathbb{R}^{2\times 2}$ and $d \in \mathbb{R}^2$.

   For each of Jacobi and Gauss-Seidel methods, answer the following. You can either calculate by hand or use Matlab.

   - What is B?
   - What is the spectral radius $\rho(B)$ of B. Here $\rho(B) = \max(|\lambda_1|, |\lambda_2|)$ where λ_1, λ_2 are the two eigenvalues of B. (If you want to calculate using matlab, you may find *eigs* useful.)
   - From the value of $\rho(B)$, decide whether the corresponding scheme (2) converges or diverges.
   - Repeat the three questions above for the following A,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}. \tag{3}$$

     Note that this A is not strictly diagonally dominant.

7. Recommended Reading: Section 2.5.2 about SOR method (this is not covered in the exam), and the comparison of three methods in Example 2.24.