HW2: MATH/CSCI-4800-02 Numerical Computing

Due 2pm on 2.7.2019 (Thursday)

1. The function $f(x) = x^5 - x^4 - x + 1$ is plotted in Figure 1. Propose an INDIRECT computational strategy to find the solution $x = 1$ using the bisection method. (You do not need to implement the proposed strategy.)

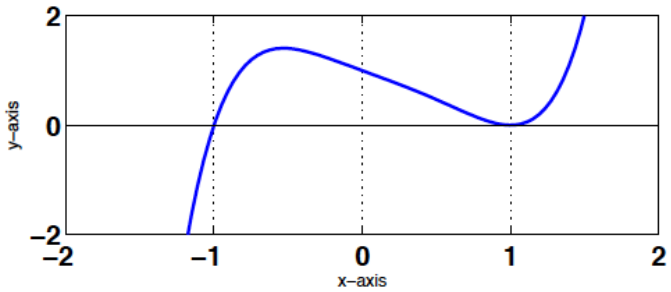

Figure 1: Plot of $f(x) = x^5 - x^4 - x + 1$

2. This problem examines how to use an iterative method to calculate the positive square root of a positive number α, by only involving four elementary arithmetic operations (addition, subtraction, multiplication and division).

   (a) Convert this problem to a root-finding problem, which can be numerically solved by bisection method and Newton's method.

   (b) To apply bisection method when $\alpha = 3$, propose an initial interval $[a_0, b_0]$ and justify your choice. With this starting interval $[a_0, b_0]$, calculate by hand c_0 and c_1.

   (c) Write down the update formula of the Newton's method. Propose an initial guess x_0, and justify your choice. With this x_0, calculate x_1 and x_2 by hand when $\alpha = 3$.

3. Given t, the value of y is determined by solving $y + t = e^{-y}$.

   (a) Sketch the left and right sides of this equation as a function of y and explain why, for any given t, there is exactly one solution.

   (b) For a given t, explain how secant method can be used to find y. Write down the update formula.

4. Computer problem: use the bisection method to calculate the smallest positive solution of $\cos(x) = 2\sin(x)$ within six correct decimal places. Tabulate the steps and your computed solutions. Make sure to state the initial interval and the stopping criterion. (*Refer to Section 1.1.2 of the textbook, especially Definition 1.3.*)

5. Computer problem: write your code to implement Newton's method, and apply it to problem 8 on page 63 (*if you use 2nd edition of the book, it is problem 8 on page 60*). And tabulate the steps and your computed solutions. Make sure to state your initial guess and the stopping used in your algorithm.

6. Text problem on page 53: 4 (*if you use 2nd edition of the book, it is text problem 4 on page 50*). (Additional reference: Section 1.3 of the textbook)

7. Use (1.21) in the textbook to approximate the root of $f(x) = (x-1)(x-2)(x-3)(x-4) - \epsilon x^6$ near $x = 4$, where $\epsilon = 10^{-6}, 10^{-8}$. Use fzero in Matlab to check your approximations.

Instructions:

- Justify your results and answers with sufficient details.
- For computer problems, in addition to the results and/or tables and/or plots (if required in the problem description), you are required to include the codes you write (such as scripts, functions etc). A log of your Matlab session can be included if you find it important to support your results.