HW1: MATH/CSCI-4800-02 Numerical Computing

Due 4pm on 1.22.2019 (Tuesday)

Note: Page numbers below are with respect to the textbook in 3rd edition. The actual problem descriptions are also given below, in case you are using a different edition of the textbook.

1. Text exercises on page 5: 6.(b)
   Explain how to evaluate the polynomial for a given point x, using as few operations as possible. How many multiplications and how many additions are required?
   $$P(x) = a_7x^7 + a_{12}x^{12} + a_{17}x^{17} + a_{22}x^{22} + a_{27}x^{27}$$

2. Computer problem on page 5: 2 (*either use nest.m or myPolyEval.m shared by Prof. Li*).
   Evaluate $P(x) = 1 - x + x^2 - x^3 + ... + x^{98} - x^{99}$ at $x = 1.00001$. Find a simpler, equivalent expression, and use it to estimate the error of the nested multiplication.

3. Convert the following binary numbers to base 10: (a) 101.101, (b) $10.\overline{101}$

4. Text exercises on page 16: 6.(a) (check your answer using matlab)
   Do the following sum by hand in IEEE double precision computer arithmetic, using the Rounding to Nearest Rule.
   $$(1 + (2^{-51} + 2^{-52} + 2^{-54})) - 1$$

5. Text exercises on page 17: 14.(c)
   Do the following operation by hand in IEEE double precision computer arithmetic, using the Rounding to Nearest Rule. (check your answer using matlab)
   $$(4.9 - 3.9) - 1$$

6. Let $x = 2$. To avoid subtraction of nearly equal numbers, find an alternative form to evaluate
   $$f(h) = \frac{x^4 - (x-h)^4}{h} \qquad (1)$$
   for small h. Compute $f(h)$ using matlab based on $f(h)$ in (1) and the alternative form of $f(h)$ you propose, and report your results for $h = 10^{-1}, 10^{-2}, \cdots, 10^{-15}$. Summarize and comment on your observations.

7. To find the roots of $ax^2 + bx + c = 0$, one can use the quadratic formula
   $$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \qquad (2)$$
   Consider an example with $a = 1$, $b = 1.234 \times 10^5$, and $c = 4.567 \times 10^3$.

   (a) Use 4-digit, base 10, floating-point arithmetic to compute two approximated roots $\hat{x}_1$ and $\hat{x}_2$ based on (2). (*This means, all intermediate steps also involve 4-digit, base 10, floating-point arithmetic.*) In addition, compute the two roots x_1 and x_2 using Matlab and *regard* them as the exact roots. Based on your results, compute the relative errors in $\hat{x}_1$ and $\hat{x}_2$. Are the 4-digits in $\hat{x}_1$ and $\hat{x}_2$ accurate?

(b) For the one which has larger relative error, rewrite the root formula based on (0.13) or (0.14) on page 20 of the textbook, and re-compute this approximate root. Still, use 4-digit, base 10, floating-point arithmetic. Now recompute the relative error of this re-computed root.

(Reference reading: Section 0.4)

Instructions:

- Justify your results and answers with sufficient details.
- For computer problems, in addition to the results and/or tables and/or plots (if required in the problem description), you are required to include the codes you write (such as scripts, functions etc). A log of your Matlab session can be included if you find it important to support your results.