HW6: MATH/CSCI-4800-02 Numerical Computing

(Not collected)

1. Given $A \in \mathbb{R}^{m\times n}$ with $m > n$, and $b \in \mathbb{R}^m$. Consider to solve $Ax = b$ for $x \in \mathbb{R}^n$.

   - Is $Ax = b$ always solvable? if yes, justify your answer. if no, for what b is $Ax = b$ solvable?
   - If $Ax = b$ is solvable for some given $b \in \mathbb{R}^m$, when will the solution $x \in \mathbb{R}^n$ be unique?
   - Give an example of A and b, such that $Ax = b$ is solvable yet the solution x is not unique.

2. Find the quadratic fit of the following data in the least squares sense, and find the corresponding *root mean squared error* (RMSE, refer to Section 4.1.1 of the textbook).

j	x_j	y_j
1	1	2
2	3	2
3	4	1
4	6	3

3. Given data points $(x, y, z) = (0, 0, 3)$, $(0, 1, 2)$, $(1, 0, 3)$, $(1, 1, 5)$, $(1, 2, 6)$, find the plane in three dimensions (model $z = c_1 + c_2x + c_3y$) that best fits the data (in the least squares sense).

4. Fit the following data (in the least squares sense) to the periodic model $y = f(t) = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$. Find the 2-norm error and the RMSE (refer to Section 4.1.1 and Section 4.2.1 of the textbook).

t	y
0	3
1/2	1
1	3
3/2	2

(1)