Exploratory Data Analysis

Lecture 9

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TECHNICAL UNIVERSITY
OF MOLDOVA

What is Hypothesis Testing

To understand hypothesis testing let us start with nonstatistical applications of it, the best known of which is a criminal trial.

When a person is accused of a crime, he or she faces a trial. The prosecution presents its case, and a jury must make a decision on the basis of the evidence presented.

In fact, the jury conducts a <u>test of hypothesis</u>.

- The first is called the null hypothesis and is represented by H_0:
  H_0: The defendant is innocent.
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There are only two possible decisions:

- Convicting the defendant $\rightarrow$ rejecting the null hypothesis (H_0) in favor of the alternative H_A.
  - There was enough evidence to conclude that the defendant was guilty.
- Acquitting the defendant $\rightarrow$ not rejecting the null hypothesis (H_0) in favor of the alternative H_A
  - There was not enough evidence to conclude that the defendant was guilty.

Notice that we do not say that we accept the null hypothesis!

This would be equal to saying that the defendant is innocent, and this is not a decision that the justice system is allowed to make.

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Type I error & Type II error

There are two possible errors:

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  - The probability of a Type I error $= \alpha$ (also called the *significance level*)
- Type II error $\rightarrow$ not rejecting a false H_0.
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  - The probability of a Type II error $= \beta$

The error probabilities α and β are inversely related $\rightarrow$ any attempt to reduce one will increase the other.

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Hypothesis Testing in Regression Analysis

The most often testsed hypothesis in regression analysis is:

$$H_0: \beta = 0$$
$$H_A: \beta \neq 0$$

It tells about the importance of the predictors included in the model:

- Conclude that there is enough evidence to support H_A $\rightarrow$ reject H_0 $\rightarrow$ there is a correlation between X and y and $\hat{\beta}$ is a statistically significant coefficient $\rightarrow$ X is an important predictor of y and we should include X into our model.
- Conclude that there is not enough evidence to support H_A $\rightarrow$ **cannot reject** H_0 $\rightarrow$ no correlation between X and y could be identified and $\hat{\beta}$ is not a statistically significant coefficient $\rightarrow$ X is not an important predictor of y and we should not include X into our model.

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How to decide whetherH_0 should or should not be rejected in favor of H_A?

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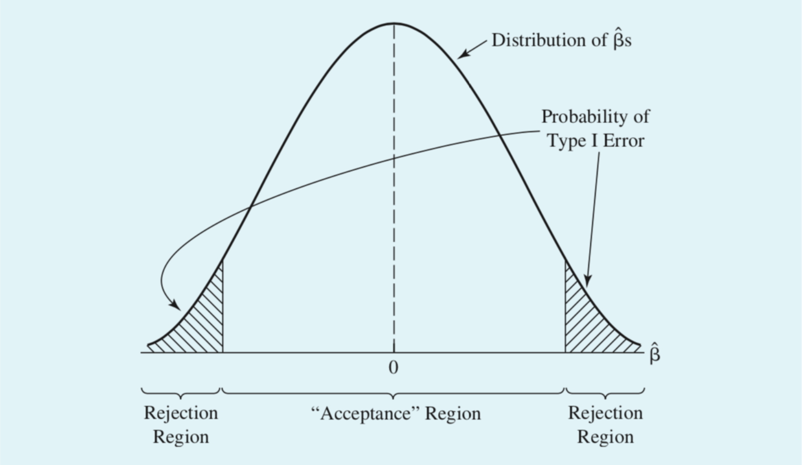

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$$\hat{Y}_i = 102{,}192 - 9075N_i + 0.3547P_i + 1.288I_i$$
$$\qquad\qquad (2053) \quad (0.0727) \quad (0.543)$$
$$t = -4.42 \qquad 4.88 \qquad 2.37$$
$$N = 33 \qquad \bar{R}^2 = .579$$

Question: Is P_i an important predictor?
In other words is β_P significantly different from zero?
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Decide whether to reject or not to reject H_0 based on the critical t-value which distinguishes the acceptance region from the rejection region.

Let's inspect the t-table

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2) Decide on the **Significance Level** - α

The significance level measures the amount of Type I Error implied by a particular t-critical.

For example: If the level of significance is 10%, and we reject H_0 at 10% significance level $\rightarrow$ 10% percent of the time H_0 was correct, but we rejected it.

Think of it as a margin of error that you accept.

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Inspect the table to find t-critical

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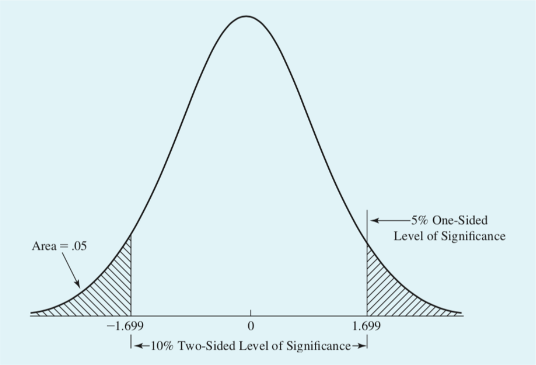

Decision Rule:

Reject H_0 if $-1.699 < t_p > 1.699$ else do not reject.

$t_p = 4.88 > 1.699 \rightarrow$ Reject $H_0 : \beta_P = 0$ in favor of $H_A : \beta_P \neq 0$

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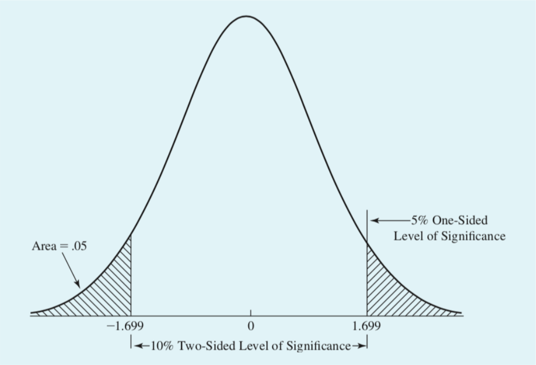

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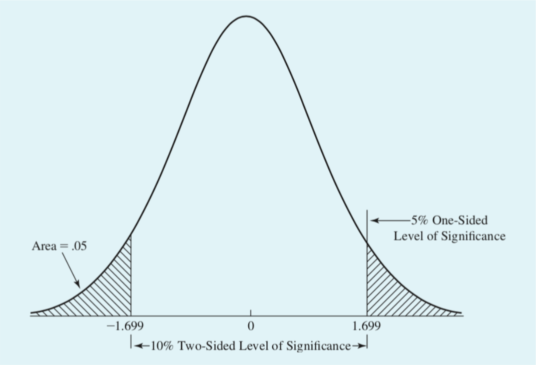

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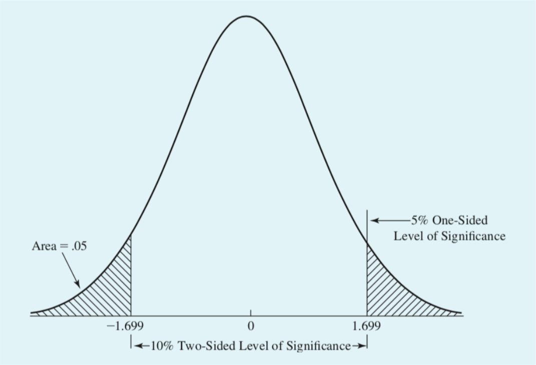

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Let's get Started!

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