FINA 279, Spring 2018
Assignment
Due Date:
May 3 (Thursday section), May 4(Friday section), May 7 (Monday section)

Submit the assignment by 5pm on due date. Be sure to include all relevant calculations to get partial credit. Email one copy to me (sagca@gwu.edu) and one copy to course assistant, Fuhong Li (fli15@gwmail.gwu.edu). I will notify you when I got your assignment. If you do not get any notification, it means that I haven't received it. Assignments that are not received on time will not be graded. Please plan ahead.

Part I:
You have Treasury bill and bond quotes on March 15, 2018. The dataset is at the course homepage named the *hw1data spring18.xls*. The Treasury bills are quoted in discount yields and Treasury bond prices are **full prices in decimals**. There are three sets of the data: First set has 8 Treasury securities, second part has 5 Treasuries and third set has 4 Treasuries. Note that quoted prices are full prices.

A. For all Treasuries in the data (i.e. both first, second and third set Treasuries):

   1. What are the bid and ask prices of the Treasuries for a $100 face value. Use **ask** prices for below questions.
   2. For the Treasury bills, why is bid discount yield higher than ask discount yield? Please explain.

B. For this part, use only the 8 Treasuries in the *first set*.

   2. a) Using bootstrap method long way, extract zero coupon bonds.
      b) Calculate spot rates of every six months up to 4 years
      c) Plot the spot rate curve

   3. a) Using short way of matrix inversion extract zero coupon bonds.
      b) Do you find the same zeros as in 2(a)? Why? Why not?

C. For this part, use *first and second set*, i.e. total of 13 Treasuries

   4.
      a) Using regression methods, extract zero coupon bonds. (Note: Force the intercept to zero in the regression).
      b) Calculate 6 month spot rates
      c) Plot the spot rate curve
      d) Compare the spot rates with the ones you found by bootstrapping. Are they different? Why? Why not?

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D. For this part, use all 17 Treasuries*, i.e. first, second, and third sets*.

5. Using regression methods, find zeros semiannually until year 4 and the annually until year 6. (Note: Force the intercept to zero in the regression).

6. Using *polynomial spline method of order 3*

a) Fit the zero curve to find the zeros for year 4.5 and 5.5.
b) Calculate 6 month spot rates for 6 years and plot the spot rate curve

7. Using *exponential spline method of order 3*

a) a) Fit the zero curve to find the zeros for year 4.5 and 5.5. Take initial values of α, β_1, β_2, β_3, and λ as **0.01** while using the Solver.
b) Calculate 6 month spot rate for 4 years and plot the spot rate curve

8. Using *Nelson-Siegel method*

a) Fit the spot curve to find the spot rate for year 4.5 and 5.5. Take initial values of α, β_1, β_2, and λ as **1** while using the Solver.
b) What are the 6 month spot rates for 6 years? What are the 6 months zeros? Plot the spot rate curve.
c) Using the spot rates, find 6 month forward rates after 6 months, 1 year, 1.5 years, .. up to 6 years.
d) You want to take a two year loan after three years. How much is the minimum that a bank will charge you considering the current market rates?

Part II:

Suppose on 03/15/2018, you are expecting that the yield curve will flatten. You want to enter into a spread trade using $100 million face value of 20-year bond and a corresponding amount of 2-year bond. The quotes related to these bonds are below. Note that quoted bid and ask prices are clean prices. Take one year as 360 days. Also note that you buy at the ask and sell at the bid.

Maturity	**Coupon (%)**	**Bid**	**Asked**	**Chg**
3/15/2020	1.625	98.7109	98.7266	0.0078
3/15/2038	4.375	122.2344	122.2969	0.7656

1. Note that above quotes are clean prices. Find the full prices of each bond.
2. Find yield to maturity of each bond using full ask prices (ask yield)
3. Find the PVBP of each bond using ask prices and ask yields.
4. How much will you buy or sell of 2-year bond on 3/15/2018 for the spread trade? Will you buy or sell $100 million face value of 20-year bond? Why did you buy/sell of each bond? (Use full prices)

To finance this transaction you decided to enter into a 3-day repo transaction. The repo dealer offers 1% on reverse repo and 1.25% on repo transaction. Also, the repo dealer gets 0.5% haircut. After 3 days the quotes are as follows:

Maturity	**Coupon (%)**	**Bid**	**Asked**	**Chg**
3/15/2020	1.675	98.0859	98.1016	0.0156
3/15/2038	4.375	122.4063	122.4688	0.7891

5.

a) Show the details of the repo transaction. What is the profit (loss) on repo transaction?
b) Show the details of the reverse repo transaction. What is the profit (loss) of the reverse repo transaction?
c) What is the total profit (loss) of these two transactions?
d) What is the return on your own capital from these transactions?
e) Were you correct at predicting flatenning yield curve? Why? Why not?

Overall Hint:
You can do all this assignment in Excel. In fact in most of the parts, it is easier to do it in Excel.

Part III:

Suppose, using market coupon bond data, you extracted different maturity annual zeros. They are as follows:

Z(0,0.5)	0.9908
Z(0,1)	0.9795
Z(0,1.5)	0.9680
Z(0,2)	0.9558
Z(0,2.5)	0.9432
Z(0,3)	0.9301
Z(0,3.5)	0.9153
Z(0,4)	0.8997

Since you have prices available semiannually, you should take Δt=0.5 while forming the tree. Take up and down probability as 0.5. Use above zeros for Part I and Part II below

1. Calibrate Vasicek (1977) model up to 2 years using the discount factors above.
   - Write the stochastic process of the model
   - Write the discretized model. (*Hint: You will use the discretized model and a binomial tree to calibrate Vasicek (1977)*)
   - Use r(0,0.5) you as the starting point of the binomial tree.
   - Use σ=0.006
   - Choose **initial values** for k and b of the Vasicek model as **k=0.01 and b=0.01**, form the semiannual spot rate binomial tree using the initial values. Form corresponding discount factor tree.
   - Calculate 0.5 year, 1 year, 1.5 year, 2 year zeros of the model using the binomial tree.
   - Use solver to find the model parameters k and b that gives the closest values to the market zeros. **While using solver, put a constraint on k such that k cannot be negative.**
   - What are the model parameters (k and b) of Vasicek? What is the interpretation of k and b, i.e. what do they tell us about the interest rate?

Suppose, you analyzed the historical volatility of the spot rates and you find that spot rate volatility follows below function:

$$\sigma(t+\Delta t) = \sigma(0)\times(1+0.001\sqrt{\Delta t})\times\exp(-0.05\times\Delta t)$$

$$\sigma(0) = 0.004$$

Your aim is to calibrate these prices to binomial trees for different models.
Since you have prices available for every year, you should take Δt=0.5 while forming the tree.

2. Fit the zeros to a normally distributed spot rate using Excel Solver. In other words, form annual normally distributed spot rate and corresponding discount factor trees that are consistent with the market value of zeros. While doing this, use Excel Solver. (*Hint: use Pascal triangle to find how many ways exist to go to a particular node.*)

Pascal Triangle:

							1
						1	
					1		7
				1		6	
			1		5		21
		1		4		15	
	1		3		10		35
1		2		6		20	
	1		3		10		35
		1		4		15	
			1		5		21
				1		6	
					1		7
						1	
							1

3. Calibrate generalized Ho and Lee model to market prices. What is the stochastic process of the interest rate in this model? Estimate the parameters of the stochastic process.

4. Calibrate Hull and White-generalized Vasicek model to market prices. Estimate the parameters of the stochastic process of the interest rate in this model. Use below stochastic process:

$$dr(t)=(\gamma(t)-k(t)r(t))dt+\sigma(t)dW(t)$$

5. Fit the zeros to a lognormally distributed spot rate using Excel solver. In other words, form annual lognormally distributed spot rate and corresponding discount factor trees that are consistent with the market value of zeros. While doing this, use Excel solver.

6. Calibrate Black-Derman-Toy model to market prices. Assume that $\sigma(t+\Delta t)$ are forward volatilities. What is the stochastic process of the interest rate of this model? Estimate the parameters of the stochastic process.

7. Calibrate Black Karasinski model to market prices. Assume that $\sigma(t+\Delta t)$ are forward volatilities. What is the stochastic process of the interest rate in this model? Estimate the parameters of the stochastic process. (*Hint: This is similar to Hull and White generalized Vasicek model. The only difference is that spot rates are lognormally distributed. So, you need to force the tree to recombine similar to the way we forced it in Hull and White*.)

Part IV:

1. Suppose you calibrated a recombining Hull and White model to market data and you found that

$$\sigma(t + \Delta t) = \sigma(0) * \exp(-0.07 \times \Delta t)$$
$$r(0) = 0.02$$
$$\sigma(0) = 0.006$$

   a) Form an 8 period spot rate binomial tree. Take $\Delta t = 0.5$. So each period are semiannual periods and therefore you are forming a 4-year tree.
   b) Determine the zero tree according to the spot rate tree you found in (a)
   c) Find the value of zeros today that matures in 6 months, 1 year, 1.5 years, 2 years, 3 years, 3.5 years and 4 years.

2. Use the tree you formed in (1) to answer below questions.
   a) Determine the tree of the ex-coupon price of a 4% coupon bond that matures in 4 years. What is the price of the bond today?
   b) What is the price of the bond in (a) if it is a callable bond and the redemption value of the call is $104. Suppose that the bond is not callable for the first two years.

3. Use the tree you formed in (1) to answer below questions. Take notional principal as $1 whenever you need a notional principal.
   a) Determine the fixed interest rate of FRAs that mature in 1 year, 2.5 years and 3 years.
   b) Determine the fixed rates on a 6 month swap, a 2 year swap, and a 3 year swap. Swap payments occur every six months. Note that, since you are using semiannual tree, swap rates you found will be semiannual rates. Multiply it by 2 to find the annual swap rate.
   c) What is the value of a 4 year floor if the strike is 7%?
   d) What is the price of a receiver swaption on a 2 year swap if the swaption matures in 1 year and has the strike of 2%? Swap payments occur every six months.