CIVIL-557
Extensions of latent classes and model comparison

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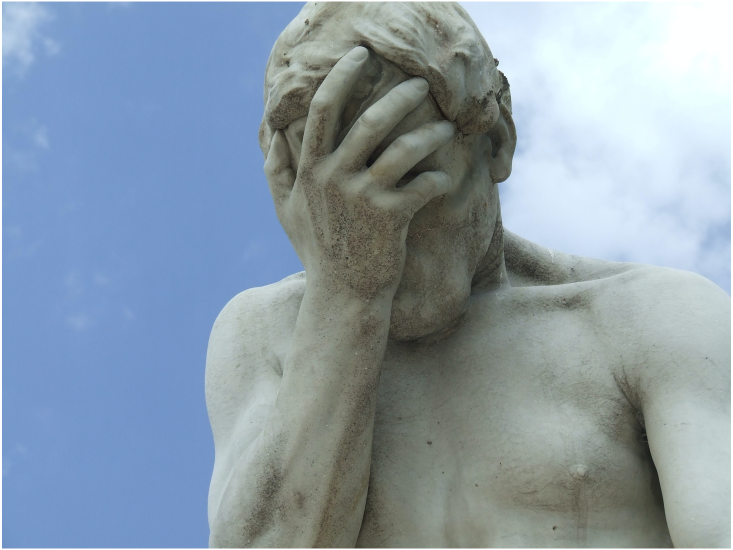
Previously on Decision Aid Methodologies...

Previous lectures

- Fundamentals of statistical modelling
- Introduction to driving behaviour models
- Car-following models
- Random effects
- Latent classes

Today it's the end!!

But first..



Lecture overview

- Metrics of model comparison (goodness-of-fit) and transferability (revision)
- Lane-changing models
  - Extension of latent classes
- Project

Goodness-of-fit

Likelihood ratio (LR) test

- Log-likelihood is a metric of how well the model fits the data
- The LR test can be used as a metric of relative improvement between a model (unrestricted model) and a simpler version of it (restricted model)
- We can evaluate the improvement of model fit (if any) after the addition of time of day using the likelihood ration test as: $LR = -2(LL^{restricted} - LL^{unrestricted})$

Goodness-of-fit

– For instance, I estimated a model and got $LL^{restricted} = -6434.891$

– Then, I added 3 new independent variables and received $LL^{unrestricted} = -6177.035$

$$LR = -2(LL^{restricted} - LL^{unrestricted}) = -2(-6434.891 - [-6177.035]) = 515.712$$

We must evaluate this result for 3 degrees of freedom (DoF are equal to the number of newly added parameters) at the 0.05 level of significance

– We find the critical values in the χ^2-distribution table If LR $> \chi^2$ critical value

then the addition of the new variable is significantly improving model fit In our

case this value is $515.712 > 7.81$ hence, model fit is significantly improved

⚠ LR test can be only used if the old model is a simplified version of the new model. E.g. if all new parameters were estimated with 0 value then the two models would have identical specification.

Goodness-of-fit

Chi-square Distribution Table

d.f.	.995	.99	.975	.95	.9	.1	.05	.025	.01
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41

Goodness-of-fit

To compare models of different specification (same dependent variable)

Akaike Information Criterion (AIC):

$\mathrm{AIC} = -2\ln(L) + 2k$

where L is the likelihood of the model and k is the number of parameters in the model.

Bayesian Information Criterion (BIC):
$\mathrm{BIC} = -2\ln(L) + k\ln(n)$

where n is the number of observations in the data.

Smaller AIC and BIC values suggest better fit. AIC and BIC penalise the number of parameters and observations allowing their use on different model specifications.

Comparison of individual parameters

t-test of individual parameter equivalence (Galbraith and Hensher, 1982)

– For each of the model parameters it was calculated the term

$$t_{diff} = \frac{\beta_k - \beta_{k*}}{\sqrt{\sigma_k^2 + \sigma_{k*}^2}} \tag{1}$$

where:

β_k: parameter estimates of model k

β_{k*}: parameter estimates of model k*

σ_k and σ_{k*}, standard errors

If $|t_{diff}| > 1.96$ then significant difference

Comparison of models overall

– The test can be always used for the continuous models that we have seen so far.

– The test of individual parameters cannot be used in logit models

– The logit model parameters include a scale coefficient μ which cannot be estimated and differs across models

– Alternatively evaluate model overall:

- We have Model 1 and Model 2 and want to see how close they are
- Model 1 is estimated from dataset1 and Model 2 is estimated from dataset2
- We use the results of Model 2 as starting values on dataset1 (or the opposite)
- We take note of the initial log-likelihood value

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Comparison of models overall - continued

- We implement the LR test with degrees of freedom equal to the number of parameters (of Model 2)
- The $LL^{unrestricted}$ is the LL of Model 1
- The $LL^{restricted}$ is the initial LL when the results of Model 2 are implemented as starting values on dataset 1

Lane-changing models

Two types of lane-changing models

- Modelling lane-changing decision making process
- Modelling the impact on the surrounding traffic

Modelling lane-changing decision making process

- Rule based (Gipps'-type) models
- Utility based models
- Cellular automata models
- Hazard-based models
- Fuzzy logic models
- Game theory models
- Machine learning models

Rule-based models

Gipps' model (1986), deterministic rules for lane-changing

- Risk of collision
- Obstructions
- Heavy vehicles
- Special purpose lanes
- Intention for taking an exit
- Speed advantage

Rule-based models

Yang and Koutsopoulos (1996), probabilistic approach

- Discretionary lane-changing
- Mandatory lane-changing

For mandatory lane-changing

- A probability for a mandatory lane-change to take place
- A gap-acceptance model based on deterministic critical gap rules

For discretionary lane-changing

- It is checked whether lane-changing will improve current travel attributes (e.g allow for higher speed)
- A gap-acceptance model is also considered

Utility-based models

– Ahmed (1999) developed a more flexible version of the lane-changing model of Yang and Koutsopoulos (1996)

– Toledo (2003), merged the discretionary and mandatory lane-changing models in the same specification

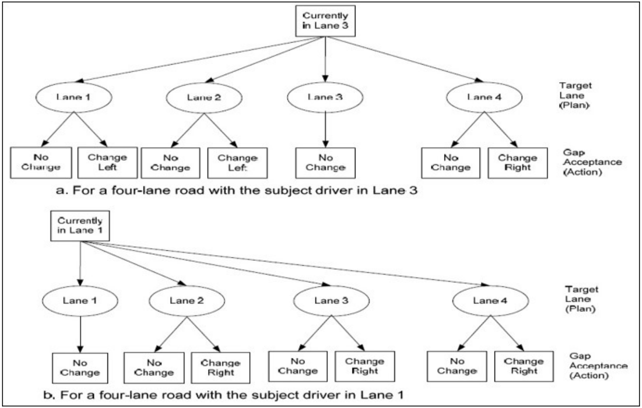
– Choudhury (2007) introduced the concept of latent plans

Latent plans lane-changing

– We observe the actions of a driver (lane-change or not) but not the intention Examples:

- A driver executes a lane-changing manoeuvre aiming to reach an even further lane
- A driver stays in a lane not because it is the target lane but because traffic conditions do not allow for a lane-changing manoeuvre

Latent plans lane-changing model



The latent plan framework of Choudhury's (2007) lane-change model

Latent plans lane-changing model

Model components:

- Target lane model
- Gap-acceptance model

Latent plans lane-changing model

Target lane model:

- A driver chooses the lane with the highest utility as the target lane
  The typical linear utility function $U_{nt} = V_{nt} + \epsilon_{nt}$ can be used to determine utility
- The factors affecting target lane utility can be:
  - Lane attributes
  - Neighbouring traffic attributes
  - Path-plan attributes

Latent plans lane-changing model

Target lane model:

- The probability for a lane to be the target lane is

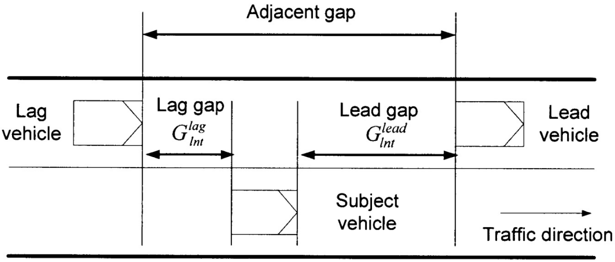
$$P_n(l_t) = \frac{e^{V_{lnt}}}{\sum_{l' \in L_n} e^{V_{l'nt}}}$$

Latent plans lane-changing model

Gap-acceptance model:

- The direction of lane-change is determined by the lane with the higher utility
- Then, a driver evaluates whether the traffic conditions allow for a lane-changing manoeuvre towards that direction

Latent plans lane-changing model



Latent plans lane-changing model

Gap-acceptance model:

- Critical gap: The minimum gap size a driver would accept to perform a lane-changing manoeuvre
- Critical gap is not observed - approximated via explanatory variables
- Usually assumed to follow a log-normal distribution

$$G_{lnt}^{gcr} = e^{\beta X_{lnt}^{g} + \epsilon_{lnt}^{g}} \; , g \in (lead, lag)$$

Latent plans lane-changing model

Gap-acceptance model:

$$P_n[G^g_{lnt} > G^{gcr}_{lnt}] = P_n[ln(G^g_{lnt}) > ln(G^{gcr}_{lnt})] = \Phi\left[\frac{ln(G^g_{lnt}) - \beta X^g_{lnt}}{\sigma_g}\right]$$

$\epsilon^g_{lnt} \sim N(0, \sigma^2_g)$

The use of log-normal distribution (else using $e^{\beta X}$) also ensures positivity. A gap is not allowed to be estimated as negative.

Latent plans lane-changing model

Gap-acceptance model:

The total gap-acceptance probability takes into account both the lead and the lag gaps as:

$$P_n^{accept} = P_n[ln(G_{lnt}^{lead}) > ln(G_{lnt}^{cr,lead})]P_n[ln(G_{lnt}^{lag}) > ln(G_{lnt}^{cr,lag})] =$$

$$\Phi\left[\frac{ln(G_{lnt}^{lead}) - \beta X_{lnt}^{lead}}{\sigma_{lead}}\right]\Phi\left[\frac{ln(G_{lnt}^{lag}) - \beta X_{lnt}^{lag}}{\sigma_{lag}}\right]$$

The probability of rejecting any of the adjacent gaps is $P_n^{reject} = 1 - P_n^{accept}$

Latent plans lane-changing model

The total probability of a lane-changing manoeuvre is:

$$P_n^{total} = P_n(l_t)P_n^g \quad, g \in (lead, lag)$$

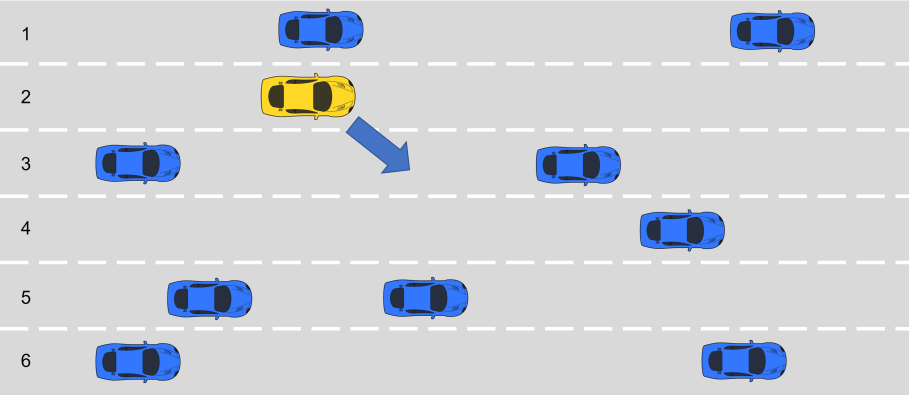
where:

$$P_n(l_t) = \sum_{l \in L_n} P_n$$

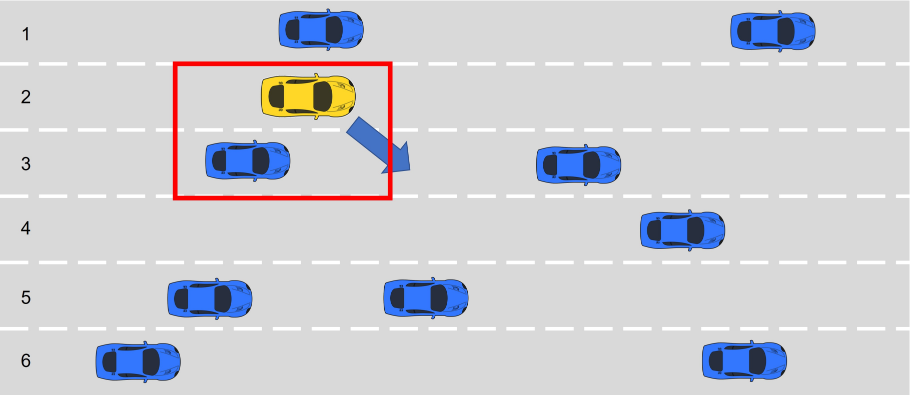
towards the direction of the target lane

Now let's see how to implement this in code...

Typical lane-changing situation



Negative gap situation in the data



Lab task

We want to simplify the lane-changing model. In particular, we want to model the decision of changing lane towards left or right (or do nothing) without considering the impact of all possible target lanes. This means:

- The discrete outcome has now only three alternatives (left, right, do-nothing)
- The utility of each outcome is relative to the current lane of a driver
- The probability of a lane-change is the product of a probability to move towards that lane and the probability of accepting the available gap
- The total probability of do-nothing is a function the probability of doing nothing and the probability of changing lane to any direction but the available gap is rejected

Lab task

Estimate the model considering the following variables/parameters

- Estimate four (4) parameter for the choice utility: lane density, lane speed, space headway and relative speed with the lead vehicle
- Estimate a lane-specific constant only for lane 1
- Do not forget to add two (2) constants to the model (e.g. a constant for left and right lane-changing probability)
- Nothing changes in the specification of the gap-acceptance probabilities compared to the existing lane-changing code
- In practice you just need to work on the existing lane-changing code with some slight modifications