

# 1 OPERATIONS RESEARCH

**Question 1.** (20 marks)

**Modeling:** The classical capacitated Vehicle Routing Problem (VRP) is a problem where a set of delivery requests (that is,  $N$ ) have to be fulfilled from a single central depot with a fleet of vehicles with the same capacity of  $Q$ .

- (i) In the course, we saw 3 models for the VRP. That is, the 2 index, 3 index and set partitioning formulations. Write your favorite formulation, and properly define the variables, each set, constraints, and objective function. **(12 marks)**
- (ii) In the Vehicle Routing Problem, sometimes routes have intra-route constraints that each route must satisfy. Describe 2 intra-route constraints for the VRP and explicitly write down the corresponding equation for each constraint. The constraints do not necessarily have to be part of the previous formulation. **(8 marks)**

Continuation Answer Q1:

Continuation Answer Q1:

**Question 2.**

(20 marks)

**Metaheuristics**

- (i) What are heuristic methods and why are they used instead of exact methods? Explain and justify your answer.  
**(5 marks)**
- (ii) How can we evaluate the quality of a heuristic method?  
**(5 marks)**
- (iii) In the course, we saw different heuristics to solve the Traveling Salesman Problem. For example, 2-opt, Simulated annealing, nearest-neighbor, etc. Describe clearly each step of your favorite heuristic, with pseudo code.  
**(10 marks)**

Continuation Answer Q2:

Continuation Answer Q2:

**Question 3.**

(20 marks)

**Shortest Path Problems**

- (i) What is the difference between the shortest path problem, shortest path problem with resource constraints and the elementary shortest path problem with resource constraints. Explain and justify your answer. **(3 marks)**
- (ii) In the context of the shortest path problem, describe explicitly what is the “principal of optimality”. **(7 marks)**
- (iii) To solve shortest path problems we can use a labeling algorithm, what is a label?**(4 marks)**
- (iv) In a labeling algorithm what are the feasibility checks and dominance rules used for? Explain and justify your answer.**(6 marks)**

Continuation Answer Q3:



**Question 4.**

(20 marks)

**Exact methods:**

In the following multiple choice questions (4.1 and 4.2) mark with an  $\mathcal{X}$  the correct answer.

**Question: 4.1**

In a combinatorial optimization problem like the Vehicle Routing Problem, what is a relaxation of the problem. **(3 marks)**

- ☐ A relaxation is when we replace the objective function with a linear expression and eliminate some feasible solutions.
- ☐ A relaxation is a new problem that results from removing some constraints from the original problem without eliminating feasible solutions.
- ☐ A relaxation is a new problem that adds new variables that are not binary to eliminate some feasible solutions.
- ☐ A relaxation is a new problem obtained by removing some variables and adding them at a later step.

**Question: 4.2**

What is an ng-route: **(3 marks)**

- ☐ A route that cycles only with close neighbors.
- ☐ A route that has no cycles.
- ☐ A route that has cycles only when distant neighbors are in the cycle.
- ☐ A route that violates resource constraints.

**Question: 4.3**

At each iteration of a Branch-and-Bound algorithm we have to select a node of the enumeration tree to evaluate, one way to select a node is to select one randomly. However, there are other more intelligent ways to select a node in the enumeration tree. Write down and explain a node selection strategy that would be better than just randomly selecting a node. **(14 marks)**

Continuation Answer Q4: