A Research of Confidence Level for Edge Detection

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Abstract

Edge detection is widely used in many fields, including image processing, magnetic resonance imaging (MRI), and radar reconnaissance. A crucial composition of images, edge is one of the very basic while complicated features in image processing. The challenge is then to find the discontinuity and address the location in the global Fourier data blurred by the noise from background. The edge detection method from Professor Anne Gelb [6] provides the concept of concentration kernel in order to recover the jump response function. In this paper, we are interested in finding the probability of an edge in a specific interval under the condition of concentration factor method and furthermore discuss the reliability of the probability using hypothesis test.

1 Introduction

Edges play a key role in signal and image processing, and their detection provide details for many types of problems – such as surface orientation, material properties and scene illumination. Detecting edges amounts to recovering the corresponding jump function of a piecewise smooth function. This problem is inherently challenging due to the Gibbs phenomenon when data are acquired in the Fourier domain, and it is further exasperated when there is background noise. Many algorithms have been developed to recover jump functions from Fourier data from [2, 6, 8, 12]. In this investigation we focus on the *concentration factor edge detection method* , originally designed in [6]. The convergence properties of the concentration factor method are well understood and the method has been successfully used in the presence of noise [1, 4]. However, since the acquired data are global Fourier coefficients, and edges are local features, the results obtained by the concentration factor method are either oscillatory or blurred, depending on whether and how much filtering is used in the jump function reconstruction. Therefore it can be difficult to distinguish between a true edge and an artificial oscillation. Indeed in [11] and [13], hypothesis testing was used to determine whether or not a particular grid point in an image domain was a true detect or false alarm. While such information is useful, there are other circumstances for which we are interested in knowing the probability that a given interval in the image domain contains an edge. This paper presents a method that evaluates the probability that an edge occurs in an interval defined by a specific value that is less than the maximum value of the peak of the oscillations produced by the concentration factor method and to measure the reliability of the probability via hypothesis testing. Receiver operating characteristic (ROC) curves are used to demonstrate our results.

For convenience, we only consider a piecewise smooth function containing only one edge. The background noise is assumed to be white Gaussian noise (WGN) with zero mean and finite detectable variance received by sending a sequence of blank signal to the target. We start by defining the jump function that records the value of all discontinuities. We limit our studies to just one type of concentration factor, but note that the same experiments can be performed more generally. The concentration factor method is oscillatory and forms the peak at discontinuities. As the oscillations die out away from the discontinuities, we study the behavior of the "peaks", and by assigning a threshold t on the image domain y-axis that cuts across the peak, we obtain the interval in which the Fourier partial sums are greater that t. Such intervals become candidates of possibly containing the actual edge. We use the central limit to compute the probability of an edge in a given potential candidate interval. Whereas the probability is not accurate enough to make decision, we then construct the hypothesis test with null hypothesis as edge absent and alternative hypothesis as edge detected. The false alarm rate (type one error) is substituted by the probability value that is complementary to the one computed from probability distribution deduction which describes the likelihood of edge in the interval. The last step is to utilize Neymar-Pearson Theorem (NP Theorem) to maximize the probability of positive detection. The direct result of NP theorem is presented in terms of ROC curve where the area underneath illustrates the reliability of potential edge-contained-interval under the circumstance of the chosen Gaussian concentration factor.

The rest of this paper is organized as follows. In section 2, we review the concepts of the concentration factor and processing of piecewise function in Fourier domain. In section 3, we would demonstrate the probability of an edge being in the interval determined by a value t that cuts across the peak. By introducing central limit theorem in the deduction, we are able to illustrate how such distribution is dependent on both t and variance of noise. In section 4, we would use probabilities of variance and Gaussian distribution to compute the variance of the probability distribution model in section 3. In section 5, we introduce the hypothesis test and NP theorem in calculation of the reliability of current probability for the final decision making. Computation examples would be given in section 6.

2 Preliminaries

Consider a piecewise smooth function $f : \mathbb{R} \to \mathbb{R}$ supported on $[-1, 1]$. Its corresponding jump function $[f] : \mathbb{R} \to \mathbb{R}$ is defined as:

$$[f] = f(x^+) - f(x^-). \tag{1}$$

Suppose we are given the first $2N+1$ Fourier coefficients of f, where

$$\hat{f}_k = \frac{1}{2}\int_{-1}^{1} f(x)e^{-ik\pi x}dx, -N \leq k \leq N. \tag{2}$$

While the signal is interfered with noise, each Fourier coefficient is a linear combination of actual signal Fourier coefficient, denoted y_k, and noise Fourier coefficient, denoted n_k,

$$\hat{f}_k = y_k + n_k. \tag{3}$$

Since f is a piecewise smooth function, its partial Fourier sum

$$S_N f = \sum_{k=-N}^{N} \hat{f}_k e^{ikx} \longrightarrow f, \quad as \quad N \to \infty.$$

converges to the original function uniformly almost everywhere but has a large oscillations near the jump discontinuity. To reduce the effects from Gibbs phenomenon during the jump discontinuity we introduced a method in [6] to modify the original Fourier partial sum by convolving the concentration factor σ, by which

$$S_N{}^{\sigma} f(x) = i\sum_{k=-N}^{N} \hat{f}_k \sigma(\frac{|k|}{N})sgn(k)e^{ikx} \longrightarrow [f](x), \quad as \quad N \to \infty. \tag{4}$$

$$S_N{}^{\sigma} f(x) \in L^2.$$

The convergence performance depends on the choice of concentration factor which need to satisfy the following conditions:

$$1. \frac{\sigma(x)}{x} \in C^2[0,1]$$
$$2. \lim_{N\to\infty}\int_{\frac{1}{N}}^{1} \frac{\sigma(x)}{x}\, dx \to -\pi$$

Below are some examples of concentration factors [7] that satisfy the above conditions. In this paper, we choose Gaussian concentration factor [5] to compute Fourier series

1. *Trigonometric Concentration Factor*

$$\sigma^T(k) = \frac{\pi sin\pi k}{Si(\pi)},\ \textit{with}\ \ \mathrm{Si}(\pi) = \int_0^\pi \frac{sin(\eta)}{\eta} d\eta.$$

2. *Polynomial Concentration Factor*

$$\sigma_p{}^p(k) = \pi p k^p.$$

3. *Exponential Concentration Factor*

$$\sigma_\alpha{}^{exp}(k) = \pi \mathrm{C} \cdot k e^{\frac{1}{\alpha k(k-1)}},\ \mathrm{C} = \int exp^{\frac{-1}{\alpha\eta(\eta-1)}} d\eta.$$

4. *Gaussian Concentration Factor*

$$\begin{aligned}\sigma^G(k) &:= \sigma_\epsilon{}^G(k) \\ &= ike^{ik\xi} \int_{-\pi}^{\pi} e^{-\frac{(x-\xi)^2}{2\epsilon^2}}\ e^{-ikx} dx \\ &= ike^{ik\xi}\left(\sqrt{\frac{\pi}{2}}\ \epsilon e^{-ik\xi} e^{-\frac{1}{2}\ \epsilon^2 k^2}\right) erf\left(\frac{\sqrt{2}(x+\xi-ik\epsilon^2)}{2\epsilon}\ \right)\Big|_{-\pi}^{\pi}\end{aligned}$$

With the convolution of concentration factors, the new Fourier partial sum tends to concentration in the neighborhood of the jump discontinuity, ξ, reaching the height of $[f](\xi)$, while the oscillations die out away from the jump discontinuity. In the following theorem, we then discuss in what rate $S_N{}^\sigma f(x)$ is convergent to the jump function $[f](x)$ and split into two cases.

Theorem.1 [4] Assume f is a function of Bounded Variation and is piecewise C^2 smooth. Its concentration factor convolving Fourier sum satisfied has the following property

$$\begin{aligned}|S_N{}^\sigma f(x) - [f](\xi)| &\lesssim \frac{logN}{N} \qquad \text{if} \quad |x-\xi| \lesssim \frac{logN}{N} \\ |S_N{}^\sigma f(x)| &\lesssim \frac{logN}{Nd_x} \qquad \text{if} \quad \min_k |x-\xi_k| \geq \frac{1}{N}\end{aligned}$$

3 Edge reconstruction

After the original function is processed by the convolution of gaussian concentration factor, it can be seen from the graph that the oscillations die out away the discontinuities and enhance in forming of peaks around discontinuities. By specifying a t on y-axis which cuts across the absolute value of $S_N{}^\sigma f(x)$, the intervals corresponding are then the candidates intervals which have large likelihood that contain edges.

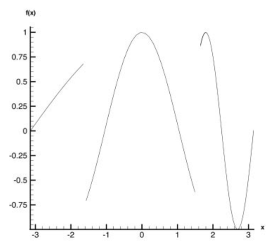


Figure 1: Original Function

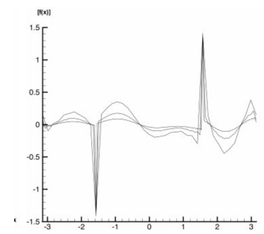


Figure 2: Detection of Edges using concentration factor method

The interval I_t is considered as

$$I_t = \{x \mid |S_N{}^{\sigma} f(x)| > t\}$$

Hence, the probability of an edge contained in I_t is:

$$\mathbf{P}(\xi \in I_t) = \mathbf{P}(|S_N{}^{\sigma} f(x)| > t) = 1 - \mathbf{P}(|S_N{}^{\sigma} f(x)| \leq t)$$

In order to find the lower bound such that given a number t an edge is contained in I_t, it's equivalent to compute the upper bound of $\mathbf{P}(|S_N{}^{\sigma} f(x)|) \leq t$.

According to eq.(2), the Fourier partial sum convolving noise and concentration factor is

$$S_N{}^{\sigma} f(x) = i \sum_{k=-N}^{N} (y_k + n_k)\sigma(\frac{|k|}{N})sgn(k)e^{ikx} \tag{5}$$

As the actual data and noise are independent, their expectations follow the rule of linearity

$$\mathbf{E}(S_N{}^{\sigma} f(x)) = \mathbf{E}(S_N{}^{\sigma} y_k(x)) + \mathbf{E}(S_N{}^{\sigma} n_k(x))$$

$$\mathbf{E}({S_N}^{\sigma} f(x)) = i \sum_{k=-N}^{N} y_k \sigma(\frac{|k|}{N})sgn(k)e^{ikx} \tag{6}$$

since the background noise is assumed to be White Gaussian noise, its expectation is zero, $\mathbf{E}({S_N}^{\sigma} n_k(x)) = 0$.
The series $\mathbf{E}({S_N}^{\sigma} f(x))$ converges to $[f](\xi)$ as $\mathcal{O}(\frac{logN}{N})$ [3], by Theorem 1, we evaluate the lower bound of $|\mathbf{E}({S_N}^{\sigma} f(x)) - [f](\xi)|$ as the following where we split into two cases. [4]
case.1 x is near discontinuity point with $d_x := |x - \xi| \lesssim \frac{logN}{N}$

$$|\mathbf{E}({S_N}^{\sigma} f(x)) - [f](\xi)| \leq \frac{logN}{N} \tag{7}$$

case.2 x is bounded away from the discontinuity point $d_x := |x - \xi| \gg |\frac{1}{N}|$

$$|\mathbf{E}({S_N}^{\sigma} f(x))| \leq \frac{logN}{Nd_x} \tag{8}$$

By T.I

$$|\mathbf{E}({S_N}^{\sigma} f(x))| - |\mathbf{E}({S_N}^{\sigma} f(x)) - {S_N}^{\sigma} f(x)| \leq |{S_N}^{\sigma} f(x)| \leq t \tag{9}$$

the inequality is equivalent to

$$|\mathbf{E}({S_N}^{\sigma} f(x)) - {S_N}^{\sigma} f(x)| \geq |\mathbf{E}({S_N}^{\sigma}(f)(x))| - t \tag{10}$$

with the left side of the last inequality is in the gaussian distribution with mean $\mu = 0$ and variance δ^2. It can be deducted from eq.(8) (9) (10) and Theorem.1 that

$$|\mathbf{E}({S_N}^{\sigma} f(x)) - {S_N}^{\sigma}(f)(x)| \geq |[f](\xi)| - \mathcal{O}(\frac{logN}{N}) - t \tag{11}$$

$$\Leftrightarrow$$

$$|\mathbf{E}({S_N}^{\sigma} f(x)) - {S_N}^{\sigma} f(x)| \geq \begin{cases} |[f](\xi)| - \frac{logN}{N} - t & \text{if } d_x \lesssim \frac{logN}{N} \\ \frac{logN}{Nd_x} & \text{if } d_x \gg |\frac{1}{N} \end{cases} \tag{12}$$

Next we apply Central Limit Theorem (CLT) to eq.(11)

$$\begin{aligned}
&\mathbf{P}(\frac{|\mathbf{E}(S_N{}^{\sigma} f(x)) - S_N{}^{\sigma} f(x)| - 0}{\delta} \geq \frac{a}{\delta}) \\
\Leftrightarrow\ & 1 - \mathbf{P}(\frac{|\mathbf{E}(S_N{}^{\sigma} f(x)) - S_N{}^{\sigma} f(x)| - 0}{\delta} \leq \frac{a}{\delta}) \sim \mathcal{N}(0,1) \\
\Leftrightarrow\ & 1 - \mathbf{P}(-\frac{a}{\delta} \leq \frac{\mathbf{E}(S_N{}^{\sigma} f(x)) - S_N{}^{\sigma} f(x)}{\delta} \leq \frac{a}{\delta}) \sim \mathcal{N}(0,1)
\end{aligned} \tag{13}$$

Follow by the linearity of variance between two independent random variable and the variance of constant is zero:

$$\mathbf{Var}(\mathbf{E}(S_N{}^{\sigma} f(x)) - S_N{}^{\sigma} f(x)) = \mathbf{Var}(S_N{}^{\sigma} f(x))$$

$$\mathbf{Var}(S_N{}^{\sigma} f(x))) = \mathbf{Var}(i \sum_{k=-N}^{N} (y_k)\sigma(\frac{|k|}{N})sgn(k)e^{ikx}) + \mathbf{Var}(i \sum_{k=-N}^{N} (n_k)\sigma(\frac{|k|}{N})sgn(k)e^{ikx})$$

As y_k, the Fourier coefficient of the actual signal function is a constant and the variance of $\mathbf{Var}(S_N{}^{\sigma}(y_k))$ is the variance of the characteristic function of a constant, $\mathbf{Var}(i \sum_{k=-N}^{N} (y_k)\sigma(\frac{|k|}{N})sgn(k)e^{ikx})$ is zero according to the property of variance

$$\mathbf{Var}(S_N{}^{\sigma} f(x)) = \mathbf{Var}(S_N{}^{\sigma} n(x)) = \mathbf{Var}(i \sum_{k=-N}^{N} (n_k)\sigma(\frac{|k|}{N})sgn(k)e^{ikx}) \tag{14}$$

4 Estimation of variance of noise

4.1 Variance Computation

By reducing signal output to zero that is to block the instrument from sending signals and instead only receiving the signals from the environment, we are then able to acquire the noise floor from the background. Consider the background noise in gaussian distribution with zero mean and finite variance and then compute the total variance for the distribution model in eq(13). The complete proof of variance calculation is as follows:

Define the noise variance as ρ^2, the variance of concentration convolving Fourier partial sum is:

$$\mathbf{E}({S_N}^{\sigma}(n_k)) = \mathbf{E}(n_k) = 0$$

$$\begin{aligned}
\mathbf{Var}[{S_N}^{\sigma}(n_k)] &= \mathbf{E}(|i\sum_{k=-N}^{N}(n_k)\sigma(\frac{|k|}{N})sgn(k)e^{ikx}|^2) - \mathbf{E}^2({S_N}^{\sigma}(n_k)) \\
&= \mathbf{E}[(i\sum_{k=-N}^{N}(n_k)\sigma(\frac{|k|}{N})sgn(k)e^{ikx})(-i\sum_{j=-N}^{N}(n_j)\sigma(\frac{|j|}{N})sgn(j)e^{-ijx})] - 0 \\
&= \mathbf{E}[\sum_{k=-N}^{N}(\sigma^2(\frac{|k|}{N}){n_k}^2)] + 0 \\
&= \sum_{k=-N}^{N}\sigma^2(\frac{|k|}{N})\mathbf{E}({n_k}^2) \\
&= \rho^2\sum_{k=-N}^{N}\sigma^2(\frac{|k|}{N})
\end{aligned} \tag{15}$$

As the partial sums of different n are independent, $\mathbf{E}({S_N}^{\sigma}(n_k){S_N}^{\sigma}(n_j)) = 0$ if $k \neq j$.

4.2 Flatness Correlation

The specification of Gaussian concentration factor is introduced in eq.(4). The flatness of the peak, which is formed by Gaussian concentration factor and is used as the Gaussian-regularized indicator function, is affected by both the variance from noise and that assigned to concentration factor, denoted as ϵ in eq.(4).
Fix the bound of partial sum N and variance of background noise ρ, to get the same confidence level t and variance of Gaussian concentration factor follows the condition:

$$\frac{|[f](\xi)| - \frac{logN}{N} - t_1}{|[f](\xi)| - \frac{logN}{N} - t_2} = \frac{\rho^2\sum_{k=-N}^{N}{\sigma_1}^2(\frac{|k|}{N})}{\rho^2\sum_{k=-N}^{N}{\sigma_2}^2(\frac{|k|}{N})} = \frac{{\epsilon_a}^2}{{\epsilon_b}^2}\frac{\sum_{k=-N}^{N}k^2e^{-k^2{\epsilon_a}^2}}{\sum_{k=-N}^{N}k^2e^{-k^2{\epsilon_b}^2}}$$

Define the difference between t_i and $\frac{logN}{N} - |[f](\xi)|$ to be z_i, and z_i is proportional to the function dependent on ϵ as following:

$$\frac{z_i}{z_j} = \frac{{\epsilon_i}^2}{{\epsilon_j}^2} \frac{\sum_{k=-N}^{N}(k^2 e^{-k^2})^{{\epsilon_i}^2}}{\sum_{k=-N}^{N}(k^2 e^{-k^2})^{{\epsilon_j}^2}} \tag{16}$$

4.3 Interval Estimation

The sample mean of the normal distribution we build in eq.(13) is zero, under the assumption that background noise the white Gaussian. For every t pointed to ${S_N}^\sigma$, there would be a unique probability,P_t indicating the lower bound of likelihood that an edge is in the interval which has a projective relation to the cumulative distribution function of $\mathbf{E}({S_N}^\sigma f(x)) - {S_N}^\sigma f(x)$ which has normal distribution. Thus the original interval I_t is transferred to an interval estimation in normal distribution under following conditions

1. z statistics $z_{\alpha/2} = \frac{1-P_t}{2}$
2. margin of error $= z_{\alpha/2}\frac{\mathbf{Var}({S_N}^\sigma f(x))}{N}$
3. sample mean $\mu = \mathbf{E}[\mathbf{E}({S_N}^\sigma f(x)) - {S_N}^\sigma f(x)] = 0$

$$Interval := 0 \pm z_{\alpha/2}\frac{\mathbf{Var}({S_N}^\sigma f(x))}{N} = \pm z_{\alpha/2}\frac{\rho^2 \sum_{k=-N}^{N} \sigma^2(\frac{|k|}{N})}{N} \tag{17}$$

5 Hypothesis Test for ROC Curve

From the previous work, we have already known the probability, denoted k, that an edge is in the interval defined by given t. Therefore we substitute that to the Confidence level $1-\alpha$ with k to verity the reliability of the potential edge-contained-interval with the convolution of Gaussian concentration factors.

Build the hypothesis test with the null hypothesis that an edge is not contained in interval specified by t, and the alternative hypothesis is an edge is contained in such interval. H_0 and H_1 are defined as below:

$$\begin{gathered}
H_0 : F = N \sim \mathcal{CN}[0, C_v] \\
H_1 : F = N + Y \sim \mathcal{CN}[Y, C_v] \\
F = ({S_N}^\sigma[f](x_1), ..., {S_N}^\sigma[f](x_k))^T \\
Y = ({S_N}^\sigma[y](x_1), ..., {S_N}^\sigma[y](x_k))^T \\
N = ({S_N}^\sigma[n](x_1), ..., {S_N}^\sigma[n](x_k))^T
\end{gathered}$$

$\mathcal{CN}$ represents the system of multivariate gaussian distribution
Fixing a concentration factor,σ, The covariance matrix is defined as below for each input we calculate the covariance between the Fourier series of x_a and x_b

$$
\begin{aligned}
C_v = \mathbf{Cov}^{\sigma}_{x_a,x_b} &= \mathbf{E}[(S_N{}^{\sigma}n(x_a))(S_N{}^{\sigma}n(x_b))^*] \\
&= \mathbf{E}[(i\sum_{k=-N}^{N}(n_{x_a})\sigma(\frac{|k|}{N})sgn(k)e^{ikx_a})(-i\sum_{k=-N}^{N}(n_{x_b})\sigma(\frac{|k|}{N})sgn(k)e^{-ikx_b})] \\
&= \rho^2\sum_{k=-N}^{N}\sigma^2(\frac{|k|}{N})e^{ik(x_a-x_b)}
\end{aligned}
$$

We then use Neymar-Pearson lemma [10] to calculate the likelihood-ratio test which rejects H_0 in favor of H_1 Probability of false alarm $P_{FA} = k$, in favor of H_1 if:

$$
\begin{aligned}
\mathcal{L}(x) &= \frac{\mathcal{CN}(Y, C_v)}{\mathcal{CN}(0, C_v)} > \gamma \\
&= \frac{exp(-\frac{1}{2}\ (F-Y)^T C_v{}^{-1}(F-Y))}{exp(-\frac{1}{2}\ F^T C_v{}^{-1}F)} > \gamma \\
&= exp(Y^T C_v{}^{-1}F - Y^T C_v{}^{-1}Y) > \gamma \\
&= Y^T C_v{}^{-1}F > ln\gamma + Y^T C_v{}^{-1}Y = \gamma'
\end{aligned} \tag{18}
$$

We refer $Y^T C_v{}^{-1}F$ the text statistics, denoted as $T(x)$ in the hypothesis test, and $Y^T C_v{}^{-1}Y$ is the Signal-to-noise ratio (SNR).
Then it is easy to deduct that:

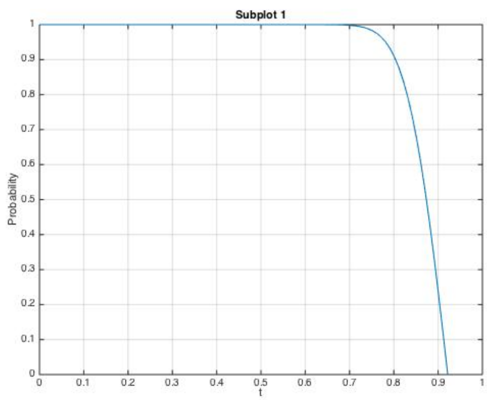
$$
P_{FA} = \int_{\gamma'}^{\infty} p(t|H_0)dt = Q(\frac{\gamma'}{\sqrt{Y^T C_v{}^{-1}Y}}) = k \tag{19}
$$

$$
P_D = \int_{\gamma'}^{\infty} p(t|H_1)dt = Q(\frac{\gamma' - Y^T C_v{}^{-1}Y}{\sqrt{Y^T C_v{}^{-1}Y}}) = Q(Q^{-1}(P_{FA}) - \sqrt{Y^T C_v{}^{-1}Y}) \tag{20}
$$

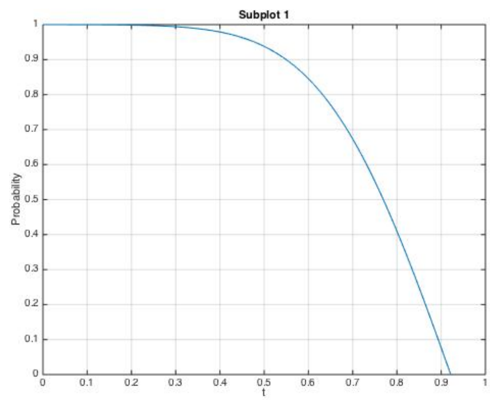
6 Result

Polynomial Function with N=50

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x+1 & \text{if } x > 0 \end{cases}$$

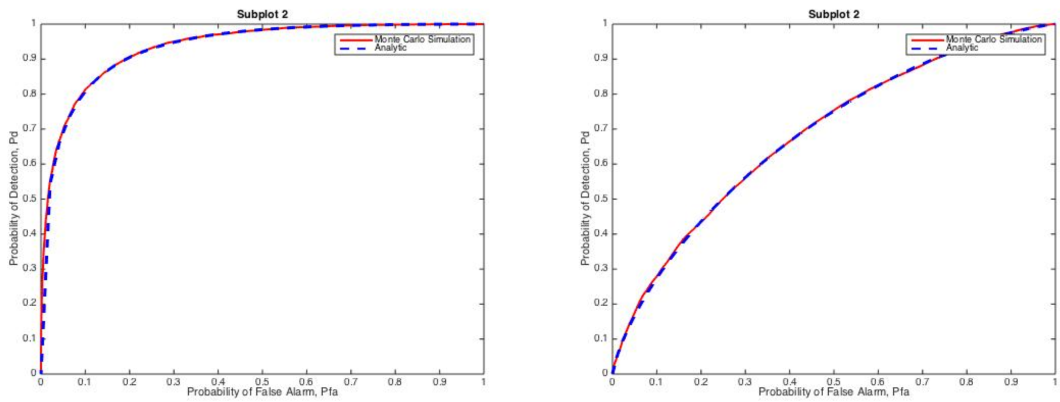


(a) $\rho^2 = 0.004$



(b) $\rho^2 = 0.04$

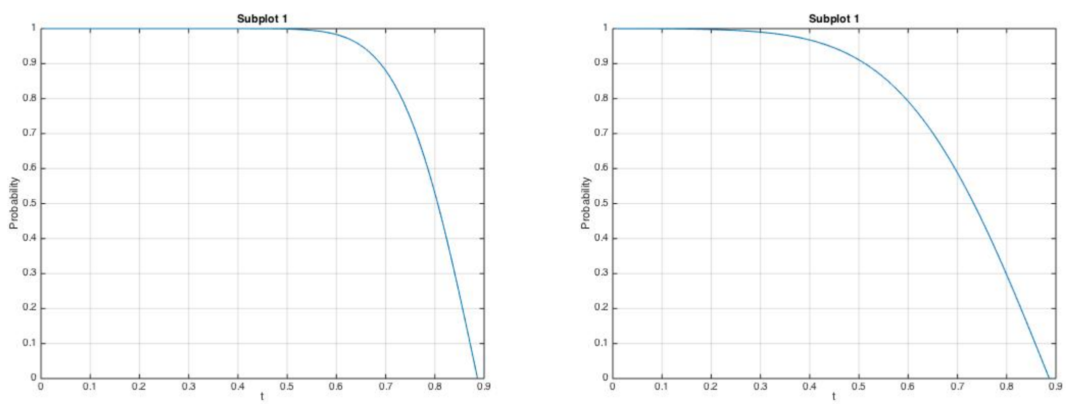
Figure 3: Probability of edge contained in interval determined by t with $f[\xi] = 0$



(a) $\rho^2 = 0.004$ (b) $\rho^2 = 0.04$

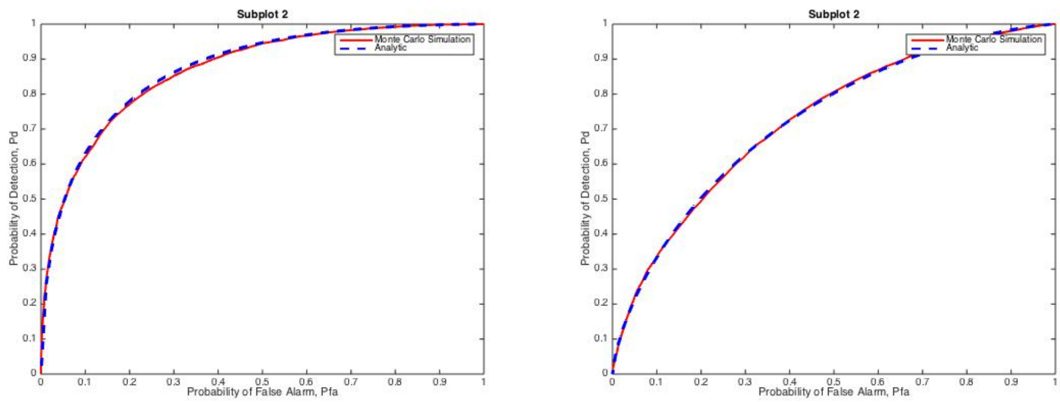
Figure 4: ROC Curve with confidence level substituted by the probability computed in section 3

Polynomial Function N=30



(a) $\rho^2 = 0.004$ (b) $\rho^2 = 0.04$

Figure 5: Probability of edge contained in interval determined by t with $f[\xi] = 0$

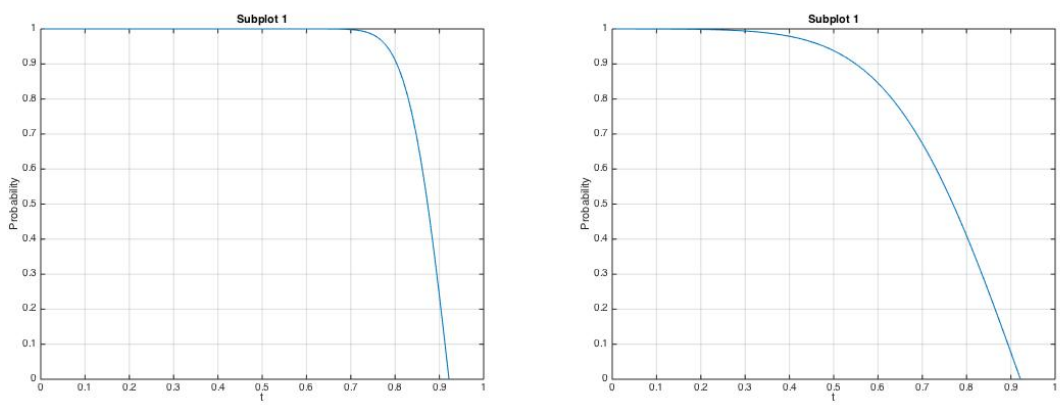


(a) $\rho^2 = 0.004$ (b) $\rho^2 = 0.04$

Figure 6: ROC Curve with confidence level substituted by the probability computed in section 3

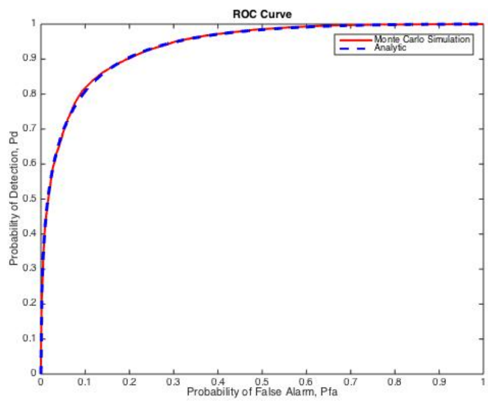
Linear Function with N=50

$$f(x) = \begin{cases} -x-1 & \text{if } x < 0 \\ x & \text{if } x > 0 \end{cases}$$

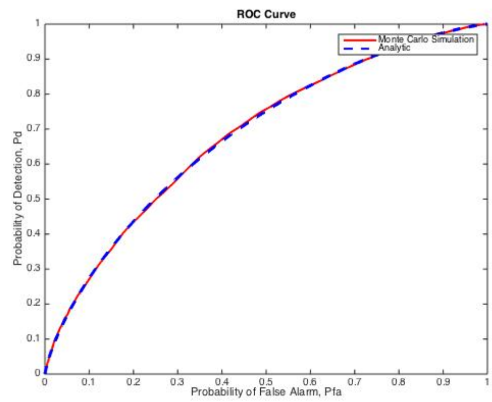


(a) $\rho^2 = 0.004$ (b) $\rho^2 = 0.04$

Figure 7: Probability of edge contained in interval determined by t with $f[\xi] = 0$



(a) $\rho^2 = 0.004$



(b) $\rho^2 = 0.04$

Figure 8: ROC Curve with confidence level substituted by the probability computed in section 3

7 Concluding remarks

In this paper, we demonstrate the probability of an edge contained in an interval determined by any value on y-axis and furthermore utilize the statistical multivariate hypothesis test in computing the reliability of such interval under specific concentration factor chosen. We note that there is a strong relationship between the noise background and the multivariate probability distribution model. However, since we start by man made value on determining the interval, it still remains unsolved for the mathematical relationship between ROC curve and the value specifying the interval. Also, while assuming the actual data and noise function are in first order relationship which may not be the case in practical life, we need consider a more general relationship and from that address a stronger probability space. These will be topics of future work.

References

[1] Rick Archibald and Anne Gelb. Reducing the effects of noise in image reconstruction. *Journal of Scientific Computing*, 17(1):167–180, 2002.

[2] Tobin A Driscoll and Bengt Fornberg. A padé-based algorithm for overcoming the gibbs phenomenon. *Numerical Algorithms*, 26(1):77–92, 2001.

[3] Shlomo Engelberg. Edge detection using fourier coefficients. *American Mathematical Monthly*, 115(6):499–513, 2008.

[4] Shlomo Engelberg and Eitan Tadmor. Recovery of edges from spectral data with noise?a new perspective. *SIAM Journal on Numerical Analysis*, 46(5):2620–2635, 2008.

[5] Anne Gelb and Taylor Hines. Detection of edges from nonuniform fourier data. *Journal of Fourier Analysis and Applications*, 17(6):1152–1179, 2011.

[6] Anne Gelb and Eitan Tadmor. Detection of edges in spectral data. *Applied and computational harmonic analysis*, 7(1):101–135, 1999.

[7] Anne Gelb and Eitan Tadmor. Detection of edges in spectral data ii. nonlinear enhancement. *SIAM Journal on Numerical Analysis*, 38(4):1389–1408, 2000.

[8] Donald Geman and George Reynolds. Constrained restoration and the recovery of discontinuities. *IEEE Transactions on pattern analysis and machine intelligence*, 14(3):367–383, 1992.

[9] Jean Jacod and Philip Protter. *Probability essentials*. Springer Science & Business Media, 2004.

[10] Steven M Kay. Fundamentals of statistical signal processing: Detection theory, vol. 2, 1998.

[11] Alex Petersen, Anne Gelb, and Randall Eubank. Hypothesis testing for fourier based edge detection methods. *Journal of Scientific Computing*, 51(3):608–630, 2012.

[12] Martin Storath, Andreas Weinmann, and Laurent Demaret. Jump-sparse and sparse recovery using potts functionals. *IEEE Transactions on Signal Processing*, 62(14):3654–3666, 2014.

[13] Adityavikram Viswanathan, Douglas Cochran, Anne Gelb, and Dennis M Cates. Detection of signal discontinuities from noisy fourier data. In *Signals, Systems and Computers, 2008 42nd Asilomar Conference on*, pages 1705–1708. IEEE, 2008.