

Notes on Taxation

Free Market Given the following supply and demand equations:

$$\begin{aligned}\text{Supply: } q_S &= \alpha_0 + \alpha_1 p \\ \text{Demand: } q_D &= \beta_0 - \beta_1 p\end{aligned}\tag{1}$$

, where q_S and q_D denote respectively quantity supplied and quantity demanded.

In the absence of tax, the equilibrium is solved by equating $q_S = q_D$.

Per-unit Tax Now suppose we have a per-unit tax T , then in equilibrium, we have:

$$\begin{aligned}\text{Supply: } q &= \alpha_0 + \alpha_1 p^s \\ \text{Demand: } q &= \beta_0 - \beta_1 p^b \\ p^b &= p^s + T\end{aligned}\tag{2}$$

, where p^b is the price that buyers pay and p^s is the price that sellers receive. Solving (2) $\Rightarrow \{p^b, p^s, q\}$.

What is the observed equilibrium market price? That depends on whether the tax is levied on the buyer or the seller.

- If levied on the buyer: $p = p^s$. The buyer pays p^s on the market, and then pays an additional T to the government.
- If levied on the seller: $p = p^b$. The seller receives p^b on the market, and then pays T to the government.

Equivalently, if the tax is levied on the buyer, then (2) can also be written as:

$$\begin{aligned}\text{Supply: } q &= \alpha_0 + \alpha_1 p \\ \text{Demand: } q &= \beta_0 - \beta_1 (p + T)\end{aligned}$$

, i.e. the buyer pays the market price $+T$.

If the tax is levied on the seller, then (2) can also be written as:

$$\begin{aligned}\text{Supply: } q &= \alpha_0 + \alpha_1 (p - T) \\ \text{Demand: } q &= \beta_0 - \beta_1 p\end{aligned}$$

, i.e. the seller receives market price $-T$.

Ad-valorem Tax Now suppose the government imposes an ad-valorem tax $\tau \in (0, 1)$ such that the seller receives a percentage $(1 - \tau)$ of what the buyer pays. Then in equilibrium, we have:

$$\begin{aligned}\text{Supply: } q &= \alpha_0 + \alpha_1 p^s \\ \text{Demand: } q &= \beta_0 - \beta_1 p^b \\ p^b (1 - \tau) &= p^s\end{aligned}\tag{3}$$

Alternatively, the government can impose an ad-valorem tax τ such that the buyer pays a percentage $(1 + \tau)$ of what the seller receives, in which case the equilibrium is given by:

$$\begin{aligned}\text{Supply: } q &= \alpha_0 + \alpha_1 p^s \\ \text{Demand: } q &= \beta_0 - \beta_1 p^b \\ p^b &= p^s (1 + \tau)\end{aligned}\tag{4}$$