UNIVERSITY OF MIAMI
Department of Electrical and Computer Engineering
EEN 204

Name: Jing Ma, Nelson Cortes, Muhammet Erkoc
Section:
Date:

EXPERIMENT 7

RESONANT CIRCUITS

PURPOSE: To study series and parallel resonant circuits. Characteristics such as *bandwidth*, *half power frequencies*, *resonant frequency*, and *quality factor* will be calculated from the circuit responses.

Equipment

1 Frequency Generator
1 D.V.M.
Resistors, Capacitors, and Inductors

Preliminary Work

a) *Derive* the expressions for the resonant frequency f_O, low half-power frequency f_L, high half-power frequency f_H, bandwidth BW, and quality factor Q for the circuit of Fig. 6.1.

b) *Derive* the expressions for the resonant frequency ω_O, low half-power frequency ω_L, high half-power frequency ω_H, bandwidth BW, and quality factor Q for the circuit of Fig. 6.2. (hint: use admittances for parallel elements and assume that $R_p >> R_s$)

Experimental Procedure

I. Series Resonant Circuit:

a) Set up the circuit shown in Fig. 6.1. Adjust the input voltage to 1 V_{rms}. Make R = 560 Ω, C = 0.047 μF, and L = 22 mH.

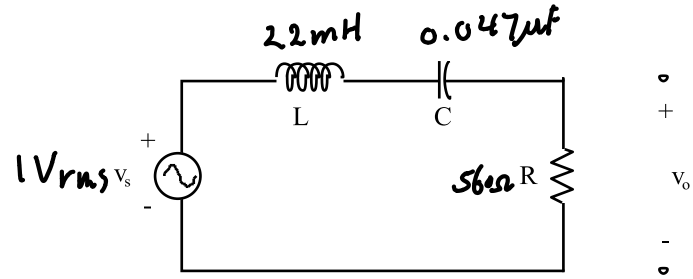


Figure 6.1 Series *RLC* resonant circuit.

b) Compute the quantities below using the given equations.

Resonant Frequency $f_o =$ 4949.48 Hz

Low Half-Power Frequency $f_L =$ 3322 Hz

High Half-Power Frequency $f_H =$ 7373 Hz

Bandwidth $BW =$ 4051 Hz

Quality Factor $Q =$ 1.22 Hz

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$f_H, f_L = \frac{1}{2\pi}\,\frac{\pm CR + \sqrt{(CR)^2 + 4LC}}{2LC}$$

$$BW = f_H - f_L = \frac{R}{2\pi L}$$

$$Q = \frac{\text{Resonant Frequency}}{\text{Bandwidth}} = \frac{f_o}{BW}$$

(*Note:* Make sure that you are consistent with your units. With the exception of Q, the formulas given above are in hertz. When calculating Q make sure that both the bandwidth and the resonant frequency are in Hz or rad/sec.)

c) Varying the frequency of the source as shown in Table 6.1, measure V_R, V_L, and V_C. Make sure the input voltage V_s remains constant at **each** frequency. Also compute the current in the circuit I, the capacitive reactance X_C, the inductor's equivalent impedance Z_L (remember that inductors have a small internal resistance, use the DVM in Ω mode to measure this), the inductive reactance X_L, and the equivalent reactive impedance X_{eq} using the following equations:

$$I = \frac{V_R}{R}, \quad X_C = \frac{V_C}{I}, \quad Z_L = \frac{V_L}{I}$$

$$X_L = \sqrt{Z_L^2 - R_L^2}$$

R_L is the resistance of the inductor. Be sure to measure this quantity.

$$X_{eq} = X_L - X_C$$

$R_L = 2.3471\ Hz$

d) From the results in part (c), plot, (***Note:*** **Use semi-log paper for your graphs.)**

(i) V_R vs f.

(ii) V_L and V_C vs f on the same graph.

(iii) X_L, X_C, and $|X_{eq}|$ vs f on the same graph.

e) Label the graph of V_R vs f and identify f_o, f_L, f_H, BW, and Q. Calculate the %error as compared to the calculated values. Tabulate your results in Table 6.2.

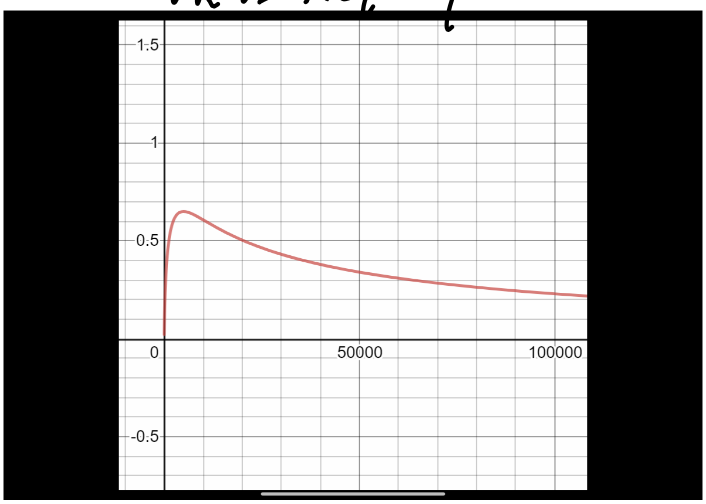
	Measured			Calculated				
f (Hz)	**V_R (V)**	**V_L (V)**	**V_C (V)**	**I (mA)**	**X_C (Ω)**	**Z_L (Ω)**	**X_L (Ω)**	**$\|X_{eq}\|$ (Ω)**
300	0.049734	0.003683	1.002	0.0881	11373.4	41.8048	41.7389	11331.7
500	0.082248	0.010275	1.007	0.147	6850.34	69.898	69.8586	6780.48
700	0.117313	0.020271	1.013	0.209	4846.89	96.9904	96.962	4749.93
1 k	0.16991	0.041944	1.027	0.303	3389.44	138.429	138.409	3251.03
2 k	0.367756	0.181613	1.112	0.657	1692.54	276.428	276.418	1416.12
3 k	0.617713	0.475766	1.244	1.103	1127.83	431.338	431.332	696.498
5 k	0.999443	1.236	1.206	1.785	675.63	692.437	692.433	16.803
5.5 k	0.996778	1.316	1.06	1.78	595.506	739.326	739.322	143.816
6 k	0.901281	1.339	0.905633	1.609	562.858	869.484	869.481	306.623
6.5 k	0.825773	1.33	0.765541	1.475	519.011	901.695	901.692	382.681
7 k	0.752753	1.306	0.64764	1.344	481.875	971.726	971.723	489.848
10 k	0.463155	1.153	0.277754	0.827	335.863	1394.2	1394.2	1058.34
20 k	0.199152	1.032	0.059107	0.356	166.031	2898.88	2898.88	2732.85
30 k	0.136383	1.013	0.027402	0.244	112.303	4151.64	4151.64	4042.34
50 k	0.080805	1.006	0.009685	0.144	67.2569	6986.11	6986.11	6978.85
70 k	0.057039	1.003	0.004841	0.102	47.4608	9833.33	9833.33	9785.87

Table 6.1 Measured and calculated data for the circuit in Fig. 6.1.

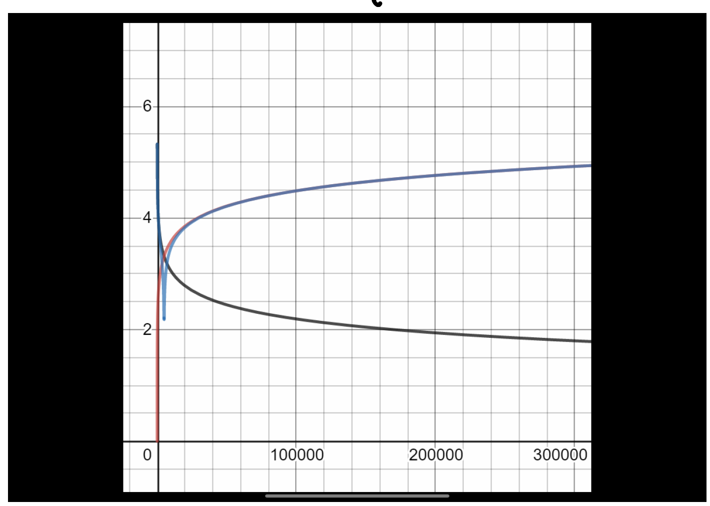
	Calculated	From Graph	% Error
f_0	4949	5000	1%
f_L	3222.2	3000	6%
f_H	4651	5000	20%
BW	4068.4	3600	22%
Q	1.22	1	18%

Table 6.2 Frequency response parameters for the circuit in Fig. 6.1.

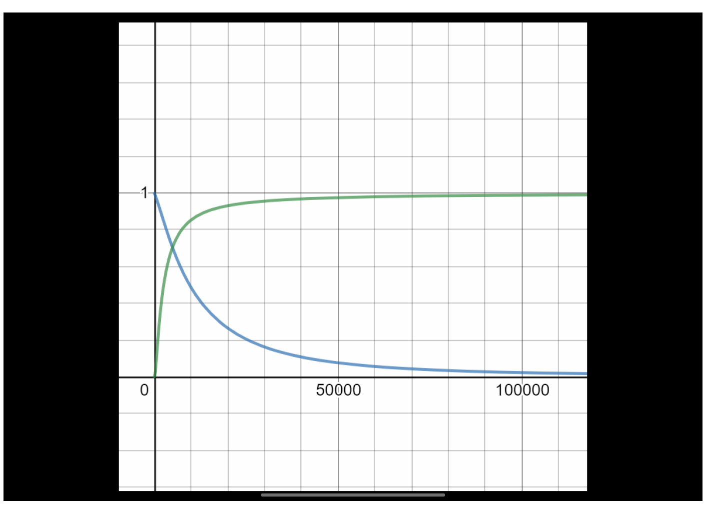
V_R vs Frequency.



Z_T, X_T, X_{eq} vs F



V_T vs F, V_C vs F



II. Parallel Resonant Circuit:

a) Set up the circuit shown in Fig. 6.2. Adjust the input voltage to 1 V_{rms}. Make R_S = 3.3 kΩ, R_p = 10 kΩ, C = 0.047 μF, and L = 22 mH.

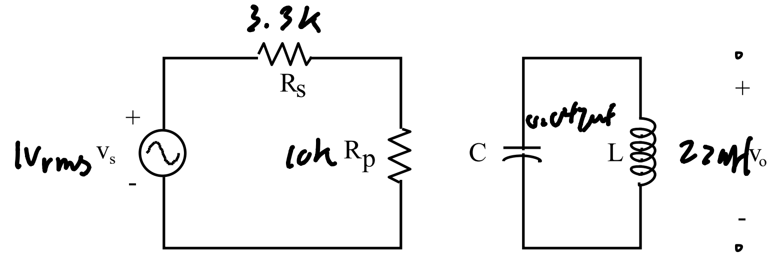


Figure 6.2 Parallel *RLC* resonant circuit.

b) Compute the quantities below using the given equations.

Resonant Frequency	f_o = 4949.48
Low Half-Power Frequency	f_L = 4313.92
High Half-Power Frequency	f_H = 5628.63
Bandwidth	BW = 1626.14
Quality Factor	Q = 4.82

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$f_H, f_L = \frac{1}{2\pi}\,\frac{\pm L + \sqrt{L^2 + 4LCR^2}}{2LCR} \quad ; \qquad R = R_S \| R_p$$

$$BW = f_H - f_L = \frac{1}{2\pi CR}$$

$$Q = \frac{\text{Resonant Frequency}}{\text{Bandwidth}} = \frac{f_o}{BW}$$

c) Varying the frequencies as given in Table 6.3, measure V_o. Make sure that V_i is 1 V_{rms} at **each** frequency.

d) From the results in part (c), plot V_O vs f.

e) In the graph of part (d), find and identify f_O, f_L, f_H, BW, and Q. Calculate the %error as compared to the calculated values. Tabulate your results in Table 6.4. Use your judgement to fill out the missing frequencies in the table given below in order to arrive at a smooth curve.

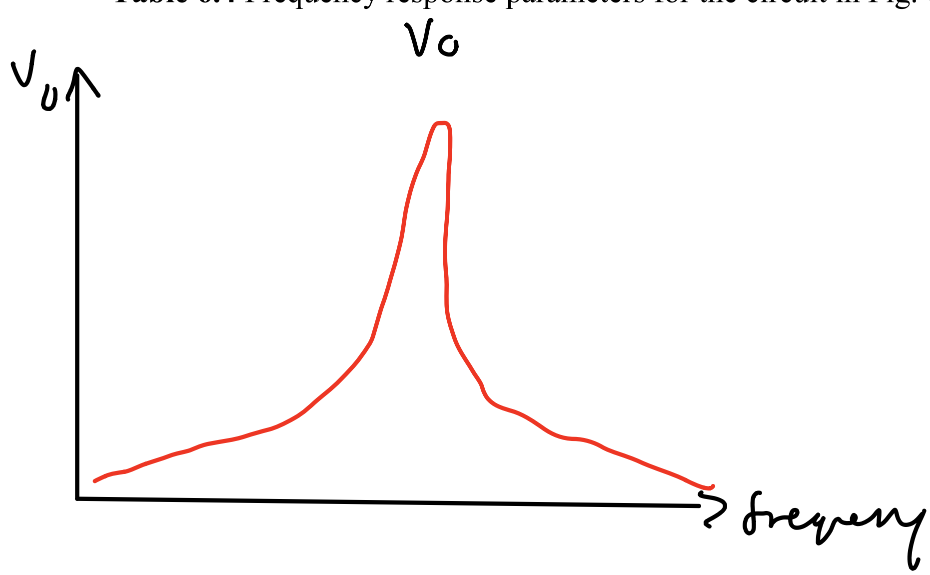
f (Hz)	V_o (V)
300	0.012611
500	0.021152
700	0.029898
1 k	0.0436
2 k	0.099291
3 k	0.192335
5 k	0.747179
5.5k	0.596553
6 k	0.435833

f (Hz)	V_o (V)
6 K	0.596553
6.5k	0.335938
7 k	0.270267
10 k	0.130744
20 k	0.052364
30 k	0.034967
50 k	0.020522
70 k	0.014466

Table 6.3 Measured data for the circuit in Fig. 6.2.

	Calculated	From Graph	% Error
f_o	4949	5000	1%
f_L	4314	4000	7%
f_H	5678	6000	5%
BW	1364	2000	46%
Q	3.63	2.5	30%

Table 6.4 Frequency response parameters for the circuit in Fig. 6.2.



Discussion of Results

a) For the parallel resonant circuit, R_p was assumed to be much larger than R_s.

(i) Which quantities were affected by this approximation? (i.e. Q, BW, f_o, f_l, f_h)
(ii) From the results in Table 6.4, do you think this was a reasonable approximation? Explain.

b) Prove that for the series resonant circuit

$$|V_L| = |V_C| = Q|V_S|$$

at resonance. (*Hint:* Write the equations for V_L and V_C in Fig. 6.1.)

c) How could V_C and V_L get higher values than V_s in part I? (*Hint:* Use the result of part (d) above.)

d) Discuss the applications of resonant circuits, particularly in,

(i) tuning of radio receivers,
(ii) use of doubly tuned circuits for wider bandwidth applications, and
(iii) narrow- and wide-band tuned circuits.

e) Write a conclusion.

a). i). Q, BW, f_L, f_H will be affected due to Rp and Rs calculation, their answers depend on R.

ii). The approximation does seem reasonable even though they are a little different from the expected value.

b). $\omega L = \frac{1}{\omega C}$, $I_{max} = \frac{V_s}{R}$

$V_L = I_{max}(j\omega_o L) = (\frac{V_s}{R})(jL)\omega_o$, $\omega_o = (BW)Q$, $BW = \frac{R}{L}$

$|V_L| = |\frac{V_s}{R}|(4|\frac{R}{L}| \times Q| = Q|V_s|$

c). V_c, the capacitance voltage can exceed the source voltage V_s when the inductor transfers energy to the capacitance in resonance. This occurs at the inductor's resonant frequency

d). i). the tunes must only resonate at 1 frequency. Resonant frequency of an inductor changes with impedance, adding a variable resistor to change the impedance, allows to alter resonance frequency.

ii). increasing impedance will decrease resonance at lower and higher frequencies from f_o, allowing a wider tone to pass using two inductors and capacitors, allows to widen the frequency range. When in parallel, the impedance can be changed individually.

iii). Can alter how the frequency range is by altering the impedance can reduce the voltage at f_0, allowing wider of range to pass.

Conclusion

The lab showed us about the effects of frequency on an inductor. This showed us how a circuit can be used as a filter to allow only certain frequencies to pass. This is useful because changing the impendance of the circuit affects the frequency content of a signal. This lets us change or tune the circuit. Using a simpler filter such as a capacitor in parallel with a resistor doesn't allow us to alter the frequency content as easily.