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EEN 204

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EXPERIMENT 3

TRANSIENT ANALYSIS OF SIMPLE RC CIRCUITS

PURPOSE: In this experiment we shall study two simple circuits containing resistors and capacitors. When circuits contain only capacitors and inductors and no active elements (i.e., diodes, transistors, op-amps, etc) they are referred to as *RC* or *RL* circuits. The application of Kirchhoff's laws to these circuits gives rise to first order differential equations. The solutions give insight into the behavior of circuits in response to the sudden application of current or voltage signals. In general, these responses tell us how fast a circuit can respond to external inputs. When sources are suddenly connected to these circuits, they produce two kinds of responses. The first is the *natural response* that is characterized by the nature of the circuit itself and is of exponential nature for *RC* and *RL* circuits. The second response is the *forced response* and depends on the forcing or driving function and the steady state behavior of capacitors and inductors. From the natural responses, we will be able to determine charging and discharging times (*time constants)* for a particular circuits. When the circuit is pure RC or RL, the charging and discharging times are generally equal (see parts a and b in the preliminary work). Some circuits utilize active components in order to have different charging and discharging times.

Equipment

2 Variable resistance boxes
1 0.1 μF capacitor
1 Digital Oscilloscope
1 Frequency Generator
1 1N4001 Diode

Preliminary Work

a) Assuming the source of Fig. 3.1 to be a step voltage of V_o volts, show that the capacitor voltage builds up according to the relationship

$$V_c(t) = V_o\,(1 - e^{-t/\tau})$$

$v_c(t) = V_o(-e^{-t/\tau})$ $\tau = R \cdot C$, $t = \infty$

$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau}$

$v_c(t) = V_o + ($

$v_c(t) = V_o(1 - e^{-t/RC})$

Where the initial capacitor voltage is $V_c(0^+) = 0$ V and $\tau = RC$ is the time constant.

b) After a sufficient time, a short circuit is placed across the source. Show that the capacitor voltage decreases according to

$$V_c(t) = V_o\,e^{-t/\tau}.$$

c) Show that the charging and discharging time constants are given by the expressions $V_c(\tau_c)=0.632V_o$ and $V_c(\tau_d)=0.368V_o$ respectively.

Experimental Procedure

I. RC Circuit with Square Wave Input:

a) Set up the circuit shown in Fig. 3.1. Adjust the input voltage to vary from **0 to 5 V** and a frequency of 500 Hz (make sure that there is no dc offset in the waveform by viewing the waveform in the DC mode of the oscilloscope and varying the DC level knob in the signal generator). Make R = 1 kΩ and C = 0.1 μF. Note that you must account for the internal resistance of the signal generator for the theoretical calculations.

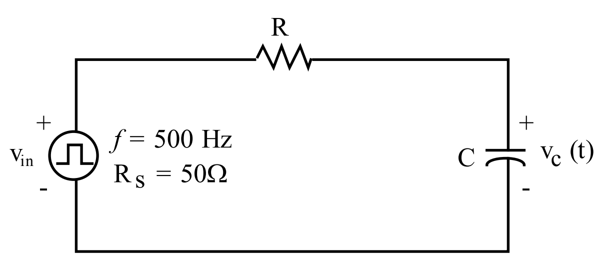
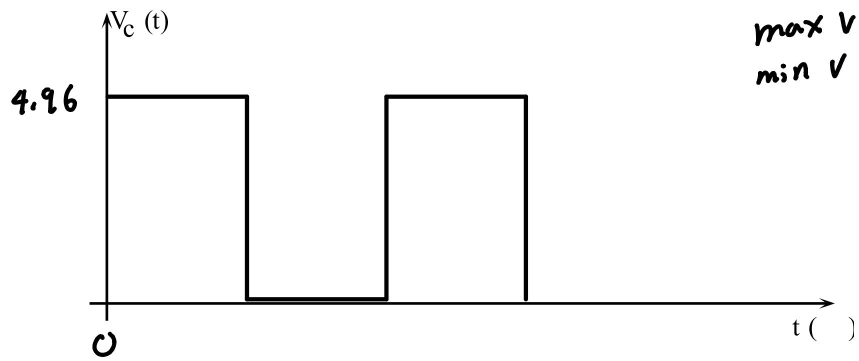


Figure 3.1 Driven *RC* network.

b) Calculate the time constant of the charging path, $(R_S+R)C$ τ_c = 1.05×10^{-4}

c) Calculate the time constant of the discharging path, RC τ_d = 1×10^{-4}

d) Sketch the waveform of the voltage across the capacitor, $V_c(t)$. Indicate the **maximum voltage, minimum voltage**, and the charging and discharging times.



max V = 4.96V
min V = 0.0V

e) Measure the charging time constant, τ_c = 1.05 x10⁻⁴ s

f) Measure the discharging time constant, τ_d = 1x10⁻⁴ s

g) Calculate the % error for τ_c, % error τ_c = 0

h) Calculate the % error for τ_d, % error τ_d = 0

i) Vary R and *f* according to the values given in Table 3.1. Measure and calculate τ_c, τ_d, and the % errors. Use fixed resistance instead of the resistance boxes.

		Measured		Calculated		% Error	
R (Ω)	*f* (Hz)	τ_c (ms)	τ_d (ms)	τ_c (ms)	τ_d (ms)	τ_c	τ_d
470	300	52	46.4	52	46.4	0	0
	500	52	46.4	52	46.4	0	0
	1000	52	46.4	52	46.4	0	0
1000	300	105	100	105	100	0	0
	500	105	100	105	100	0	0
	1000	105	100	105	100	0	0

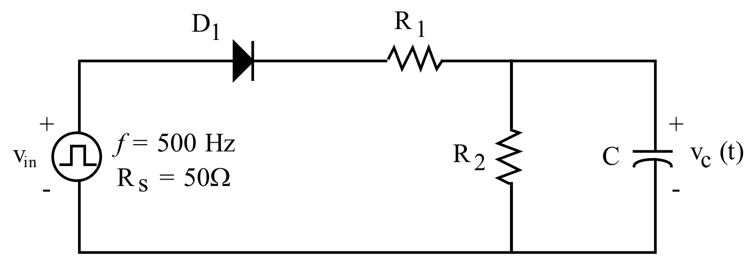
Table 3.1 Measured and Calculated data for time constants of circuit in Fig. 3.1.

Note that the time constant is independent of frequency for the range considered in this experiment.

II. RC Circuit with Active Element:

a) Set up the circuit shown in Fig. 4.2. Adjust the input voltage to vary from **0 to 5 V** and a frequency of 500 Hz (use the procedure explained in I(a) to remove any DC offsets from the signal). Choose C = 0.1 μF. Measure τ_c and τ_d in each of the cases below and sketch the waveform of the voltage across the capacitor, $V_c(t)$. Indicate the **maximum voltage, minimum voltage**, and the charging and discharging times.

$$\tau_c = \frac{(R_1 + R_2)R_2}{R_1 + R_s + R_2}$$



$$\tau_d = = R_2$$

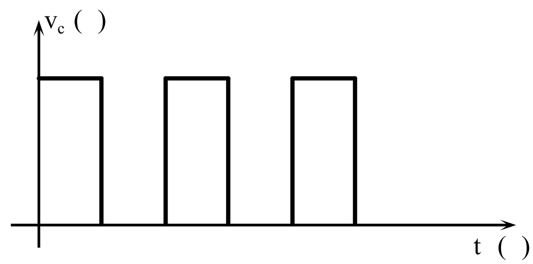
$$0.028 = \frac{X - 34}{X}$$

Figure 4.2 RC circuit wit different charging and discharging times.

(b) Set $R_1 = 470\ \Omega$, $R_2 = 470\ \Omega$.

τ_c = 26 μs

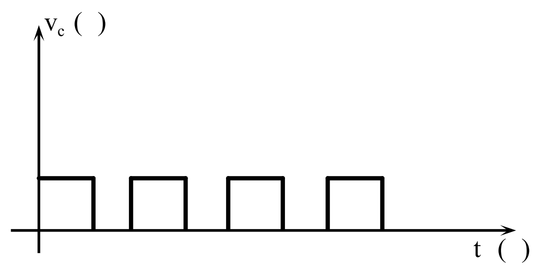
τ_d = 30 μs



(c) Set $R_1 = 1\ \mathrm{k}\Omega$, $R_2 = 470\ \Omega$.

τ_c = 30 μs

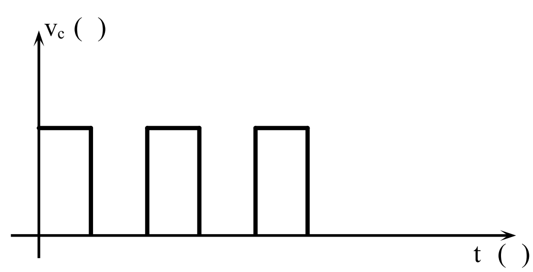
τ_d = 34 μs



(d) Set $R_1 = 470\ \Omega$, $R_2 = 1\ \mathrm{k}\Omega$.

τ_c = 36 μs

τ_d = 40 μs



Discussion of Results

a) Why would the measurements be inaccurate if higher frequencies were to be used?

b) In part II, why doesn't the capacitor voltage build up to the source maximum voltage?

c) In part II, explain the difference in τ_c and τ_d by deriving the necessary circuit equations.

d) Calculate the theoretical values of τ_c and τ_d for parts II.b, II.c, and II.d and find the % errors. Present the calculations in tabular form. Show one example calculation.

e) From the results of part II.b, estimate the forward drop across the diode. Assume a constant voltage drop across the diode when forward biased.

f) Write a conclusion.

Pre-Lab

a). $f = 500 Hz$

$R_s = 50\Omega$

$\tau = RC$

$V_c(0^+) = 0$

$Q = CV$

$\frac{dQ}{dt} = C\frac{dv}{dt}$

$i = C\frac{dv}{dt}$

"KCL"

$C\frac{dv}{dt} + \frac{V_c - V_0}{R} = 0$

$\frac{dV}{dt} = -\frac{V_c - V_0}{RC}$

$\frac{dV_c}{dt}\left(\frac{1}{V_c - V_0}\right) = -\frac{1}{RC}$

$\rightarrow \int \frac{1}{V_c - V_0}\, dV_c = -\int \frac{1}{RC}\, dt$

$\ln(V_c - V_0) = -\frac{t}{RC} + D$

$RC = \tau$

$V_c - V_0 = e^{D - \frac{t}{\tau}}$

$V_c - V_0 = e^{D} e^{-t/\tau}$

$t = 0$

$V_c(0^+) - V_0 = e^D e^0$

$e^D = -V_0$

$\longrightarrow$

$V_c(t) - V_0 = -V_0 e^{-t/\tau}$

$V_c(t) = V_0 - V_0 e^{-t/\tau}$

$V_c(t) = V_0(1 - e^{-t/\tau})$

b). $\frac{dV_c(t)}{dt} + \frac{V_c(t)}{RC} = 0$

$\frac{dV_c(t)}{V_c(t)} = \frac{-dt}{RC}$

$\ln(V_c(1)) = -\frac{t}{RC} + D$

$V_c(t) = e^D e^{-t/\tau}$

at $t = 0$, $V_c(0^+) = V_0$

$V_0 = e^D e^0$

$V_0 = e^D$

$V_c(t) = V_0 e^{-t/\tau}$

c). $V_c(\tau_c) = 0.632 V_0$, $V_c(\tau_d) = 0.368 V_0$

Charging $\longrightarrow$ $V_c(\tau_c) = V_0(1 - e^{-1})$

$V_c(\tau_c) = 0.632 V_0$

Discharging $\longrightarrow$ $V_c(\tau_D) = V_0 e^{-1}$

$V_c(\tau_D) = 0.368 V_0$

Discussion

a). Measurements would be inaccurate if higher frequencies were used due to the fact that higher frequencies would be attenuated by the capacitor and won't be displayed properly on the oscilloscope. Another reason is that voltage is lost as frequency increases and the decreasing reactance would cause the capacitor to act like a short circuit.

b). Capacitor voltage does not build up to the source maximum voltage because the reactance of the capacitor causes some of the current to pass, not allowing it to be completely stored in the capacitor.

c). $\tau_c = (R_s + R)C$

$\tau_d = RC$

When charging, τ_c is dependent on R_s and R because resistance in the capacitor prevents the charge from building up instantaneously and the charge needs to be built up to reach full voltage.

During discharge, nothing is stopping the capacitor from discharging the current in its entirity, except the resistance after the capacitor and it discharges quickly.

d).

	measured		calculated		% error	
	t_c	t_d	t_c	t_d	t_c	t_d
b	26	30	23	23	13%	30%
c	30	34	33	32	10%	3%
d	36	40	32	32	13%	25%

f). In conclusion, by varying the circuit configuration, important concepts in circuit theory was observed. One concept observed was the effect of frequency on the reactance of the capacitor. When frequency is low (or when V_{dc} is applied), the reactance of the capacitor is able to reach a value close to V_s. when frequency is high, the reactance of the capacitor is not observed and the capacitor acts like a short circuit. This quality allows capacitors to be used to filter low and high frequencies.