

# Iterative deblending of simultaneous-source seismic data using shaping regularization

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## SUMMARY

We introduce a new iterative estimation scheme for separation of blended seismic data from simultaneous sources. We first construct an augmented estimation problem, then use shaping regularization to constrain the model when iteratively solving the problem. Both model and data in the estimation problem correspond to different seismic sources. In the case of two sources, the forward operator is a block operator composed of two identity operators, one blending operator and one inverse blending operator. The shaping operator used in our method is soft thresholding in the seislet domain, where useful seismic signals stand compact and sparse. Numerically-blended synthetic and field datasets demonstrate an excellent performance of the proposed algorithm.

## INTRODUCTION

Wide azimuth acquisition (WAA) can improve the illumination of subsalt structures, which helps in turn improve the quality of seismic imaging (Michell et al., 2006). However, WAA suffers from the low efficiency problem, resulting from the large temporal shooting interval between two consecutive shots (Beasley et al., 1998; Beasley, 2008; Berkhout, 2008). The large shooting interval ensures that no interference exists between adjacent shots. An alternative way to avoid interference is by increasing the spatial shooting interval, which results in serious spatial aliasing problems. The simultaneous-source technique aims at removing the limitation of no interference between adjacent shots, by allowing more than one sources shot simultaneously. Thus it can reduce acquisition time and increase spatial sampling (Berkhout, 2008). Because of its economic benefits and technical challenges, this technique has attracted the attention of researches in both industry and academia (Moore et al., 2008; Mahdad et al., 2011; Wapenaar et al., 2012; Huo et al., 2012; Abma et al., 2012). The biggest issue involved in simultaneous-source processing is intense crosstalk noise between adjacent shots, which poses a challenge for conventional processing. One way to deal with simultaneous-source seismic data is using a first-separate and second-process strategy, which is also known as "deblending" (Doulgeris et al., 2012). The other is by direct imaging and waveform inversion. In this paper, we focus on the first approach.

Different filtering and inversion methods have been used previously to deblend seismic data. Filtering methods utilize the property that the coherency of the simultaneous-source data is not the same in different domains, thus we can get the unblended data by filtering out the randomly distributed blending noise in one special domain, where one source record is coherent and the other is not (Hampson et al., 2008; Mahdad et al., 2012; Huo et al., 2012). Inversion methods treat the separation problem as an estimation problem which aims at estimating the desired unknown unblended data. Because of

the ill-posed property of such estimation problems, a regularization term is usually required (Doulgeris and Bube, 2012). Borselen et al. (2012) proposed to distribute all energy in the shot records by reconstructing the individual shot records at their respective locations. Mahdad et al. (2012) introduced an iterative estimation and subtraction scheme that combines the property of filtering and inversion methods and exploits the fact that the characteristics of the blending noise differs in different domains. One choice is to transform seismic data from the common-shot domain to common-receiver, common-offset or common-midpoint domain. Beasley et al. (2012) proposed a separation technique called the alternating projection method (APM), which was demonstrated to be robust in the presence of aliasing.

In this paper, we propose an iterative estimation scheme for the separation of blended seismic data. We construct an augmented estimation problem, then use shaping regularization (Fomel, 2007, 2008) to constrain the characteristics of the model during the iteration process to get a suitable estimation result. We use soft thresholding in the seislet domain (Fomel and Liu, 2010) as the shaping operator to remove the crosstalk noise and at the same time to preserve useful components of seismic data. Shaping regularization helps constrain the model and can solve the estimation problem with a small number of iterations. We use numerically-blended data to test the proposed approach.

## THEORY

### Iterative deblending of seismic data

Let us suppose that the seismic record is blended using two independent sources, which correspond to two shooting vessels in the ocean bottom nodes (OBN) acquisition (Mitchell et al., 2010). Here one source means a collection of shots. Each shot in one source has a random dithering compared with the corresponding shot in another source. Thus the forward problem can be formulated as follows:

$$\mathbf{d} = \mathbf{d}_1 + \mathbf{T}\mathbf{d}_2. \quad (1)$$

Here,  $\mathbf{d}_1$  and  $\mathbf{d}_2$  denote the seismic record to be separated out.  $\mathbf{d}$  is the blended data and  $\mathbf{T}$  denotes the forward dithering operator. Note that we perform the processing in the common-receiver domain, because each blended record in this domain is coherent for one source and incoherent for another (Hampson et al., 2008). We augment equation 1 with another equation through the inverse dithering operator  $\mathbf{T}^{-1}$ :

$$\mathbf{T}^{-1}\mathbf{d} = \mathbf{T}^{-1}\mathbf{d}_1 + \mathbf{d}_2. \quad (2)$$

By combining equation 1 and 2, we formulate an augmented estimation problem:

$$\mathbf{F}\mathbf{m} = \tilde{\mathbf{d}}, \quad (3)$$

where

$$\tilde{\mathbf{d}} = \begin{bmatrix} \mathbf{d} \\ \mathbf{T}^{-1}\mathbf{d} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \mathbf{I} & \mathbf{T} \\ \mathbf{T}^{-1} & \mathbf{I} \end{bmatrix}, \quad \mathbf{m} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \end{bmatrix}. \quad (4)$$

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An estimate of the model  $\mathbf{m}$  can be obtained by a simple iteration as follows:

$$\mathbf{m}_{n+1} = \mathbf{m}_n + \mathbf{B}\tilde{\mathbf{d}} - \mathbf{B}\mathbf{F}\mathbf{m}_n, \quad (5)$$

where  $\mathbf{B}$  is the backward operator that provides an inverse mapping from data space to model space. If  $\mathbf{B}$  is taken as the adjoint of  $\mathbf{F}$ , iteration 5 is known as the *Landweber iteration* (Landweber, 1951). The Landweber iteration solves the system of normal equations and converges to the least-squares estimate of  $\mathbf{m}$ :

$$\min_{\mathbf{m}} \|\mathbf{F}\mathbf{m} - \tilde{\mathbf{d}}\|_2^2, \quad (6)$$

where  $\|\cdot\|_2^2$  denotes the squared  $L_2$  norm of a function. The least-squares optimization problem 6 are usually regularized by adding a regularization term:

$$\min_{\mathbf{m}} \|\mathbf{F}\mathbf{m} - \tilde{\mathbf{d}}\|_2^2 + \mathbf{R}(\mathbf{m}), \quad (7)$$

where  $\mathbf{R}(\cdot)$  is a regularization function. Alternatively, with a shaping operator  $\mathbf{S}$  (Fomel, 2008), iteration 5 can be modified to the following equation:

$$\mathbf{m}_{n+1} = \mathbf{S}[\mathbf{m}_n + \mathbf{B}(\tilde{\mathbf{d}} - \mathbf{F}\mathbf{m}_n)]. \quad (8)$$

Regularized iteration 8 shapes the estimated model into the space of admissible models at each iterations (Fomel, 2007, 2008). It has been proved that if  $\mathbf{S}$  is a nonlinear thresholding operator (Donoho and Johnstone, 1994),  $\mathbf{B} = \mathbf{F}^T$  where  $\mathbf{F}^T$  is the adjoint operator of  $\mathbf{F}$ , iteration 8 converges to the solution of problem 7 with  $L_1$  regularization term (Daubechies et al., 2004). A better choice for  $\mathbf{B}$  is the pseudoinverse of  $\mathbf{F}$ :  $\mathbf{B} = (\mathbf{F}^T \mathbf{F})^{-1} \mathbf{F}^T$  (Daubechies et al., 2008).

### Backward operator

In equations 5 and 8,  $\mathbf{B}$  is an approximate inverse of  $\mathbf{F}$ . Considering that the dithering operator  $\mathbf{T}$  is unitary:

$$\mathbf{T}^T = \mathbf{T}^{-1}, \quad (9)$$

where  $\mathbf{T}^T$  is the transpose of  $\mathbf{T}$ , we derive that:

$$\mathbf{F}^T = \begin{bmatrix} \mathbf{I} & \mathbf{T} \\ \mathbf{T}^{-1} & \mathbf{I} \end{bmatrix}^T = \begin{bmatrix} \mathbf{I} & \mathbf{T}^{-T} \\ \mathbf{T}^T & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{T} \\ \mathbf{T}^{-1} & \mathbf{I} \end{bmatrix} = \mathbf{F},$$

and:

$$\mathbf{F}^T \mathbf{F} = \begin{bmatrix} \mathbf{I} & \mathbf{T} \\ \mathbf{T}^{-1} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{T} \\ \mathbf{T}^{-1} & \mathbf{I} \end{bmatrix} = 2 \begin{bmatrix} \mathbf{I} & \mathbf{T} \\ \mathbf{T}^{-1} & \mathbf{I} \end{bmatrix} = 2\mathbf{F}. \quad (10)$$

The least-squares solution of equation 3 is therefore:

$$\hat{\mathbf{m}} = (\mathbf{F}^T \mathbf{F})^{-1} \mathbf{F}^T \tilde{\mathbf{d}} = \frac{1}{2} \mathbf{F}^{-1} \mathbf{F}^T \tilde{\mathbf{d}} = \frac{1}{2} \tilde{\mathbf{d}}. \quad (11)$$

According to equation 11, an appropriate choice for  $\mathbf{B}$  is simply  $\frac{1}{2} \mathbf{I}$ .

### Shaping operator

Fomel (2006) and Fomel and Liu (2010) proposed a digital wavelet-like transform, which is defined with the help of the wavelet-lifting scheme (Sweldens, 1995) combined with local plane-wave destruction. The wavelet-lifting utilizes predictive

property of even components from odd components and find a difference  $\mathbf{r}$  between them, which can be expressed as:

$$\mathbf{r} = \mathbf{o} - \mathbf{P}[\mathbf{e}], \quad (12)$$

where  $\mathbf{P}$  is the prediction operator. A coarse approximation  $\mathbf{c}$  of the data can be achieved by updating the even component:

$$\mathbf{c} = \mathbf{e} + \mathbf{U}[\mathbf{r}], \quad (13)$$

where  $\mathbf{U}$  is the updating operator.

The digital wavelet transform can be invertible by reversing the lifting-scheme operators as follows:

$$\mathbf{e} = \mathbf{c} - \mathbf{U}[\mathbf{r}], \quad (14)$$

$$\mathbf{o} = \mathbf{r} + \mathbf{P}[\mathbf{e}]. \quad (15)$$

The above prediction and update operators can be defined, for example, as follows:

$$\mathbf{P}[\mathbf{e}]_k = \left( \mathbf{P}_k^{(+)}[\mathbf{e}_{k-1}] + \mathbf{P}_k^{(-)}[\mathbf{e}_k] \right) / 2, \quad (16)$$

and

$$\mathbf{U}[\mathbf{r}]_k = \left( \mathbf{P}_k^{(+)}[\mathbf{r}_{k-1}] + \mathbf{P}_k^{(-)}[\mathbf{r}_k] \right) / 4, \quad (17)$$

where  $\mathbf{P}_k^{(+)}$  and  $\mathbf{P}_k^{(-)}$  are operators that predict a trace from its left and right neighbors, correspondingly, by shifting seismic events according to their local slopes. The predictions need to operate at different scales, which means different separation distances between traces. Taken through different scales, equations 12-17 provide a definition for the 2D seislet transform.

In the seislet domain, useful events and blending noise reside at different scales and the useful signal tends to form more compact region than other well-known transforms for seismic data such as Fourier transform and wavelet transform (Fomel and Liu, 2010). Figures 1(a) and 1(b) show the seislet domain for clean seismic data and blended seismic data respectively. We see that the seislet domain for clean data is compact and sparse while the seislet domain for noisy data spreads across different scales. Thus, compressing the seislet domain can help suppress blending noise.

There are two ways to compress data in the seislet domain. One is by applying a mask operator, the other is by applying a thresholding operator. A mask operator can preserve the small scales and remove large scale data completely. Thresholding operators are divided into two types: soft and hard thresholding. Soft thresholding or shrinkage aims to remove data whose value is smaller than a certain level and decrease the other data value by this certain level (Donoho, 1995). Hard thresholding aims to remove data with certain small value directly. Figures 1(c) and 1(d) show the seislet domain after applying soft thresholding and mask operator.

Mask operator is more efficient because it is linear and requires only the scale coefficient below which data values are preserved. Thresholding operator needs to compare the value of each transformed domain point with a predefined coefficient. In the case of deblending, we prefer to use thresholding operator rather than mask operator as the shaping operator because

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some abnormal-amplitude features may appear on the seismic profile which needs to be carefully selected in the seislet domain rather than ignored by a statistical selection.

With  $\mathbf{S}$  set to be soft thresholding in the seislet domain and  $\mathbf{B} = \frac{1}{2}\mathbf{I}$ , combining equations 3, 4 and 8, we get:

$$\begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \end{bmatrix}_{n+1} = \frac{1}{2}\mathbf{S} \left[ \begin{bmatrix} \mathbf{d} \\ \mathbf{T}^{-1}\mathbf{d} \end{bmatrix} + \begin{bmatrix} \mathbf{I} & -\mathbf{T} \\ -\mathbf{T}^{-1} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \end{bmatrix}_n \right], \quad (18)$$

where  $\mathbf{S}$  is a block operator which takes the form  $\begin{bmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \end{bmatrix}$ ,  $\mathbf{S}_1$  and  $\mathbf{S}_2$  correspond to the shaping operators for  $\mathbf{d}_1$  and  $\mathbf{d}_2$  respectively and consist of the seislet transform, thresholding, and inverse seislet transform. Using 18 and performing appropriate iterations, we aim to get an estimate of unblended seismic data.

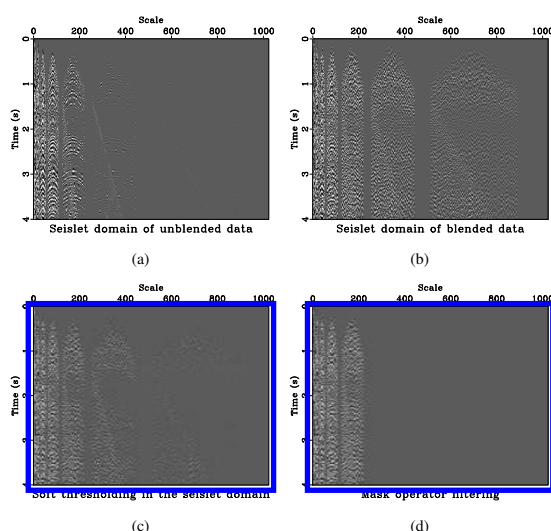


Figure 1: Demonstration of the seislet tranform. (a) Seislet domain of unblended seismic data shown in Figure 3(a). (b) Seislet domain of blended seismic data shown in Figure 3(b). (c) Seislet domain after applying soft thresholding. (d) Seislet domain after applying mask operator.

## EXAMPLES

### Numerically-blended synthetic data

We create a synthetic data set to model the simultaneous problem involved in OBN acquisition in order to test our proposed algorithm. In our synthetic survey, one node is placed on the ocean bottom. Two vessels travel along two predefined lines. The two vessels shoot pseudo-synchronously, which means one vessel shoots by applying a random dithering to the corresponding shot of another vessel. By precisely picking the shooting time of each shot of one vessel at the long seismic record on the node, we get one common receiver profile. By the same way, we get another profile.

For concisity, we only present one of the two blended profiles and its debrending results. Figures 3(a) and 3(b) are the un-

blended and blended data respectively. The synthetic model we use is composed of three parts. Hyperbolic events correspond to seismic reflections, the dipping events are actually coherent noise for the model. There are also some synthetic ground roll in the model. The target of debrending is getting the most similar estimate as Figure 3(a) from Figure 3(b).

Figures 3(c) and 3(d) show the debrending results after 2 and 20 iterations respectively. Even though we can still see much blending noise remained after 2 iterations, we barely see any noise after 20 iterations. The result after 20 iterations are nearly perfectly same as the original unblended one. Figures 3(e) and 3(f) show the estimated blending noise and the estimation error respectively.

To test the convergence performance and accuracy of our approach, we introduce two measures:  $c(n)$  and  $e(n)$ .  $c(n)$  denotes the data error, defined by:

$$c(n) = 10\log_{10} \left( \frac{\|\mathbf{m}_n - \mathbf{m}_{n-1}\|_2^2}{N} \right), \quad (19)$$

and  $c(n)$  denotes the model error, defined by:

$$e(n) = 10\log_{10} \left( \frac{\|\mathbf{m}_n - \mathbf{m}\|_2^2}{N} \right), \quad (20)$$

where  $\mathbf{m}_n$  denotes the estimated model after  $n$  iterations,  $\mathbf{m}$  denotes the true model,  $N$  denotes total data points of the profile and  $\|\cdot\|_2^2$  denotes the squared  $L_2$  norm of a function.

The convergence diagram and estimation error diagram are shown in Figure 2, which show that our approach converge to an suitable accuracy level after a small number of iterations.

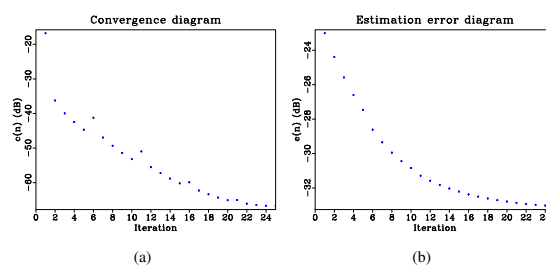


Figure 2: (a) Convergence diagram. (b) Estimation error diagram.

### Numerically-blended field data

We use two field common receiver gathers acquired by the OBN technique to generate the numerically-blended field data. According to equation 3, our debrending scheme follows equation 18. Here we only present the debrending procedures for one of the two gathers (see Figure 4). From Figure 4(a) we see the crosstalk is very intense and impossible for direct processing. After 20 iterations following equation 18, we obtain the debrended data shown in Figure 4(b), which is much cleaner than Figure 4(a). From the difference section shown in Figure 4(c) we can conclude that the debrending result is satisfactory



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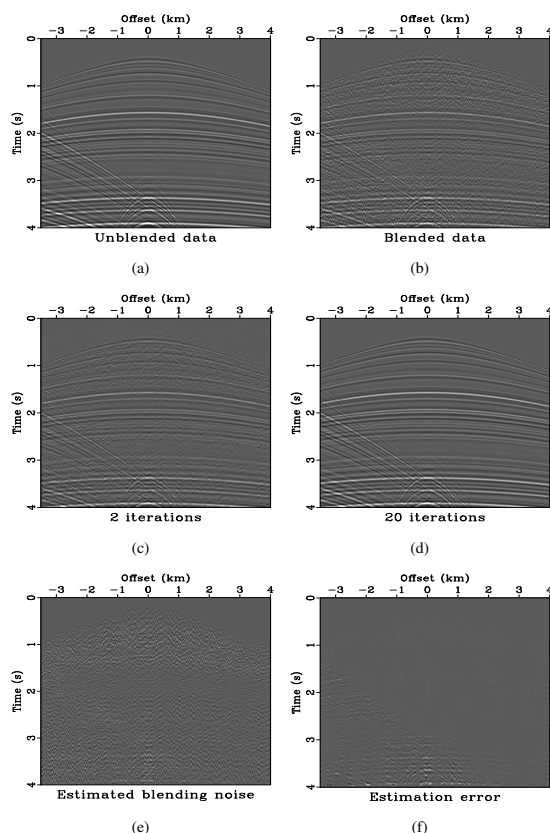


Figure 3: Estimated results using iterative shaping regularization. (a) Unblended data. (b) Blended data. (c) Estimated result after 2 iterations. (d) Estimated result after 20 iterations. (e) Estimated blending noise. (f) Estimation error.

because the section is mainly composed of blending noise and a small amount of useful energy. Note that, in the case of deblending, ground roll and coherent dip events in the original unblended sections are both considered to be useful energy.

## CONCLUSIONS

We have presented an iterative approach to deblend seismic data from simultaneous sources. The principle of our approach is treating the problem of deblending as an estimation problem, and then using shaping regularization in the seislet domain to constrain the models when iteratively solving the problem. We have derived the mathematical formulation of this method and used both numerically-blended synthetic example and field data example to test it. Results indicate that it is possible to get accurate results within a small number of iterations.

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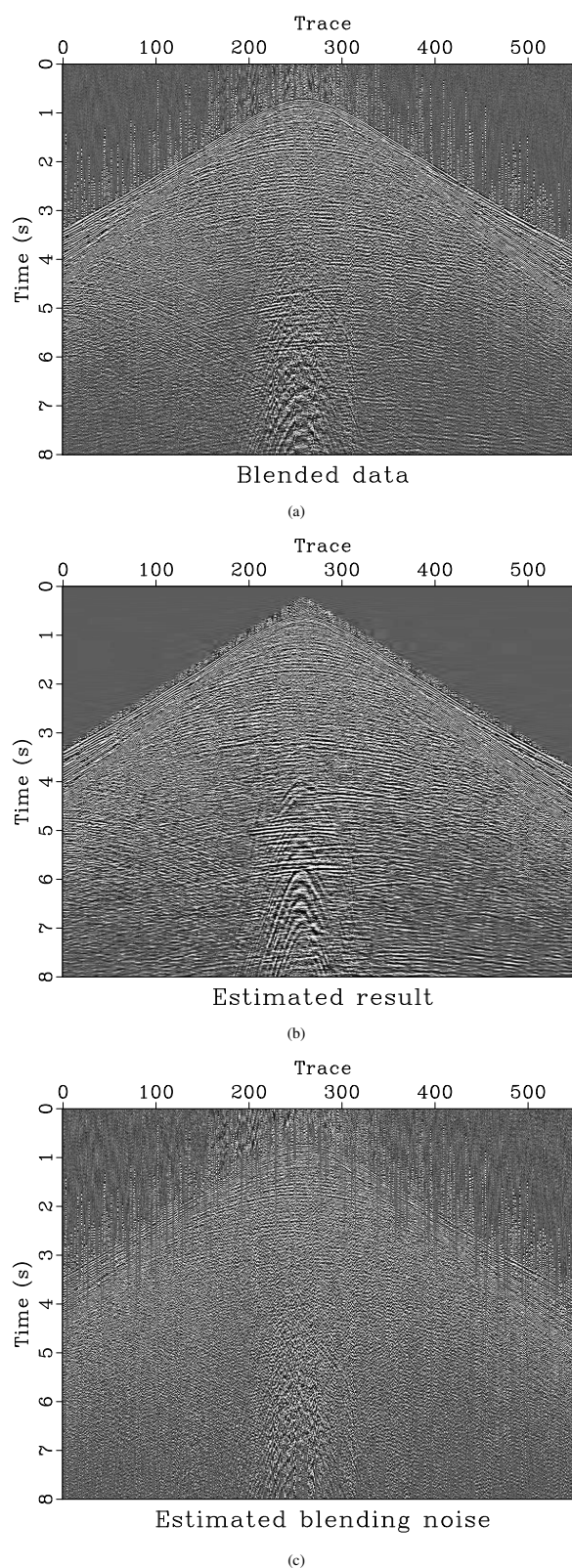


Figure 4: Deblending result for the field data test. (a) Original blended data. (b) Deblended data using shaping regularization. (c) Difference section between the deblended result and the original blended data.

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