A model for the dynamics of human weight cycling

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Background

Hopf Bifurcation (briefly)

Michaelis–Menten kinetics

Hopf Bifurcation

A fundamental concept in nonlinear dynamical systems describing how oscillations arise from a previously stable steady state.

A Hopf bifurcation occurs in a nonlinear dynamical system when:

- A pair of complex conjugate eigenvalues of the Jacobian matrix at a fixed point crosses the imaginary axis from left to right
- The steady state loses stability
- A stable limit cycle emerges (supercritical case) or vanishes (subcritical case)

This leads to oscillations in system variables.

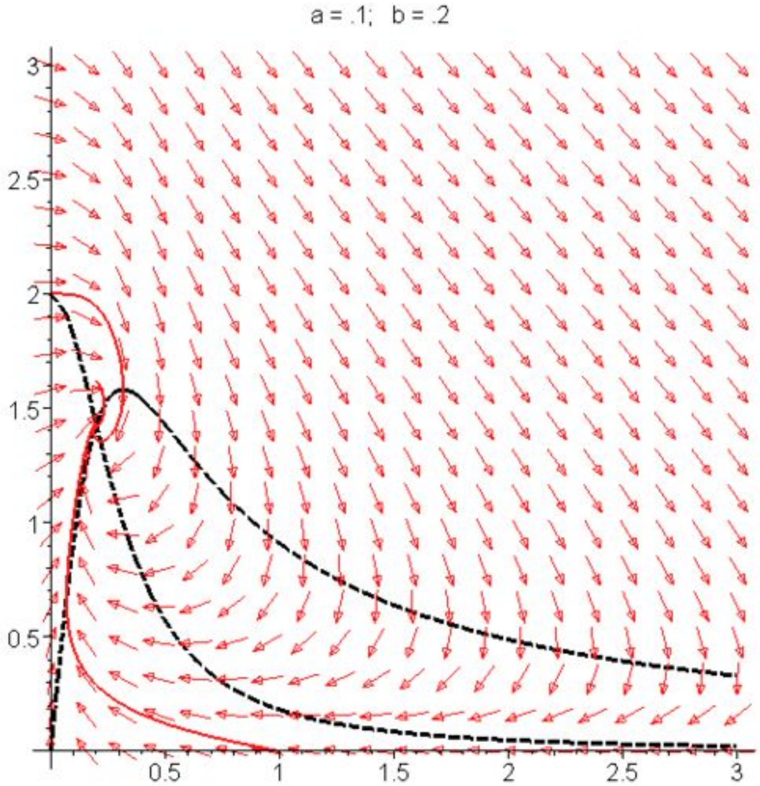
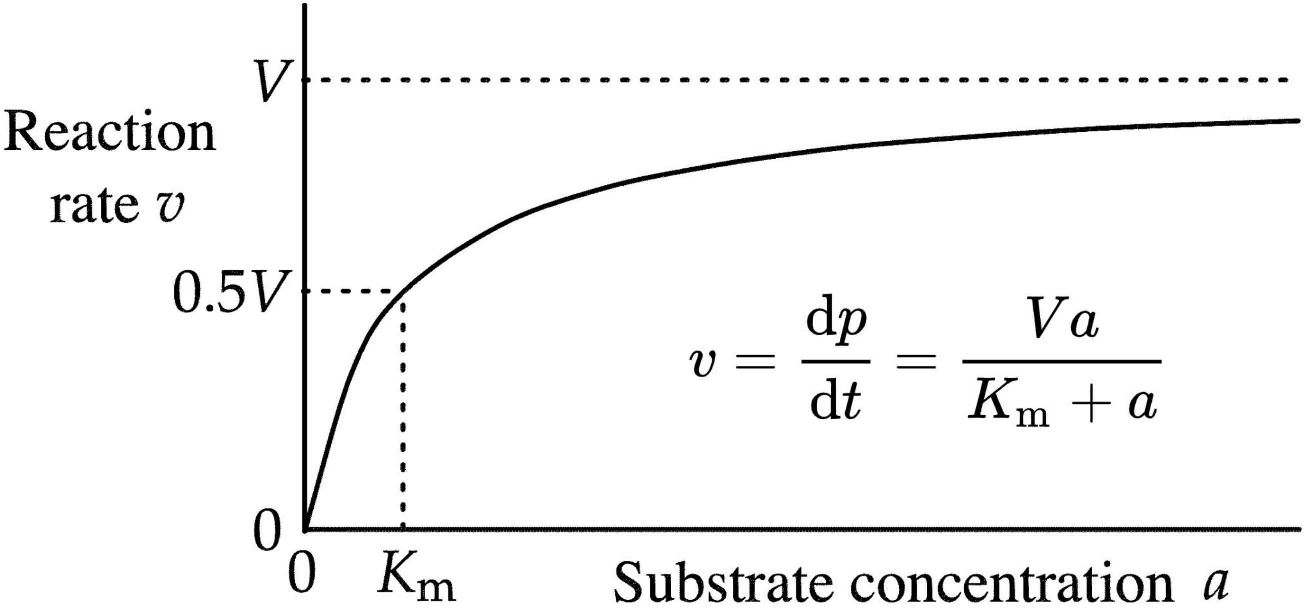


Image from: https://en.wikipedia.org/wiki/Hopf_bifurcation

Michaelis–Menten kinetics

Mostly see in biochemistry

➔ continuous
➔ differentiable
➔ non-linear



Model

ODEs

Parameters

Threshold

PQR model: Weight cycling model with 3 ODEs

$$\frac{d P}{dt} = a \cdot Q - b \cdot \frac{P}{K + P}$$

Michaelian function

Q – substrate

$$\frac{d Q}{dt} = \underbrace{V_1 \cdot \frac{(1-Q)}{K_1 + (1-Q)}}_{\text{Positive term}} - \underbrace{V_2 \cdot R \cdot \frac{Q}{K_2 + Q}}_{\text{Negative term}}$$

Positive term

The rate Q↑

Negative term

Q↓ (max rate: V_2R)

Threshold function for the dependency of Q on R

$$\frac{d R}{dt} = \underbrace{P \cdot V_3 \cdot \frac{(1-R)}{K_3 + (1-R)}}_{\text{Positive term}} - \underbrace{V_4 \cdot \frac{R}{K_4 + R}}_{\text{Negative term}}$$

Positive term

R↑

Negative term

R↓

Threshold function for the dependency of R on P

Variables: P – Weight

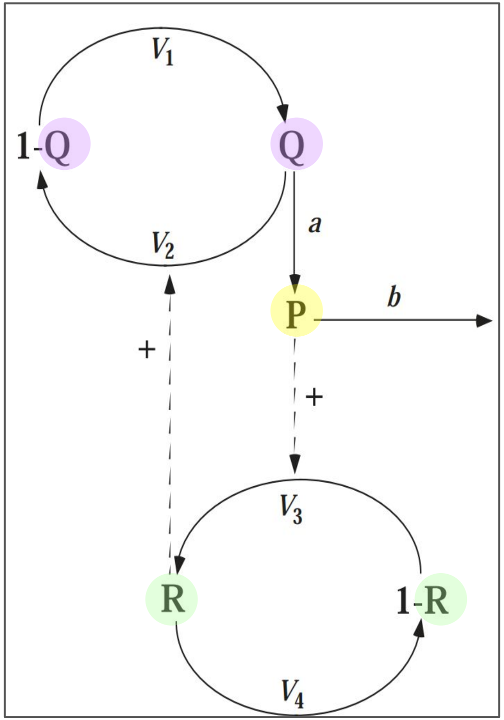
Q – Dietary Intake

R – Cognitive restraint

P = excess

= P↑ – basal (ref.) value

$Q, P \in [0, 1]$



PQR model - in detail

Variables:

P: excess body weight

Q: dietary intake

R: cognitive restraint (psychological motivation to diet)

Mathematical structure:

System of 3 ODEs with Michaelis-Menten-like kinetics

Threshold behavior in R(P) and Q(R)

Parameters (3+4+4=11):

a	Conversion rate from food intake to weight gain
b	Maximal rate of weight loss (energy dissipation)
K	Weight value at which loss rate is half-maximal
V1	Craving strength
V2	Restraint strength
K1	Sensitivity of craving increase
K2	Sensitivity of restraint effect on intake
V3	Motivation gain
V4	Motivation decay
K3	Sensitivity of restraint increase with weight
K4	Sensitivity of restraint decay with time

PQR model - parameters

dP/dt - on the change of weight

Parameter	Role	Meaning
a	Weight gain efficiency	Converts dietary intake Q to weight gain P; affected by metabolic efficiency
b	Max weight loss rate	Maximal energy dissipation (e.g., via metabolism or physical activity)
K	Saturation constant in weight loss	Value of P at which weight loss rate is half-maximal; affects how fast weight is lost when P is high

$$\frac{d}{dt}P = a \cdot Q - b \cdot \frac{P}{K + P}$$

PQR model - parameters

dQ/dt - on the change of diet

Parameter	Role	Meaning
V1	Max rate of increase in food intake	Models craving or orexigenic drive (e.g., via ghrelin)
V2	Max rate of decrease in intake due to restraint	Reflects anorexigenic signals (e.g., leptin, cognitive control)
K1	Michaelis constant for intake growth	How fast Q increases as craving dominates (small K1 → steep increase)
K2	Michaelis constant for intake suppression	How fast Q decreases when restraint R is strong

$$\frac{d}{dt}Q = V_1 \cdot \frac{(1-Q)}{K_1 + (1-Q)} - V_2 \cdot R \cdot \frac{Q}{K_2 + Q}$$

PQR model - parameters

dR/dt - on the change of cognitive restraint

Parameter	Role	Biological/Psychological Meaning
V3	Max rate of increase in restraint	Higher values mean stronger motivation to diet when weight is high
V4	Max rate of decline of restraint	Measures habituation or loss of motivation over time
K3	Michaelis constant for rise of R	Controls how quickly restraint grows with weight P
K4	Michaelis constant for decay of R	Controls how quickly restraint wanes over time

$$\frac{dR}{dt} = P \cdot V_3 \cdot \frac{(1-R)}{K_3 + (1-R)} - V_4 \cdot \frac{R}{K_4 + R}$$

Thresholds

$$P^* = \frac{V_4}{V_3} \frac{(1 + 2K_3)}{(1 + 2K_4)}$$

$$R^* = \frac{V_1}{V_2} \frac{(1 + 2K_2)}{(1 + 2K_1)}$$

How can the author derived these threshold? We don’t know

Results

Threshold dependence

Time evolution of P, Q, R

Phase plane of P vs R

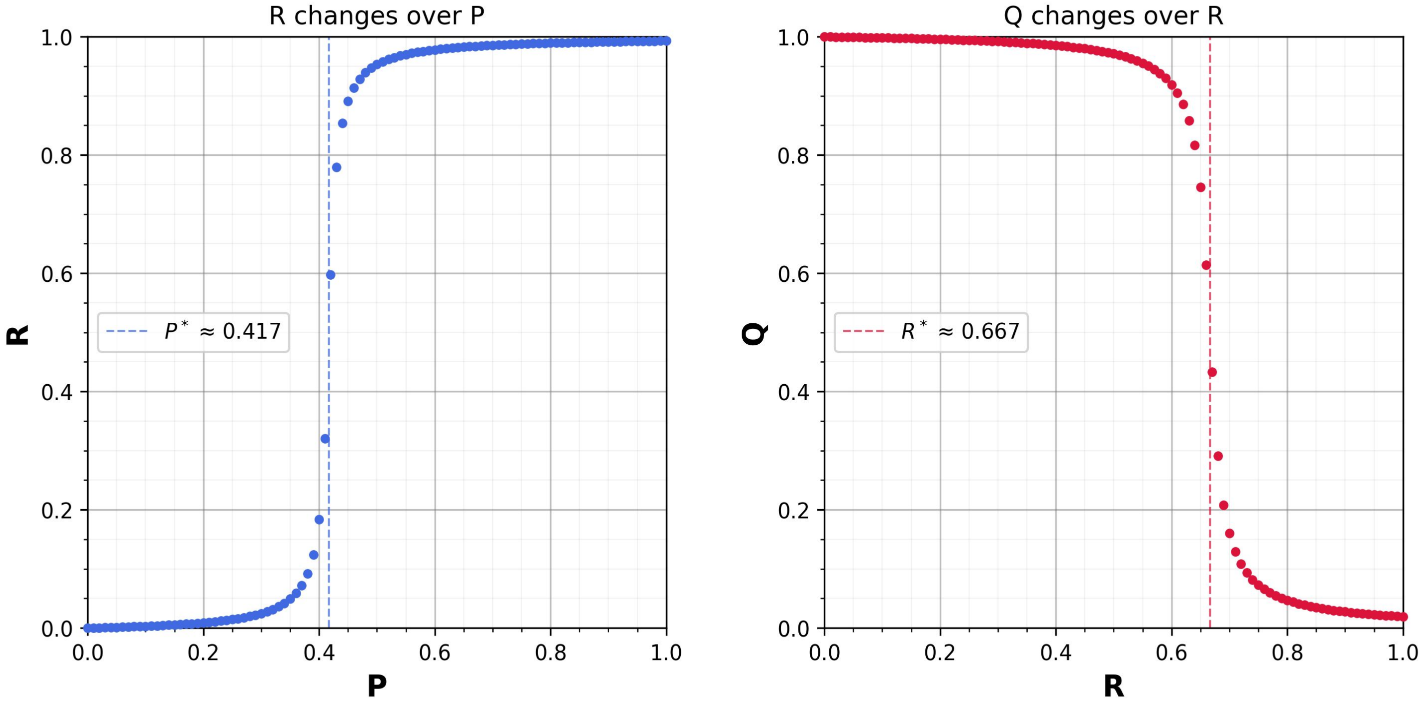
Time evolution of P for parameter V4

Time evolution of P for parameter Ki

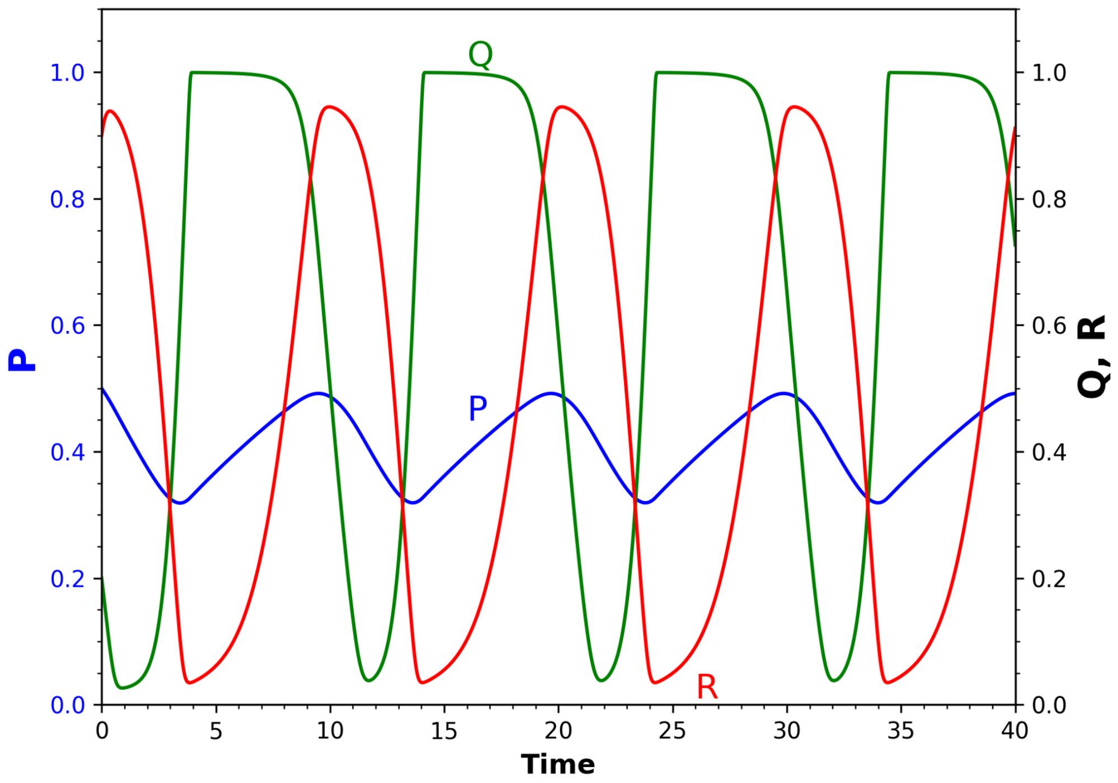
Domain of weight cycling

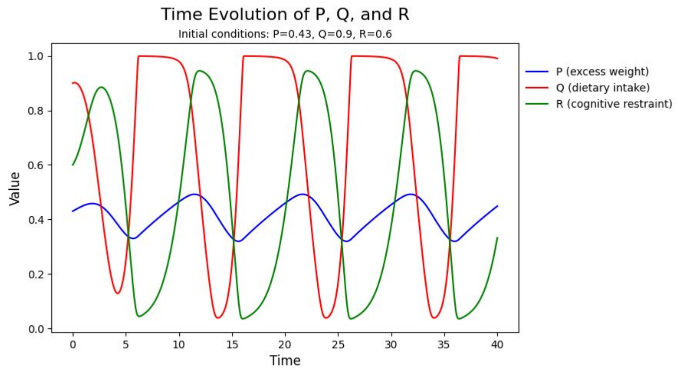
Threshold dependence of R on P and of Q on R

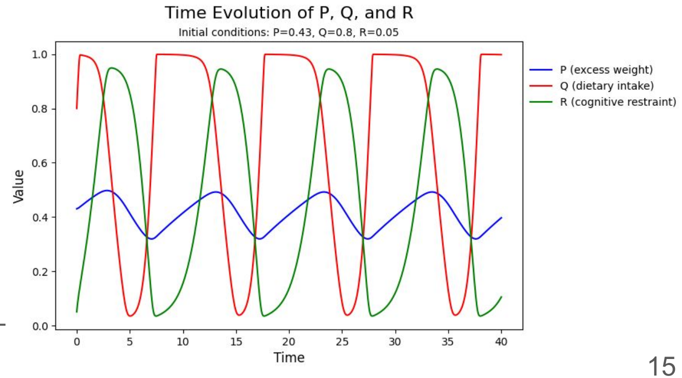
"switch" type nonlinear feedback

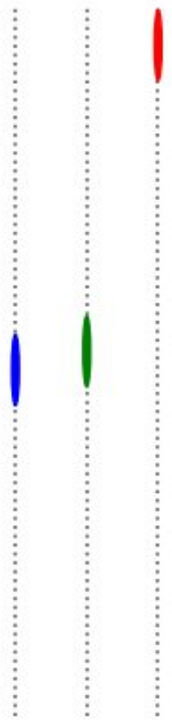


Time evolution of P, Q, R









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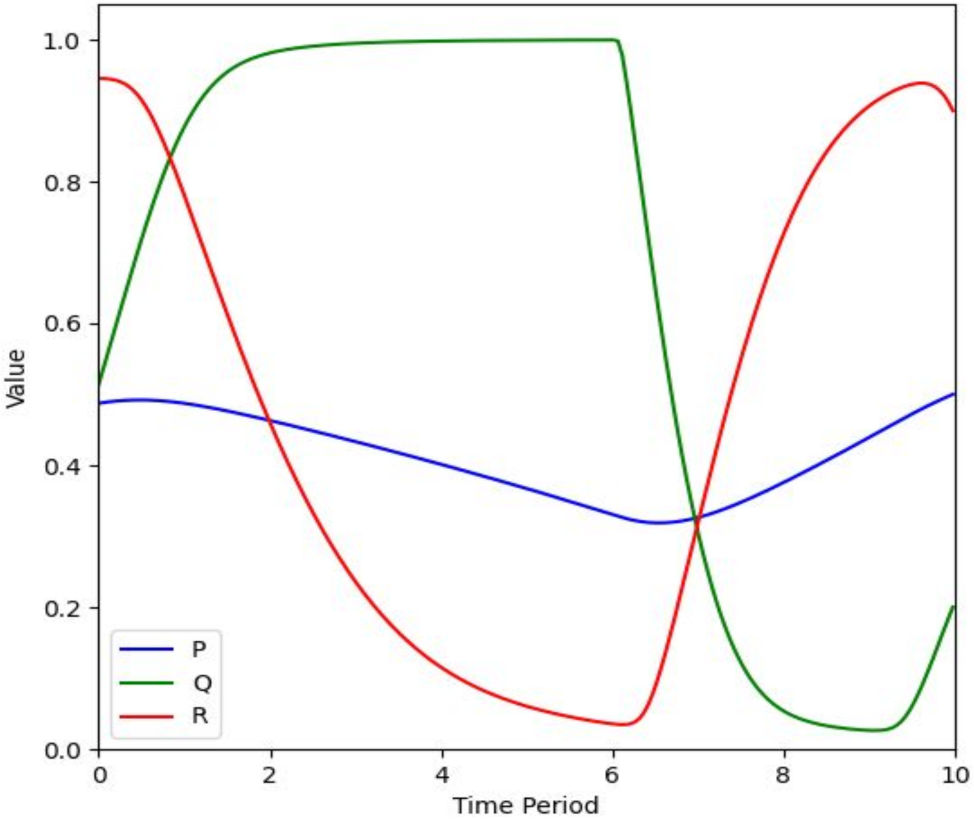
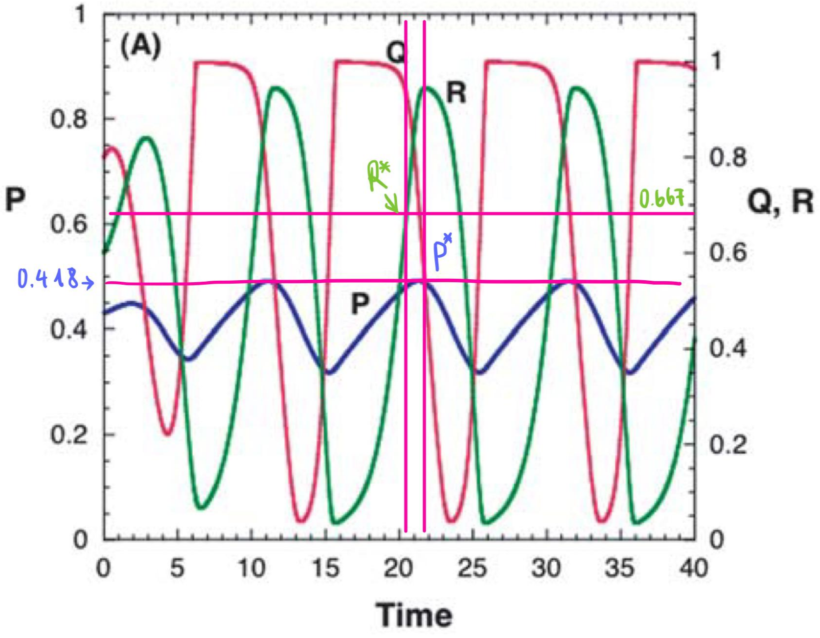
you can see it here:



System oscillation in relation to P*, R* thresholds



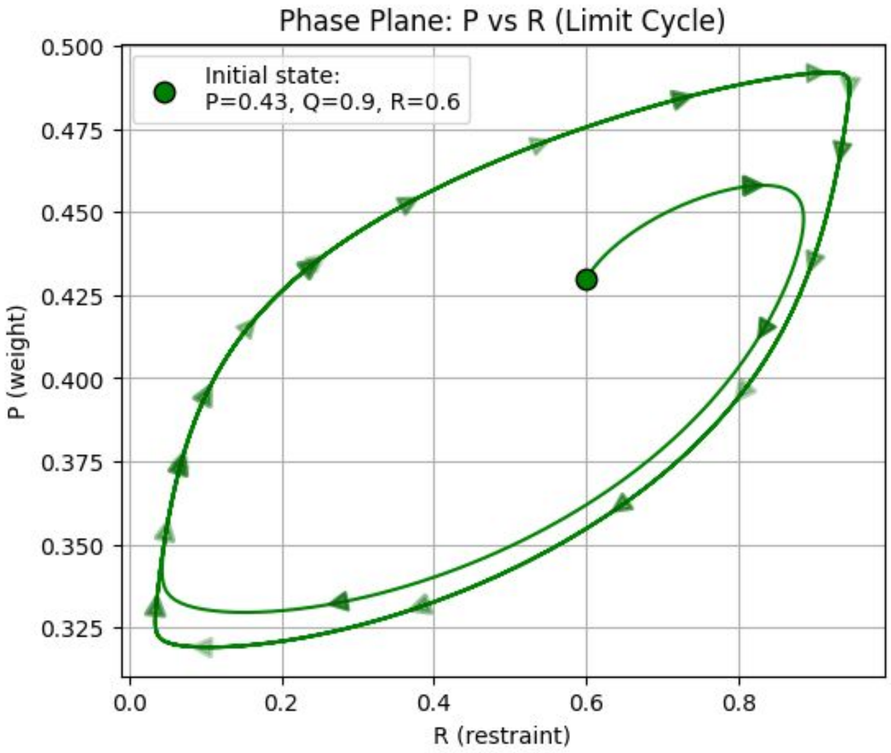
Parameter values:

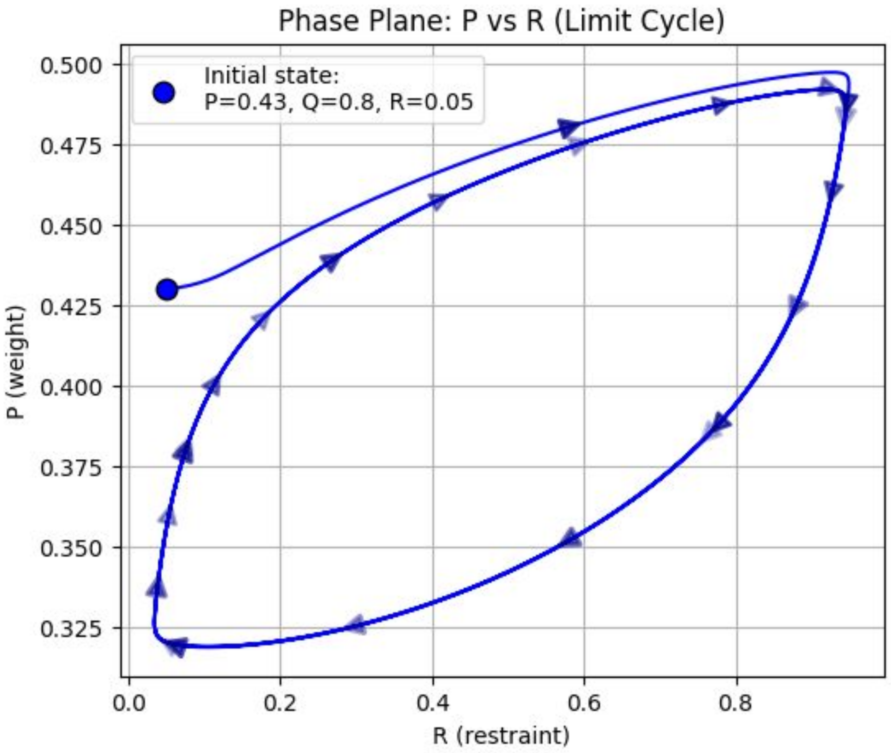
```
a, b = 0.1, 0.1
V1, V2, V3, V4 = 1.0, 1.5, 6.0, 2.5
K, K1, K2, K3, K4 = 0.2, 0.01, 0.01, 0.01, 0.01
```

$$P^* = \frac{V_4}{V_3} \frac{(1+2K_3)}{(1+2K_4)} = 0.418$$

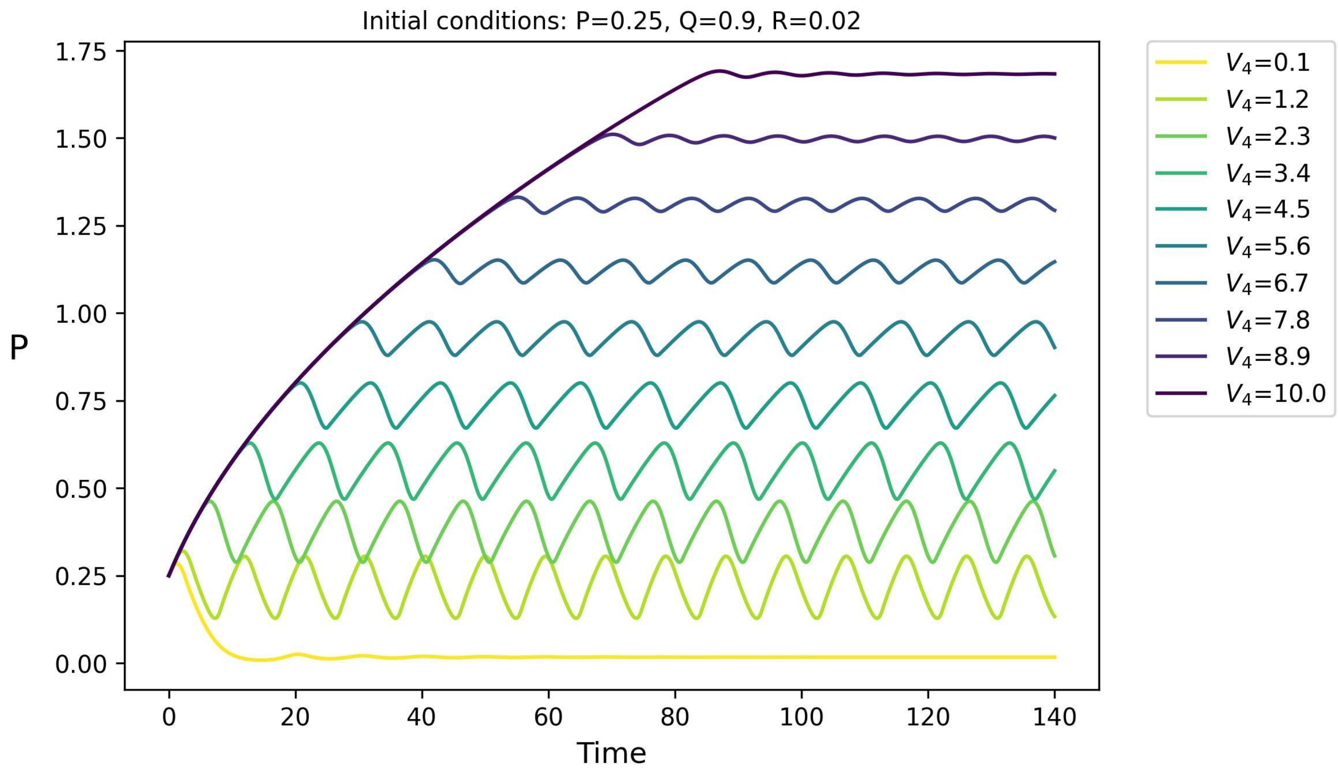
$$R^* = \frac{V_1}{V_2} \frac{(1+2K_2)}{(1+2K_1)} = 0.667$$

The oscillations correspond to the evolution towards a limit cycle

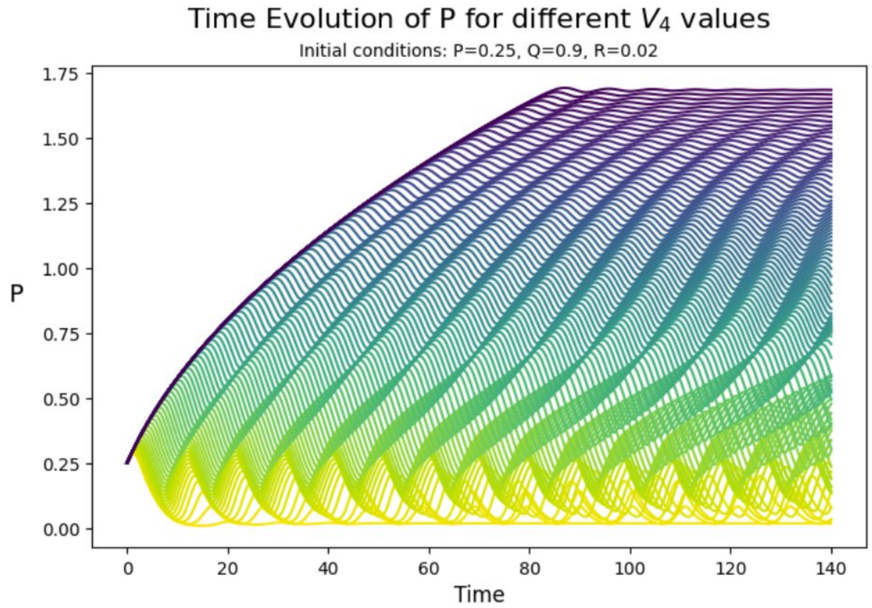


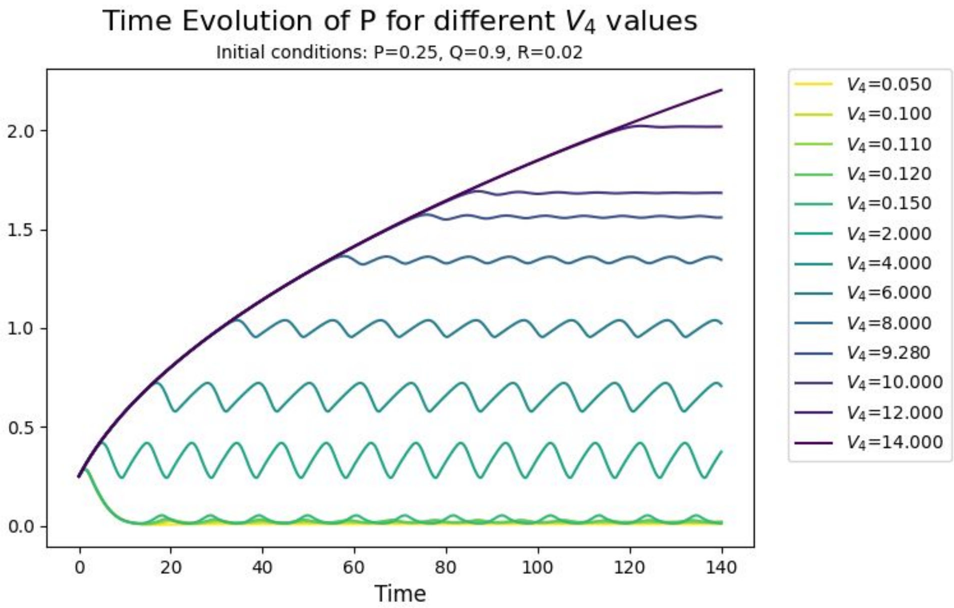


Time evolution of P for parameter V4 (Motivation decay)

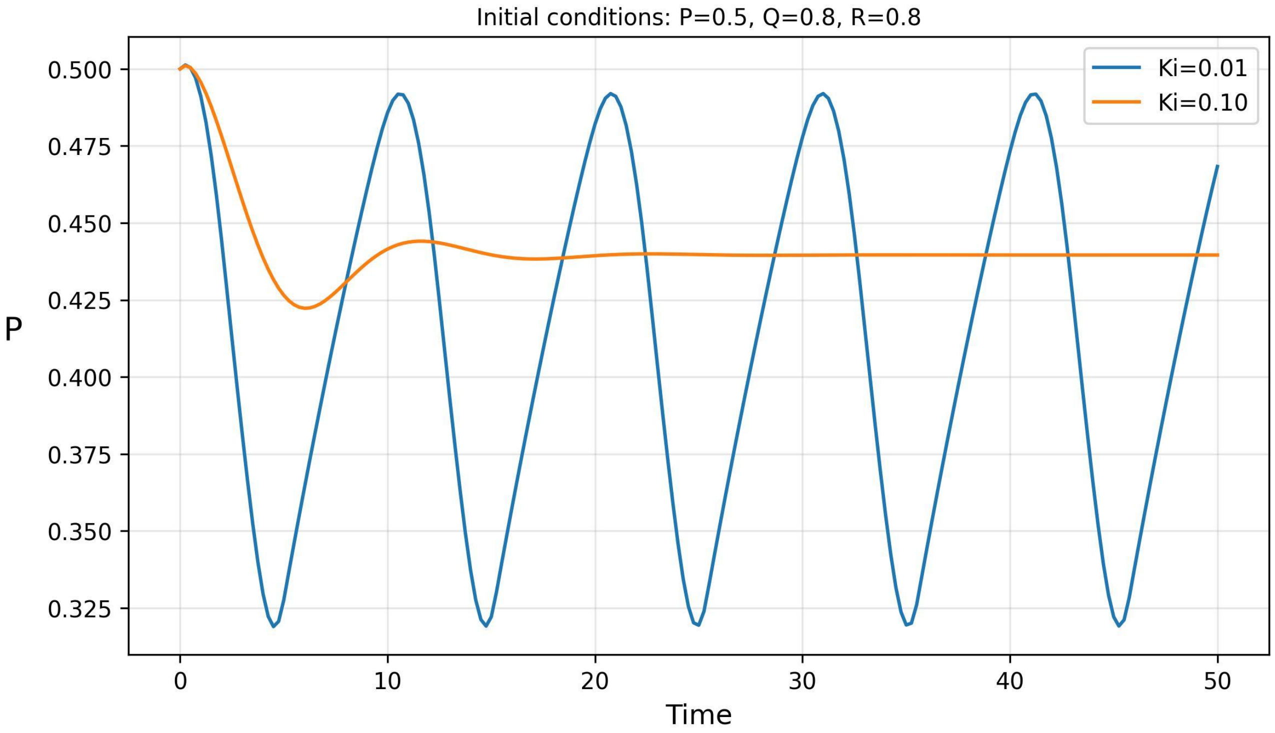


Time evolution of P for parameter V4 (Motivation decay)

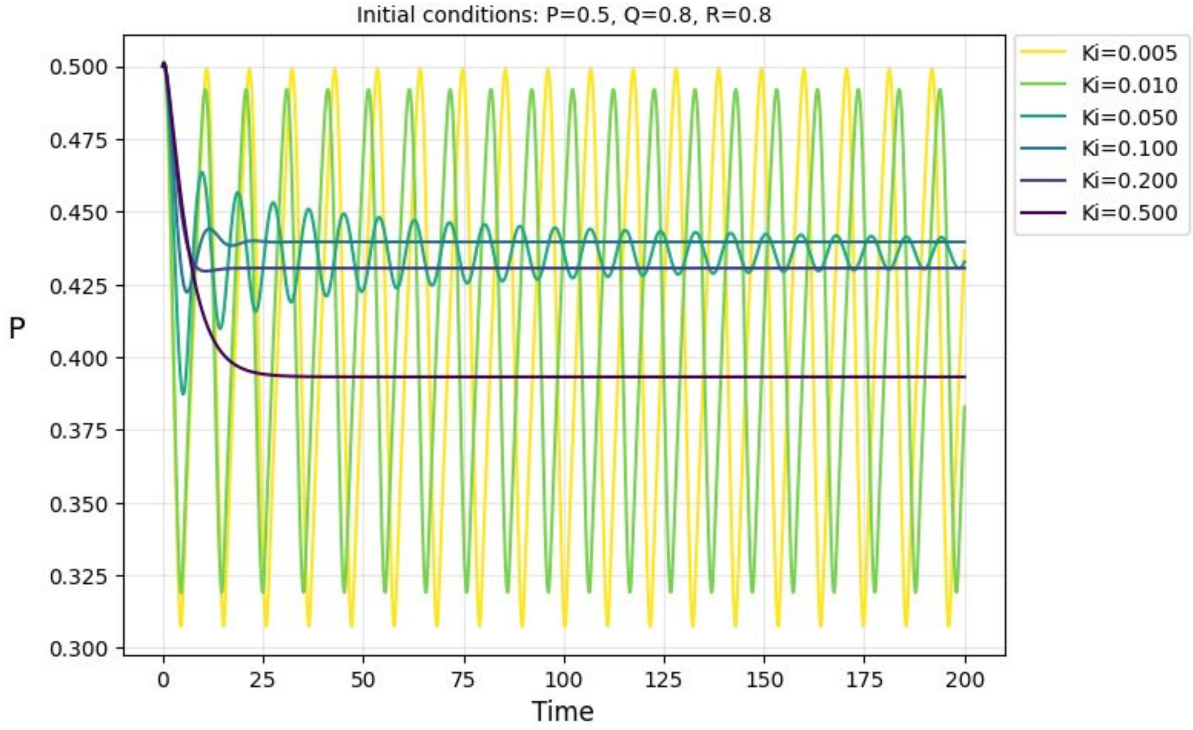




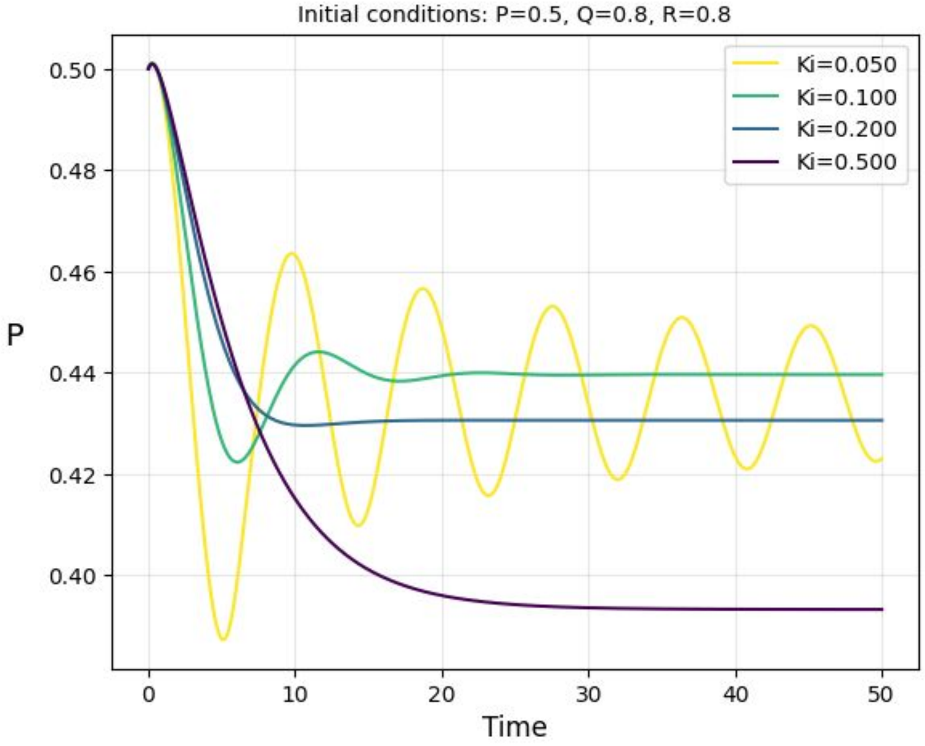
Time evolution of P for parameter Ki

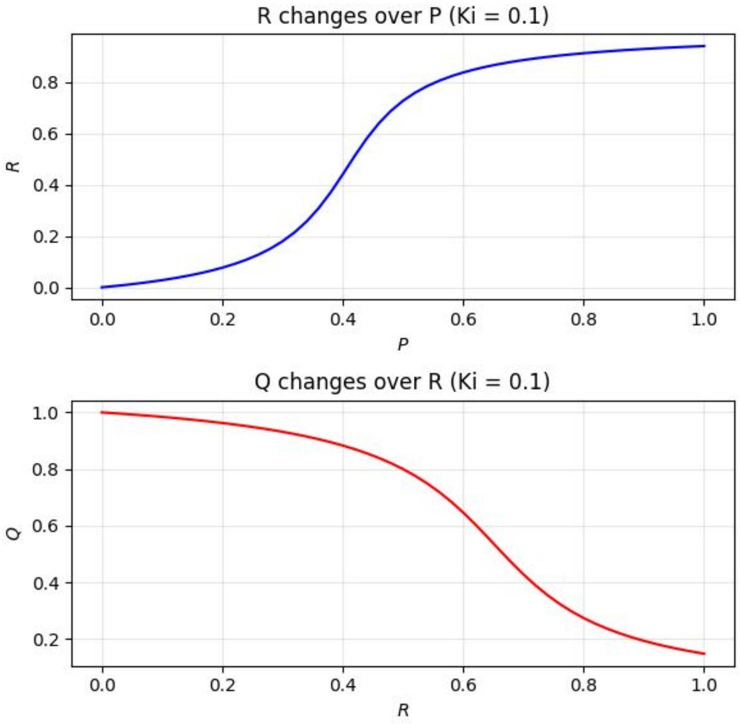


Time evolution of P for parameter Ki



Oscillations disappear when Ki …



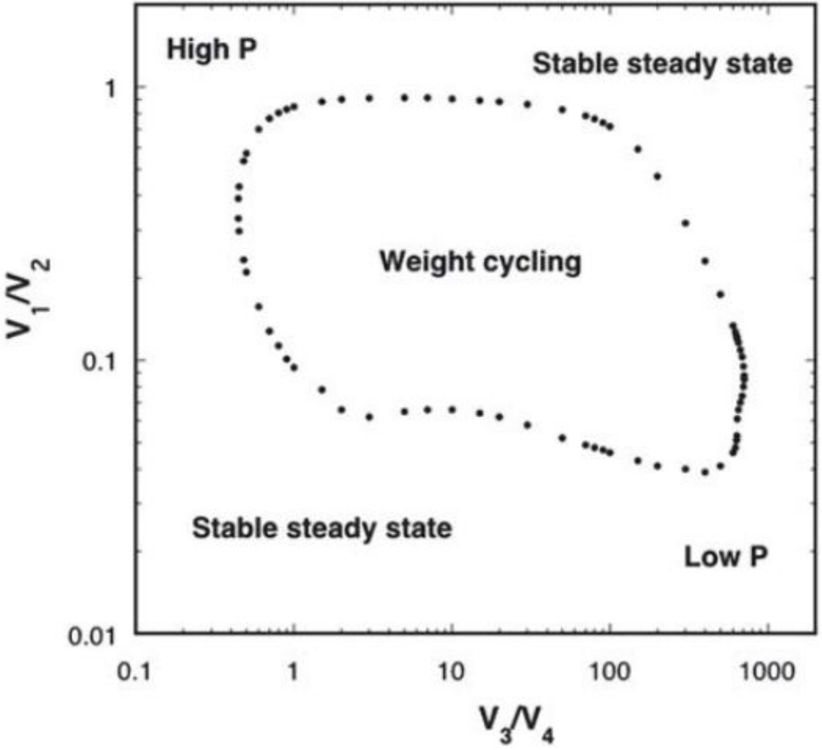


Domain of weight cycling upon varying parameter ratios

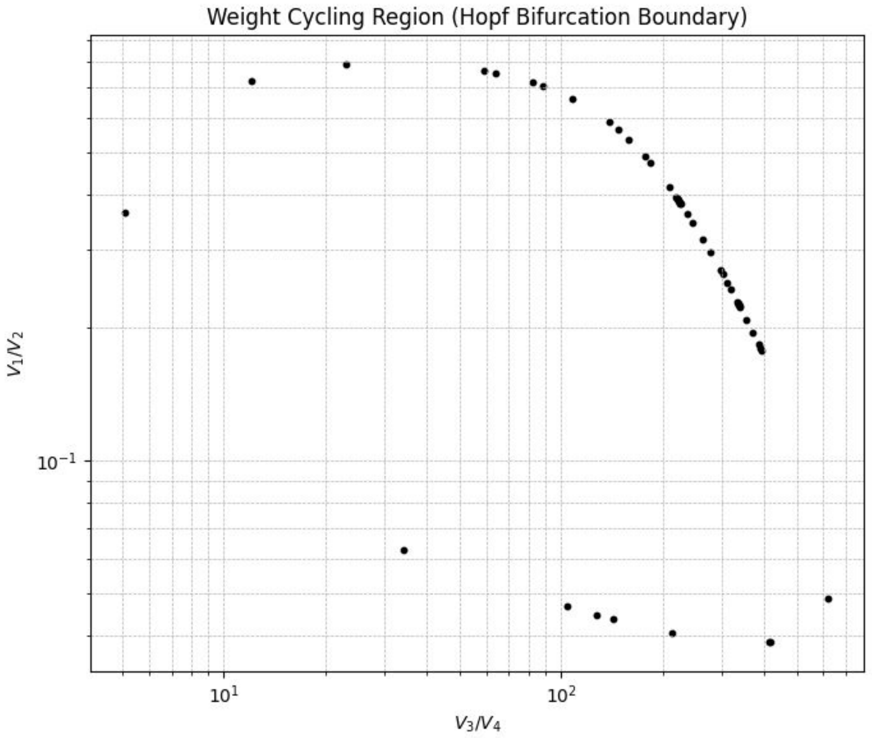
1. compute Jacobian matrix

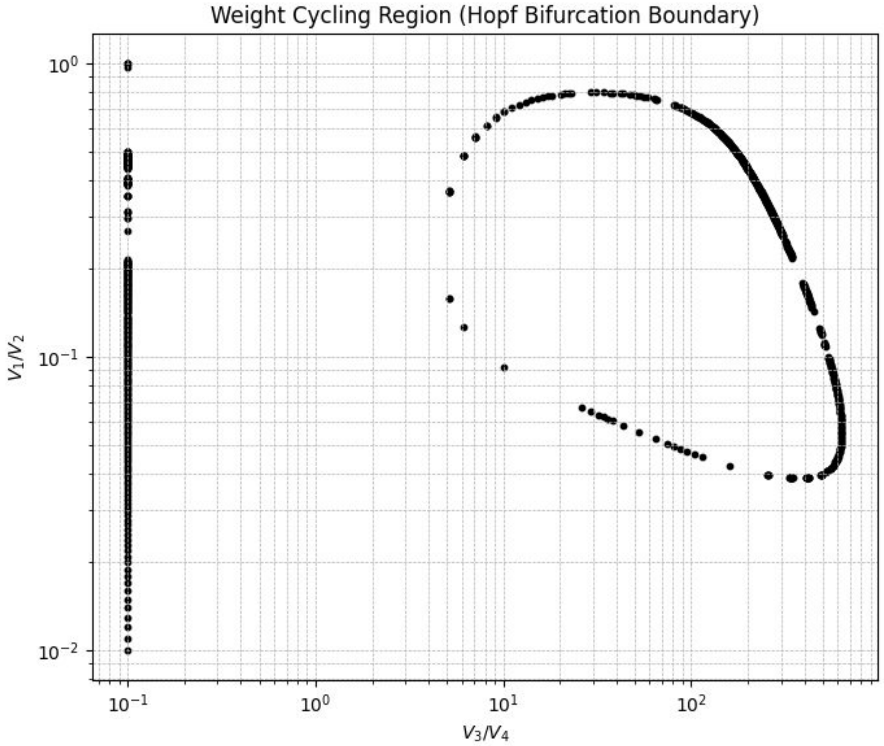
2. record combination of (V1/V2,V3/V4), where real part of eigenvalue ≈ 0

```
eigenvalues = eigvals(J)
max_real = np.max(np.real(eigenvalues))
if abs(max_real) < 5e-4:
    boundary_points.append([V3_V4, V1_V2])
```



Domain of weight cycling upon varying parameter ratios





What are the drawbacks?

- Simplified psychological factors

- Stochastic reality

- Some parameters are likely to change over time

Conclusion

1. Sustained oscillations only occur in a precise window bounded by critical values of a particular control parameter.

2. Outside the weight cycling region, P reaches a stable steady state.

3. Weight cycling occurs only when the thresholds in the dependence of R on P and of Q on R are sufficiently sharp.

Questions and Discussion

Some questions from our group

1. Michaelis-Mendel equation and its relation to each terms (in 1b, 1c eq.)?
2. Analytical solution for the threshold, how it was derived?
3. Why period = 10 time unit? (this has been discussed in the lecture session by our professor 😉)

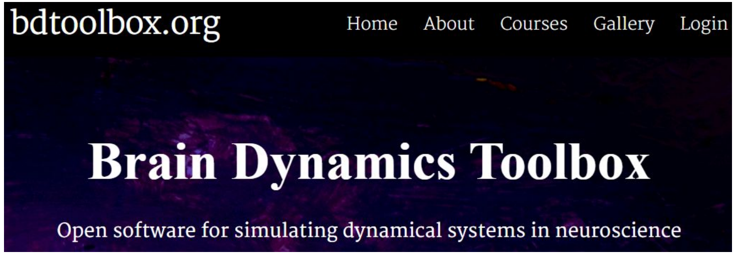
Some questions to discuss

1. What conclusion this paper/model give us to lead the dieting?

→ Therapeutic approaches, to change metabolism and psychological parameters, instead of just change the eating portion

Fun experiment: change parameters and observe behavior in 3D phase plane

MATLAB model



Example: