Deep Learning for Computer Vision: Assignment 3

Computer Science: COMS W 4995 006

Due: March 6, 2018

Problem

You are given two dimensional input from three separate classes. Your task is to implement a multi-layer perceptron (MLP) 3-class classifier with multiple hidden layers and a regularization on the weights. For the activiation function of the hidden units use ReLU or leaky ReLU. For the predictions use softmax on a linear output layer as we did in class. Your loss layer should compute $-\log P(y = i\,|\mathbf{x})$ where i is the correct label according to the training data.

a) Implement each layer type (hidden, output, and loss) as separate python classes, each with methods for initialization, forward propagation, and backpropagation.

b) Implement a MLP as its own class, with separate methods for initialization, adding a layer, forward propagation, backpropagation, training and prediction.

c) Let the layer dimensions be parameters passed when the network is created.

d) Let the number of training epochs, the mini-batch size, and the regularization parameter be parameters that are passed when training the network.

e) Build and run your network using your own constructs. The code for doing this might look like:

In [ ]:
```
NN = MLP()
NN.add_layer('Hidden', dim_in=2, dim_out=16)
NN.add_layer('Hidden', dim_in=16, dim_out=16)
NN.add_layer('Hidden', dim_in=16, dim_out=16)
NN.add_layer('Output', dim_in=16, dim_out=3)
NN.add_layer('Loss', dim_in=3, dim_out=3)

loss = NN.train(X, y, epochs=100, bsize=8, alpha=0.0)
plot_loss(loss)
plot_decision_regions(NN)
```

f) Show the decision regions of the trained classifier by densely generating points in the plane and color coding these points with the three different labels.

g) Repeat varying the number of hidden units (3, 8, 16), the number of hidden layers (1 and 3), and the regularization value (0 and some other value of your choosing).

h) Now replace your ReLU activation function with a softplus function and repeat.

Grading: a-g=90%, h=10%.

NOTE: Do not to use keras, tensorflow, pytorch, sklearn, etc. to do this. You must build the machine learning components from scratch.

YOUR CODE MUST BE YOUR OWN.

Let's start by importing some libraries.

In [1]:
```
import numpy as np
import random
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
```

Let's make up our 2D data for our three classes.

In [2]:
```
data = pd.DataFrame(np.zeros((5000, 3)), columns=['x1', 'x2', 'y'])

# Let's make up some noisy XOR data to use to build our binary classifier
for i in range(len(data.index)):
    x1 = random.randint(0,1)
    x2 = random.randint(0,1)
    if x1 == 1 and x2 == 0:
        y = 0
    elif x1 == 0 and x2 == 1:
        y = 0
    elif x1 == 0 and x2 == 0:
        y = 1
    else:
        y = 2
    x1 = 1.0 * x1 + 0.20 * np.random.normal()
    x2 = 1.0 * x2 + 0.20 * np.random.normal()
    data.iloc[i,0] = x1
    data.iloc[i,1] = x2
    data.iloc[i,2] = y

for i in range(int(0.25 *len(data.index))):
    k = np.random.randint(len(data.index)-1)
    data.iloc[k,0] = 1.5 + 0.20 * np.random.normal()
    data.iloc[k,1] = 1.5 + 0.20 * np.random.normal()
    data.iloc[k,2] = 1

for i in range(int(0.25 *len(data.index))):
    k = np.random.randint(len(data.index)-1)
    data.iloc[k,0] = 0.5 + 0.20 * np.random.normal()
    data.iloc[k,1] = -0.75 + 0.20 * np.random.normal()
    data.iloc[k,2] = 2

# Now let's normalize this data.
data.iloc[:,0] = (data.iloc[:,0] - data['x1'].mean()) / data['x1'].std()
data.iloc[:,1] = (data.iloc[:,1] - data['x2'].mean()) / data['x2'].std()

data.head()
```

Out[2]:

	x1	x2	y
0	-0.587753	-1.184344	2.0
1	-1.353896	0.866423	0.0
2	1.165150	0.657960	2.0
3	0.651693	-0.045370	0.0
4	0.392429	-0.789940	0.0

Let's message this data into a numpy format.

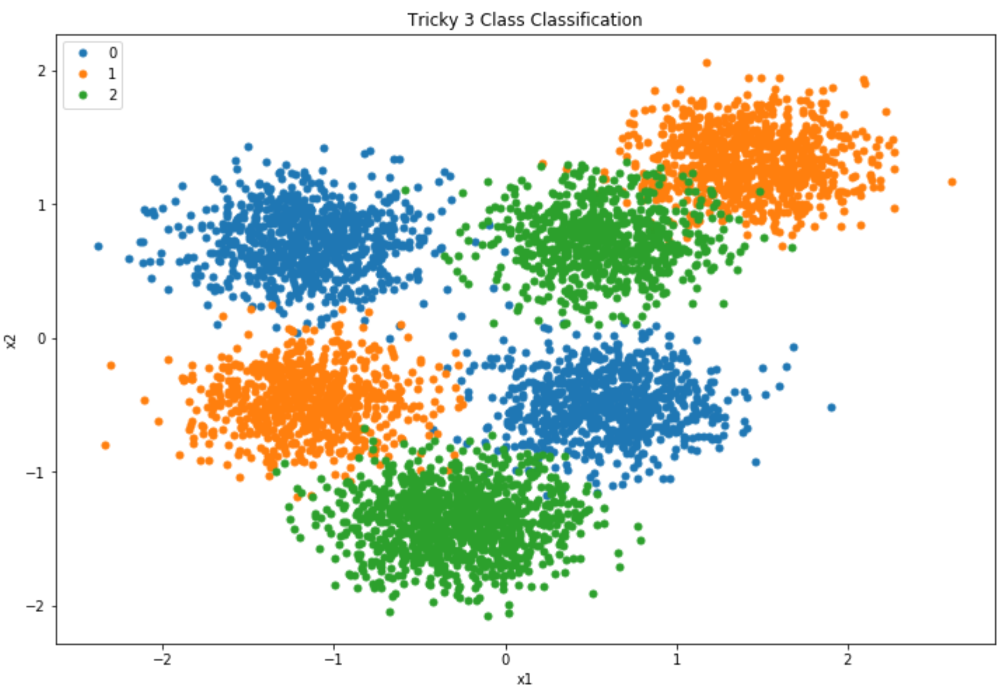
In [3]:
```
# set X (training data) and y (target variable)
cols = data.shape[1]
X = data.iloc[:,0:cols-1]
y = data.iloc[:,cols-1:cols]

# The cost function is expecting numpy matrices so we need to convert X and y before we can use them.
X = np.matrix(X.values)
y = np.matrix(y.values)
```

Let's make a sloppy plotting function for our binary data.

In [4]:
```
# Sloppy function for plotting our data
def plot_data(X, y_predict):

    fig, ax = plt.subplots(figsize=(12,8))
    ax.margins(0.05) # Optional, just adds 5% padding to the autoscaling

    indices_0 = [k for k in range(0, X.shape[0]) if y_predict[k] == 0]
    indices_1 = [k for k in range(0, X.shape[0]) if y_predict[k] == 1]
    indices_2 = [k for k in range(0, X.shape[0]) if y_predict[k] == 2]

    ax.plot(X[indices_0, 0], X[indices_0,1], marker='o', linestyle='', ms=5, label='0')
    ax.plot(X[indices_1, 0], X[indices_1,1], marker='o', linestyle='', ms=5, label='1')
    ax.plot(X[indices_2, 0], X[indices_2,1], marker='o', linestyle='', ms=5, label='2')

    ax.legend()
    ax.legend(loc=2)
    ax.set_xlabel('x1')
    ax.set_ylabel('x2')
    ax.set_title('Tricky 3 Class Classification')
    plt.show()
```

Now let's plot it.

In [5]:
```
plot_data(X, y)
```



Now build your network. Good luck! 😉

a) - g) Use ReLU as activation function and plot 12 graphs (parameter values are in the title of each graph)

In [6]:
```
X = X.A
y = y.A
```

In [70]:
```
# my plot functions

def plot_loss(loss):
    fig, ax = plt.subplots(figsize=(12,8))
    ax.margins(0.05) # Optional, just adds 5% padding to the autoscaling

    ax.plot(loss)
    ax.set_xlabel('number of iterations')
    ax.set_ylabel('loss')
    ax.set_title('{} hidden layer ({} units per layer), learning rate: {}, batch size: {}, epoch: {}, regulari
zation: {}, accuracy: {}'\
                 .format(len(NN.hidden_layer_list), NN.hidden_layer_list[0].dim_out, NN.eta, NN.batch_size, NN
.epoch, \
                        NN.regularization, NN.get_accuracy(X, y)))
    plt.show()

def plot_decision_regions(NN):

    fig, ax = plt.subplots(figsize=(12,8))
    ax.margins(0.05) # Optional, just adds 5% padding to the autoscaling

    # draw boundaries
    X_generated = np.random.uniform(-3, 3, (50000, 2))
    y_predicted = NN.predict(X_generated)
    indices_0 = [k for k in range(0, X_generated.shape[0]) if y_predicted[k] == 0]
    indices_1 = [k for k in range(0, X_generated.shape[0]) if y_predicted[k] == 1]
    indices_2 = [k for k in range(0, X_generated.shape[0]) if y_predicted[k] == 2]
    ax.plot(X_generated[indices_0, 0], X_generated[indices_0,1], marker='o', c='r', linestyle='', ms=5, label=
'0-region', zorder=1)
    ax.plot(X_generated[indices_1, 0], X_generated[indices_1,1], marker='o', c='m', linestyle='', ms=5, label=
'1-region', zorder=2)
    ax.plot(X_generated[indices_2, 0], X_generated[indices_2,1], marker='o', c='k', linestyle='', ms=5, label=
'2-region',zorder=3)

    # plot original data
    indices_0 = [k for k in range(0, X.shape[0]) if y[k] == 0]
    indices_1 = [k for k in range(0, X.shape[0]) if y[k] == 1]
    indices_2 = [k for k in range(0, X.shape[0]) if y[k] == 2]
    ax.plot(X[indices_0, 0], X[indices_0,1], marker='o', linestyle='', c='tab:blue', ms=5, label='0-sample', z
order=9)
    ax.plot(X[indices_1, 0], X[indices_1,1], marker='o', linestyle='', c='tab:orange', ms=5, label='1-sample',
 zorder=10)
    ax.plot(X[indices_2, 0], X[indices_2,1], marker='o', linestyle='', c='tab:green', ms=5, label='2-sample',z
order=11)

    ax.legend()
    ax.legend(loc=2)
    ax.set_xlabel('x1')
    ax.set_ylabel('x2')
    ax.set_title('{} hidden layer ({} units per layer), learning rate: {}, batch size: {}, epoch: {}, regulari
zation: {}, accuracy: {}'\
                 .format(len(NN.hidden_layer_list), NN.hidden_layer_list[0].dim_out, NN.eta, NN.batch_size, NN
.epoch, \
                        NN.regularization, NN.get_accuracy(X, y)))
    plt.show()
```

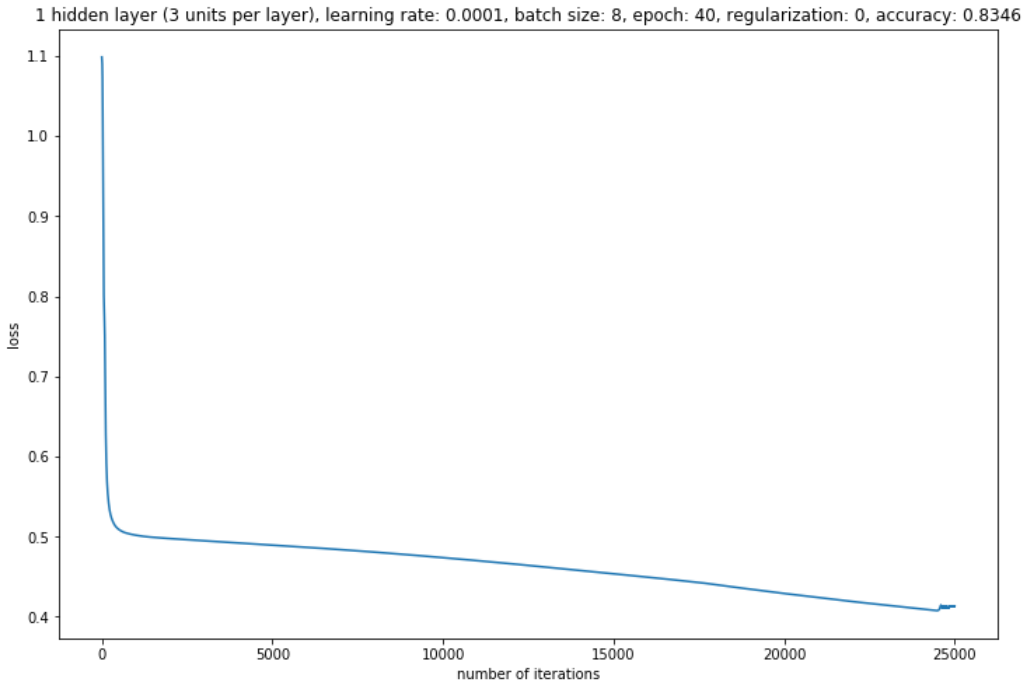
In [154]:

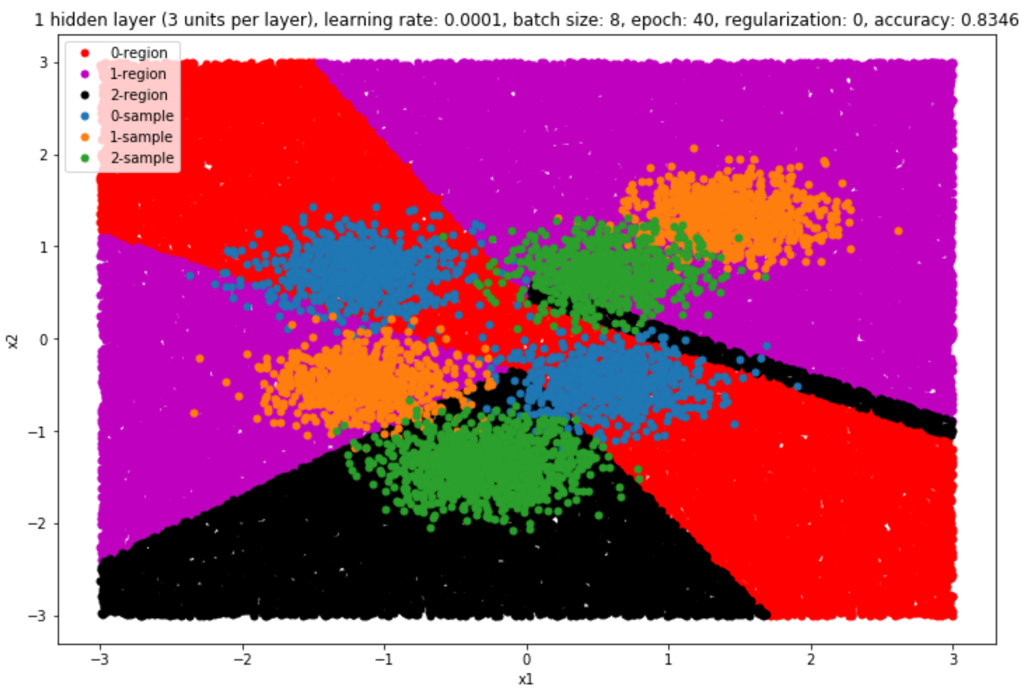
```python
class Hidden_layer:
    def __init__(self, dim_in, dim_out):
        self.dim_in = dim_in
        self.dim_out = dim_out
        self.W = np.random.normal(0, 0.1, (dim_in, dim_out))
        #self.W = self.W/np.linalg.norm(self.W)
        self.b = np.zeros((1, dim_out))

    def forward_propagation(self, X):
        r = X.dot(self.W) + self.b
        h = np.maximum(np.zeros(r.shape), r)
        return h

    def back_propagation(self, dLdh, h, X, eta, regularization):
        dhdr = np.zeros(h.shape)
        dhdr[np.nonzero(h)] = 1
        dLdr = np.multiply(dhdr, dLdh)
        dLdW = X.T.dot(dLdr) + regularization * self.W
        dLdb = np.mean(dLdr, axis=0).reshape(1, self.dim_out)
        dLdX = dLdr.dot(self.W.T)
        self.W = self.W - eta * dLdW
        self.b = self.b - eta * dLdb
        return dLdX

    def __repr__(self):
        return "dim_in: {}    dim_out: {} \nW:\n {} \n b:\n {}".format(self.dim_in, self.dim_out, self.W, self
.b)

    def __str__(self):
        return "dim_in: {}    dim_out: {} \nW:\n {} \n b:\n {}".format(self.dim_in, self.dim_out, self.W, self
.b)


class Output_layer:
    def __init__(self, dim_in, dim_out):
        self.dim_in = dim_in
        self.dim_out = dim_out
        self.W = np.random.normal(0, 0.1, (dim_in, dim_out))
        #self.W = self.W/np.linalg.norm(self.W)
        self.b = np.zeros((1, dim_out))

    def forward_propagation(self, h):
        z = h.dot(self.W) + self.b
        z = z - np.max(z, axis=1).reshape(z.shape[0], 1)
        return z

    def back_propagation(self, dLdz, h, eta, regularization):
        dLdh = dLdz.dot(self.W.T)
        dLdW = h.T.dot(dLdz) + regularization * self.W
        dLdb = np.mean(dLdz, axis=0).reshape(1, self.dim_out)
        self.W = self.W - eta * dLdW
        self.b = self.b - eta * dLdb
        return dLdh

    def __repr__(self):
        return "dim_in: {}    dim_out: {} \nW:\n {} \n b:\n {}".format(self.dim_in, self.dim_out, self.W, self
.b)

    def __str__(self):
        return "dim_in: {}    dim_out: {} \nW:\n {} \n b:\n {}".format(self.dim_in, self.dim_out, self.W, self
.b)

class Loss_layer:
    def __init__(self, dim_in, dim_out):
        self.dim_in = dim_in
        self.dim_out = dim_out

    def forward_propagation(self, z, y):
        temp = -z + np.log(np.sum(np.exp(z), axis=1)).reshape(z.shape[0], 1)
        L = temp[np.arange(z.shape[0]), y.flatten().astype(int)]
        return np.mean(L)

    def back_propagation(self, z, y):
        temp1 = np.exp(z) / np.sum(np.exp(z),axis=1).reshape(z.shape[0], 1)
        temp2 = np.zeros(z.shape)
        temp2[np.arange(z.shape[0]), y.flatten().astype(int)] = -1
        dLdz = temp1 + temp2
        return dLdz
```

In [159]:
```
class MLP:
    def __init__(self):
        self.hidden_layer_list = []
        self.output_layer = None
        self.loss_layer = None
        self.eta = None
        self.batch_size = None
        self.epoch = None
        self.regularization = None

    def add_layer(self, type, dim_in, dim_out):
        if type=="Hidden":
            self.hidden_layer_list.append(Hidden_layer(dim_in, dim_out))
        elif type=="Output":
            self.output_layer = Output_layer(dim_in, dim_out)
        elif type=="Loss":
            self.loss_layer = Loss_layer(dim_in, dim_out)
        else:
            print("Error added layer!")

    def forward(self, X, y, regularization):
        h_list = []
        for i in range(len(self.hidden_layer_list)):
            h = self.hidden_layer_list[i].forward_propagation(X)
            h_list.append(h)
            X = h
        z = self.output_layer.forward_propagation(h)
        L = self.loss_layer.forward_propagation(z, y)
        for i in range(len(self.hidden_layer_list)):
            L += regularization/2 * np.linalg.norm(self.hidden_layer_list[i].W)**2
        L = L + regularization/2 * np.linalg.norm(self.output_layer.W)**2
        return h_list, z, L

    def backward(self, h_list, z, X, y, eta, regularization):
        dLdz = self.loss_layer.back_propagation(z, y)
        dLdh = self.output_layer.back_propagation(dLdz, h_list[-1], eta, regularization)
        for i in range(len(self.hidden_layer_list)-1, 0, -1):
            dLdX = self.hidden_layer_list[i].back_propagation(dLdh, h_list[i], h_list[i-1], eta, regularizatio
n)
            dLdh = dLdX
        # first layer
        dLdX = self.hidden_layer_list[0].back_propagation(dLdh, h_list[0], X, eta, regularization)

    def train(self, eta, batch_size, epoch, regularization, X, y):
        self.eta = eta
        self.batch_size = batch_size
        self.epoch = epoch
        self.regularization = regularization

        All = np.concatenate((X, y), axis=1)
        loss = []
        for e in range(epoch):
            #print(e)
            All_permutated = np.random.permutation(All)
            eta = eta * 0.8
            for i in range(int(All.shape[0]/batch_size)):
                #print(i)
                X_batch = All_permutated[batch_size*i:batch_size*(i+1), 0:2]
                y_batch = All_permutated[batch_size*i:batch_size*(i+1), [2]]
                h_list, z, L = self.forward(X, y, regularization)
                loss.append(L)
                #print(z)
                self.backward(h_list, z, X, y, eta, regularization)
        return loss

    def predict(self, X):
        for i in range(len(self.hidden_layer_list)):
            h = self.hidden_layer_list[i].forward_propagation(X)
            X = h
        z = self.output_layer.forward_propagation(h)
        y = np.argmax(z, axis=1).reshape(z.shape[0], 1)
        return y

    def get_accuracy(self, X, y):
        y_predicted = self.predict(X)
        return np.count_nonzero(y_predicted==y)/X.shape[0]
```

(ReLU) 1 hidden layer (3 units), regularization = 0, accuracy = 0.8346

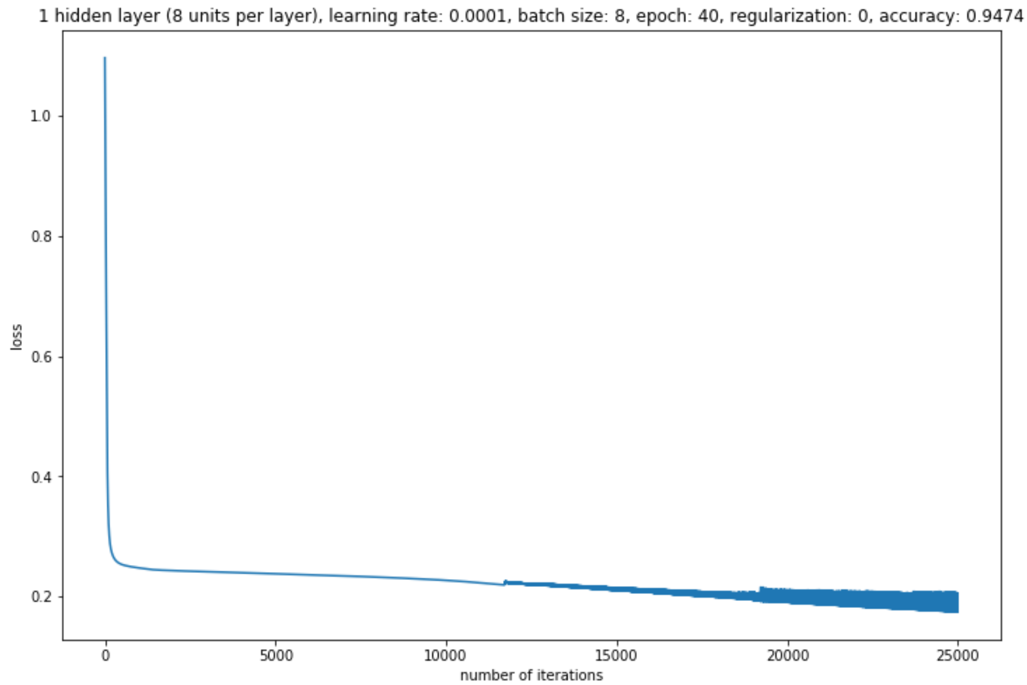
In [146]:
```
# DONE
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=3)
NN.add_layer("Output", dim_in=3, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=8, epoch=40, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

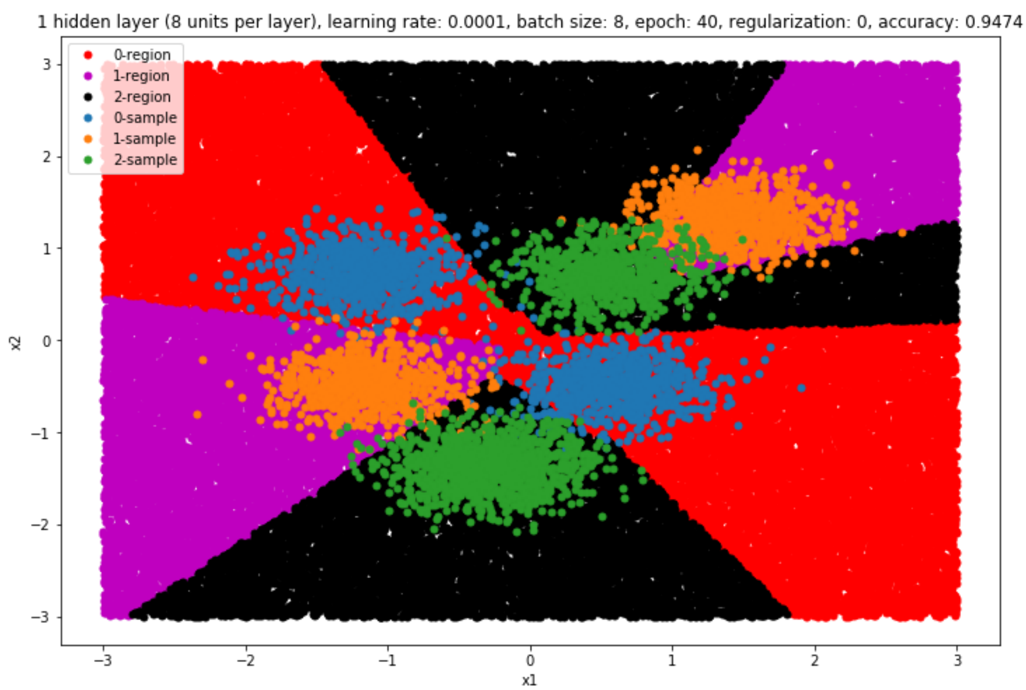




(ReLU) 1 hidden layer (8 units), regularization = 0, accuracy = 0.9474

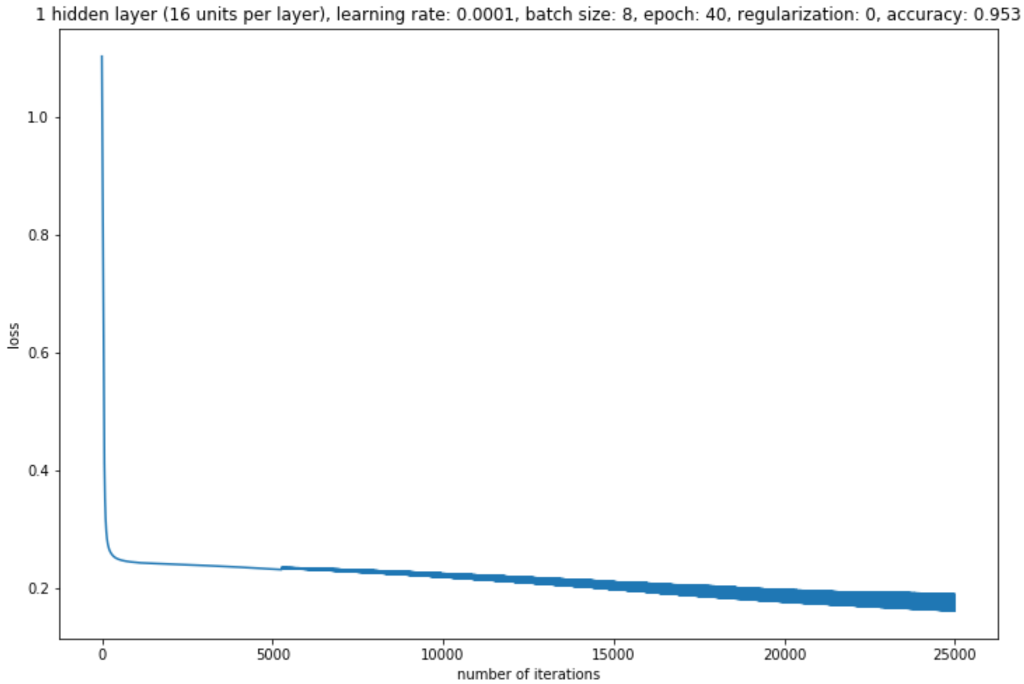
In [144]:
```
# DONE
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=8)
NN.add_layer("Output", dim_in=8, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=8, epoch=40, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

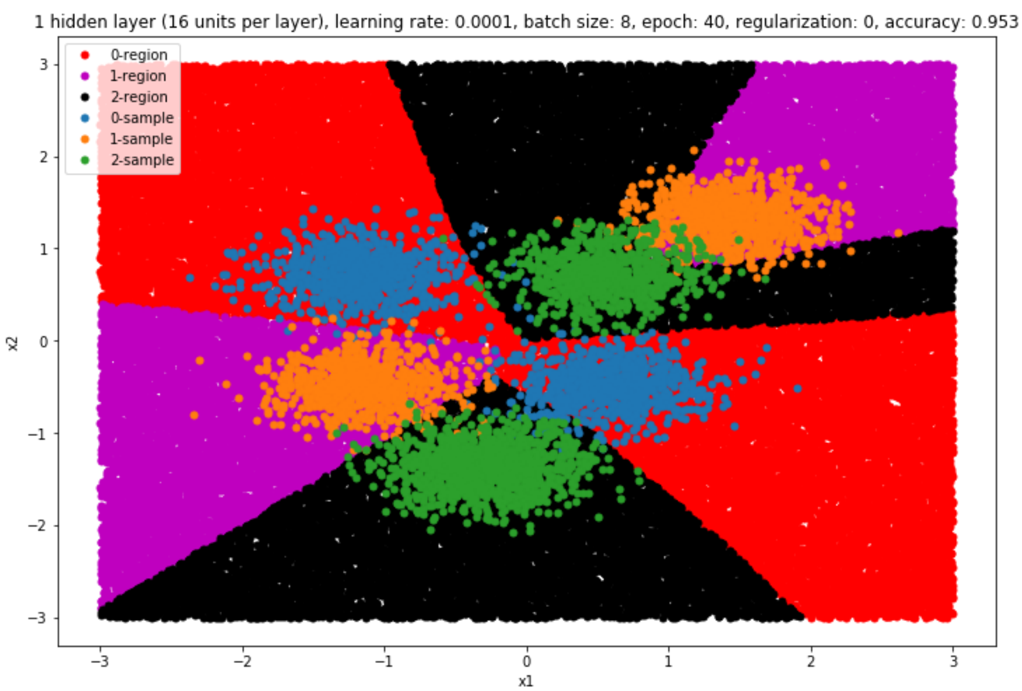




(ReLU) 1 hidden layer (16 units), regularization = 0, accuracy = 0.953

In [145]:
```
# DONE
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=16)
NN.add_layer("Output", dim_in=16, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=8, epoch=40, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

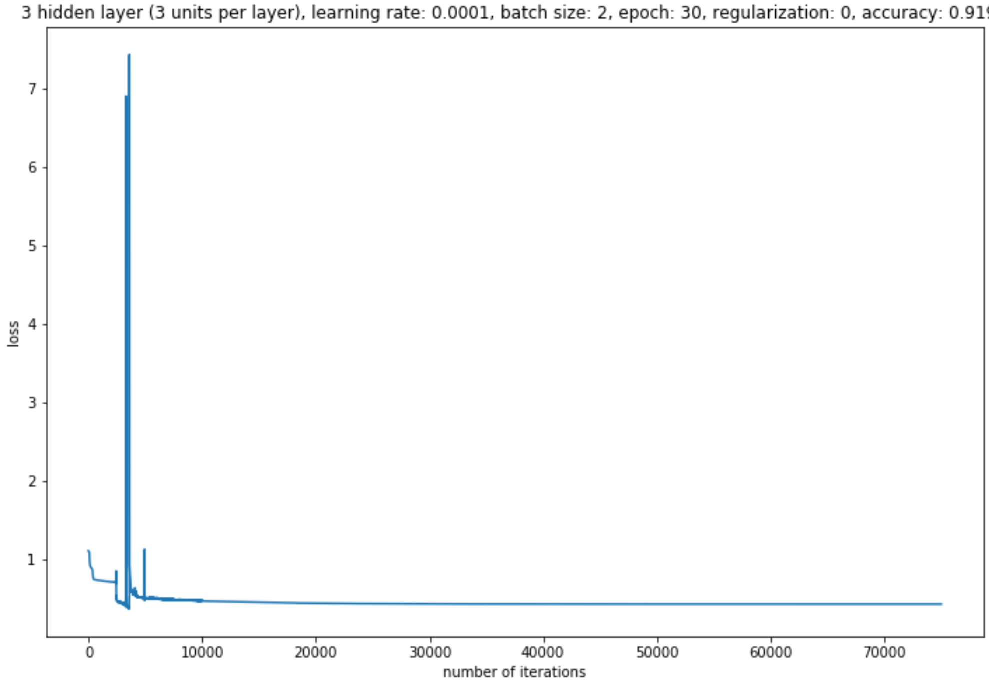




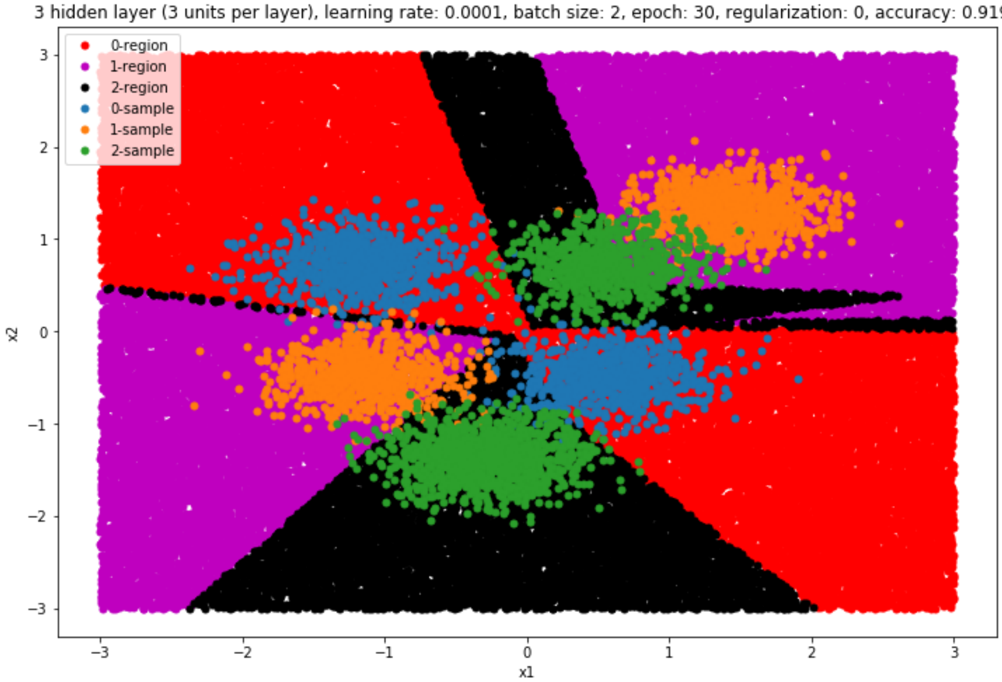
(ReLU) 3 hidden layers (3 units), regularization = 0, accuracy = 0.9198

In [135]:
```
# DONE
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=3)
NN.add_layer("Hidden", dim_in=3, dim_out=3)
NN.add_layer("Hidden", dim_in=3, dim_out=3)
NN.add_layer("Output", dim_in=3, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=2, epoch=30, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

3 hidden layer (3 units per layer), learning rate: 0.0001, batch size: 2, epoch: 30, regularization: 0, accuracy: 0.9198



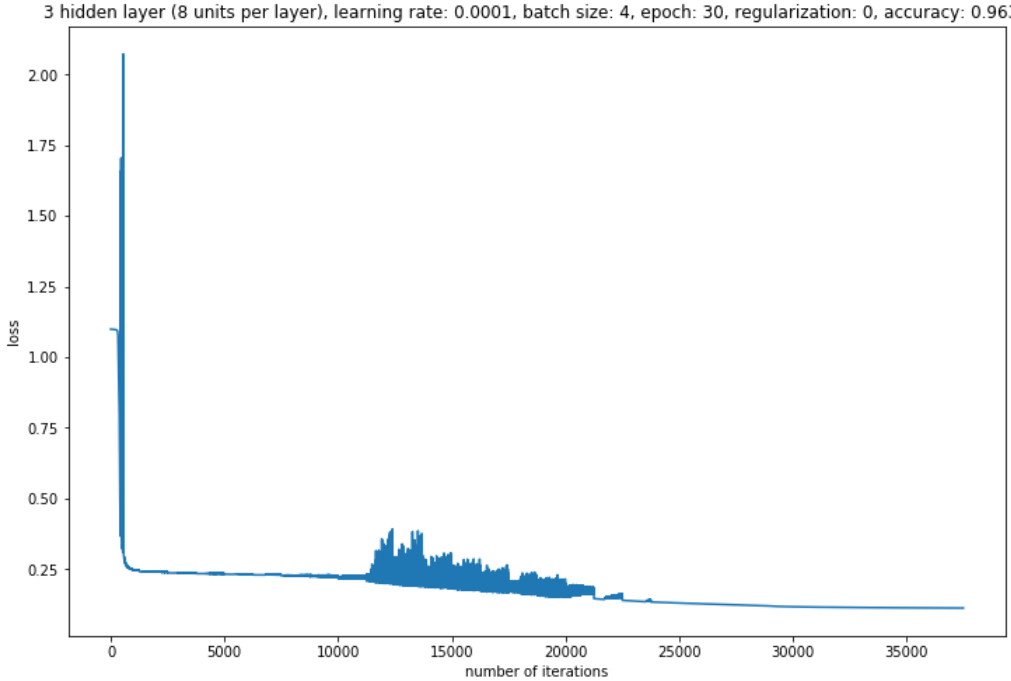
3 hidden layer (3 units per layer), learning rate: 0.0001, batch size: 2, epoch: 30, regularization: 0, accuracy: 0.9198



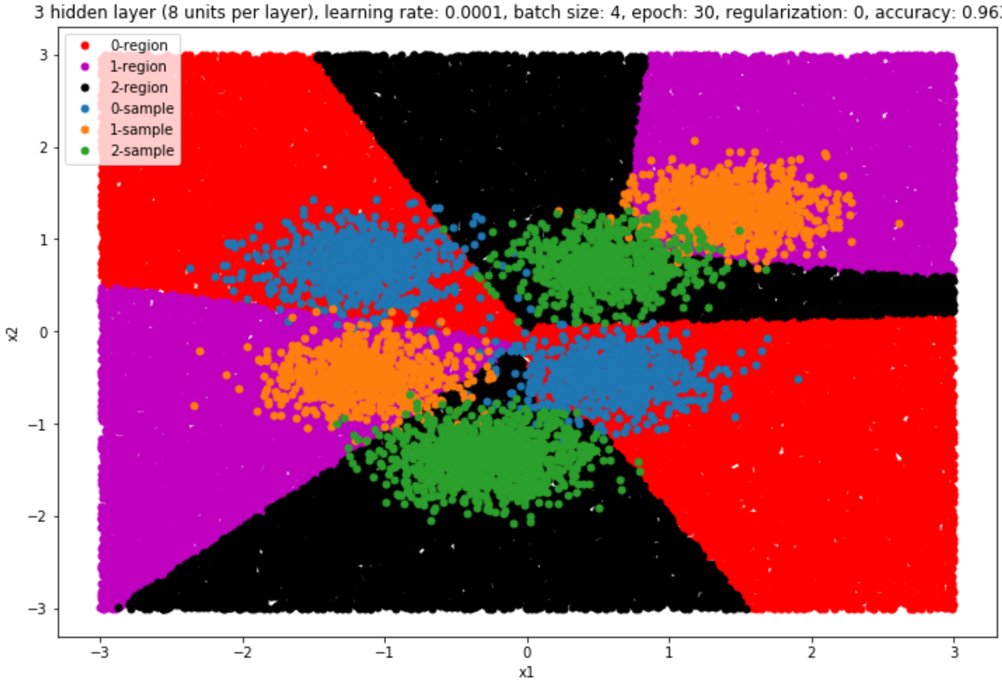
(ReLU) 3 hidden layers (8 units), regularization = 0, accuracy = 0.9632

```
In [153]: # Done
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=8)
NN.add_layer("Hidden", dim_in=8, dim_out=8)
NN.add_layer("Hidden", dim_in=8, dim_out=8)
NN.add_layer("Output", dim_in=8, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=30, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

3 hidden layer (8 units per layer), learning rate: 0.0001, batch size: 4, epoch: 30, regularization: 0, accuracy: 0.9632



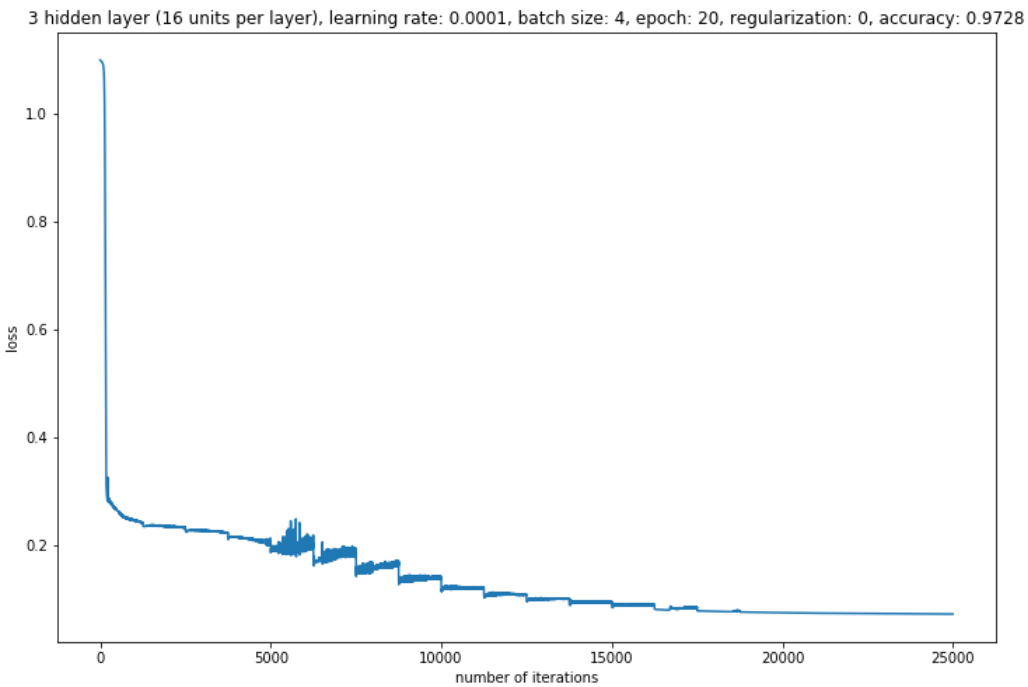
3 hidden layer (8 units per layer), learning rate: 0.0001, batch size: 4, epoch: 30, regularization: 0, accuracy: 0.9632

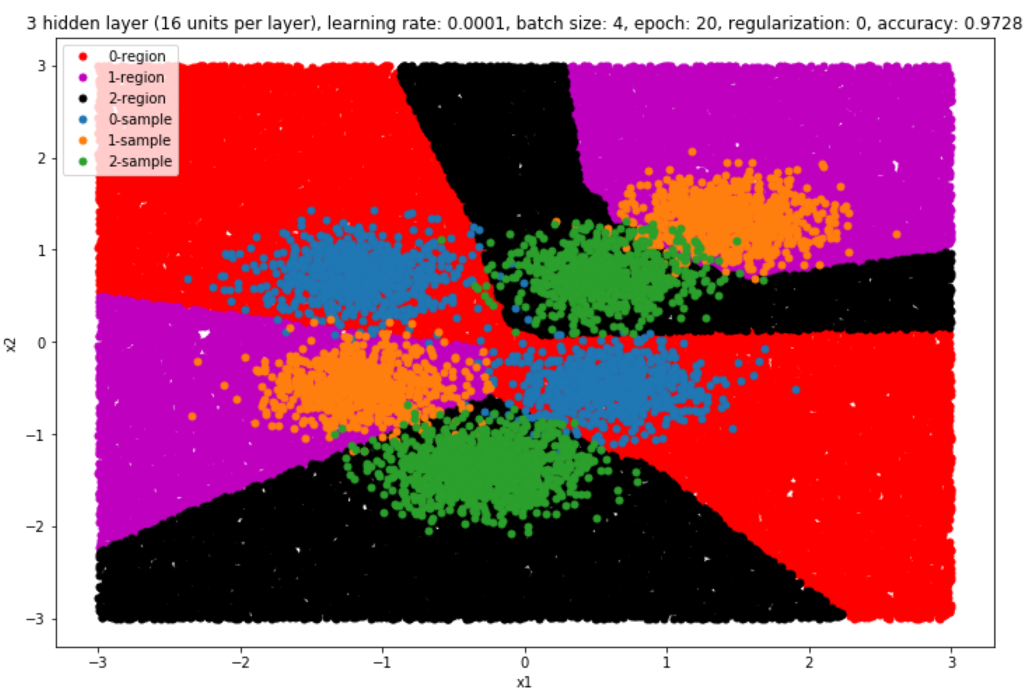


(ReLU) 3 hidden layers (16 units), regularization = 0, accuracy = 0.9728

In [152]:

```
# DONE
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=16)
NN.add_layer("Hidden", dim_in=16, dim_out=16)
NN.add_layer("Hidden", dim_in=16, dim_out=16)
NN.add_layer("Output", dim_in=16, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

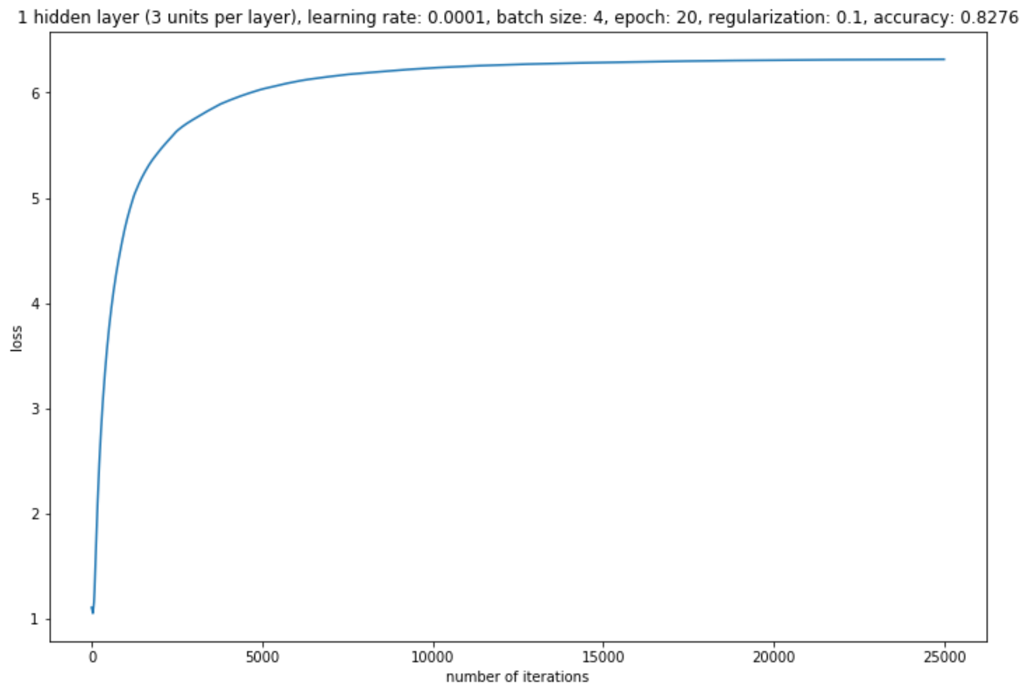


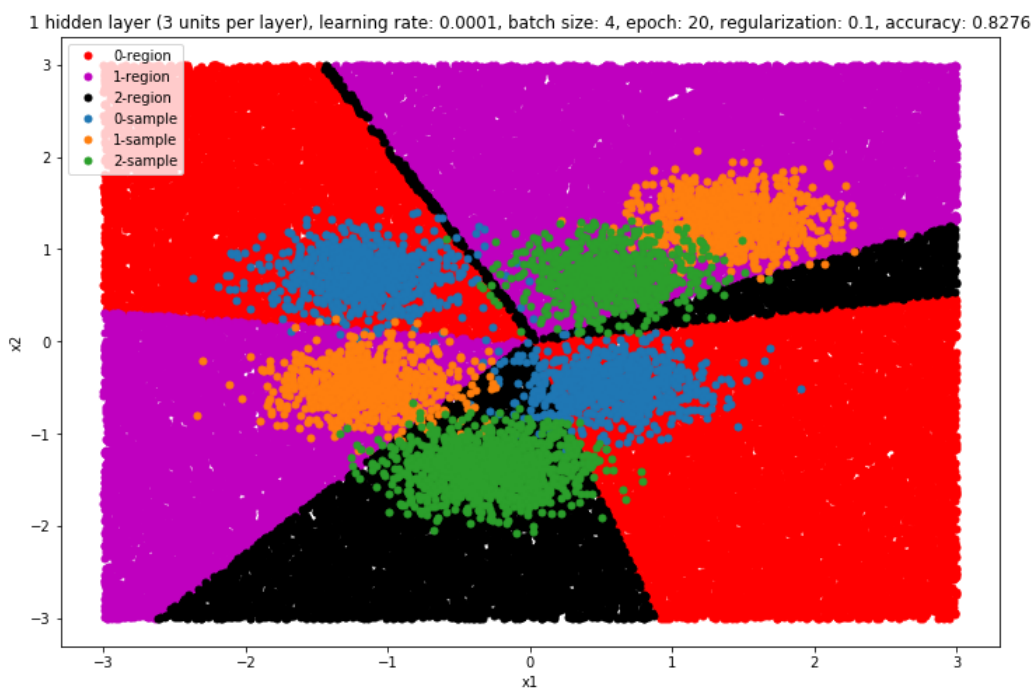


Add regularization with ReLU activation function

(ReLU) 1 hidden layer (3 units), regularization = 0.1, accuracy = 0.8276

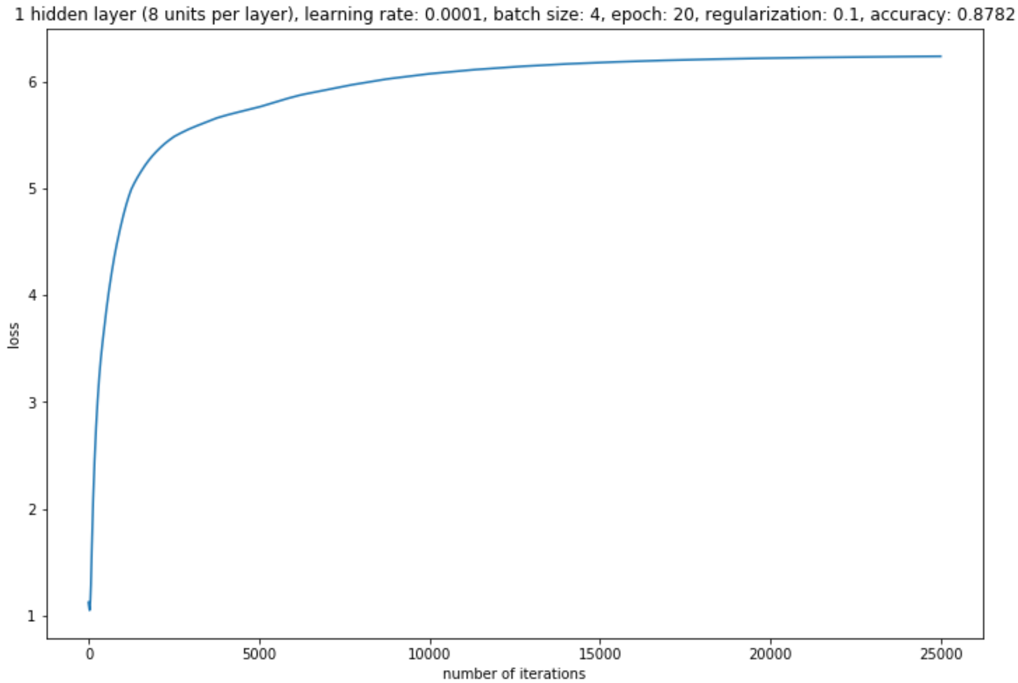
In [169]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=3)
NN.add_layer("Output", dim_in=3, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

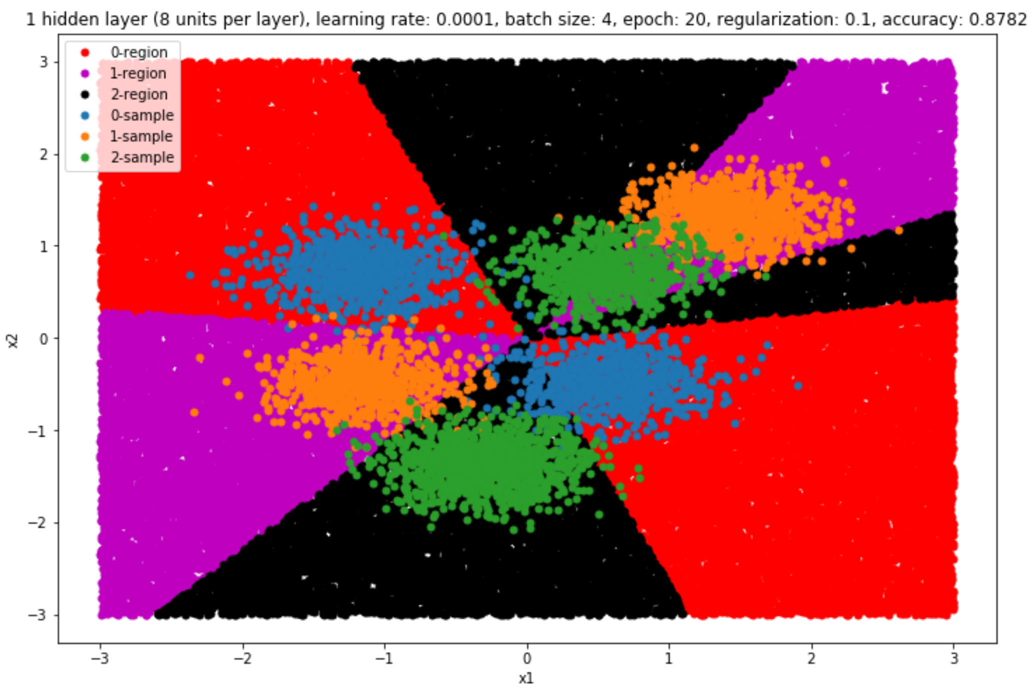




(ReLU) 1 hidden layer (8 units), regularization = 0.1, accuracy = 0.8782

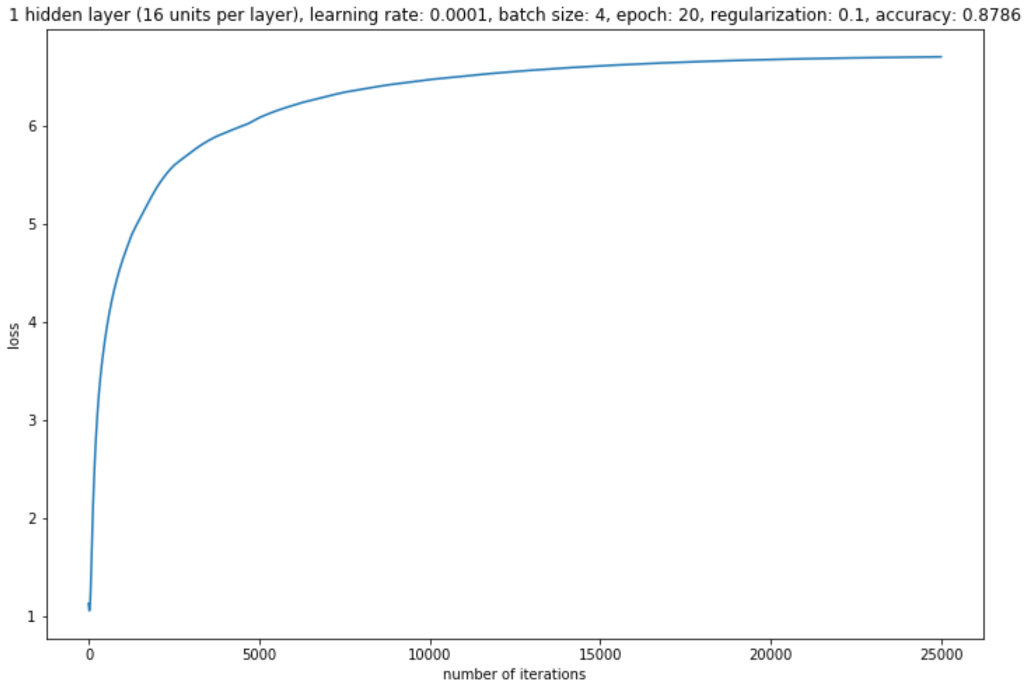
In [168]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=8)
NN.add_layer("Output", dim_in=8, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

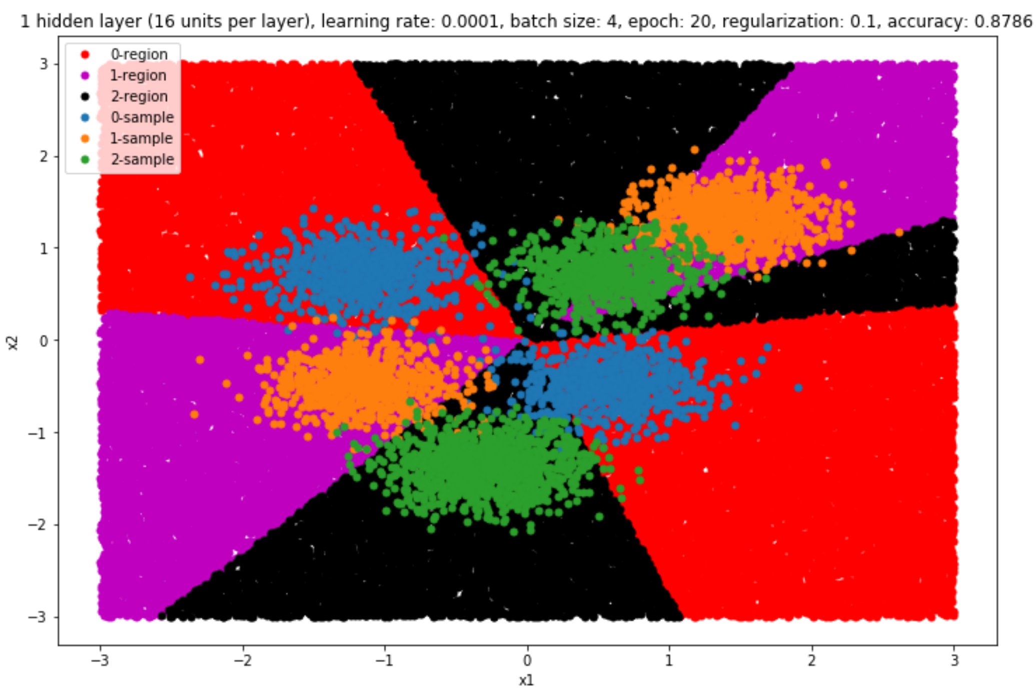




(ReLU) 1 hidden layer (16 units), regularization = 0.1, accuracy = 0.8786

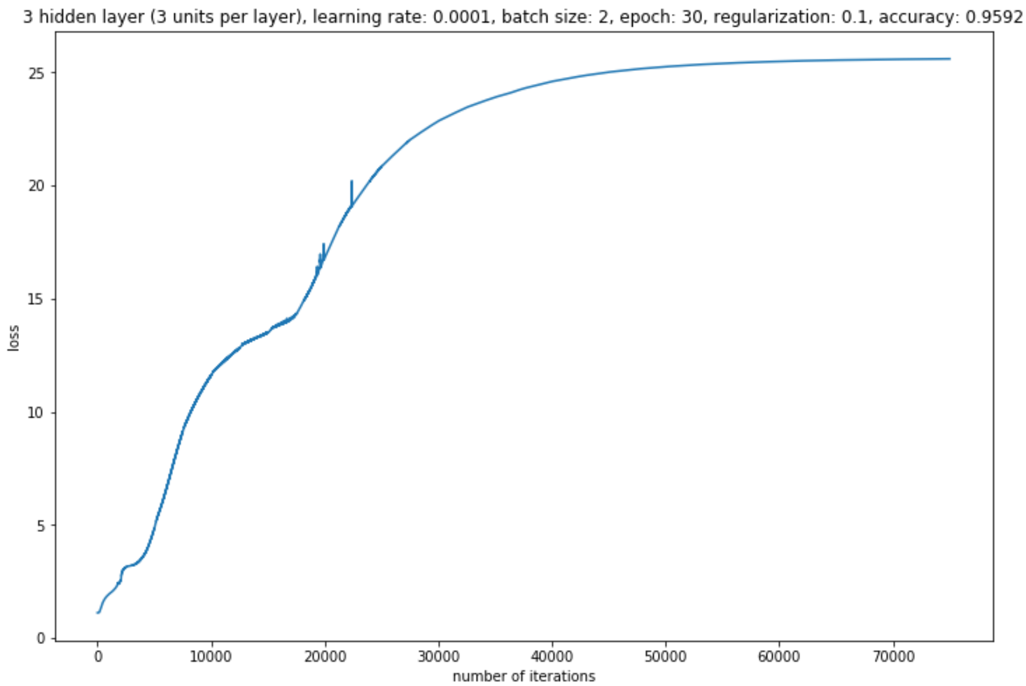
```
In [167]: NN = MLP()
          NN.add_layer("Hidden", dim_in=2, dim_out=16)
          NN.add_layer("Output", dim_in=16, dim_out=3)
          NN.add_layer("Loss", dim_in=3, dim_out=3)

          loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0.1, X=X, y=y)

          plot_loss(loss)
          plot_decision_regions(NN)
```

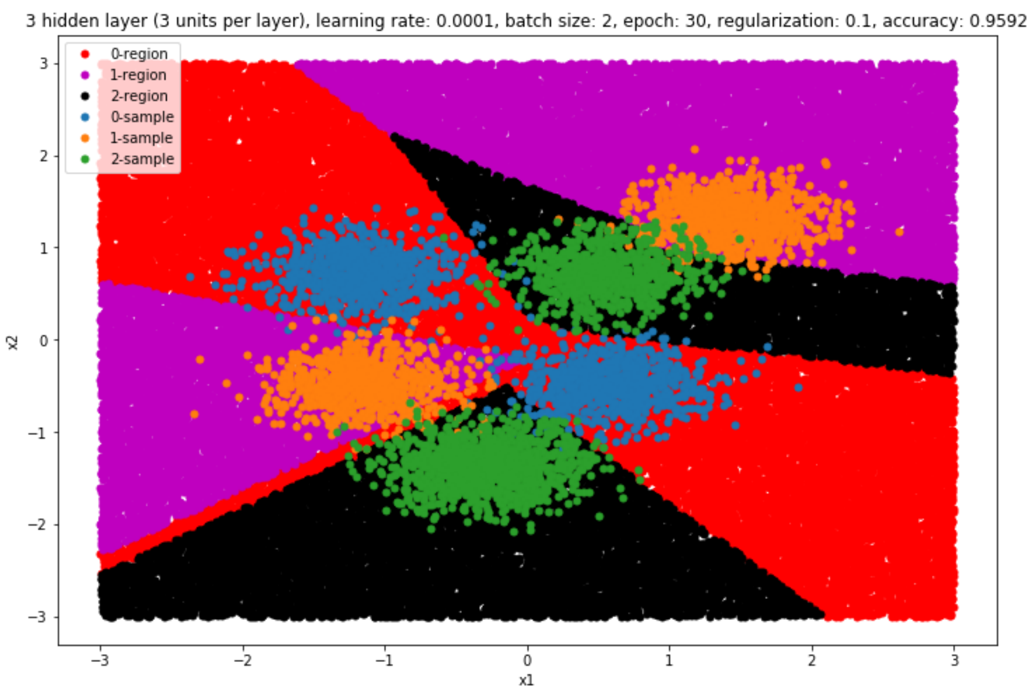




(ReLU) 3 hidden layer (3 units), regularization = 0.1, accuracy = 0.9592

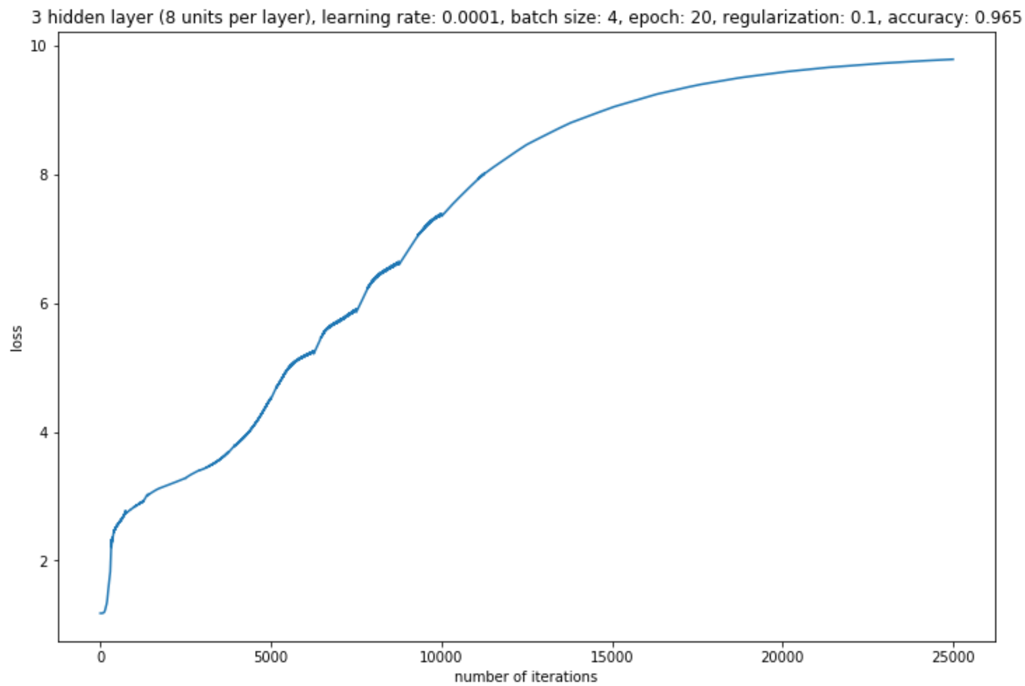
In [176]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=3)
NN.add_layer("Hidden", dim_in=3, dim_out=3)
NN.add_layer("Hidden", dim_in=3, dim_out=3)
NN.add_layer("Output", dim_in=3, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=2, epoch=30, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

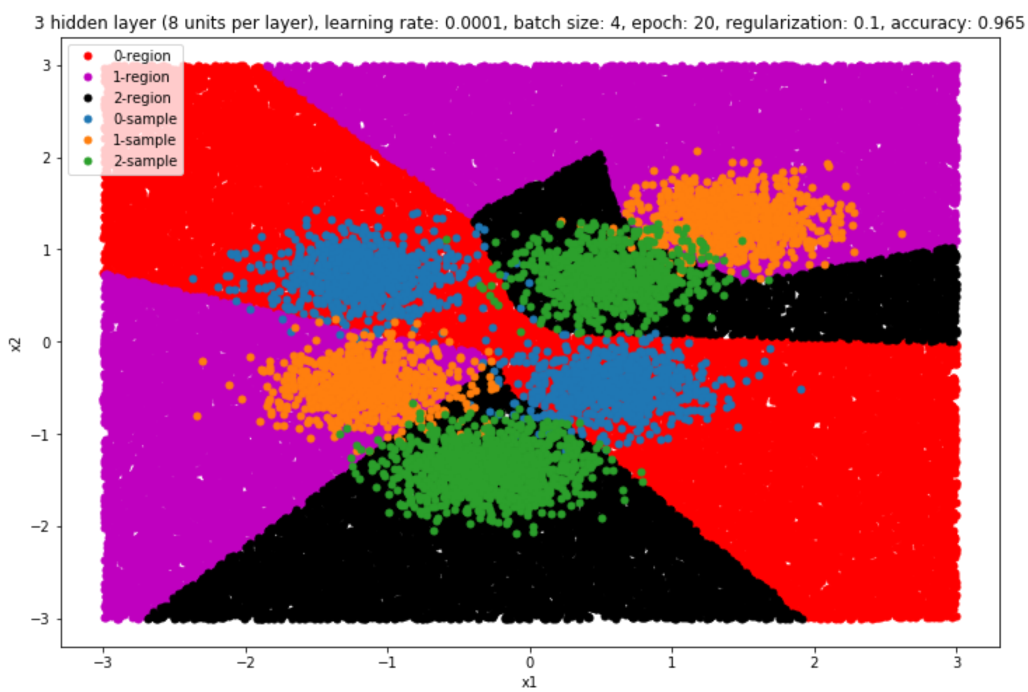




(ReLU) 3 hidden layers (8 units), regularization = 0.1, accuracy = 0.965

In [163]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=8)
NN.add_layer("Hidden", dim_in=8, dim_out=8)
NN.add_layer("Hidden", dim_in=8, dim_out=8)
NN.add_layer("Output", dim_in=8, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

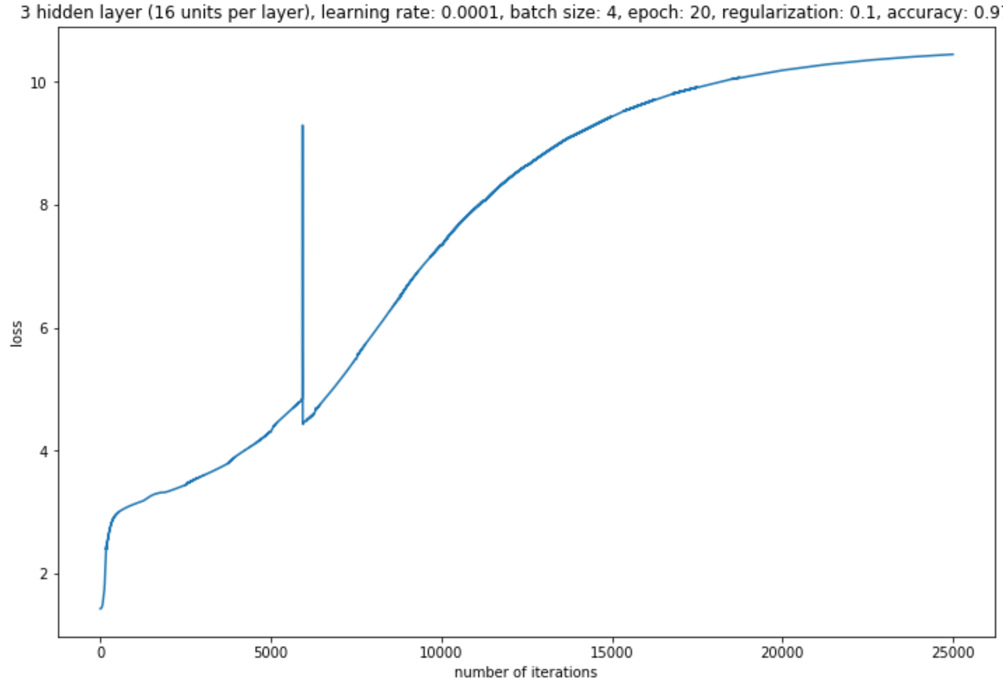




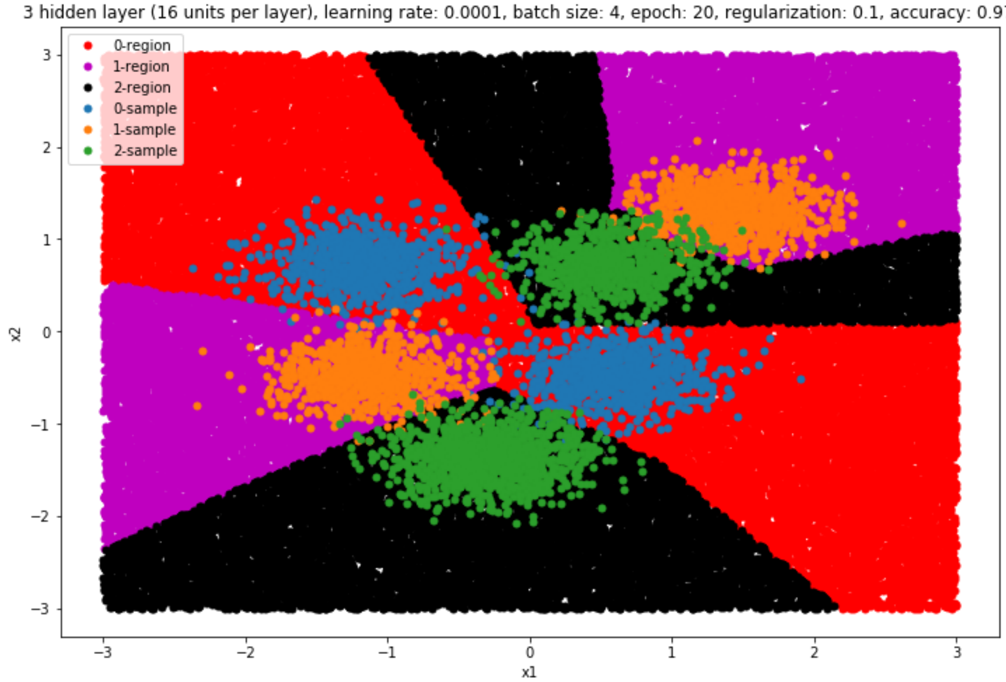
(ReLU) 3 hidden layers (16 units), regularization = 0.1, accuracy = 0.9726

In [162]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=16)
NN.add_layer("Hidden", dim_in=16, dim_out=16)
NN.add_layer("Hidden", dim_in=16, dim_out=16)
NN.add_layer("Output", dim_in=16, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

3 hidden layer (16 units per layer), learning rate: 0.0001, batch size: 4, epoch: 20, regularization: 0.1, accuracy: 0.9726



3 hidden layer (16 units per layer), learning rate: 0.0001, batch size: 4, epoch: 20, regularization: 0.1, accuracy: 0.9726



h) Use softplus as activation function and then plot 12 graphs

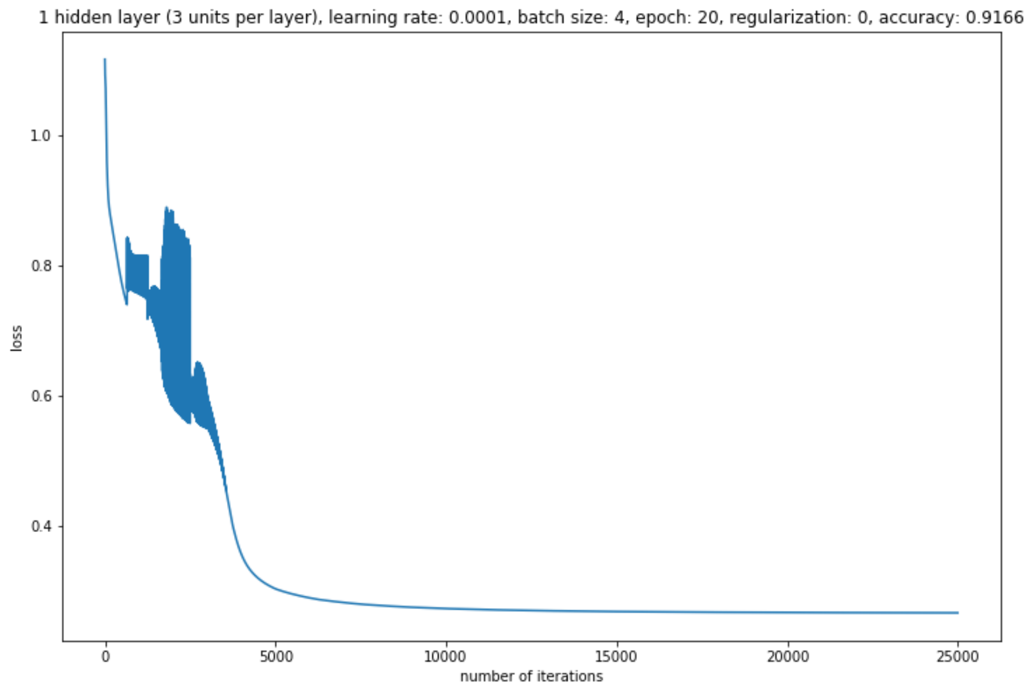
In [228]:
```
def softplus(r):
    h = np.zeros(r.shape)
    h[np.where(r>40)] = r[np.where(r>40)]
    h[np.where(r<=40)] = np.log(1 + np.exp(r[np.where(r<=40)]))
    return h

def softplus_d(r):
    dhdr = np.zeros(r.shape)
    dhdr[np.where(r<-40)] = 0
    dhdr[np.where(r>-40)] = 1/ (1 + np.exp(-r[np.where(r>-40)]))
    return dhdr
```

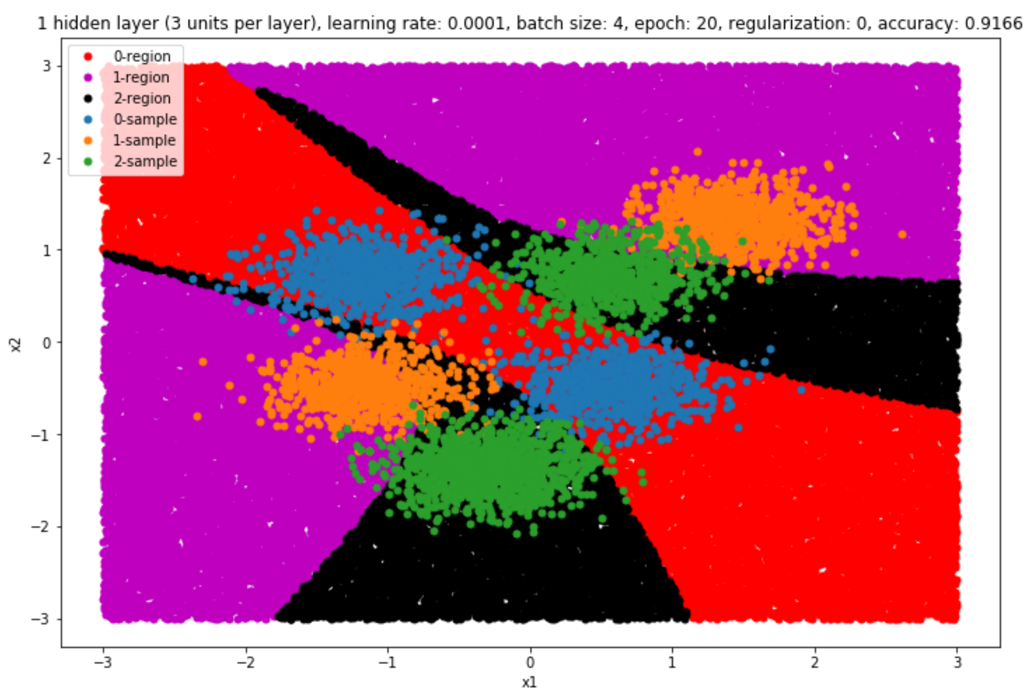
In [231]:
```
class Hidden_layer:
    def __init__(self, dim_in, dim_out):
        self.dim_in = dim_in
        self.dim_out = dim_out
        self.W = np.random.normal(0, 0.1, (dim_in, dim_out))
        #self.W = self.W/np.linalg.norm(self.W)
        self.b = np.zeros((1, dim_out))

    def forward_propagation(self, X):
        r = X.dot(self.W) + self.b
        h = softplus(r)
        return h, r

    def back_propagation(self, dLdh, r, X, eta, regularization):
        dhdr = softplus_d(r)
        dLdr = np.multiply(dhdr, dLdh)
        dLdW = X.T.dot(dLdr) + regularization * self.W
        dLdb = np.mean(dLdr, axis=0).reshape(1, self.dim_out)
        dLdX = dLdr.dot(self.W.T)
        self.W = self.W - eta * dLdW
        self.b = self.b - eta * dLdb
        return dLdX

    def __repr__(self):
        return "dim_in: {}    dim_out: {} \nW:\n {} \n b:\n {}".format(self.dim_in, self.dim_out, self.W, self
.b)

    def __str__(self):
        return "dim_in: {}    dim_out: {} \nW:\n {} \n b:\n {}".format(self.dim_in, self.dim_out, self.W, self
.b)


class Output_layer:
    def __init__(self, dim_in, dim_out):
        self.dim_in = dim_in
        self.dim_out = dim_out
        self.W = np.random.normal(0, 0.1, (dim_in, dim_out))
        #self.W = self.W/np.linalg.norm(self.W)
        self.b = np.zeros((1, dim_out))

    def forward_propagation(self, h):
        z = h.dot(self.W) + self.b
        z = z - np.max(z, axis=1).reshape(z.shape[0], 1)
        return z

    def back_propagation(self, dLdz, h, eta, regularization):
        dLdh = dLdz.dot(self.W.T)
        dLdW = h.T.dot(dLdz) + regularization * self.W
        dLdb = np.mean(dLdz, axis=0).reshape(1, self.dim_out)
        self.W = self.W - eta * dLdW
        self.b = self.b - eta * dLdb
        return dLdh

    def __repr__(self):
        return "dim_in: {}    dim_out: {} \nW:\n {} \n b:\n {}".format(self.dim_in, self.dim_out, self.W, self
.b)

    def __str__(self):
        return "dim_in: {}    dim_out: {} \nW:\n {} \n b:\n {}".format(self.dim_in, self.dim_out, self.W, self
.b)

class Loss_layer:
    def __init__(self, dim_in, dim_out):
        self.dim_in = dim_in
        self.dim_out = dim_out

    def forward_propagation(self, z, y):
        temp = -z + np.log(np.sum(np.exp(z), axis=1)).reshape(z.shape[0], 1)
        L = temp[np.arange(z.shape[0]), y.flatten().astype(int)]
        return np.mean(L)

    def back_propagation(self, z, y):
        temp1 = np.exp(z) / np.sum(np.exp(z),axis=1).reshape(z.shape[0], 1)
        temp2 = np.zeros(z.shape)
        temp2[np.arange(z.shape[0]), y.flatten().astype(int)] = -1
        dLdz = temp1 + temp2
        return dLdz
```

In [241]:
```
class MLP:
    def __init__(self):
        self.hidden_layer_list = []
        self.output_layer = None
        self.loss_layer = None
        self.eta = None
        self.batch_size = None
        self.epoch = None
        self.regularization = None

    def add_layer(self, type, dim_in, dim_out):
        if type=="Hidden":
            self.hidden_layer_list.append(Hidden_layer(dim_in, dim_out))
        elif type=="Output":
            self.output_layer = Output_layer(dim_in, dim_out)
        elif type=="Loss":
            self.loss_layer = Loss_layer(dim_in, dim_out)
        else:
            print("Error added layer!")

    def forward(self, X, y, regularization):
        h_list = []
        r_list = []
        for i in range(len(self.hidden_layer_list)):
            h, r = self.hidden_layer_list[i].forward_propagation(X)
            h_list.append(h)
            r_list.append(r)
            X = h
        z = self.output_layer.forward_propagation(h)
        L = self.loss_layer.forward_propagation(z, y)
        for i in range(len(self.hidden_layer_list)):
            L += regularization/2 * np.linalg.norm(self.hidden_layer_list[i].W)**2
        L = L + regularization/2 * np.linalg.norm(self.output_layer.W)**2
        return h_list, r_list, z, L

    def backward(self, h_list, r_list, z, X, y, eta, regularization):
        dLdz = self.loss_layer.back_propagation(z, y)
        dLdh = self.output_layer.back_propagation(dLdz, h_list[-1], eta, regularization)
        for i in range(len(self.hidden_layer_list)-1, 0, -1):
            dLdX = self.hidden_layer_list[i].back_propagation(dLdh, r_list[i], h_list[i-1], eta, regularizatio
n)
            dLdh = dLdX
        # first layer
        dLdX = self.hidden_layer_list[0].back_propagation(dLdh, r_list[0], X, eta, regularization)

    def train(self, eta, batch_size, epoch, regularization, X, y):
        self.eta = eta
        self.batch_size = batch_size
        self.epoch = epoch
        self.regularization = regularization

        All = np.concatenate((X, y), axis=1)
        loss = []
        for e in range(epoch):
            #print(e)
            All_permutated = np.random.permutation(All)
            eta = eta * 0.8
            for i in range(int(All.shape[0]/batch_size)):
                #print(i)
                X_batch = All_permutated[batch_size*i:batch_size*(i+1), 0:2]
                y_batch = All_permutated[batch_size*i:batch_size*(i+1), [2]]
                h_list, r_list, z, L = self.forward(X, y, regularization)
                loss.append(L)
                #print(z)
                self.backward(h_list, r_list, z, X, y, eta, regularization)
        return loss

    def predict(self, X):
        for i in range(len(self.hidden_layer_list)):
            h, r = self.hidden_layer_list[i].forward_propagation(X)
            X = h
        z = self.output_layer.forward_propagation(h)
        y = np.argmax(z, axis=1).reshape(z.shape[0], 1)
        return y

    def get_accuracy(self, X, y):
        y_predicted = self.predict(X)
        return np.count_nonzero(y_predicted==y)/X.shape[0]
```

(Softplus) 1 hidden layer (3 units), regularization = 0, accuracy = 0.9166

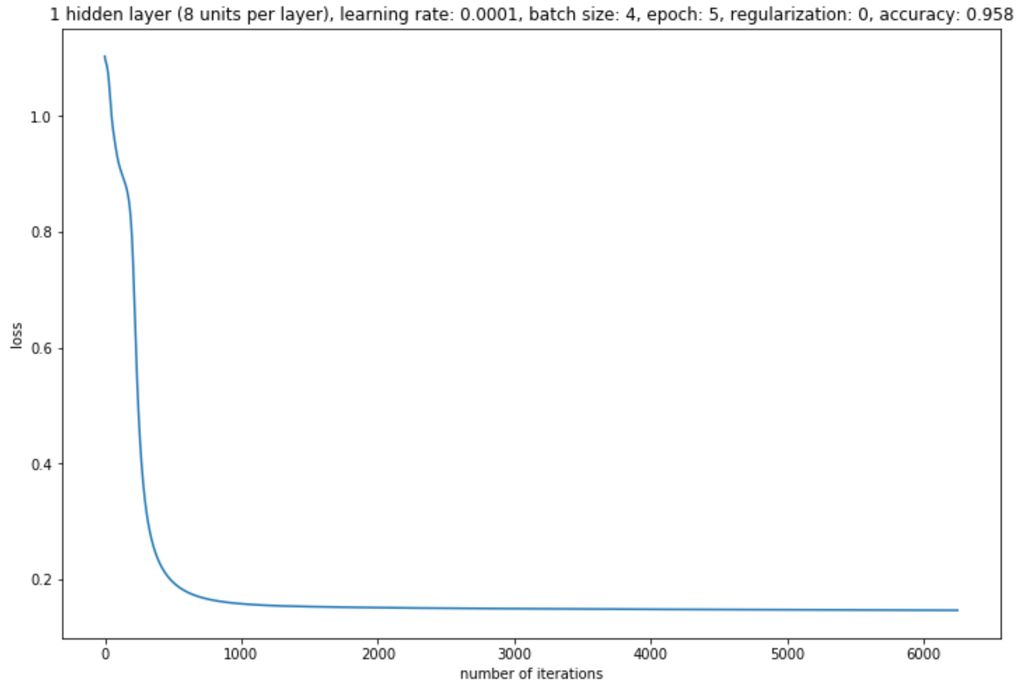
In [253]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=3)
NN.add_layer("Output", dim_in=3, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

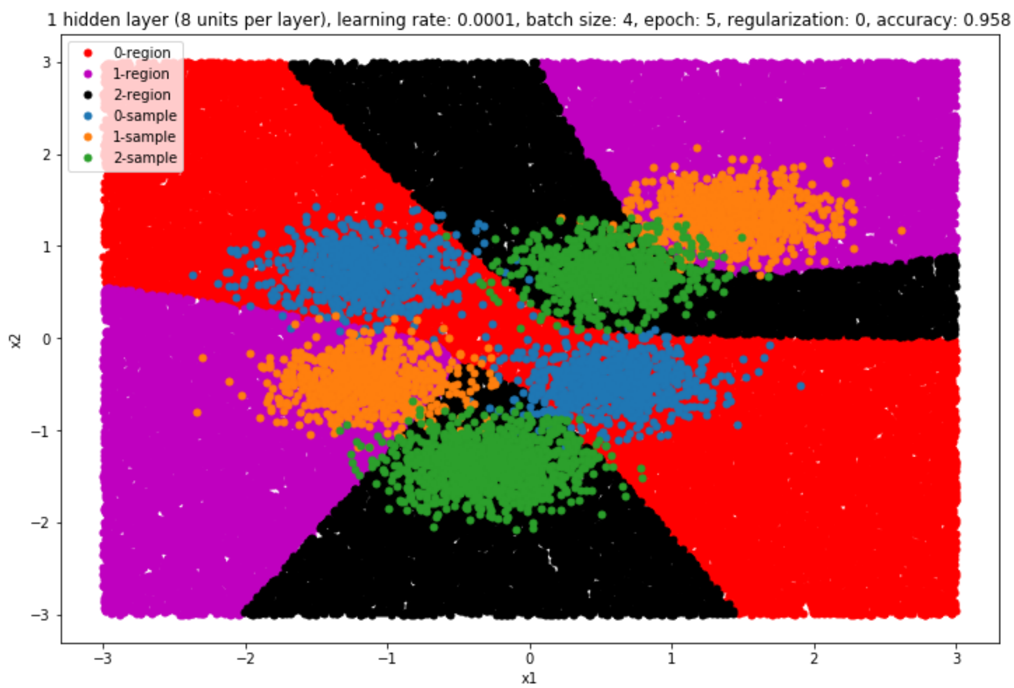




(Softplus) 1 hidden layer (8 units), regularization = 0, accuracy = 0.958

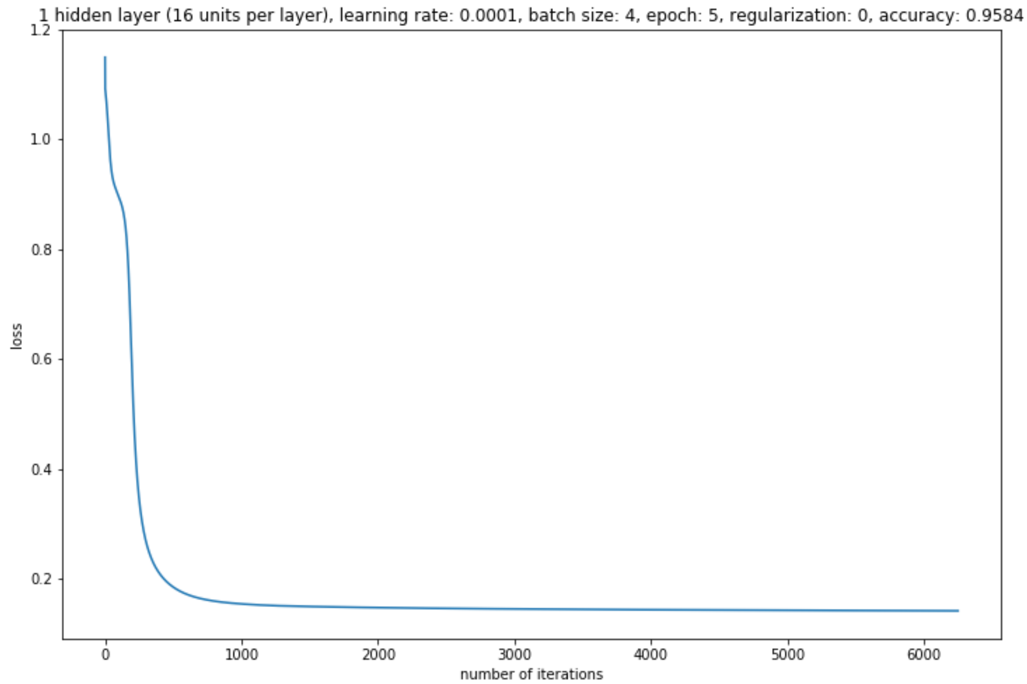
In [243]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=8)
NN.add_layer("Output", dim_in=8, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=5, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

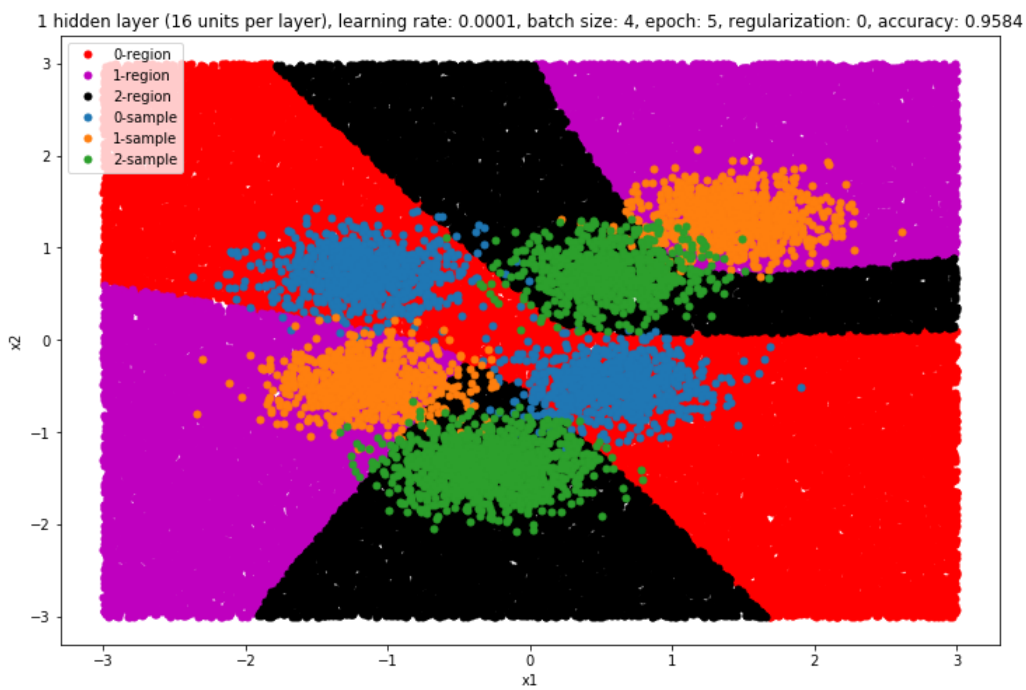




(Softplus) 1 hidden layer (16 units), regularization = 0, accuracy = 0.9584

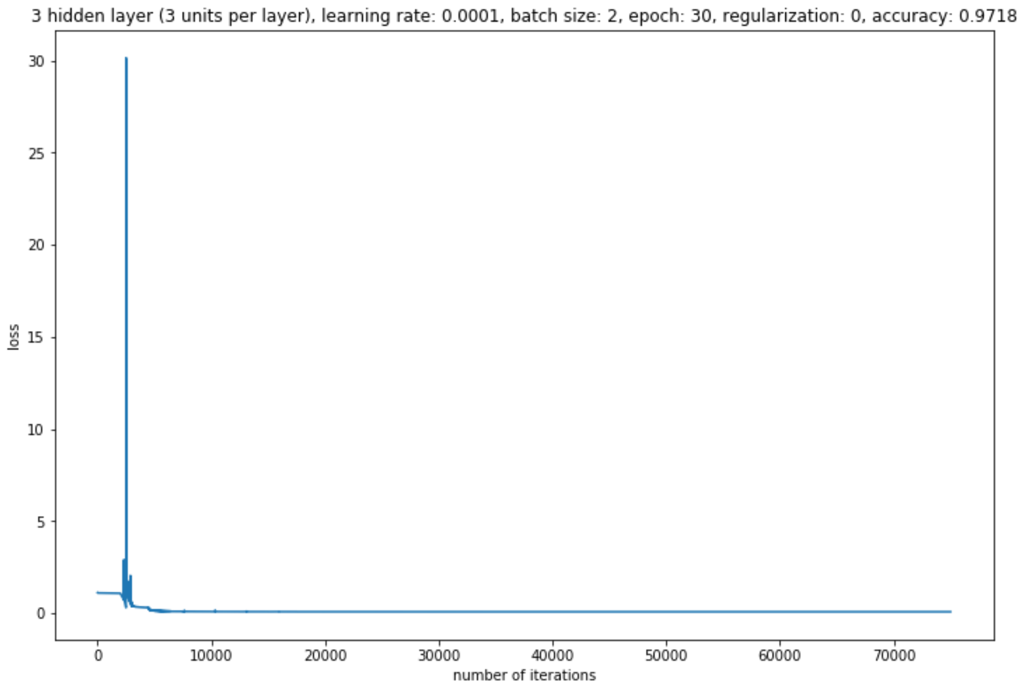
In [242]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=16)
NN.add_layer("Output", dim_in=16, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=5, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

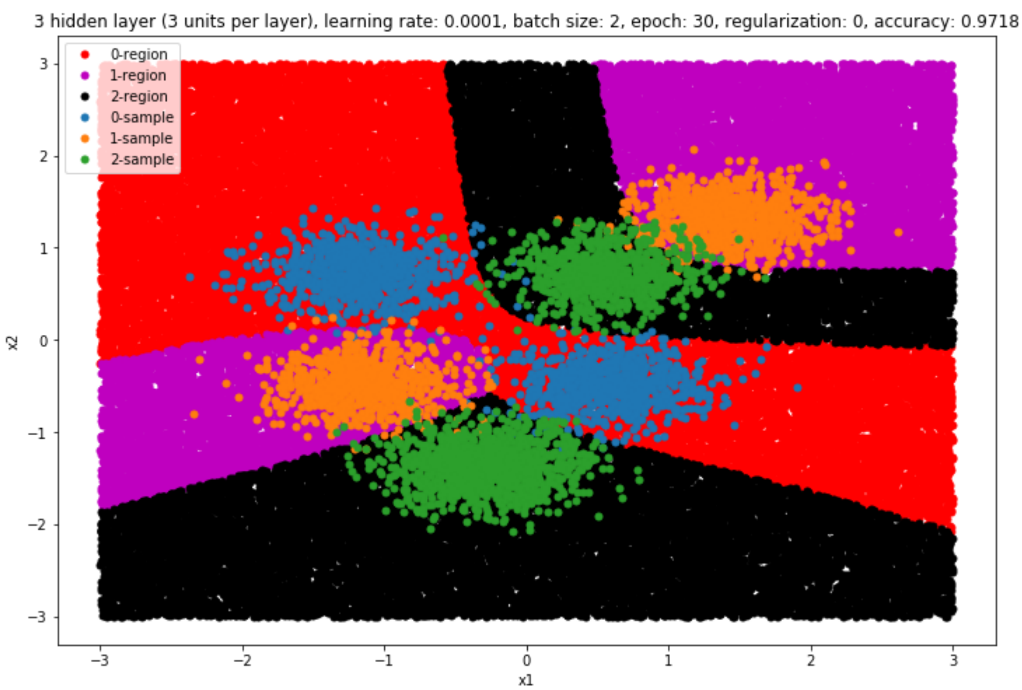




(Softplus) 3 hidden layers (3 units), regularization = 0, accuracy = 0.9718

In [247]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=3)
NN.add_layer("Hidden", dim_in=3, dim_out=3)
NN.add_layer("Hidden", dim_in=3, dim_out=3)
NN.add_layer("Output", dim_in=3, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=2, epoch=30, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

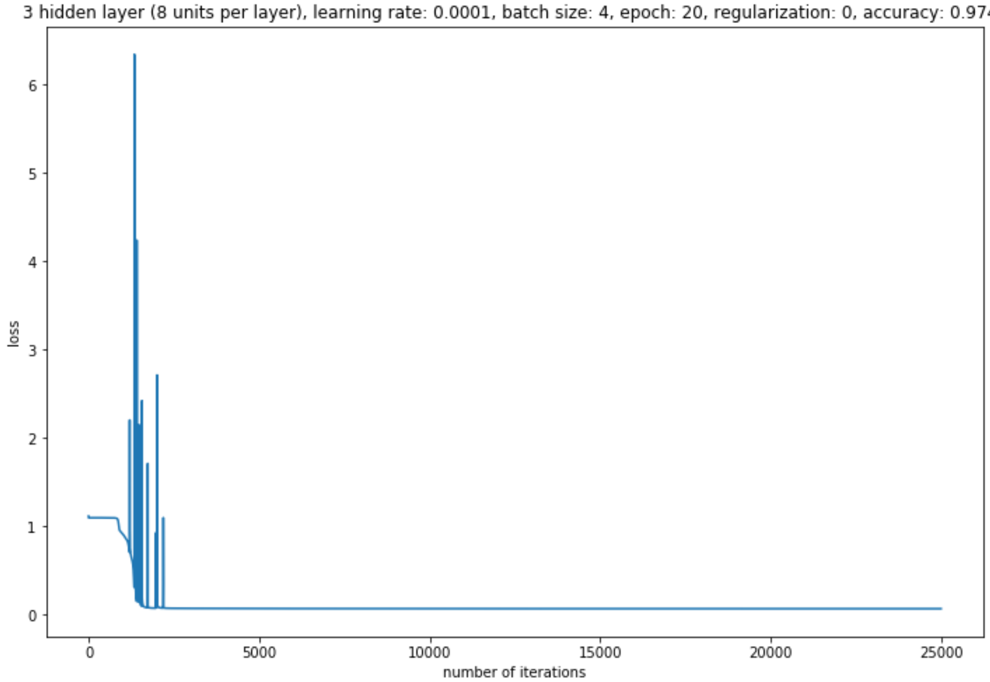




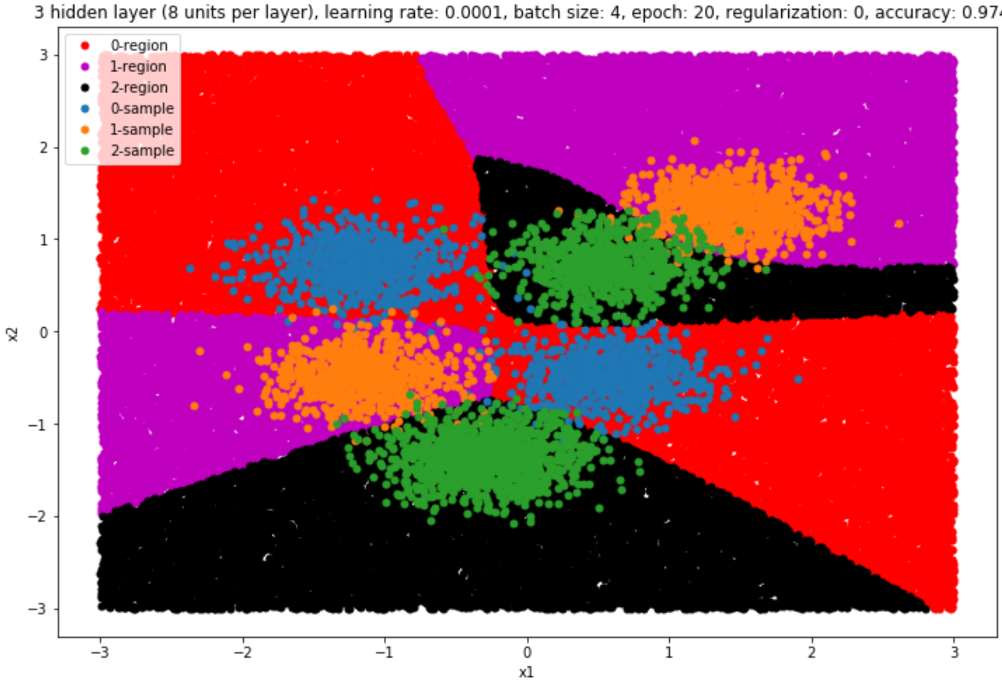
(Softplus) 3 hidden layers (8 units), regularization = 0, accuracy = 0.9746

In [245]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=8)
NN.add_layer("Hidden", dim_in=8, dim_out=8)
NN.add_layer("Hidden", dim_in=8, dim_out=8)
NN.add_layer("Output", dim_in=8, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

3 hidden layer (8 units per layer), learning rate: 0.0001, batch size: 4, epoch: 20, regularization: 0, accuracy: 0.9746

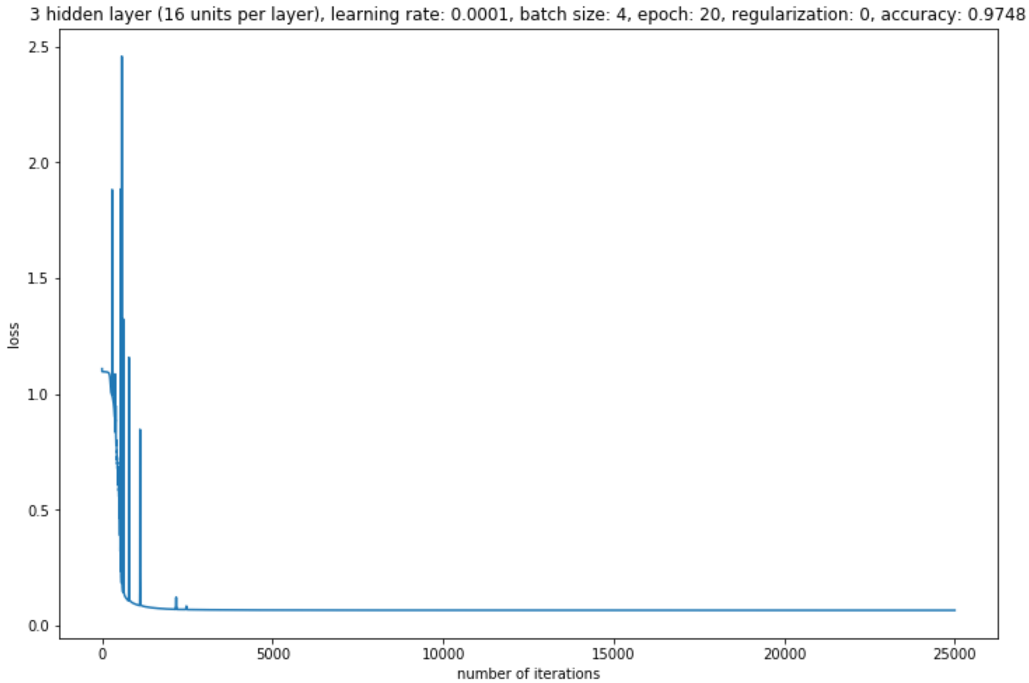


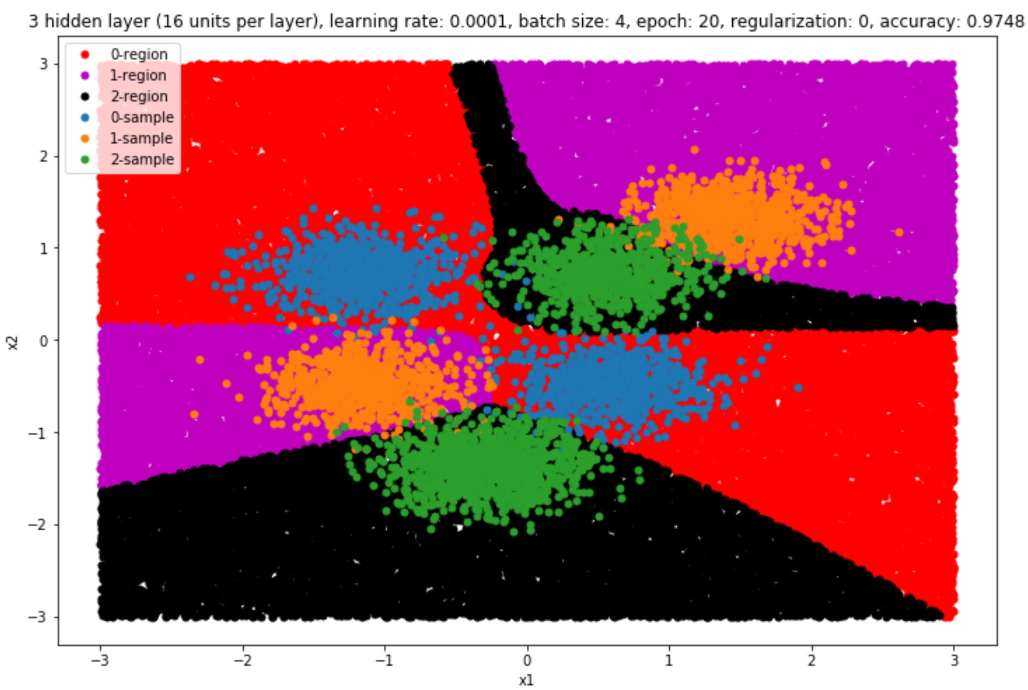
3 hidden layer (8 units per layer), learning rate: 0.0001, batch size: 4, epoch: 20, regularization: 0, accuracy: 0.9746



(Softplus) 3 hidden layers (16 units), regularization = 0, accuracy = 0.9748

In [246]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=16)
NN.add_layer("Hidden", dim_in=16, dim_out=16)
NN.add_layer("Hidden", dim_in=16, dim_out=16)
NN.add_layer("Output", dim_in=16, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

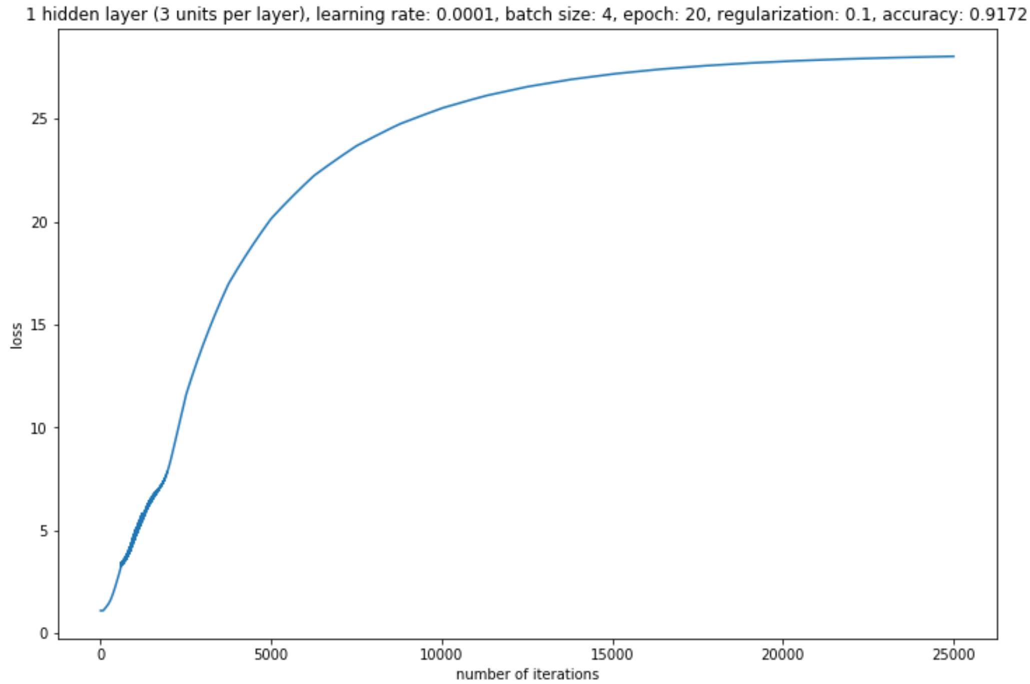


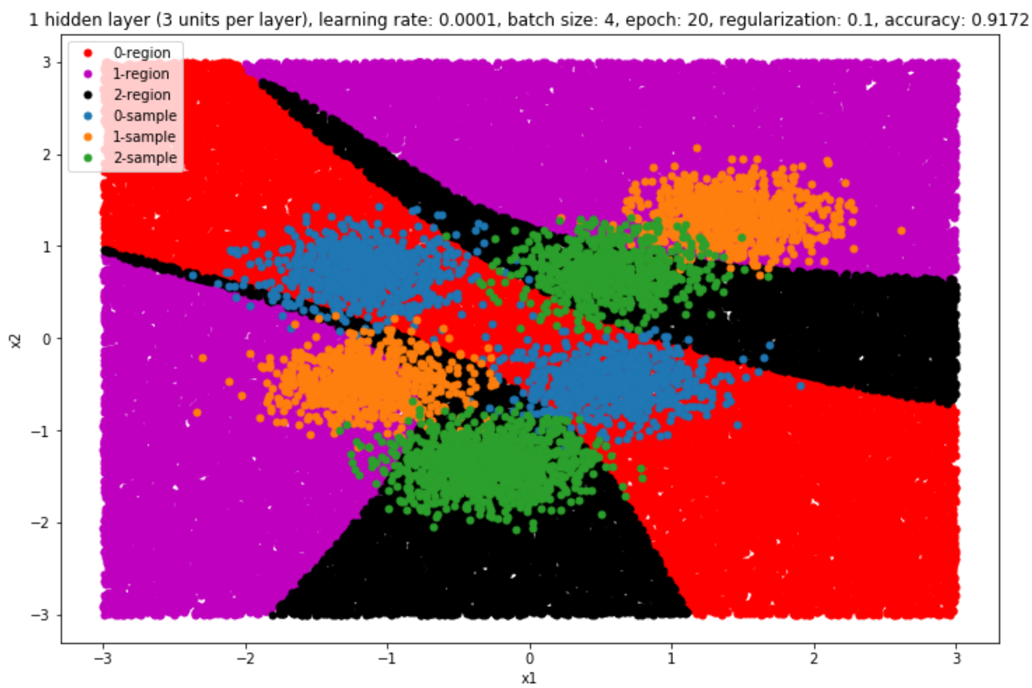


Add regularization with softplus activation function

(Softplus) 1 hidden layer (3 units), regularization = 0.1, accuracy = 0.9172

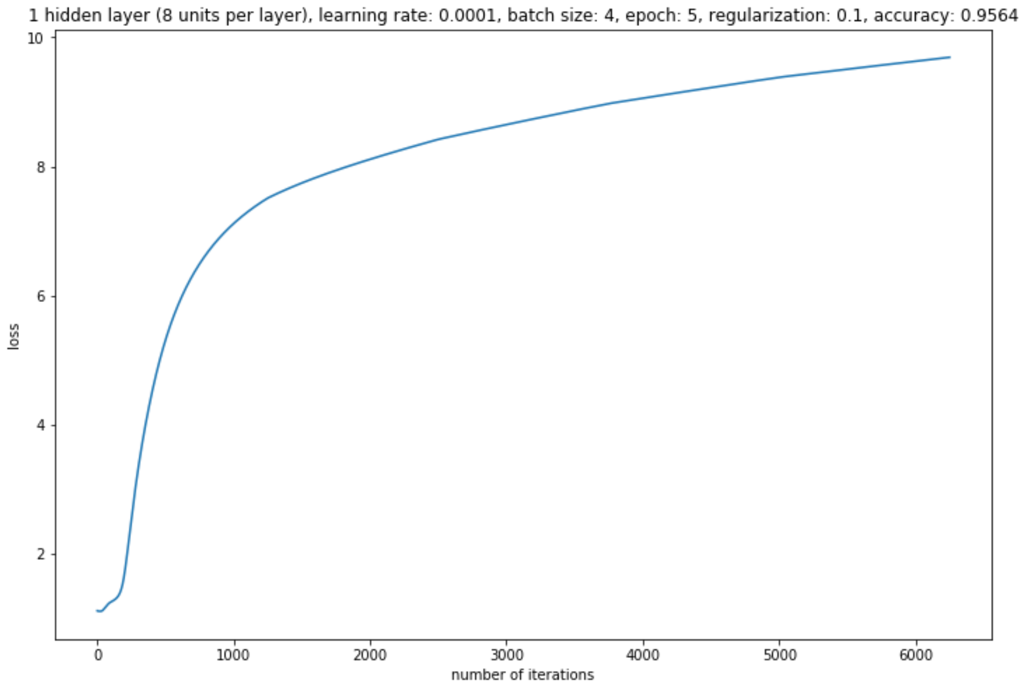
In [256]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=3)
NN.add_layer("Output", dim_in=3, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=20, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

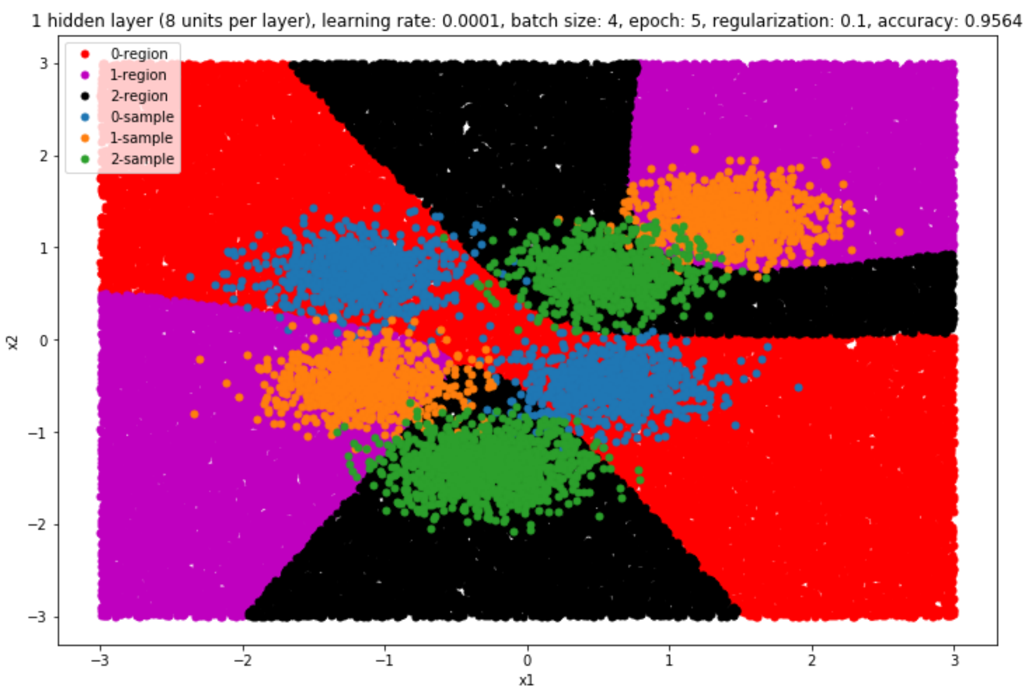




(Softplus) 1 hidden layer (8 units), regularization = 0.1, accuracy = 0.9564

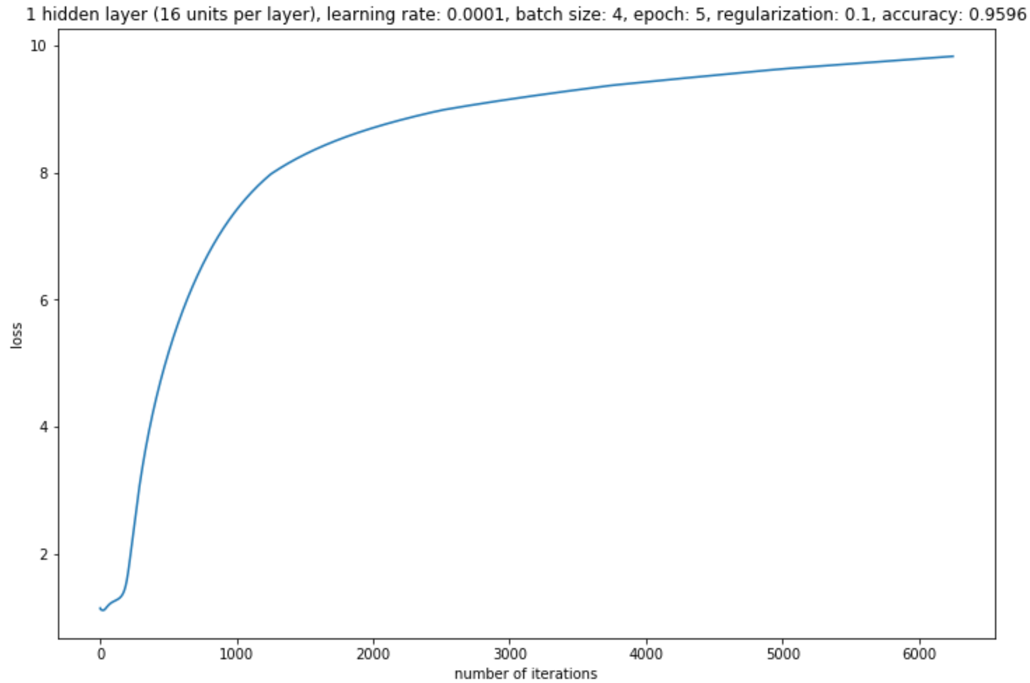
In [254]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=8)
NN.add_layer("Output", dim_in=8, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=5, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

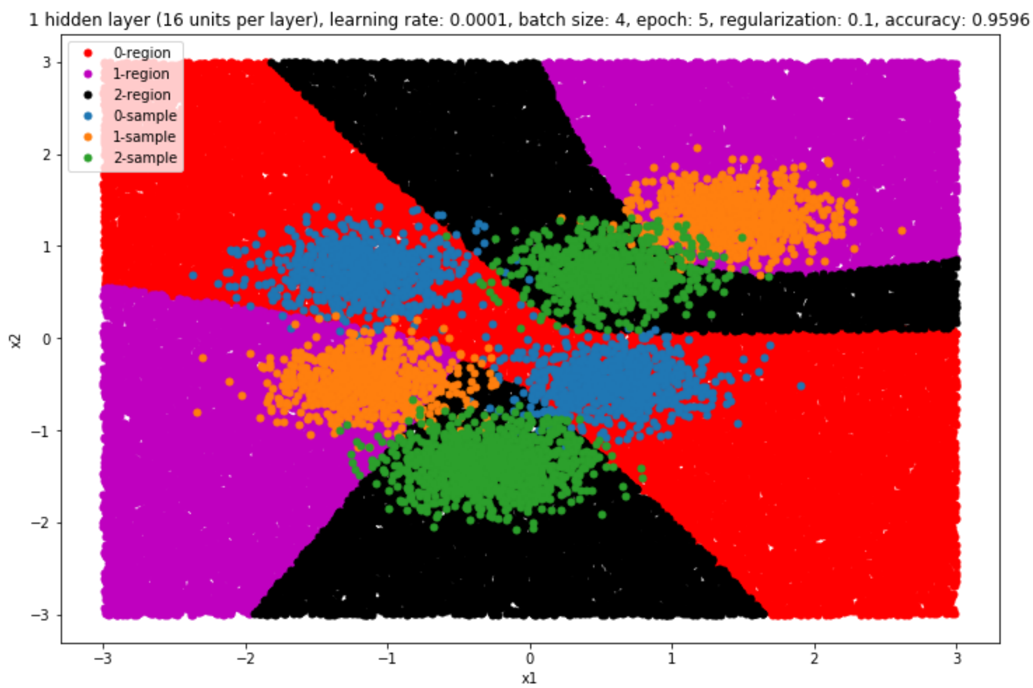




(Softplus) 1 hidden layer (16 units), regularization = 0.1, accuracy = 0.9596

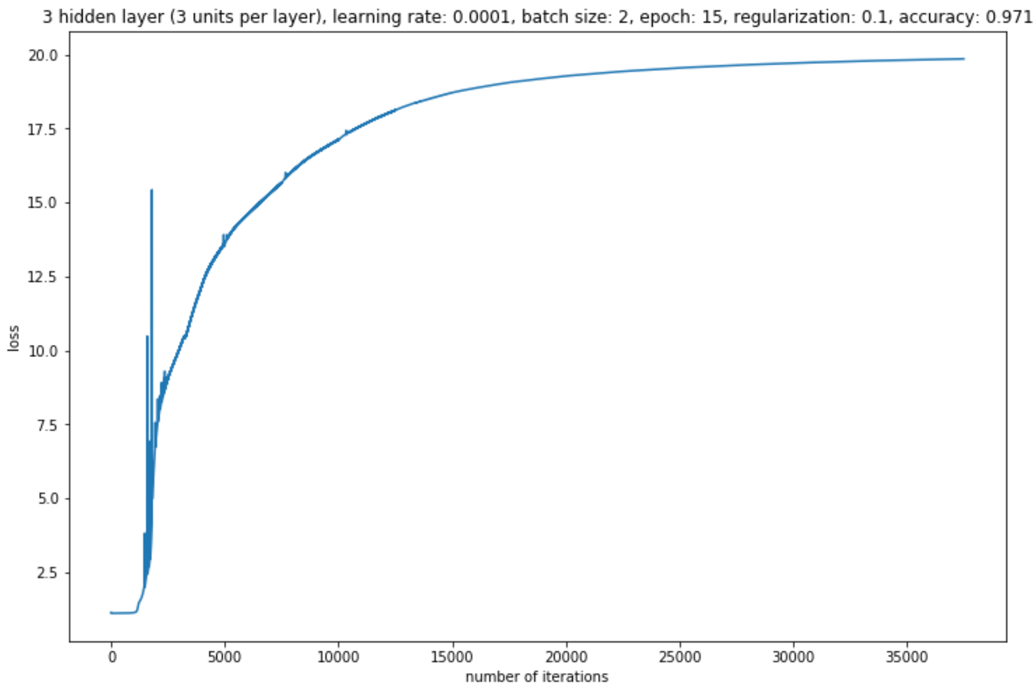
In [255]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=16)
NN.add_layer("Output", dim_in=16, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=5, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

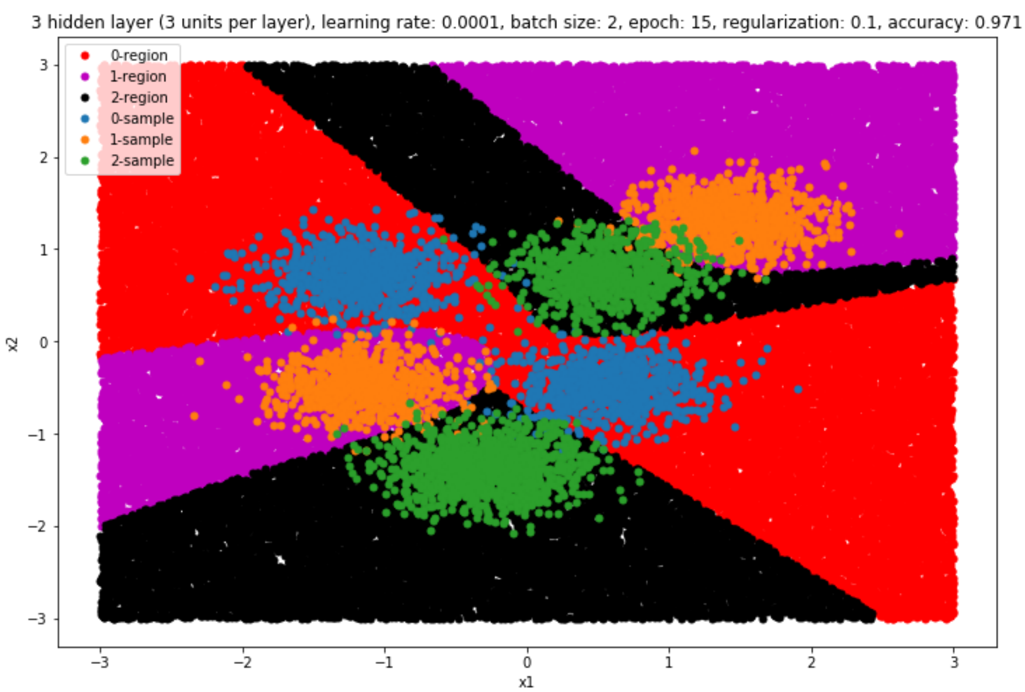




(Softplus) 3 hidden layers (3 units), regularization = 0.1, accuracy = 0.971

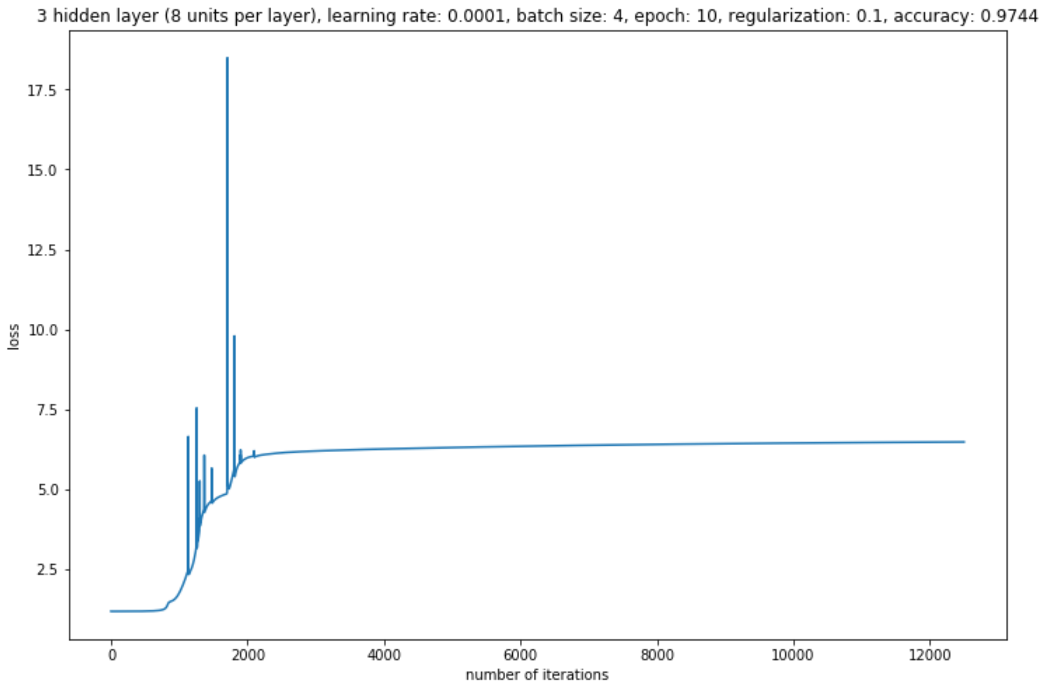
```
In [257]: NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=3)
NN.add_layer("Hidden", dim_in=3, dim_out=3)
NN.add_layer("Hidden", dim_in=3, dim_out=3)
NN.add_layer("Output", dim_in=3, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=2, epoch=15, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

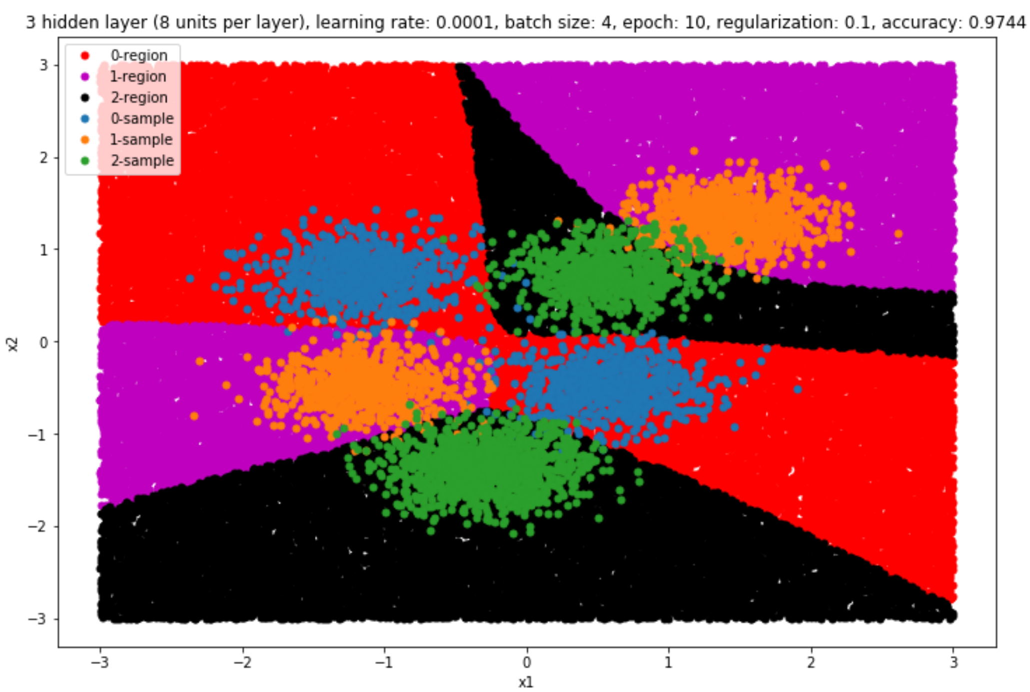




(Softplus) 3 hidden layers (8 units), regularization = 0.1, accuracy = 0.9744

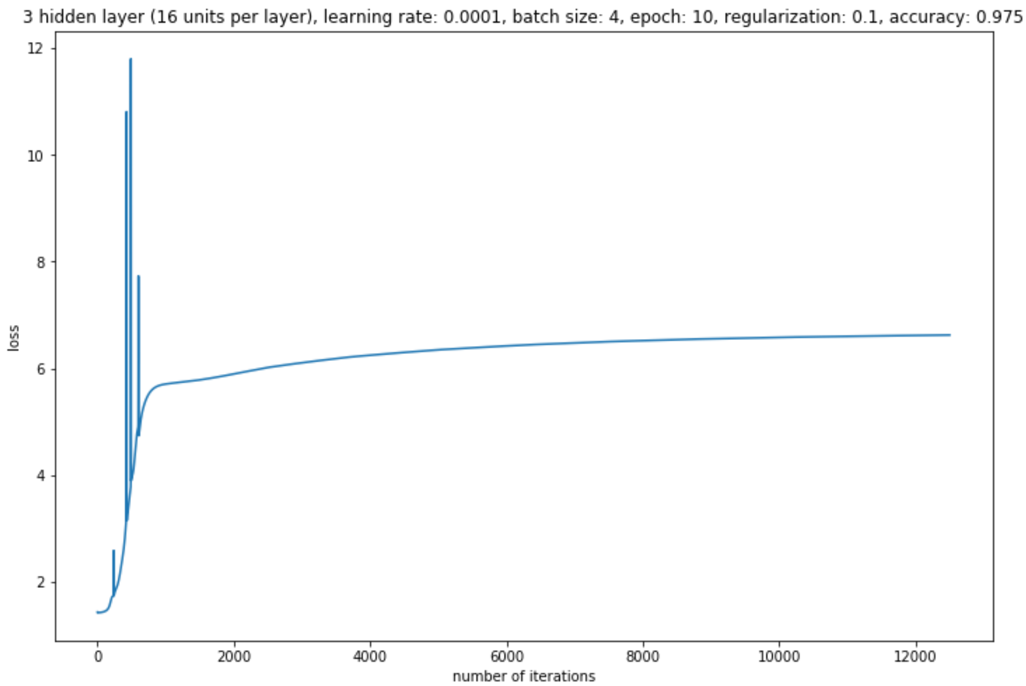
In [250]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=8)
NN.add_layer("Hidden", dim_in=8, dim_out=8)
NN.add_layer("Hidden", dim_in=8, dim_out=8)
NN.add_layer("Output", dim_in=8, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=10, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```

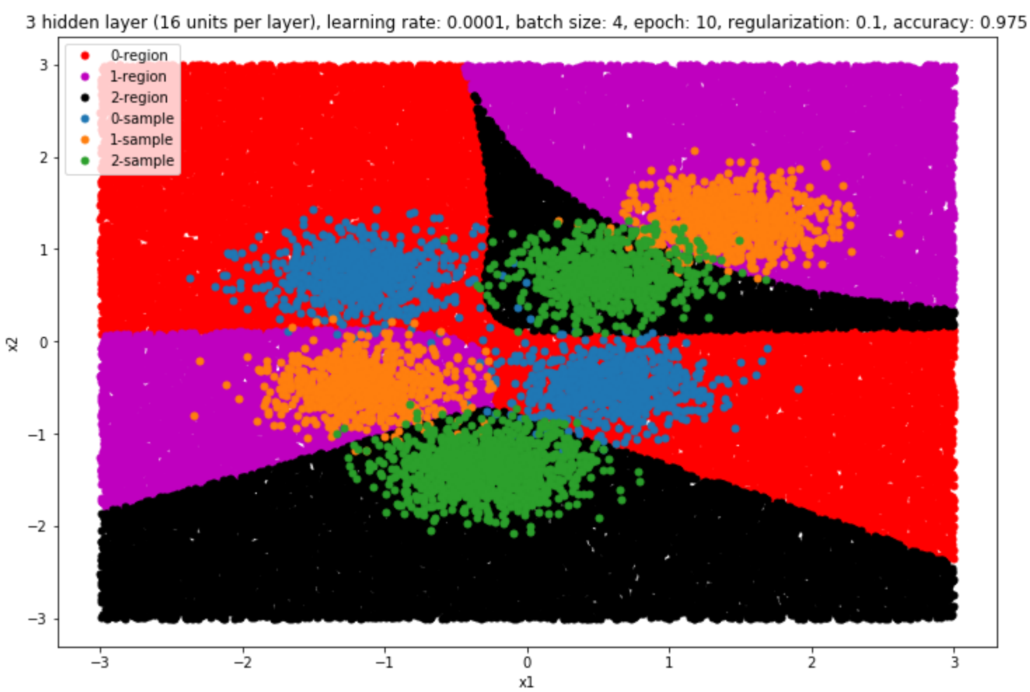




(Softplus) 3 hidden layers (16 units), regularization = 0.1, accuracy = 0.975

In [251]:
```
NN = MLP()
NN.add_layer("Hidden", dim_in=2, dim_out=16)
NN.add_layer("Hidden", dim_in=16, dim_out=16)
NN.add_layer("Hidden", dim_in=16, dim_out=16)
NN.add_layer("Output", dim_in=16, dim_out=3)
NN.add_layer("Loss", dim_in=3, dim_out=3)

loss = NN.train(eta=0.0001, batch_size=4, epoch=10, regularization=0.1, X=X, y=y)

plot_loss(loss)
plot_decision_regions(NN)
```





My test

In [ ]:
```
h, z, L = NN.forward(X, y)
NN.backward(h, z, X, y, eta)
NN.hidden_layer_list[0]
```

In [232]:

```
# softplus activation

eta = 0.001
regularization = 0.1
# forward
hidden_layer = Hidden_layer(2, 16)
output_layer = Output_layer(16, 3)
loss_layer = Loss_layer(3, 3)
h, r = hidden_layer.forward_propagation(X)
z = output_layer.forward_propagation(h)
L = loss_layer.forward_propagation(z, y)

# backward
dLdz = loss_layer.back_propagation(z, y)
dLdh = output_layer.back_propagation(dLdz, h, eta, regularization)
dLdX = hidden_layer.back_propagation(dLdh, r, X, eta, regularization)
```

In [186]:

```
# ReLU activation

eta = 0.001
regularization = 0.1
# forward
hidden_layer = Hidden_layer(2, 16)
output_layer = Output_layer(16, 3)
loss_layer = Loss_layer(3, 3)
h, r = hidden_layer.forward_propagation(X)
z = output_layer.forward_propagation(h)
L = loss_layer.forward_propagation(z, y)

# backward
dLdz = loss_layer.back_propagation(z, y)
dLdh = output_layer.back_propagation(dLdz, h, eta, regularization)
dLdX = hidden_layer.back_propagation(dLdh, h, X, eta, regularization)
```