Description of the problem

The household's utility function is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \tag{2}$$

The labor income can be characterized by

$$Y_t = \begin{cases} \tilde{Y}_t, & \text{if employed} \\ b, & \text{if unemployed} \end{cases} \tag{3}$$

where

$$\log \tilde{Y}_t = (1-\rho)\log \mu + \rho \log \tilde{Y}_{t-1} + \epsilon_t \tag{4}$$

and

$$\epsilon_t \in \mathcal{N}(0, \sigma^2) \tag{5}$$

When a household is employed, its probability of becoming unemployed next period is p. When a household is unemployed, its probability of becoming employed again next period is q.

The budget constraint is given through

$$C_t + \frac{A_{t+1}}{1+r} = Y_t + A_t \tag{6}$$

and the borrowing constraint is

$$A_t \geq 0. \tag{7}$$

The parameter values are

$$\gamma = 2, \beta = 0.98, \mu = 1, b = 0.4, \rho = 0.9, \sigma^2 = 0.05, p = 0.05, 1 = 0.25, r = 0.01. \tag{8}$$

The notation of the parameters is consistent with that used in literature.

To set up this problem, we write it as:

$$\begin{aligned}
V_e(A, Y) &= \max_{A'}\{\frac{(Y + A - \frac{A'}{1+r})^{1-\gamma}}{1-\gamma} + \beta(p \cdot \mathbb{E}V_u(A', b) + (1-p) \cdot \mathbb{E}V_e(A', Y'))\} \\
V_u(A, Y) &= \max_{A'}\{\frac{(b + A - \frac{A'}{1+r})^{1-\gamma}}{1-\gamma} + \beta((1-q) \cdot \mathbb{E}V_u(A', b) + q \cdot \mathbb{E}V_e(A', Y'))\} \\
\log Y' &= (1-\rho)\log \mu + \rho \log Y + \varepsilon'
\end{aligned}$$

The control variables are A' and Y'.

The constraints are $A' \geq 0$ and $C \geq 0 (Y + A - \frac{A'}{1+r} \geq 0)$.