CSCI576 HW2

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Q1

- Because a tile can absorb light different from its color but reflect the light of the same color as itself. Under red light, the red tiles will be bright red, almost glowing. The same as green tiles under green light, and blue tiles under blue light.
  So a red light will make blue and green tiles appear deep gray(even black), and make other tiles(white, red, orange, and yellow) appear red since the rest four colors' tiles reflect red light;
  A green light will make blue and red tiles appear deep gray(even black), and make other tiles appear green since the rest four colors' tiles reflect green light;
  A blue light will make green, red, yellow, and orange tiles appear deep gray(even black) and make other tiles appear blue since the rest two colors tiles reflect blue light.
- Red light makes it harder to solve the puzzle.
  Because red light is a component of four(white, red, orange, and yellow) colors among these six colors, these four are all distinguished as red when we solve the puzzle under red light. Under yellow light, since yellow light is made of red and green light, white tiles will be yellow; yellow tiles will be yellow; orange tiles will be orange; red tiles will be red; green tiles will be green; blue tiles will be black. So there are only two kinds of tiles that need to be distinguished—original yellow and white. This scenario is less complicated than solving the problem under red light.

Q2

2.1

As we know, $x = \frac{X}{X+Y+Z}$, $y = \frac{Y}{X+Y+Z}$, $z = \frac{Z}{X+Y+Z}$

$$\left[\frac{1-x-y}{y}\right]Y$$

$$= \left[\frac{1 - \frac{X}{X+Y+Z} - \frac{Y}{X+Y+Z}}{\frac{Y}{X+Y+Z}}\right]Y$$

$$= \frac{(X+Y+Z)-X-Y}{Y} \cdot Y$$

$$= X+Y+Z-X-Y$$

$$= Z$$

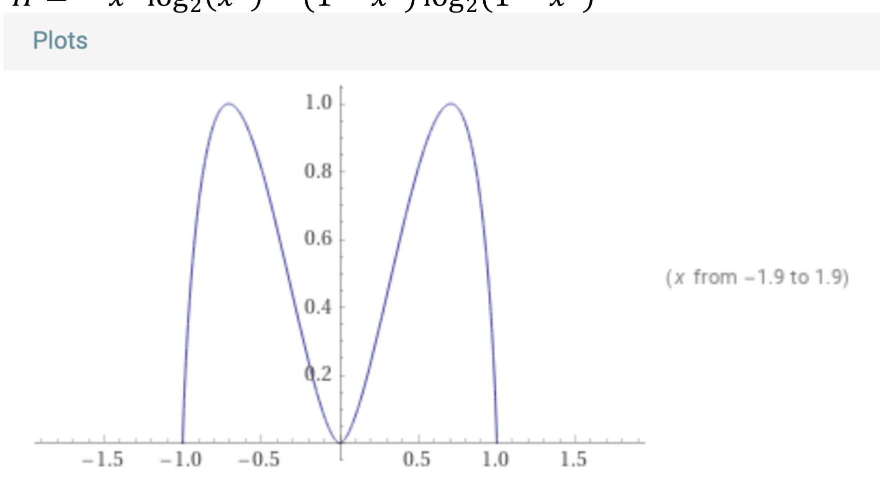
2.2

- This algorithm will work effectively in some cases, like landscape & environmental portraits, but will not work effectively when showing some scenes, like skin tones in beauty and portraits. Because different colors which are out of the printer's gamut but have the same nearest available color(eg. With constant color tone) may be mapped as the same color.

- The real image with varying color tones will be performed better. Because the printer with this algorithm can not show the differences of the colors out of the printer's gamut with the constant color tone, as these colors have the same nearest available color.
- (Perceptual) Change the colors by altering the saturation of the entire image proportionally in a balanced way so that the colors lie within the printer's gamut; (Saturation Rendering Intent) Preserve the saturated colors at the expense of change in hue and the lightness of the color.

Q3

- $H = -x^2 \log_2(x^2) - (1 - x^2) \log_2(1 - x^2)$



(using https://www.wolframalpha.com/ with "plot -x^2 log(2, x^2) - (1 - x^2) log(2, 1 - x^2)" as above)

- From my plot, when $H = 0$ becomes a minimum, $x = -1, 0, \ or\ 1$.
- When H becomes a minimum, $P(X) = x^k = 1 \ or\ P(Y) = 1 - x^k = 1$.

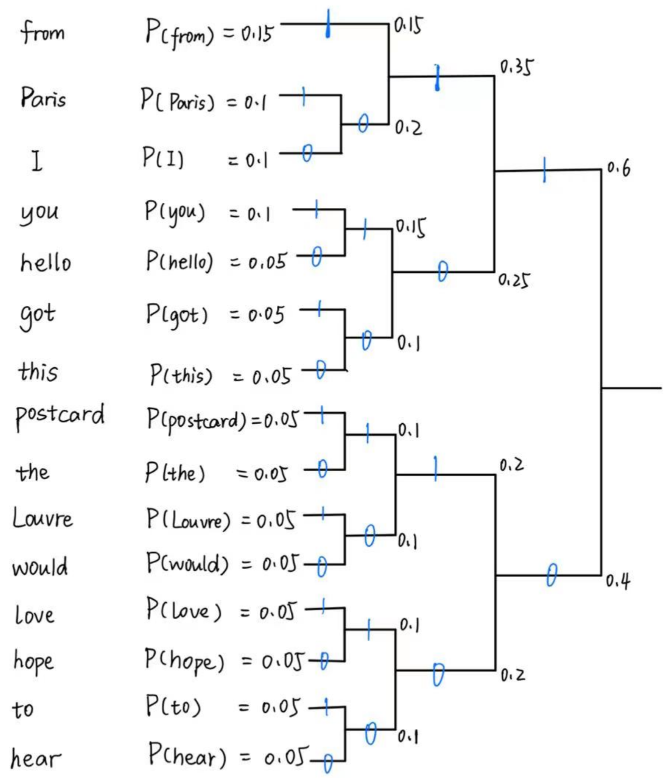
  So, $x = \begin{cases} 0\ or\ 1 \ \ (k\ mod\ 2 \neq 0) \\ 0,\ 1\ or - 1\ (k\ mod\ 2 = 0,\ k \neq 0) \\ n,\ n \neq 0 \ \ (k = 0) \end{cases}$
- From my plot, when H becomes a maximum, $x^2 = 1 - x^2$.

  So, $x = \pm\sqrt{0.5} = \pm 0.7071$
- When H becomes a maximum, $x^k = 1 - x^k$.

  So, $x = \begin{cases} \sqrt[k]{0.5} \ \ (k \neq 0) \\ -\sqrt[k]{0.5}\ (k\ mod\ 2 = 0,\ k \geq 1 \ or\ k \leq -1) \end{cases}$

Q4

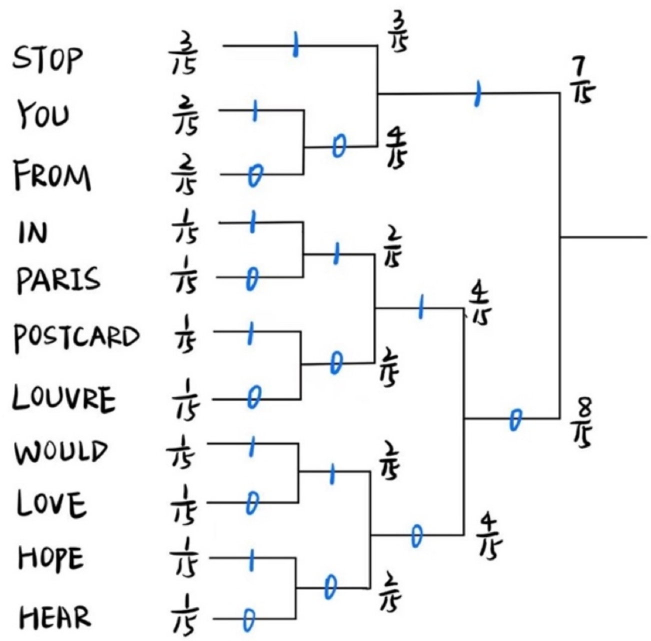
- In the body of Alice's postcard, there are 15 words used. So there are 15 symbols. The words and their corresponding Huffman codewords & probabilities are as follows:



	Symbol	Huffman codewords	Length Huffman codewords per symbol	Probability
1.	from	111	3	0.15 (3/20)
2.	Paris	1101	4	0.1 (2/20)
3.	I	1100	4	0.1 (2/20)
4.	you	1011	4	0.1 (2/20)
5.	hello	1010	4	0.05 (1/20)
6.	got	1001	4	0.05 (1/20)
7.	this	1000	4	0.05 (1/20)
8.	postcard	0111	4	0.05 (1/20)
9.	the	0110	4	0.05 (1/20)
10.	Louvre	0101	4	0.05 (1/20)
11.	would	0100	4	0.05 (1/20)
12.	love	0011	4	0.05 (1/20)
13.	hope	0010	4	0.05 (1/20)
14.	to	0001	4	0.05 (1/20)
15.	hear	0000	4	0.05 (1/20)

The average code length is $L_{Huff} = \sum_{i=1}^{i=15} P(i)l(i) = 3.85$

- In the body of Bob's telegram message, there are 11 words used. So there are 11 symbols. The words and their corresponding Huffman codewords & probabilities are as follows:



	Symbol	Huffman codewords	Length Huffman codewords per symbol	Probability
1.	STOP	11	2	3/15
2.	YOU	101	3	2/15
3.	FROM	100	3	2/15
4.	IN	0111	4	1/15
5.	PARIS	0110	4	1/15
6.	POSTCARD	0101	4	1/15
7.	LOUVRE	0100	4	1/15
8.	WOULD	0011	4	1/15
9.	LOVE	0010	4	1/15
10.	HOPE	0001	4	1/15
11.	HEAR	0000	4	1/15

The average code length is $L'_{Huff} = \sum_{i=1}^{i=11} P(i)l(i) = 3.333$
The average code of the telegram message is shorter than the code of the original letter (3.333 < 3.85).

- The postcard contains more information.
  Entropy for postcard $H_{postcard} = -\sum_{i=1}^{i=15} P(i)\log_2 P(i) = -0.15\log_2(0.15) - 3 \times 0.1\log_2(0.1) - 11 \times 0.05\log_2(0.05) = 3.784$
  Entropy for telegram $H_{talegram} = -\sum_{i=1}^{i=11} P(i)\log_2 P(i) = -\frac{3}{15}\log_2\left(\frac{3}{15}\right) - 2 \times \frac{2}{15}\log_2\left(\frac{2}{15}\right) - 8 \times \frac{1}{15}\log_2\left(\frac{1}{15}\right) = 3.323$
  Because 3.784 > 3.323, the symbols in the postcard are more equally probable, while the symbols in the telegram are much more likely to appear than other symbols. So the postcard with higher entropy contains more information.