FACULTY OF ENGINEERING
AND COMPUTER SCIENCE

Department of Computer Science
and Software Engineering

COMP498G/691G
COMPUTER VISION

LECTURE 17
PHOTOMETRIC STEREO

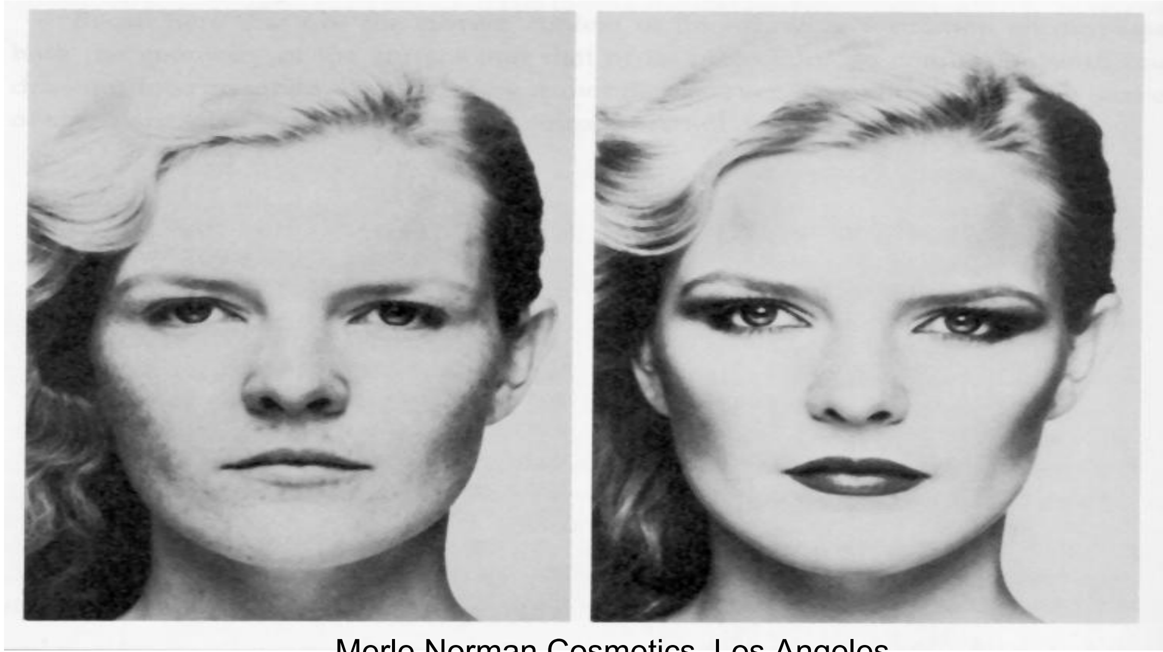
Administrative

- Assignment #3 due
- Project assigned
  - Hard deadline: 11 April 2017
  - Demo in class/tutorial
- Tonight's tutorial
  - Quiz #1 demo

Today’s Lecture

- Photometric Stereo
    - Project
- Slides acknowledgment: Lana Lazebnik, Fei-Fei Li, Rob Fergus, Antonio Torralba, and Jean Ponce
- Questions

Photometric Stereo

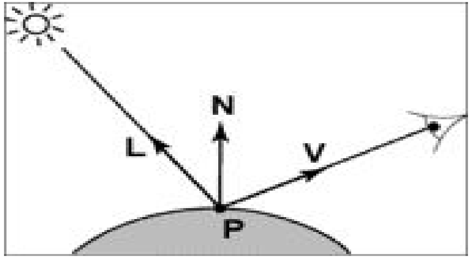


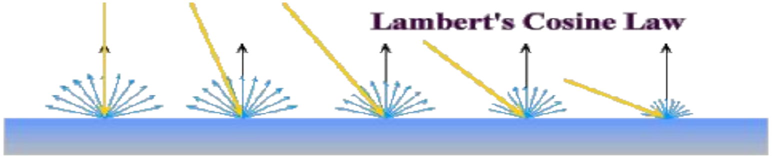
Merle Norman Cosmetics, Los Angeles

Readings

- Optional: Woodham's original photometric stereo paper
  - http://www.cs.ubc.ca/~woodham/papers/Woodham80c.pdf

Diffuse reflection





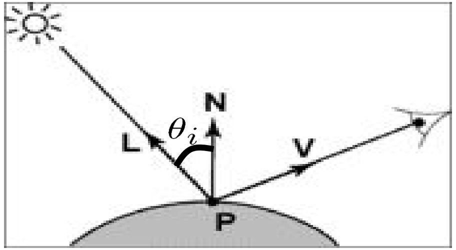
$$R_e = k_d \mathbf{N} \cdot \mathbf{L} R_i$$

image intensity of P → $I = k_d \mathbf{N} \cdot \mathbf{L}$

Simplifying assumptions

- $I = R_e$: camera response function f is the identity function:
- can always achieve this in practice by solving for f and applying f^{-1} to each pixel in the image
- $R_i = 1$: light source intensity is 1
- can achieve this by dividing each pixel in the image by R_i

Shape from shading



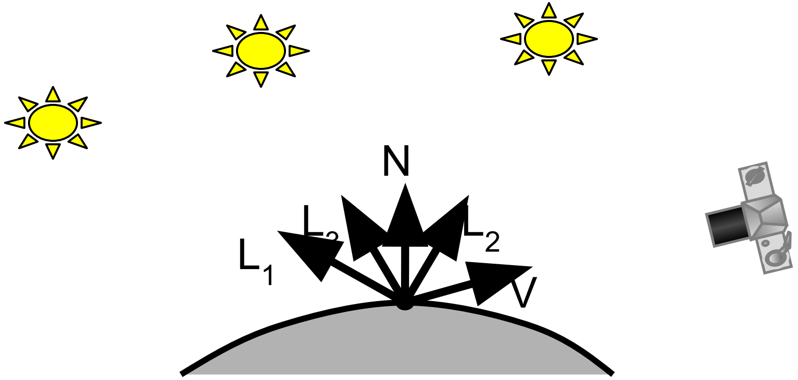
Suppose $k_d = 1$

$$\begin{aligned} I &= k_d \mathbf{N} \cdot \mathbf{L} \\ &= \mathbf{N} \cdot \mathbf{L} \\ &= cos\ \theta_i \end{aligned}$$

You can directly measure angle between normal and light source

- Not quite enough information to compute surface shape
- But can be if you add some additional info, for example
- assume a few of the normals are known (e.g., along silhouette)
- constraints on neighboring normals—“integrability”
- smoothness
- Hard to get it to work well in practice
- plus, how many real objects have constant albedo?

Photometric stereo



$$
\begin{aligned}
I_1 &= k_d \mathbf{N} \cdot \mathbf{L_1} \\
I_2 &= k_d \mathbf{N} \cdot \mathbf{L_2} \\
I_3 &= k_d \mathbf{N} \cdot \mathbf{L_3}
\end{aligned}
$$

Can write this as a matrix equation:

Solving the equations

$$\underbrace{\left[\begin{array}{ccc} I_1 & \dots & I_n \end{array}\right]}_{\substack{\mathbf{I} \\ 1\times 3}} = \underbrace{k_d \mathbf{N}^T}_{\substack{\mathbf{G} \\ 1\times 3}} \underbrace{\left[\begin{array}{ccc} \mathbf{L_1} & \dots & \mathbf{L_n} \end{array}\right]}_{\substack{\mathcal{L} \\ 3\times 3}}$$

$$\mathbf{G} = \mathbf{I}\mathbf{L}^{-1}$$

$$k_d = \|\mathbf{G}\|$$

$$\mathbf{N} = \frac{1}{k_d}\mathbf{G}$$

More than three lights

Get better results by using more lights

$$\left[\begin{array}{ccc} I_1 & \ldots & I_n \end{array}\right] = k_d \mathbf{N}^T \left[\begin{array}{ccc} \mathbf{L_1} & \ldots & \mathbf{L_n} \end{array}\right]$$

Least squares solution:

$$\begin{aligned} \mathbf{I} &= \mathbf{GL} \\ \mathbf{IL^T} &= \mathbf{GLL^T} \\ \mathbf{G} &= \mathbf{(IL^T)(LL^T)^{-1}} \end{aligned}$$

Solve for N, k_d as before

What's the size of LL^T?

Color images

The case of RGB images

- get three sets of equations, one per color channel:

$$\begin{aligned} \mathbf{I}_R &= k_{dR}\mathbf{N}^T\mathcal{L} \\ \mathbf{I}_G &= k_{dG}\mathbf{N}^T\mathcal{L} \\ \mathbf{I}_B &= k_{dB}\mathbf{N}^T\mathcal{L} \end{aligned}$$

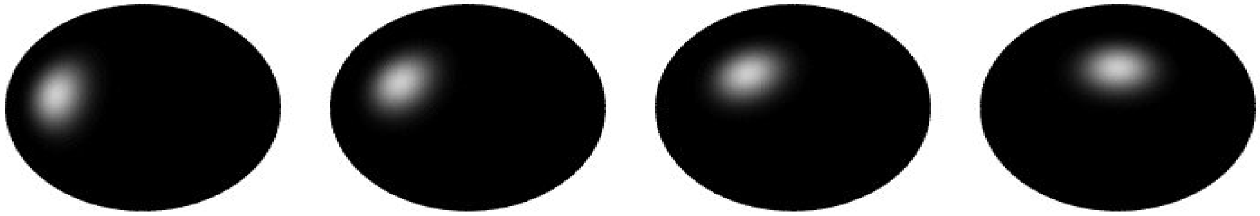
call this J

- Simple solution: first solve for N using one channel
- Then substitute known N into above equations to get k^d s:

$$\begin{aligned} \mathbf{I}_R &= k_{dR}\mathbf{J} \\ \mathbf{J}\cdot\mathbf{I}_R &= k_{dR}\mathbf{J}\cdot\mathbf{J} \\ k_{dR} &= \frac{\mathbf{J}\cdot\mathbf{I}_R}{\mathbf{J}\cdot\mathbf{J}} \end{aligned}$$

Computing light source directions

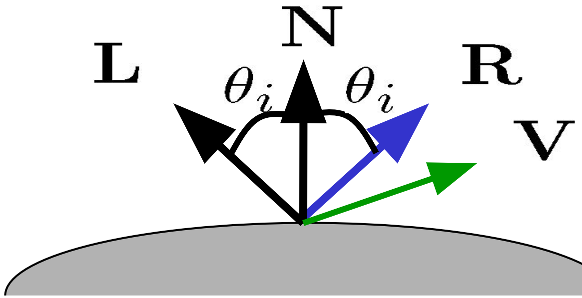
Trick: place a chrome sphere in the scene



- the location of the highlight tells you where the light source is

Recall the rule for specular reflection

For a perfect mirror, light is reflected about N



$$R_e = \begin{cases} R_i & \text{if } \mathbf{V} = \mathbf{R} \\ 0 & \text{otherwise} \end{cases}$$

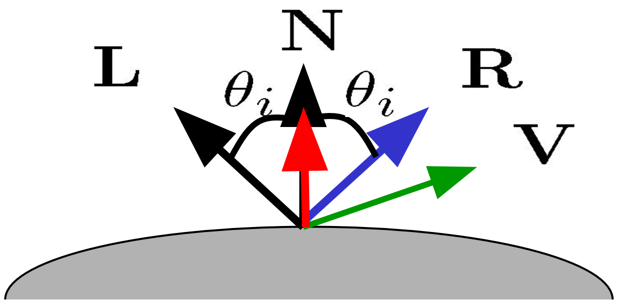
We see a highlight when V = R

- then L is given as follows:

$$\mathbf{L} = 2(\mathbf{N} \cdot \mathbf{R})\mathbf{N} - \mathbf{R}$$

Recall the rule for specular reflection

For a perfect mirror, light is reflected about N



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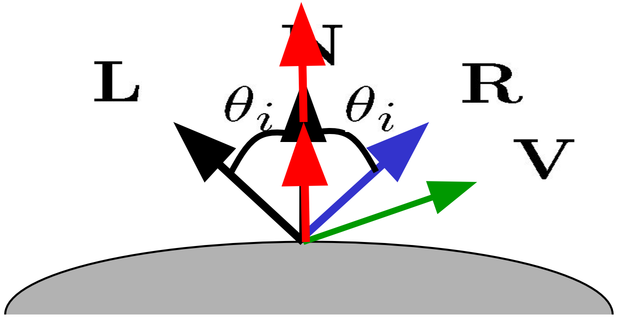

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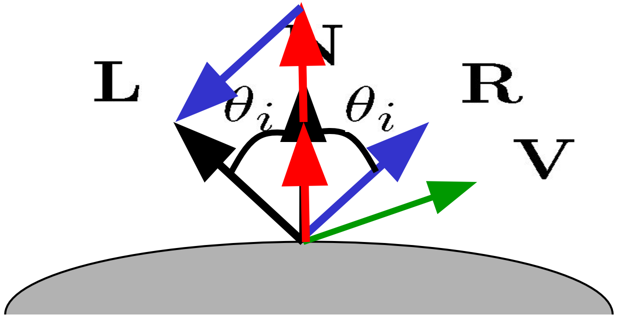

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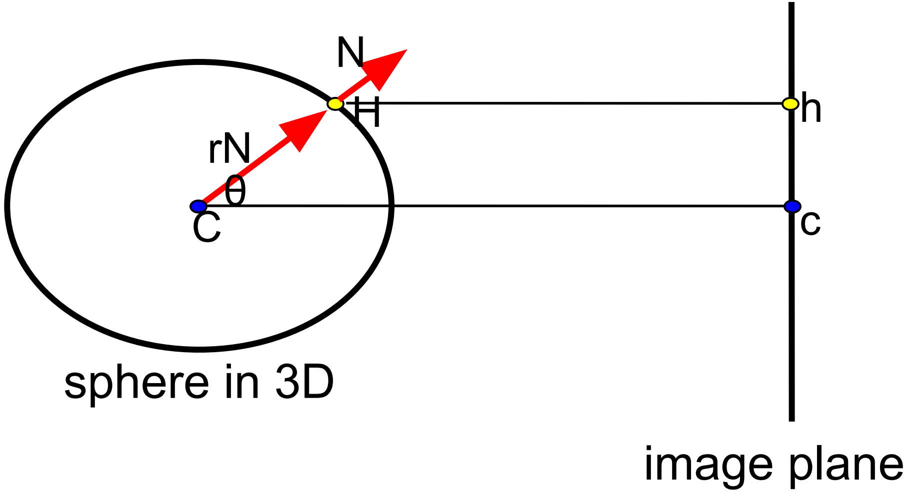
Computing the light source direction

Chrome sphere that has a highlight at position h in the image

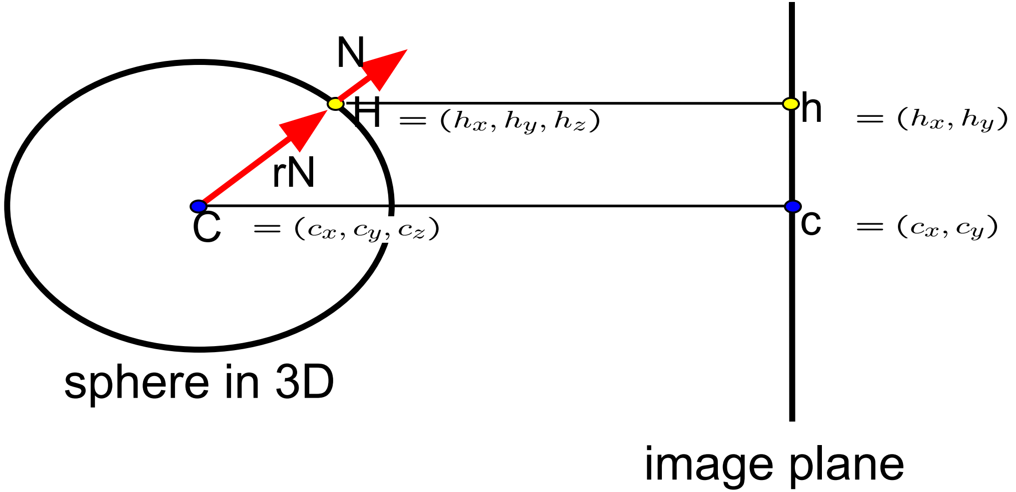


Can compute θ (and hence N) from this figure

Now just reflect V about N to obtain L

Computing the light source direction

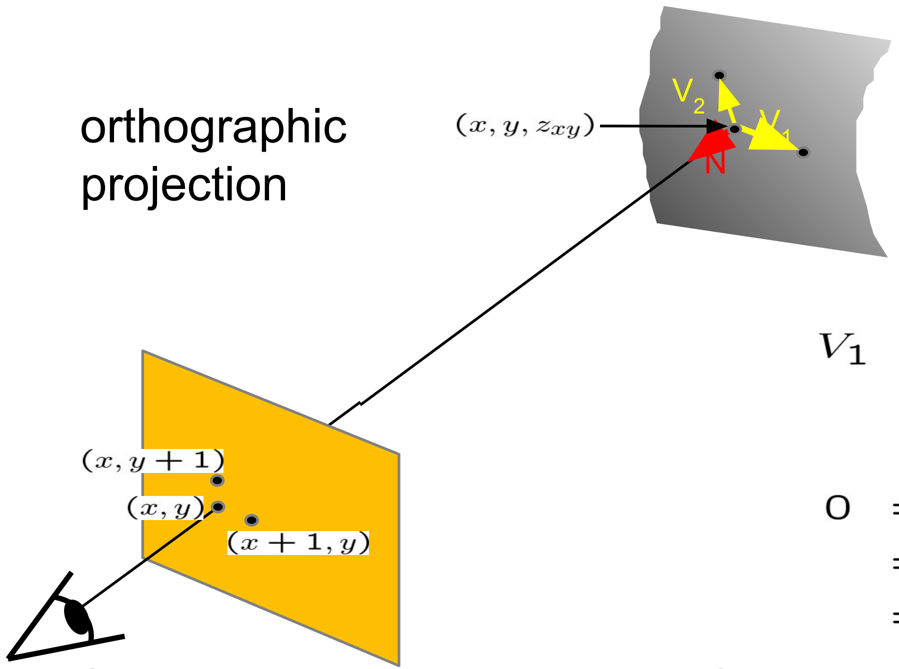
Chrome sphere that has a highlight at position h in the image



Can compute N by studying this figure

- Hints:
  - use this equation: $||H - C|| = r$
  - can measure c, h, and r in the image
  - can choose cz = 0

Depth from normals



$$
\begin{aligned}
V_1 &= (x+1, y, z_{x+1,y}) - (x, y, z_{xy}) \\
&= (1, 0, z_{x+1,y} - z_{xy})
\end{aligned}
$$

$$
\begin{aligned}
0 &= N \cdot V_1 \\
&= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\
&= n_x + n_z (z_{x+1,y} - z_{xy})
\end{aligned}
$$

Get a similar equation for V_2

- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation

Results…

Input
(1 of 12)

Normals

Normals

Shaded
rendering

Textured
rendering

Results…

from Athos Georghiades

http://cvc.yale.edu/people/Athos.html

Limitations

Big problems

- doesn’t work for shiny things, semi-translucent things
- shadows, inter-reflections

Smaller problems

- camera and lights have to be distant
- calibration requirements
  - measure light source directions, intensities
  - camera response function

Trick for handling shadows

Weight each equation by the pixel brightness:

$$I_i(I_i) \;=\; I_i[k_d\mathbf{N}\cdot\mathbf{L_i}]$$

Gives weighted least-squares matrix equation:

$$\left[\begin{array}{ccc} I_1^2 & \ldots & I_n^2 \end{array}\right] = k_d\mathbf{N}^T\left[\begin{array}{ccc} I_1\mathbf{L_1} & \ldots & I_n\mathbf{L_n} \end{array}\right]$$

Solve for N, k_d as before

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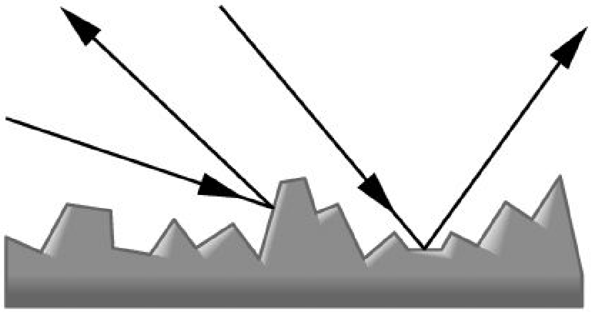
Diffuse Reflection

- Diffuse reflector scatters light equally to all directions
- Called *Lambertian* surface
- Diffuse reflection coefficient k_d, $0 \leq k_d \leq 1$

-What happens if the lighting direction is changed?
-What happens if the viewer's direction is changed?



Diffuse Reflection

- Diffuse reflector scatters light equally to all directions
- Called *Lambertian* surface
- Diffuse reflection coefficient k_d, $0 \le k_d \le 1$
- Only the angle of incoming light is important

-What happens if the lighting direction is changed?
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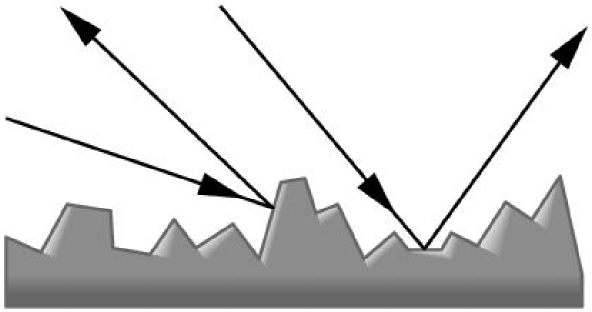
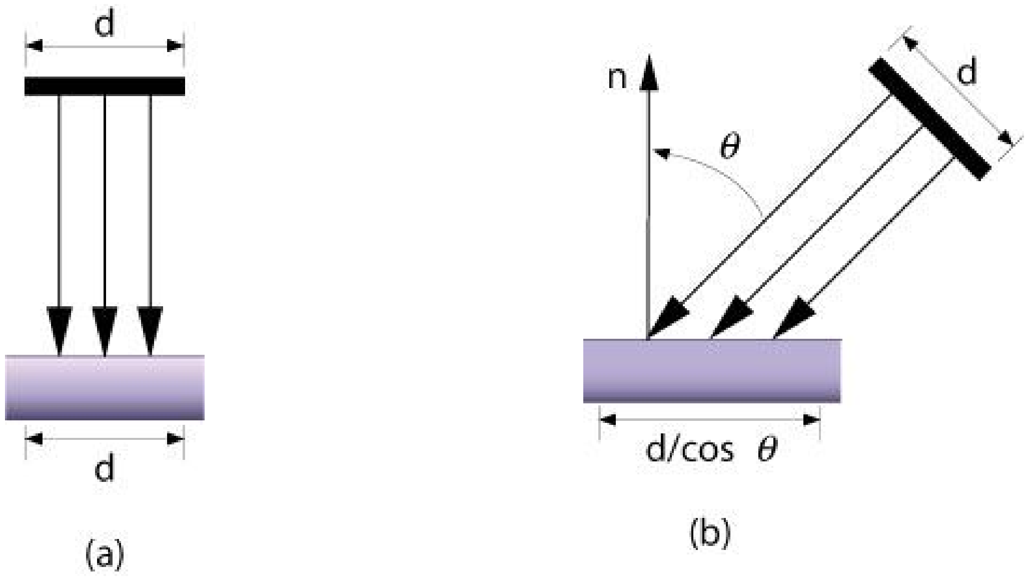


Lambert's Law

Intensity depends on angle of incoming light



Diffuse Light Intensity Depends on Angle of Incoming Light

- Recall

  l = unit vector to light

  n = unit surface normal

  θ = angle to normal

- $\cos\theta = l \cdot n$
- $I_d = k_d L_d (l \cdot n)$
- With attenuation:

$$I_d = \frac{k_d L_d}{a + bq + cq^2}(l \cdot n)$$

q = distance to light

L_d = diffuse component of light