Université Concordia University
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
Department of Computer Science and Software Engineering

COMP498G/691G COMPUTER VISION

LECTURE 8
IMAGE STITCHING

Today's Lecture

- Image stitching
  - Slides acknowledgment: L. Shapiro
- Questions

- Combine two or more overlapping images to make one larger image

Slide credit: Vaibhav Vaish

How to do it?

- Basic Procedure

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     (Rotate the camera about its optical center)

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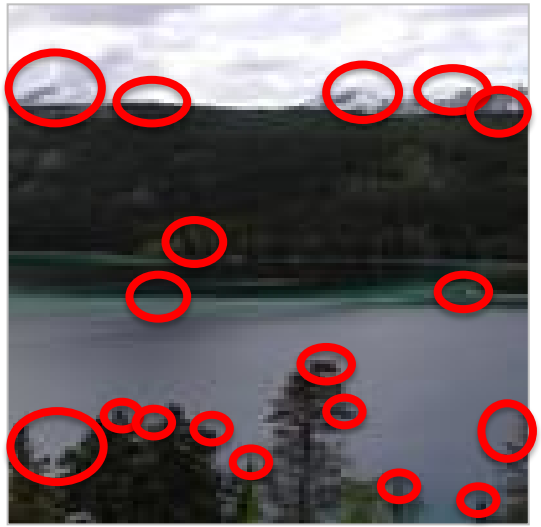
  5. If there are more images, repeat

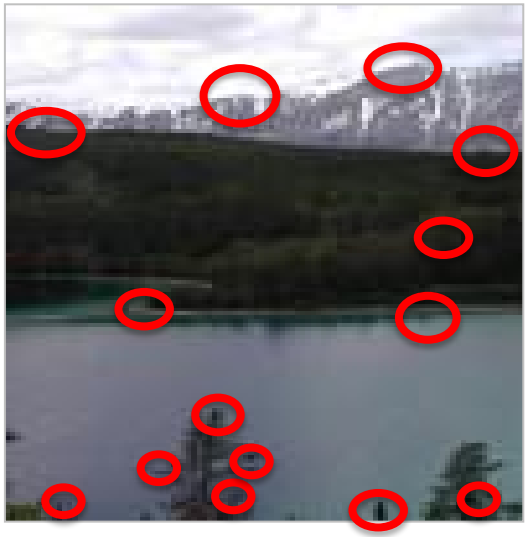
1. Take a sequence of images from the same position

- Rotate the camera about its optical center

2. Compute transformation between images

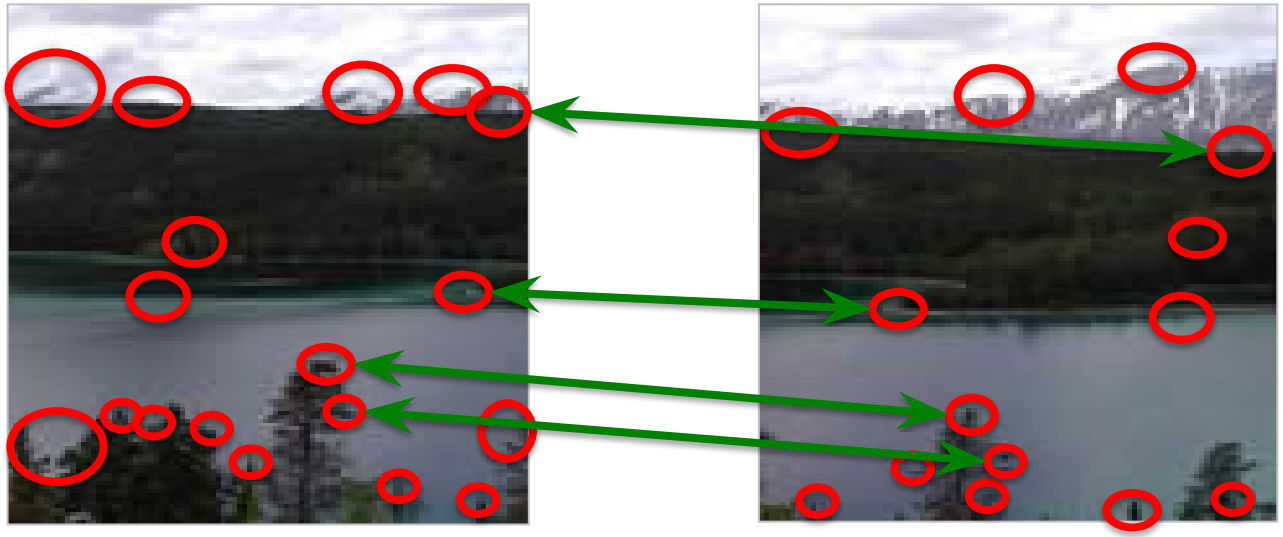
- Extract interest points





2. Compute transformation between images

- Extract interest points
- Find Matches



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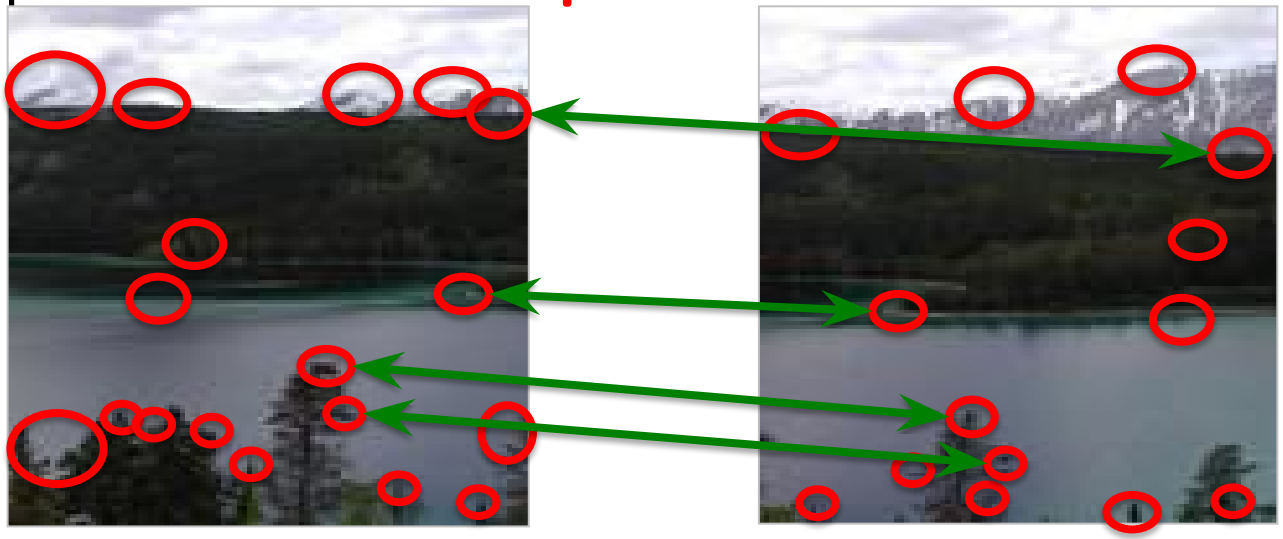
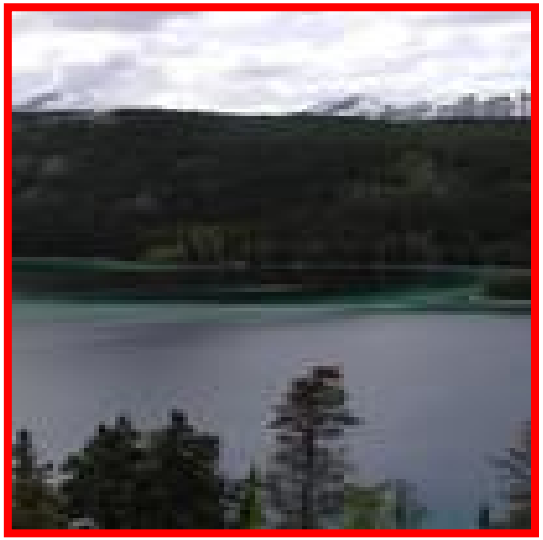
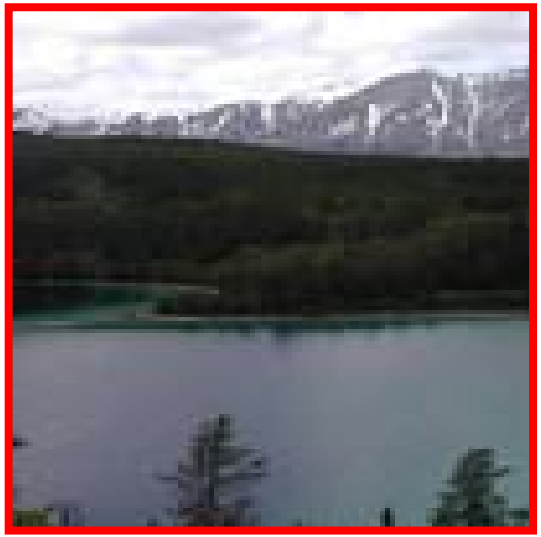


3. Shift the images to overlap





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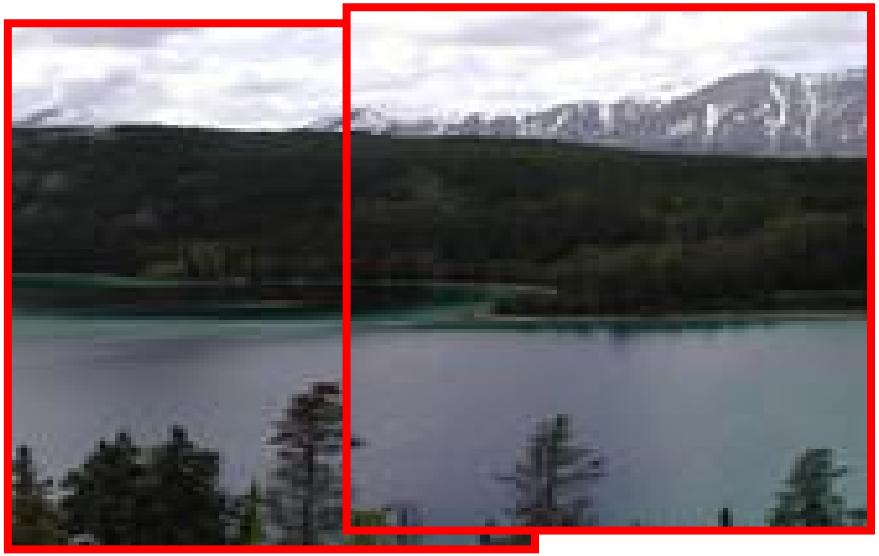
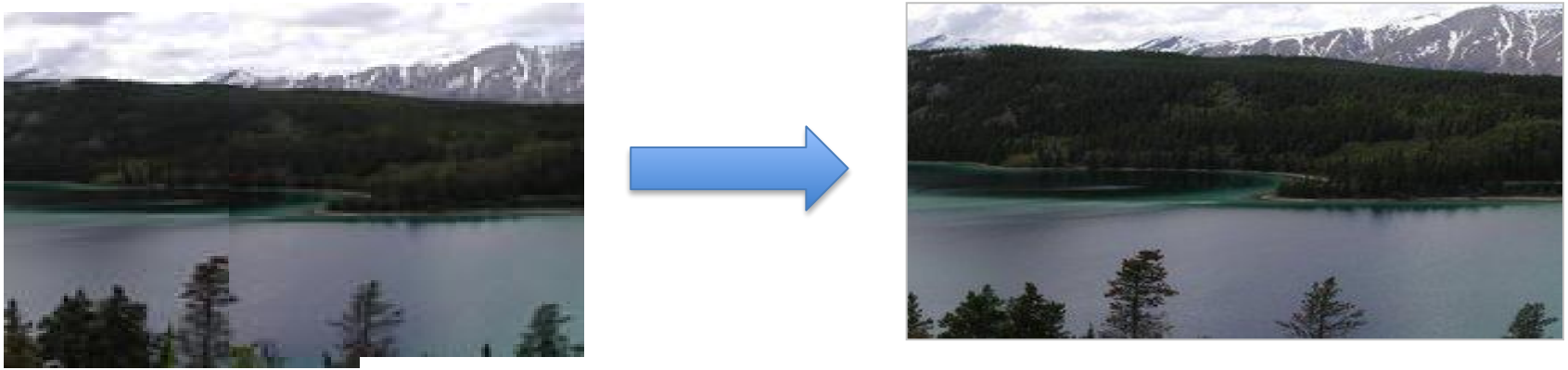


4. Blend the two together to create a mosaic



5. Repeat for all images

How to do it?

- Basic Procedure

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Rotate the camera about its optical center

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Compute Transformations

- ✓ Extract interest points
- ✓ Find good matches
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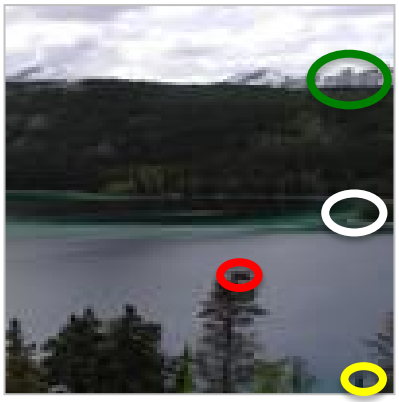
Let's assume we are given a set of good matching interest points



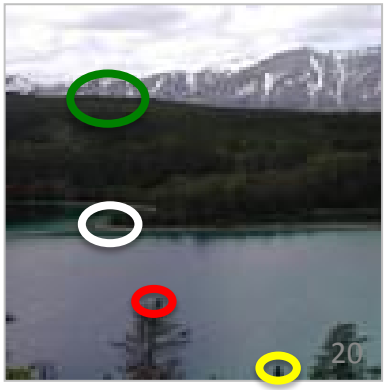
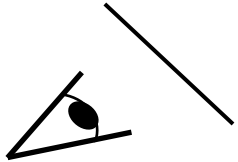


Image reprojection



- The mosaic has a natural interpretation in 3D
  - The images are reprojected onto a common plane
  - The mosaic is formed on this plane

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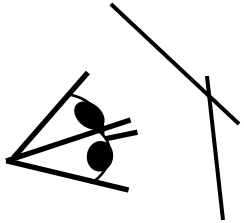

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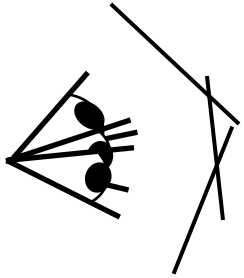

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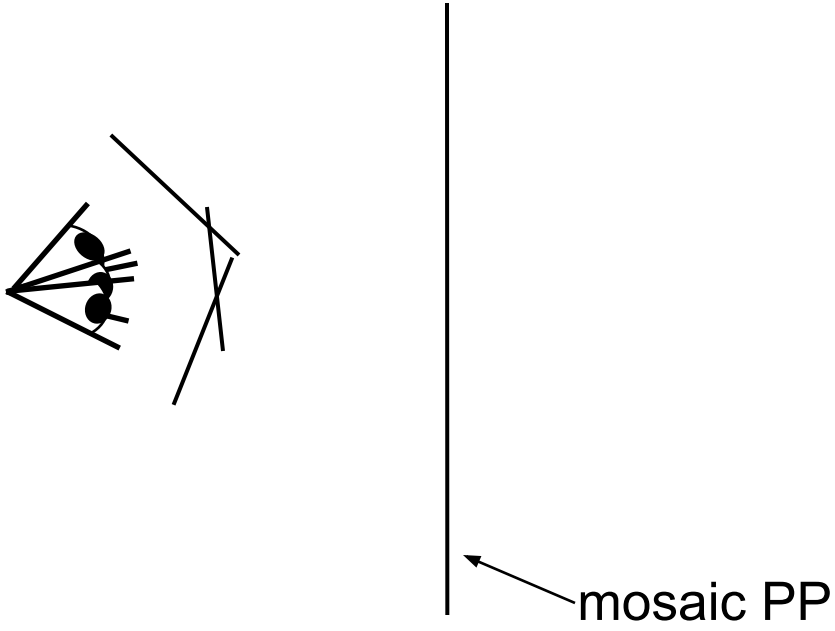

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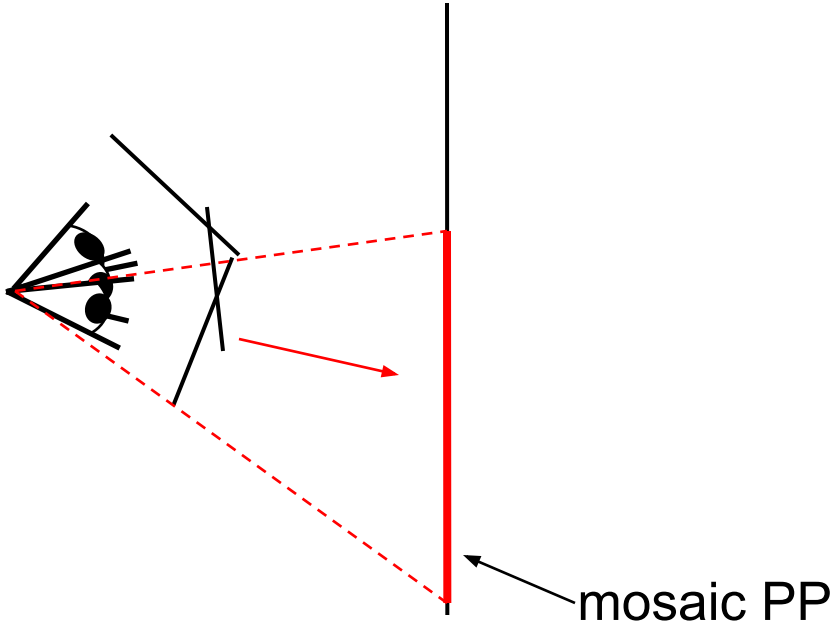

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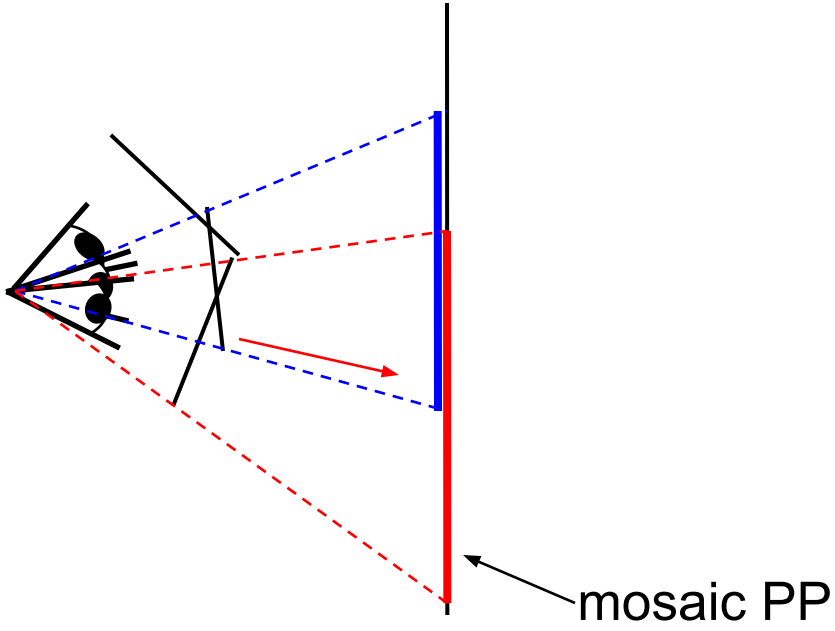

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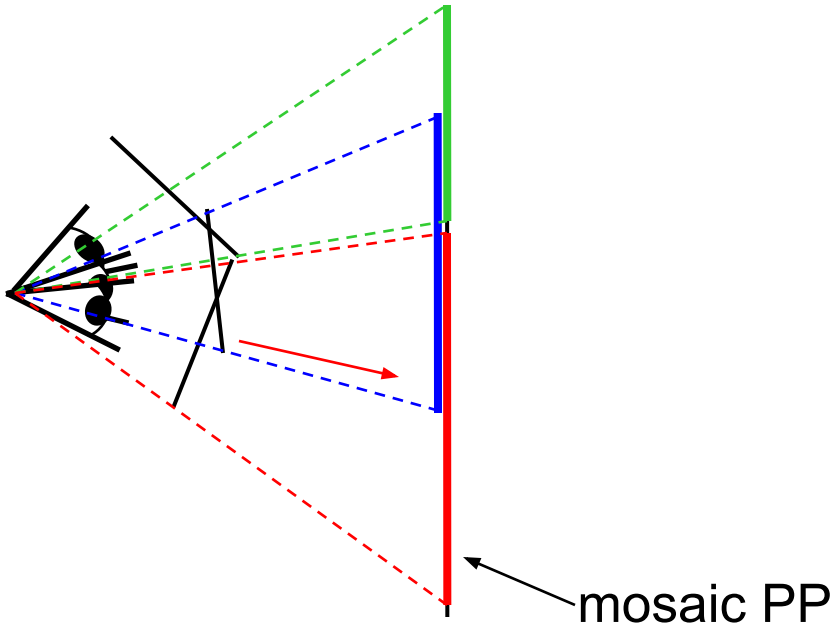

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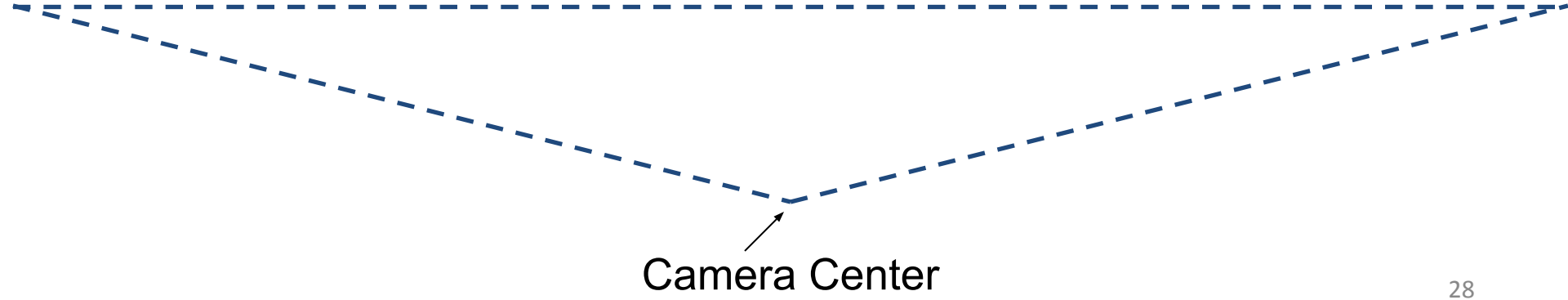
Example



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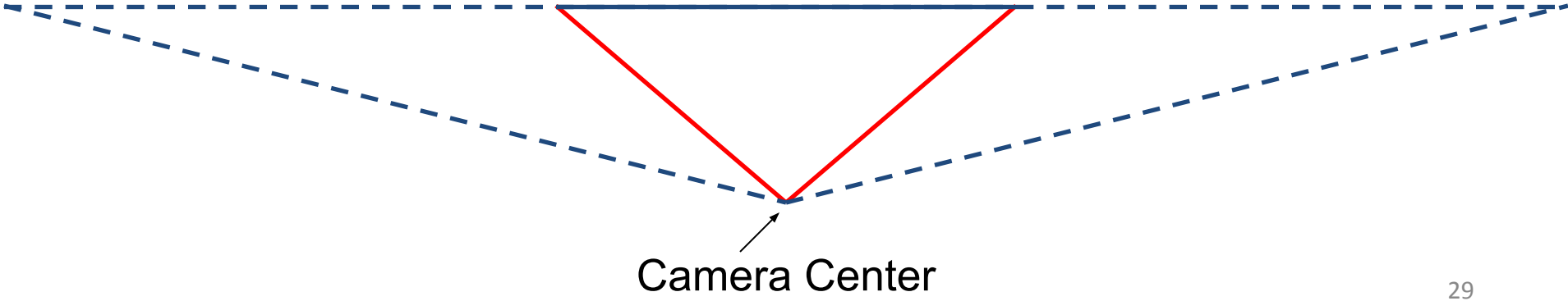

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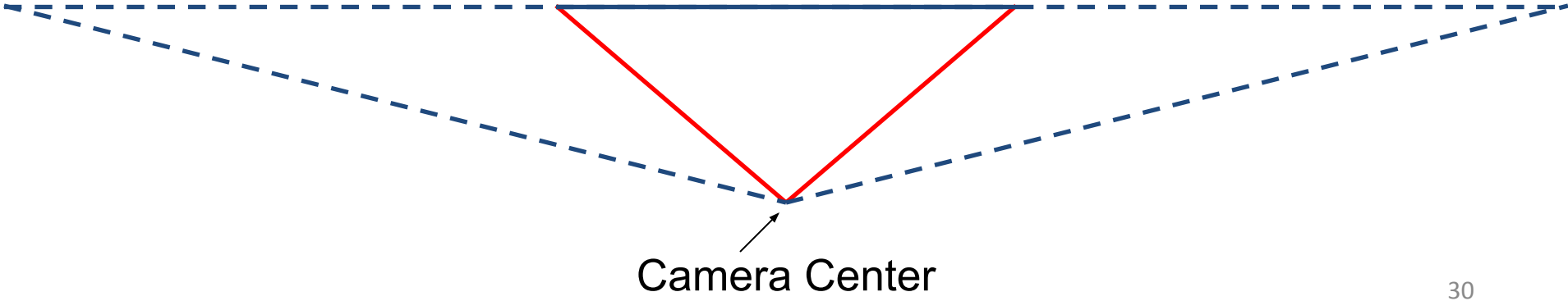

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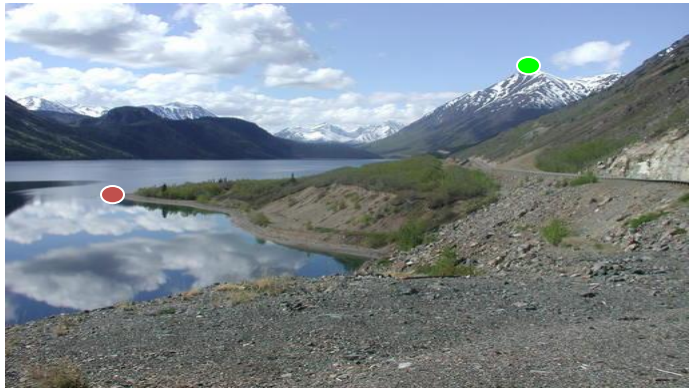
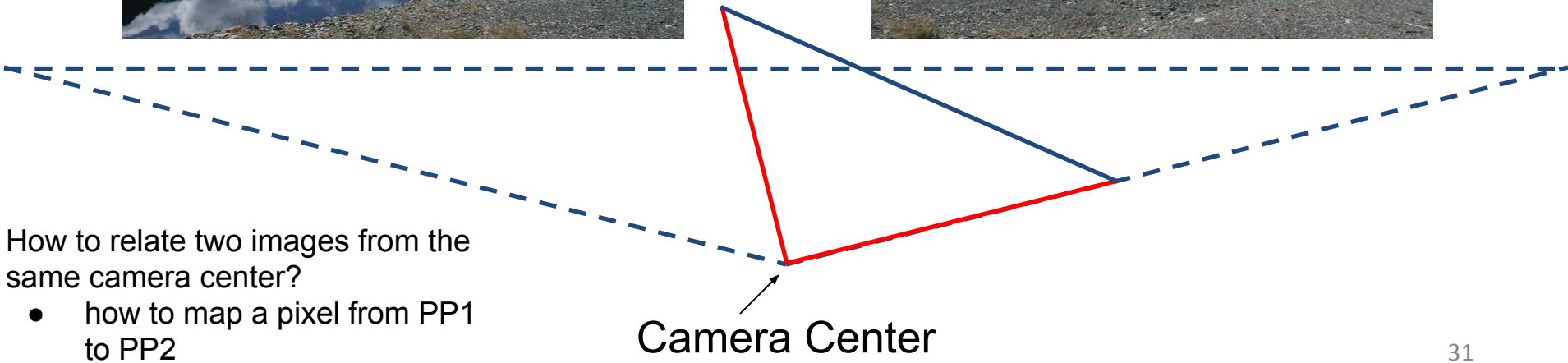
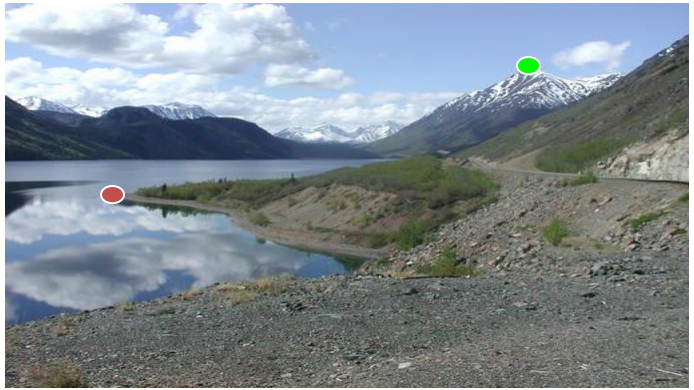


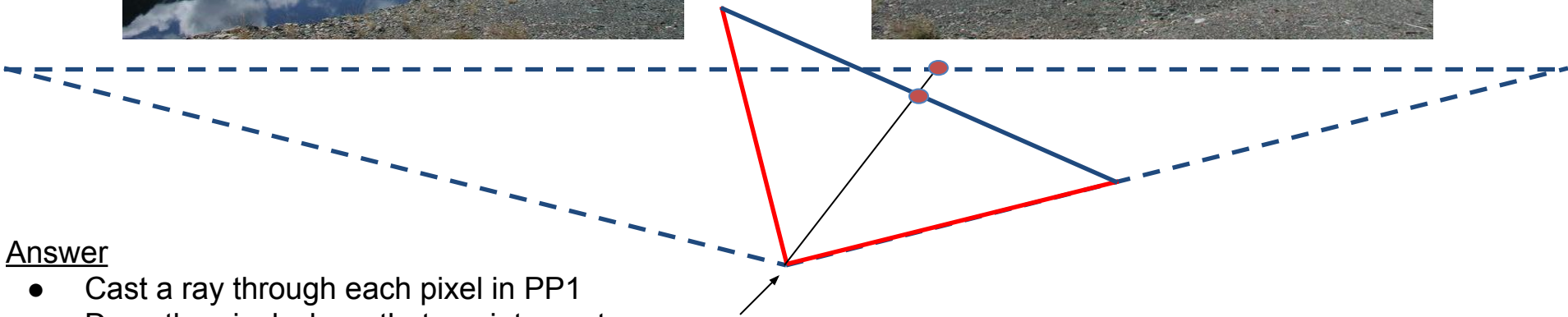


How to relate two images from the same camera center?

- how to map a pixel from PP1 to PP2

Example

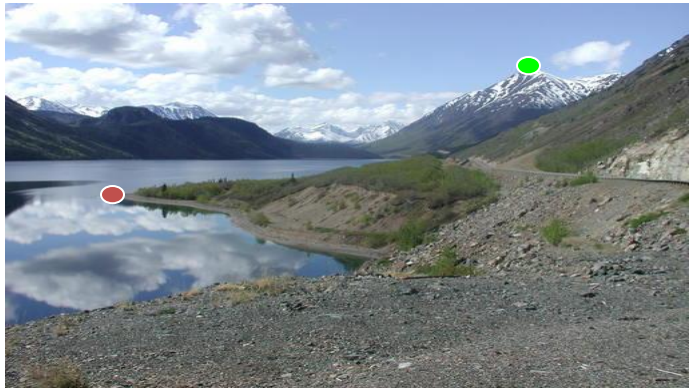


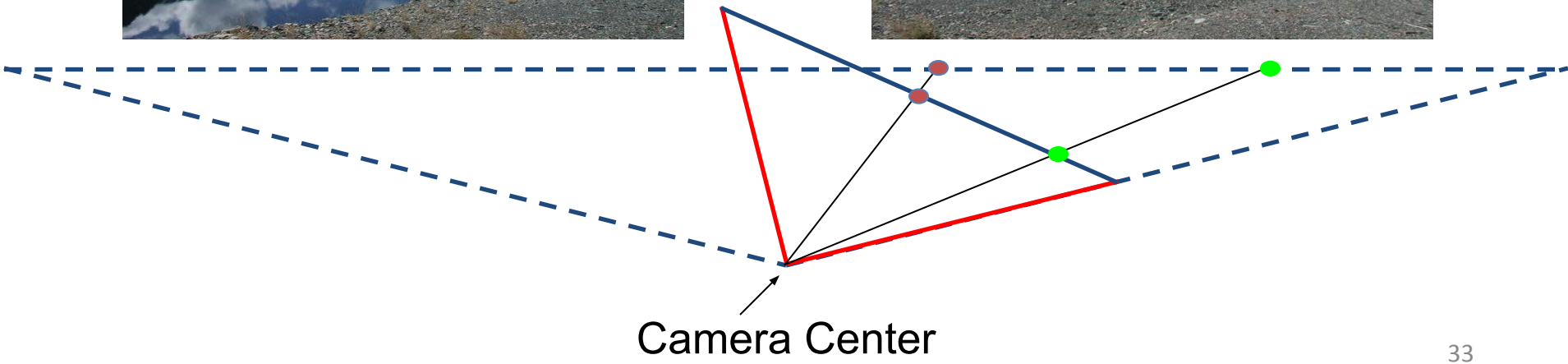


Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2

Example





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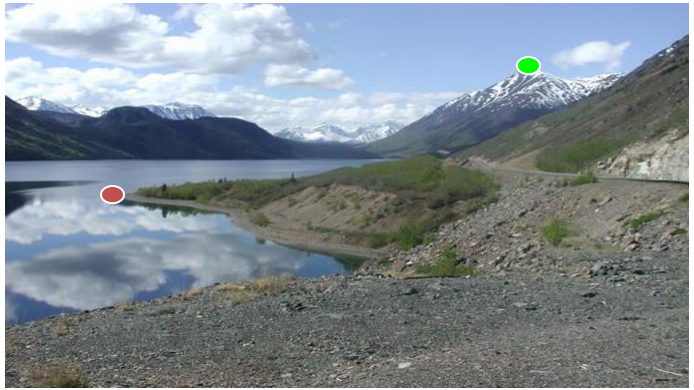


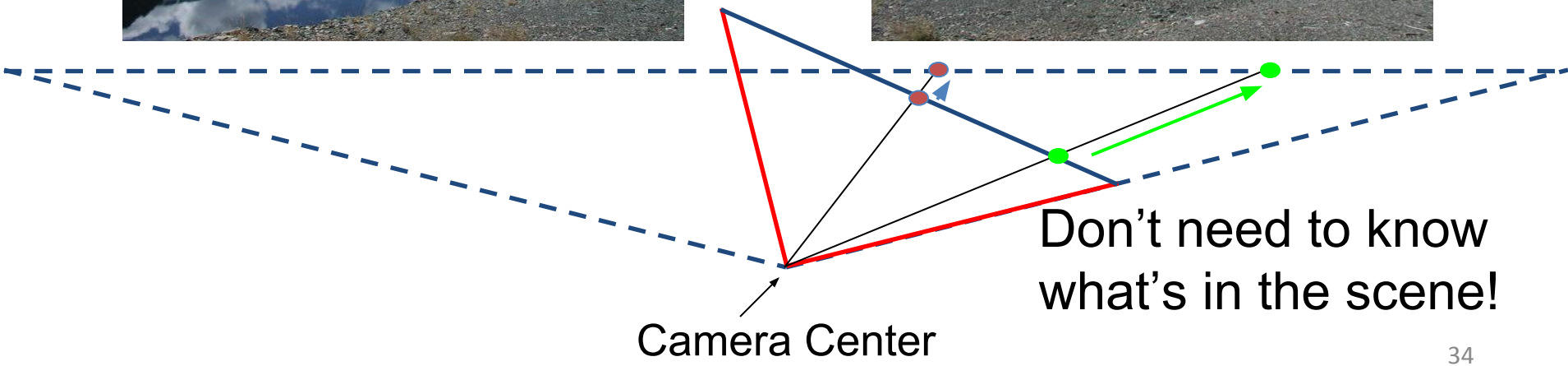
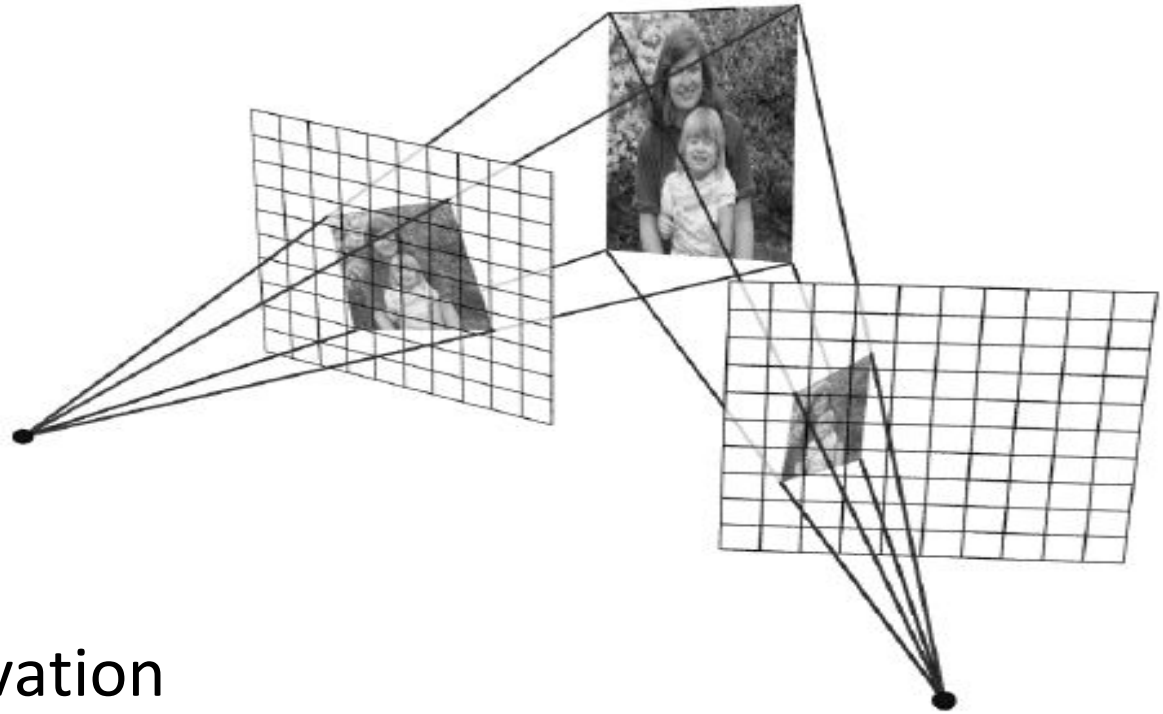


Image reprojection



- Observation
  - Rather than thinking of this as a 3D reprojection, think of it as a 2D image warp from one image to another

Motion models

- What happens when we take two images with a camera and try to align them?

Motion models

- What happens when we take two images with a camera and try to align them?
- translation?

Motion models

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- translation?
- rotation?

Motion models

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- translation?
- rotation?
- scale?

Motion models

- What happens when we take two images with a camera and try to align them?
- translation?
- rotation?
- scale?
- affine?

Motion models

- What happens when we take two images with a camera and try to align them?
- translation?
- rotation?
- scale?
- affine?
- Perspective?

Recall: Projective transformations

- (aka *homographies*)

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad \begin{aligned} x' &= u/w \\ y' &= v/w \end{aligned}$$

Parametric (global) warping

- Examples of parametric warps:

translation

rotation

aspect

affine

perspective

2D coordinate transformations

- translation: $\boldsymbol{x'} = \boldsymbol{x} + \boldsymbol{t}$ $\boldsymbol{x} = (x,y)$

2D coordinate transformations

- translation: $\boldsymbol{x'} = \boldsymbol{x} + \boldsymbol{t}$ $\boldsymbol{x} = (x,y)$
- rotation: $\boldsymbol{x'} = \boldsymbol{R}\,\boldsymbol{x} + \boldsymbol{t}$

2D coordinate transformations

- translation: $\boldsymbol{x'} = \boldsymbol{x} + \boldsymbol{t}$ $\boldsymbol{x} = (x,y)$
- rotation: $\boldsymbol{x'} = \boldsymbol{R}\,\boldsymbol{x} + \boldsymbol{t}$
- similarity: $\boldsymbol{x'} = s\,\boldsymbol{R}\,\boldsymbol{x} + \boldsymbol{t}$

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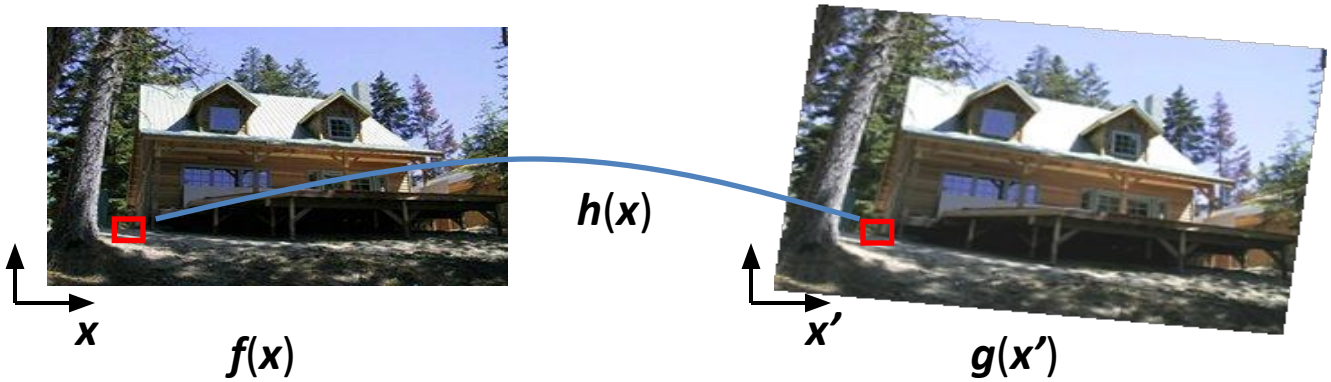
- affine: $\boldsymbol{x'} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{t}$
- perspective: $\underline{\boldsymbol{x}}' \cong \boldsymbol{H}\,\underline{\boldsymbol{x}}$ $\underline{\boldsymbol{x}} = (x,y,1)$
  ($\underline{\boldsymbol{x}}$ is a *homogeneous* coordinate)

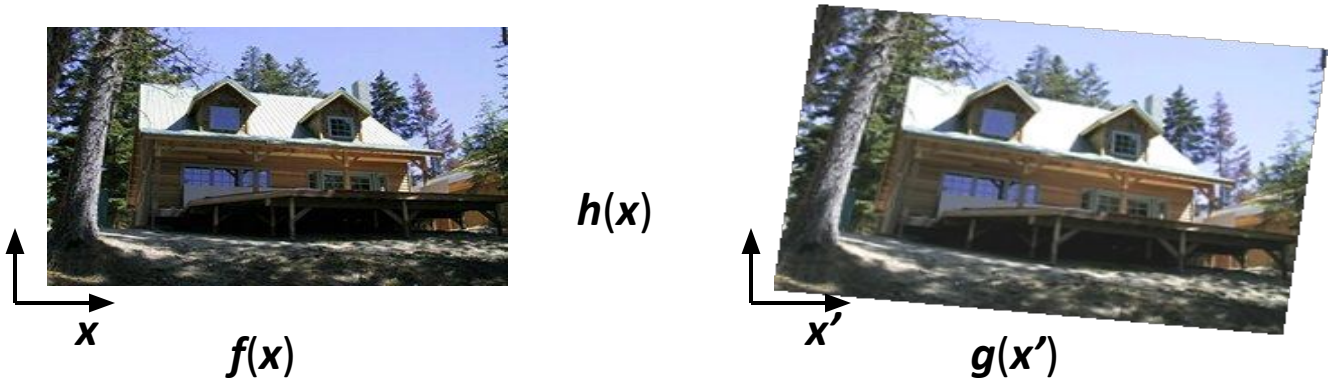
Image Warping

- Given a coordinate transform $\boldsymbol{x'} = \boldsymbol{h}(\boldsymbol{x})$ and a source image $\boldsymbol{f}(\boldsymbol{x})$, how do we compute a transformed image $\boldsymbol{g}(\boldsymbol{x'}) = \boldsymbol{f}(\boldsymbol{h}(\boldsymbol{x}))$?



Forward Warping

- Send each pixel $f(x)$ to its corresponding location $x' = h(x)$ in $g(x')$



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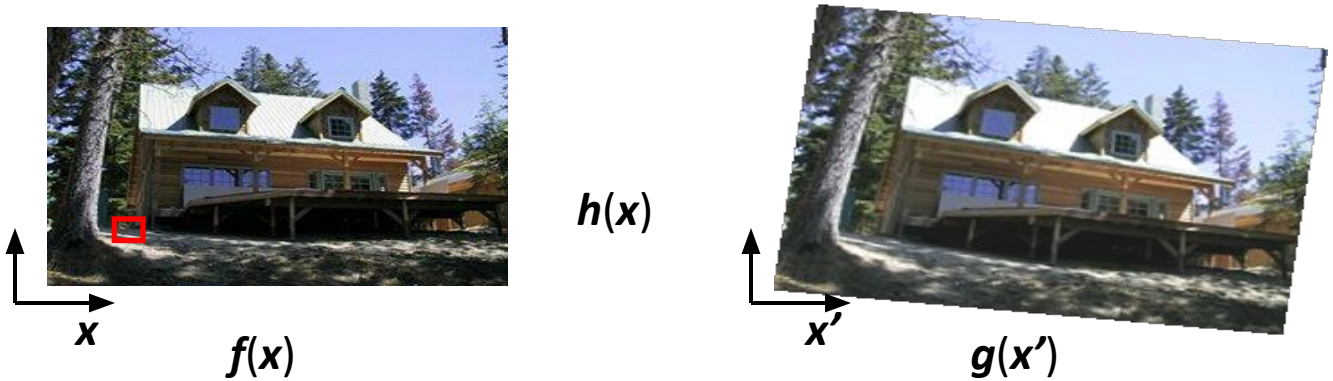

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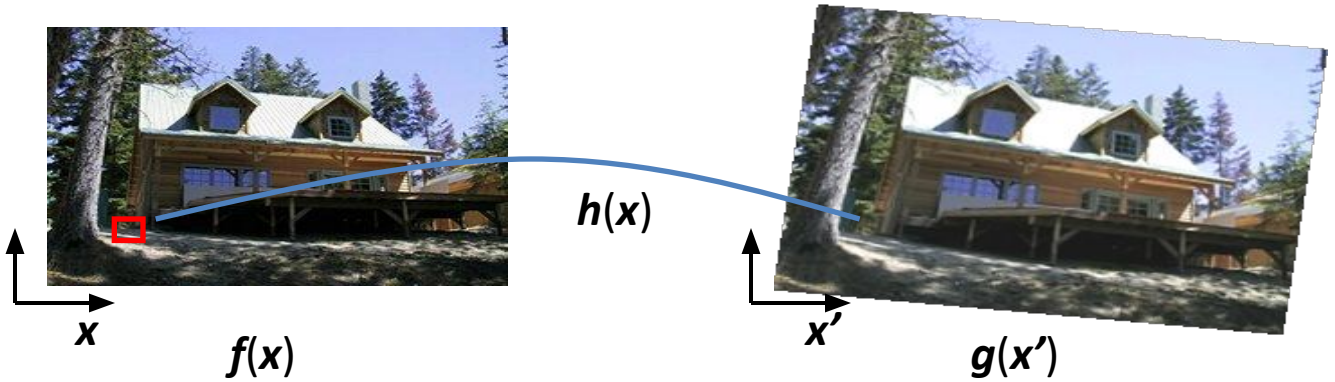

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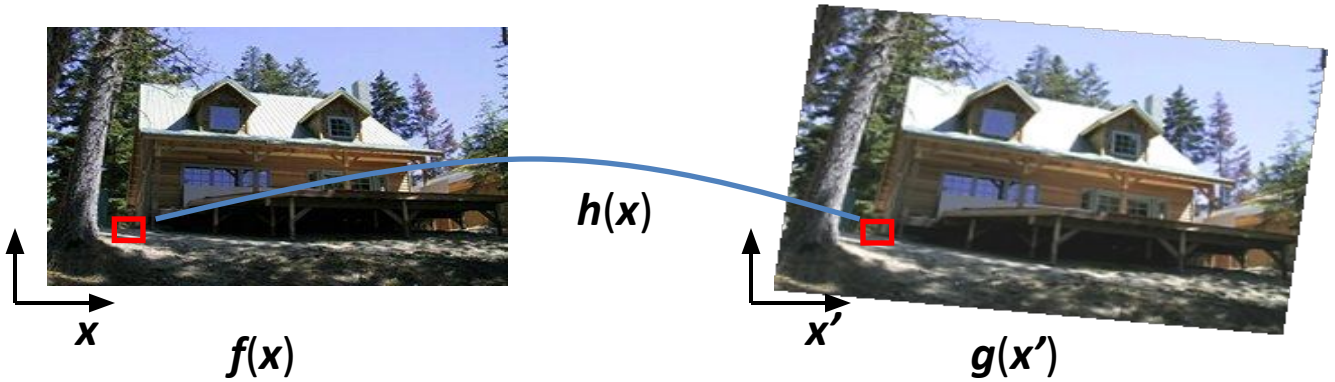
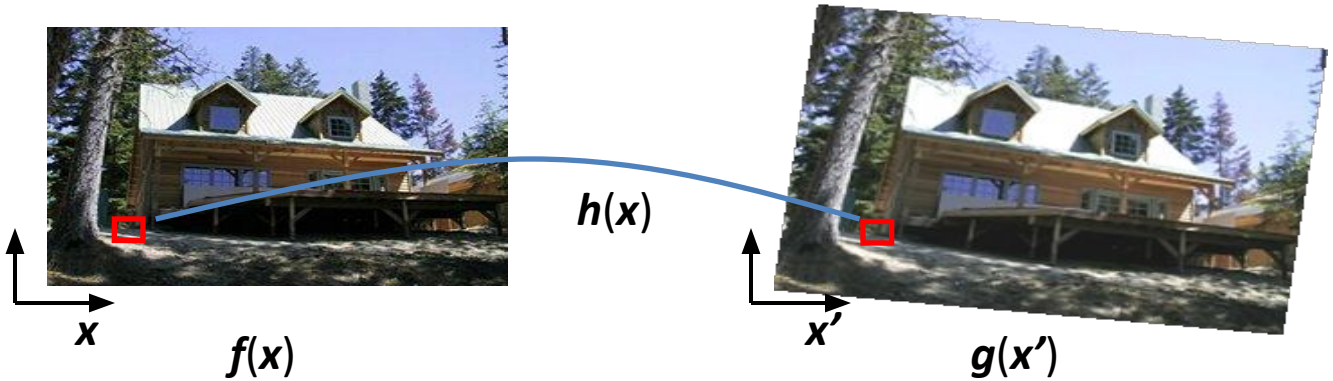


Forward Warping

- Send each pixel $f(x)$ to its corresponding location $x' = h(x)$ in $g(x')$
  - What if pixel lands "between" two pixels?



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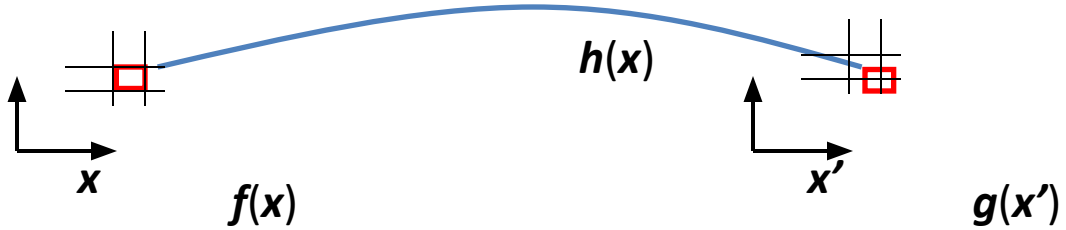
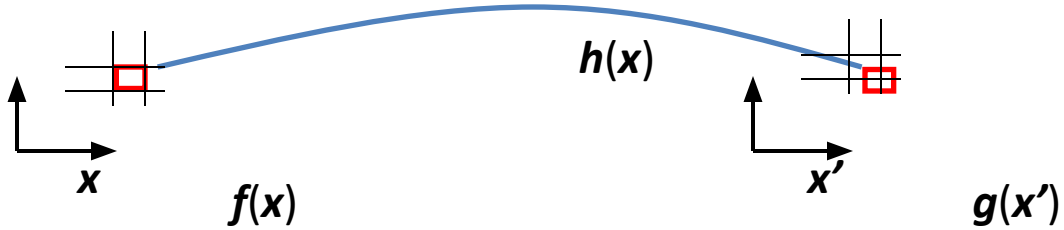


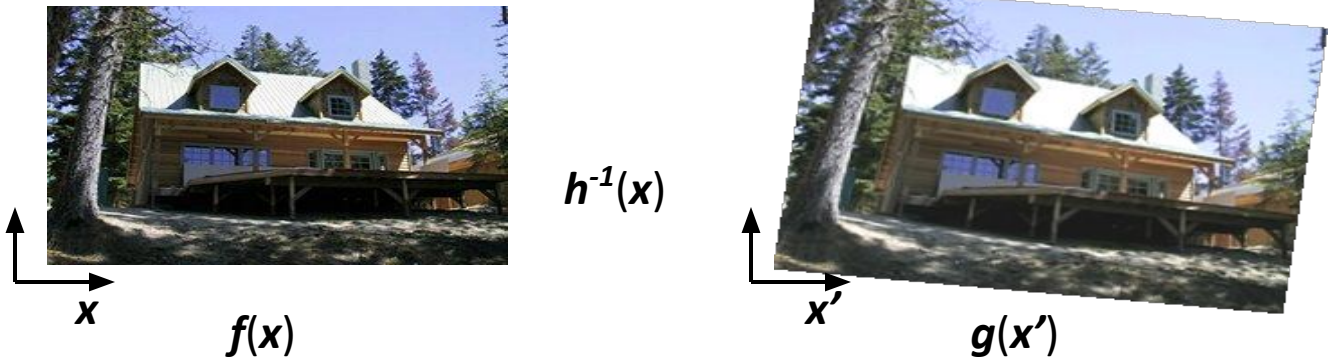
Forward Warping

- Send each pixel $f(x)$ to its corresponding location $x' = h(x)$ in $g(x')$
  - What if pixel lands "between" two pixels?
  - Answer: add "contribution" to several pixels, normalize later (*splatting*)



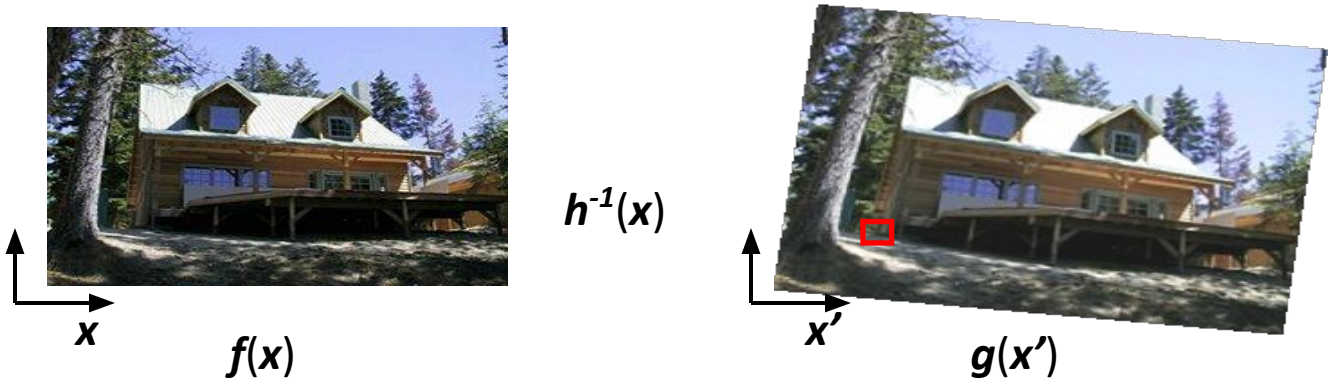
Inverse Warping

- Get each pixel $g(x')$ from its corresponding location $x' = h(x)$ in $f(x)$



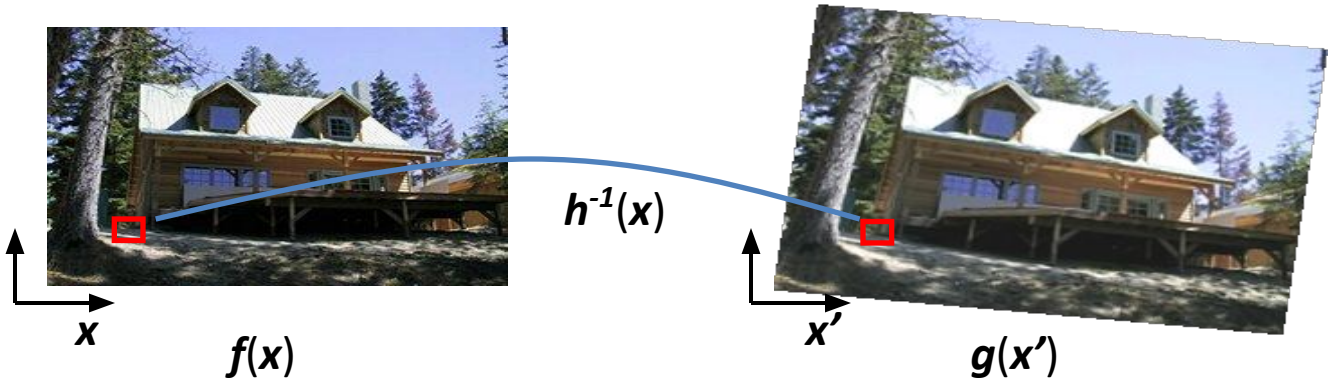
Inverse Warping

- Get each pixel $g(x')$ from its corresponding location $x' = h(x)$ in $f(x)$



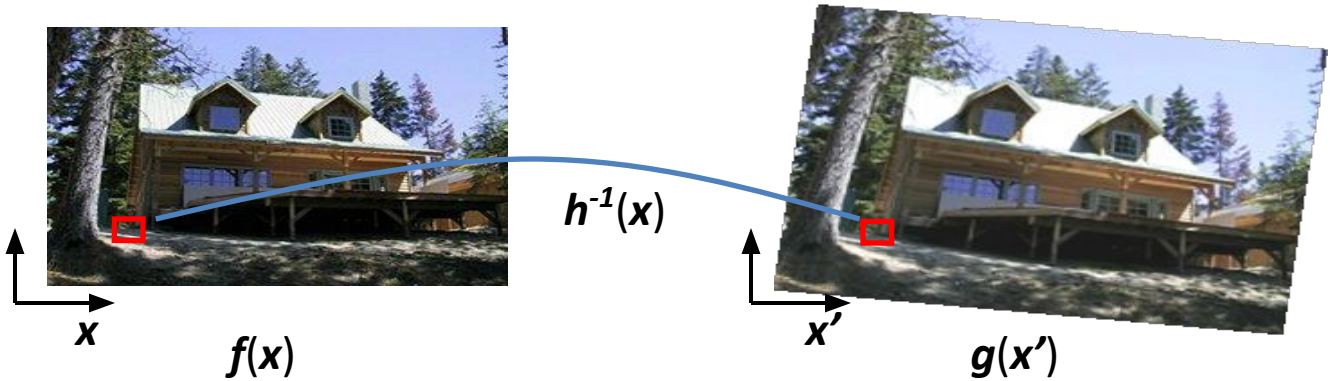
Inverse Warping

- Get each pixel $g(x')$ from its corresponding location $x' = h(x)$ in $f(x)$



Inverse Warping

- Get each pixel $g(x')$ from its corresponding location $x' = h(x)$ in $f(x)$
  - What if pixel comes from "between" two pixels?



Inverse Warping

- Get each pixel $g(x')$ from its corresponding location $x' = h(x)$ in $f(x)$
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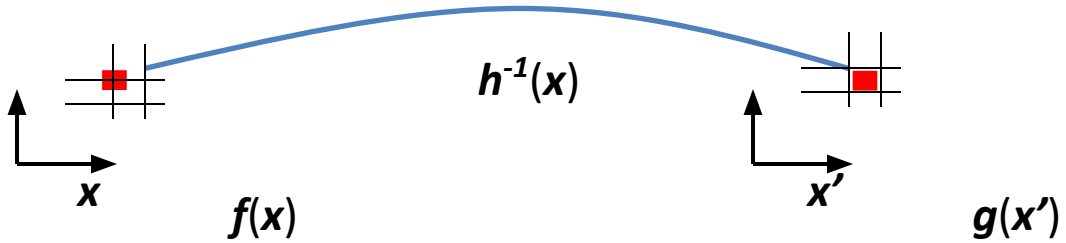
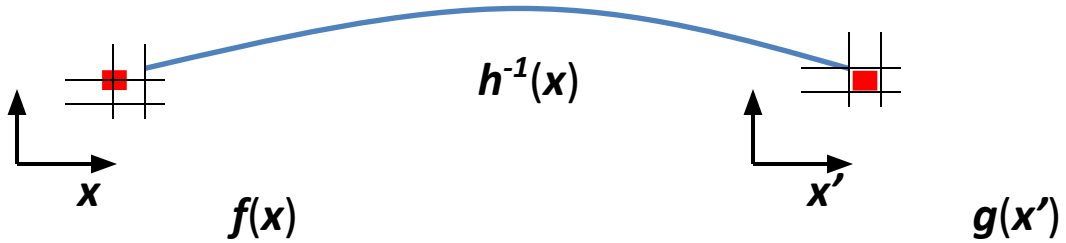


Inverse Warping

- Get each pixel $g(x')$ from its corresponding location $x' = h(x)$ in $f(x)$
  - What if pixel comes from "between" two pixels?
  - Answer: *resample* color value from *interpolated* source image



Interpolation

- Possible interpolation filters:

Interpolation

- Possible interpolation filters:
  - nearest neighbor

Interpolation

- Possible interpolation filters:
  - nearest neighbor
  - bilinear

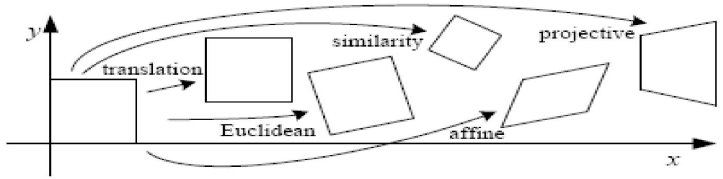
Interpolation

- Possible interpolation filters:
  - nearest neighbor
  - bilinear
  - bicubic (interpolating)

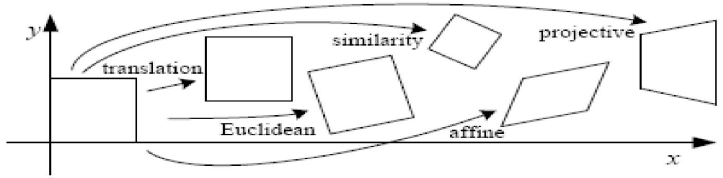
Interpolation

- Possible interpolation filters:
  - nearest neighbor
  - bilinear
  - bicubic (interpolating)

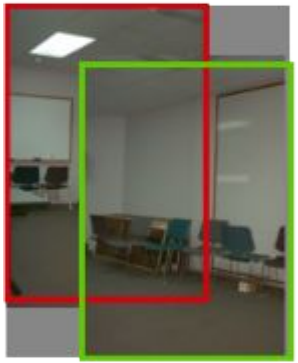
Motion models



Motion models

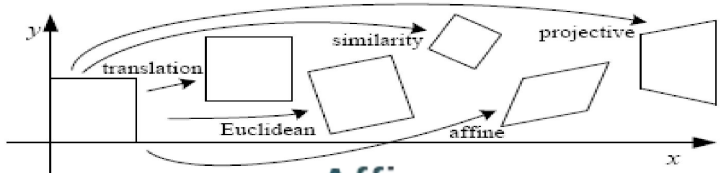


Translation



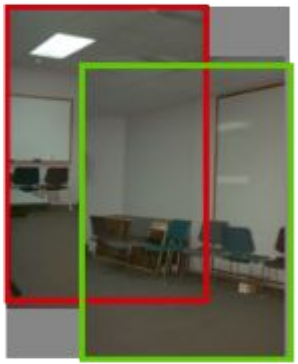
2 unknowns

Motion models



Translation

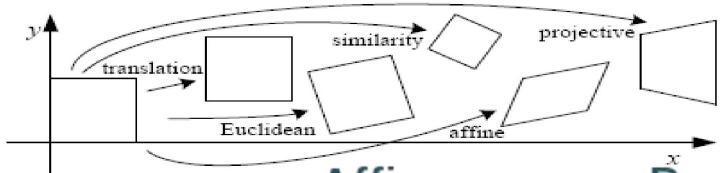
Affine



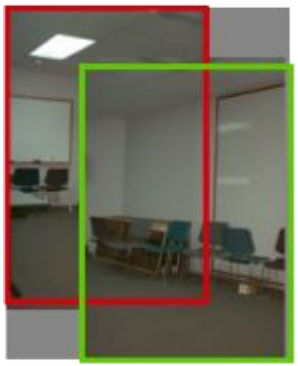
2 unknowns

6 unknowns

Motion models



Translation | Affine | Perspective



2 unknowns

6 unknowns

8 unknowns

Finding the transformation

- Translation = 2 degrees of freedom

Finding the transformation

- Translation = 2 degrees of freedom
- Similarity = 4 degrees of freedom

Finding the transformation

- Translation = 2 degrees of freedom
- Similarity = 4 degrees of freedom
- Affine = 6 degrees of freedom

Finding the transformation

- Translation = 2 degrees of freedom
- Similarity = 4 degrees of freedom
- Affine = 6 degrees of freedom
- Homography = 8 degrees of freedom

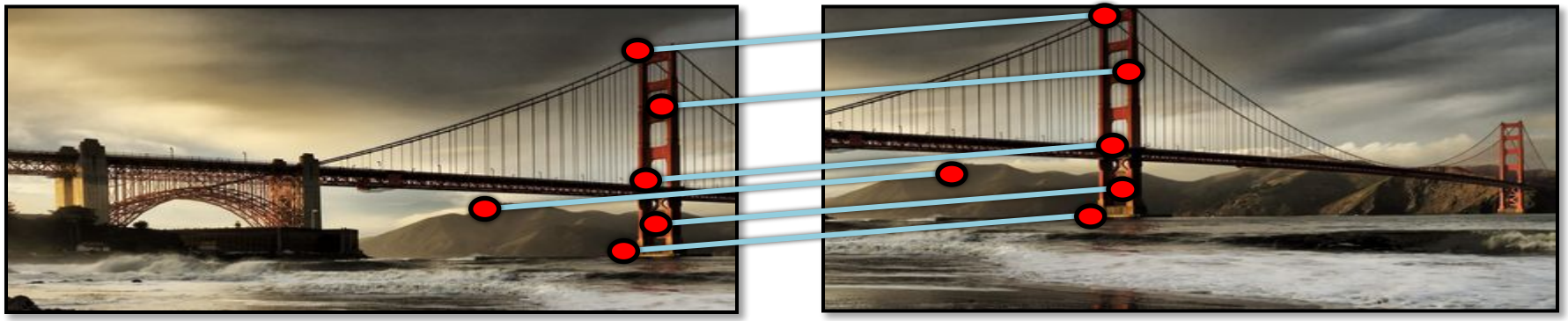
Finding the transformation

- Translation = 2 degrees of freedom
- Similarity = 4 degrees of freedom
- Affine = 6 degrees of freedom
- Homography = 8 degrees of freedom

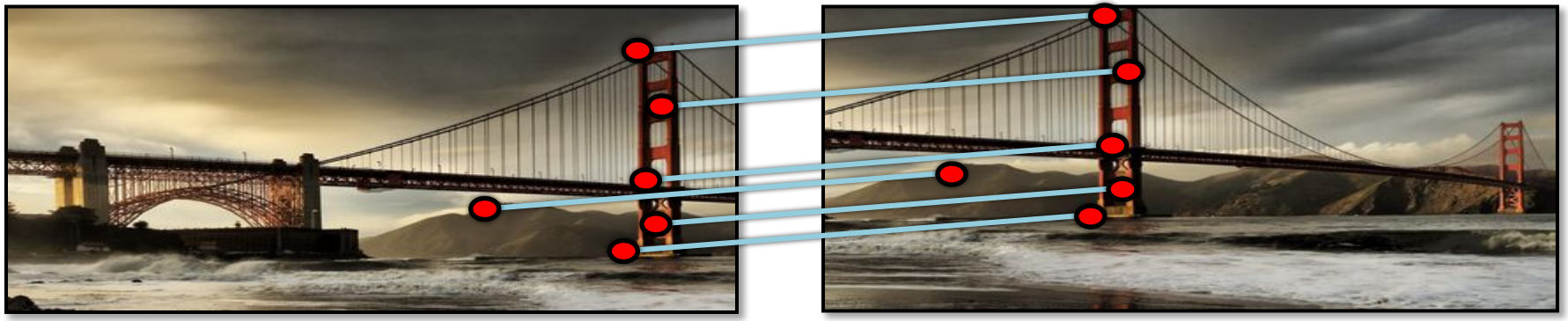
- How many corresponding points do we need to solve?

Simple case: translations

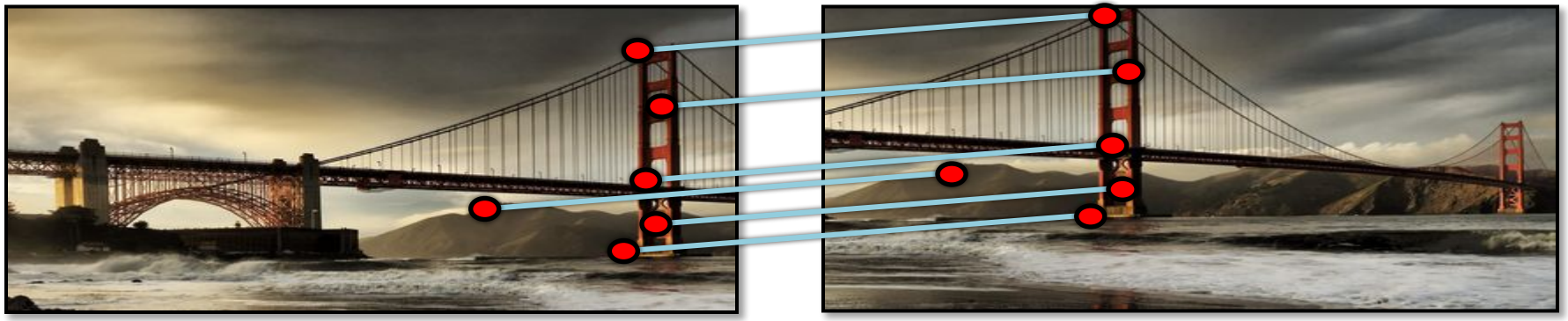
Simple case: translations



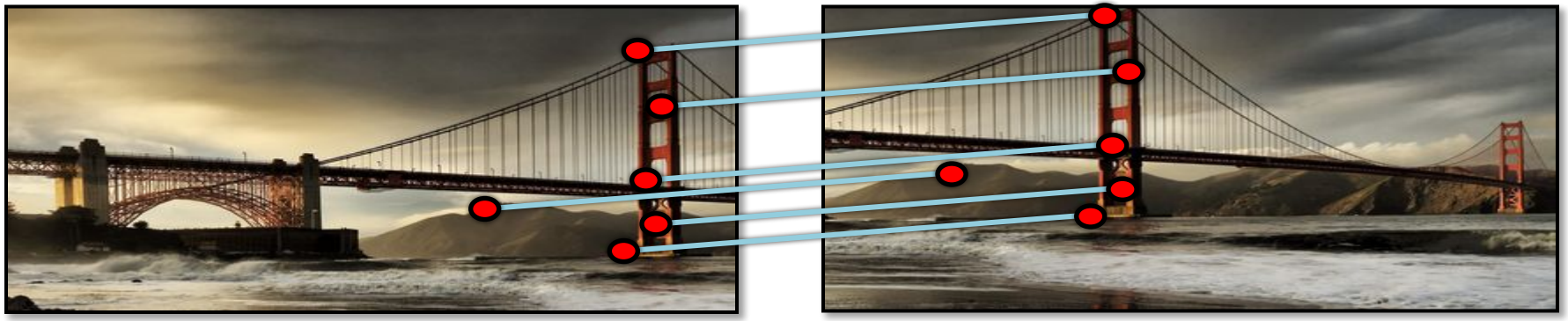
Simple case: translations



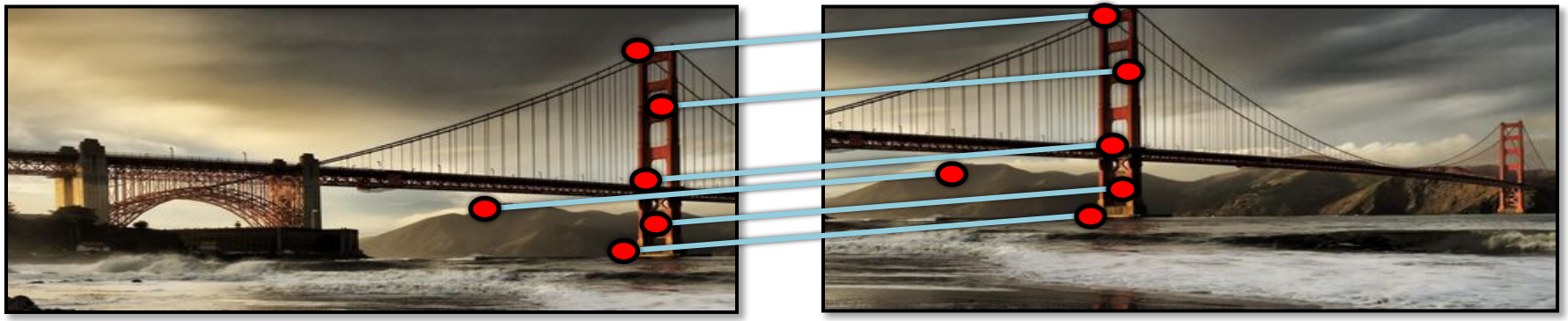
Simple case: translations



Simple case: translations

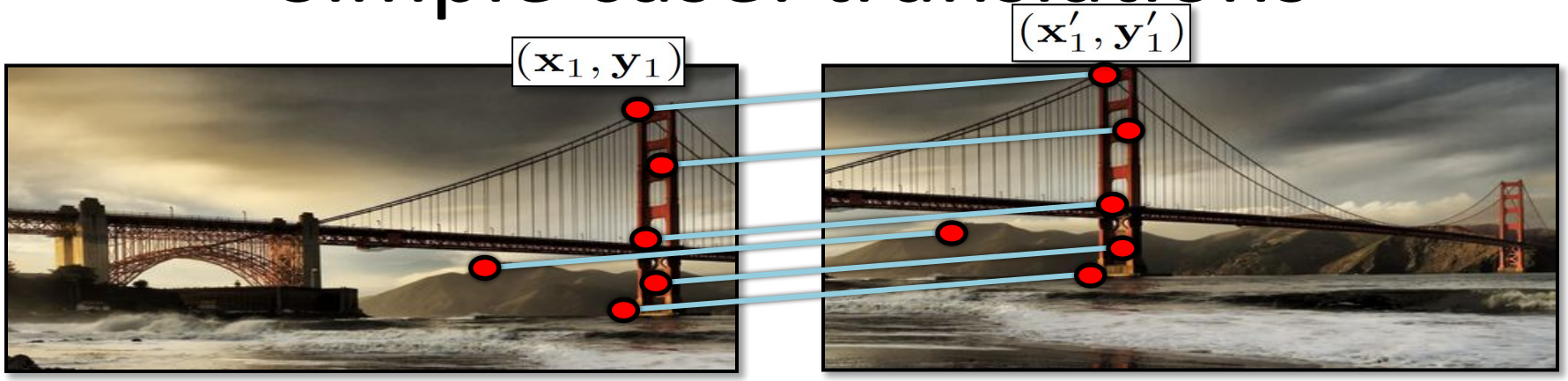


Simple case: translations

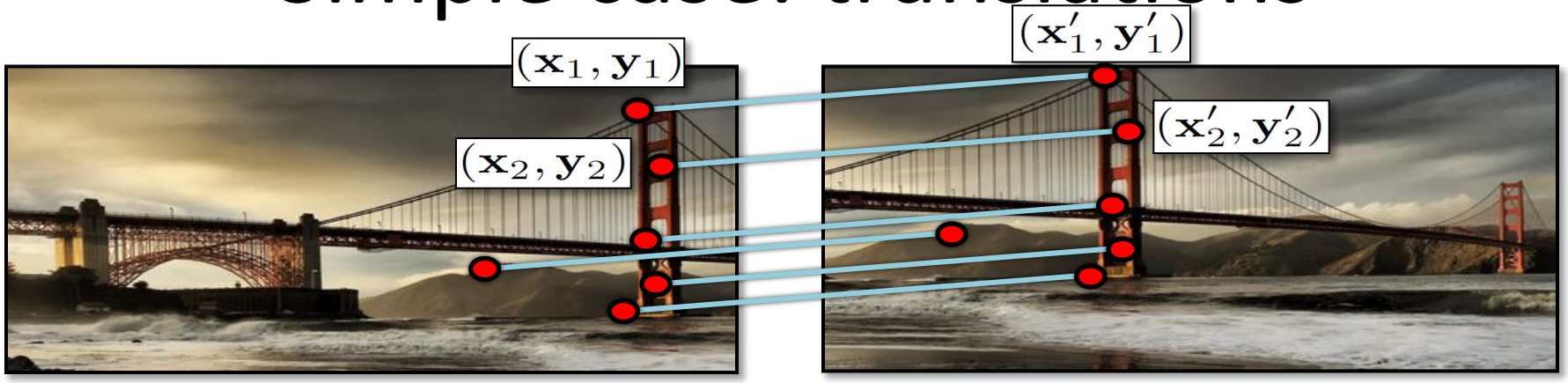


How do we solve for $(\mathbf{x}_t, \mathbf{y}_t)$?

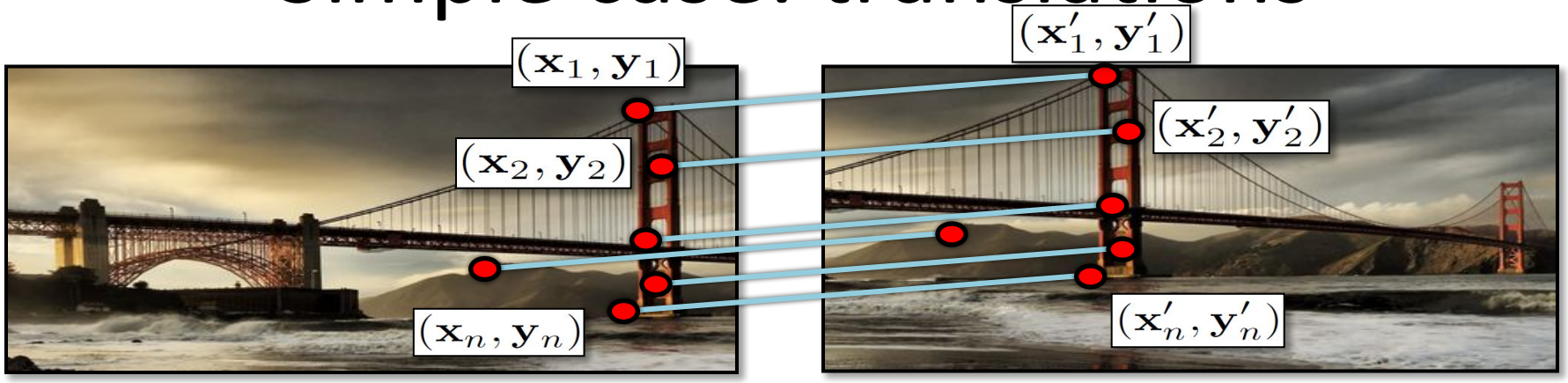
Simple case: translations



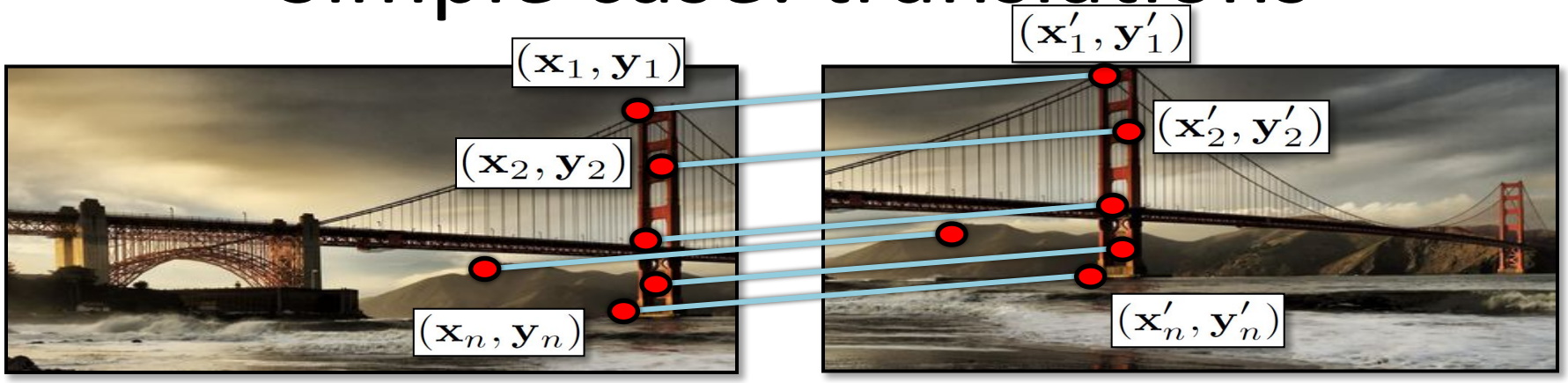
Simple case: translations



Simple case: translations

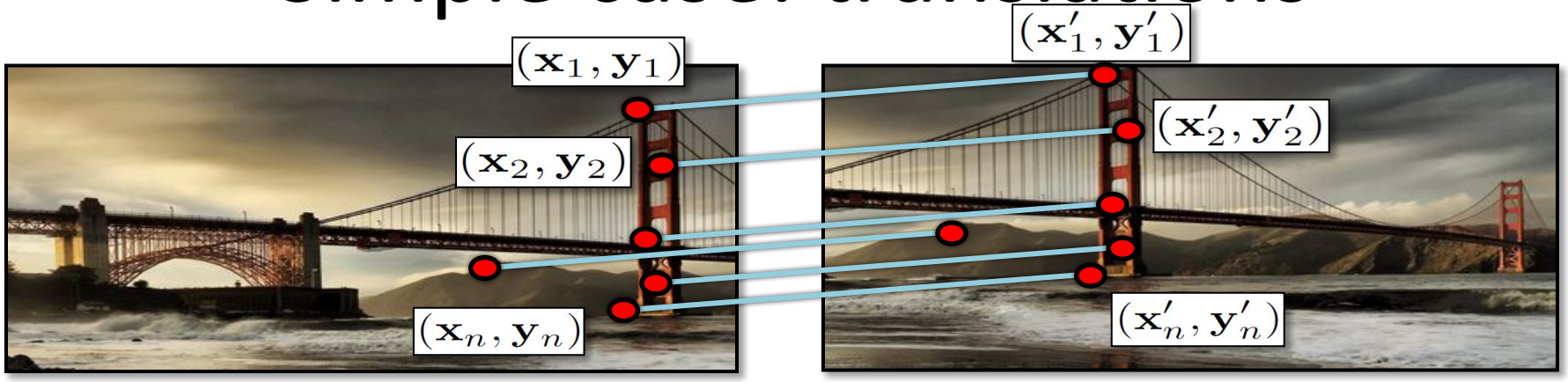


Simple case: translations



Displacement of match *i* = $(\mathbf{x}'_i - \mathbf{x}_i, \mathbf{y}'_i - \mathbf{y}_i)$

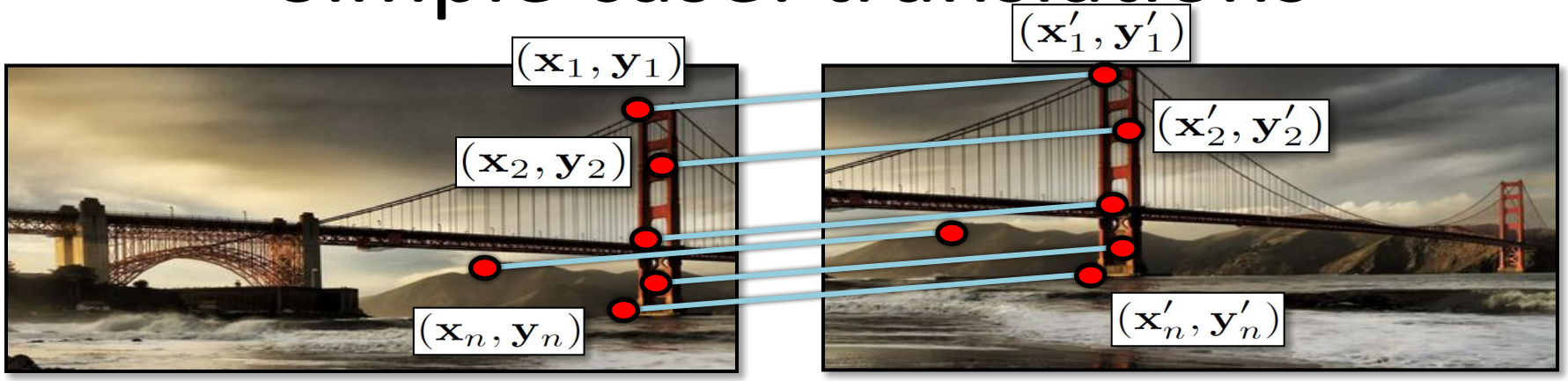
Simple case: translations



Displacement of match *i* = $(\mathbf{x}'_i - \mathbf{x}_i, \mathbf{y}'_i - \mathbf{y}_i)$

Mean displacement = $\left(\frac{1}{n} \sum_{i=1}^{n} \mathbf{x}'_i - \mathbf{x}_i, \frac{1}{n} \sum_{i=1}^{n} \mathbf{y}'_i - \mathbf{y}_i \right)$

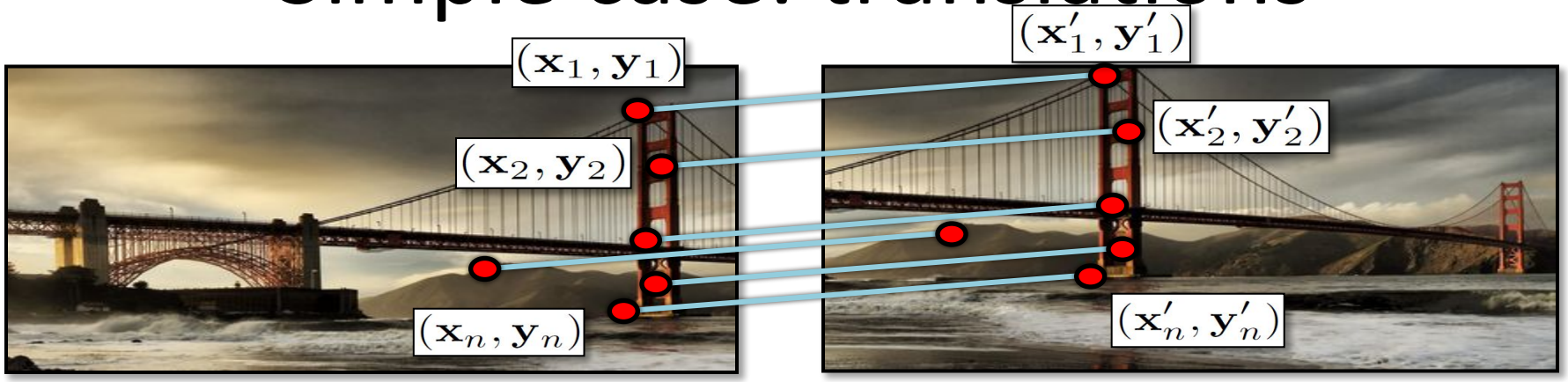
Simple case: translations



Displacement of match *i* = $(\mathbf{x}'_i - \mathbf{x}_i, \mathbf{y}'_i - \mathbf{y}_i)$

$$(\mathbf{x}_t, \mathbf{y}_t) = \left(\frac{1}{n} \sum_{i=1}^{n} \mathbf{x}'_i - \mathbf{x}_i, \frac{1}{n} \sum_{i=1}^{n} \mathbf{y}'_i - \mathbf{y}_i \right)$$

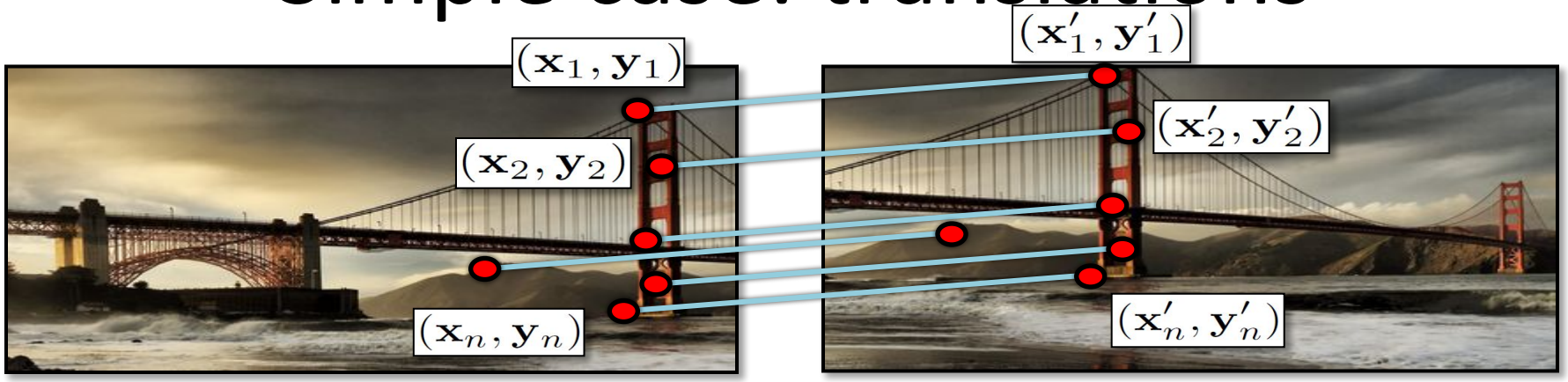
Simple case: translations



$$\mathbf{x}_i + \mathbf{x_t} = \mathbf{x}'_i$$

$$\mathbf{y}_i + \mathbf{y_t} = \mathbf{y}'_i$$

Simple case: translations

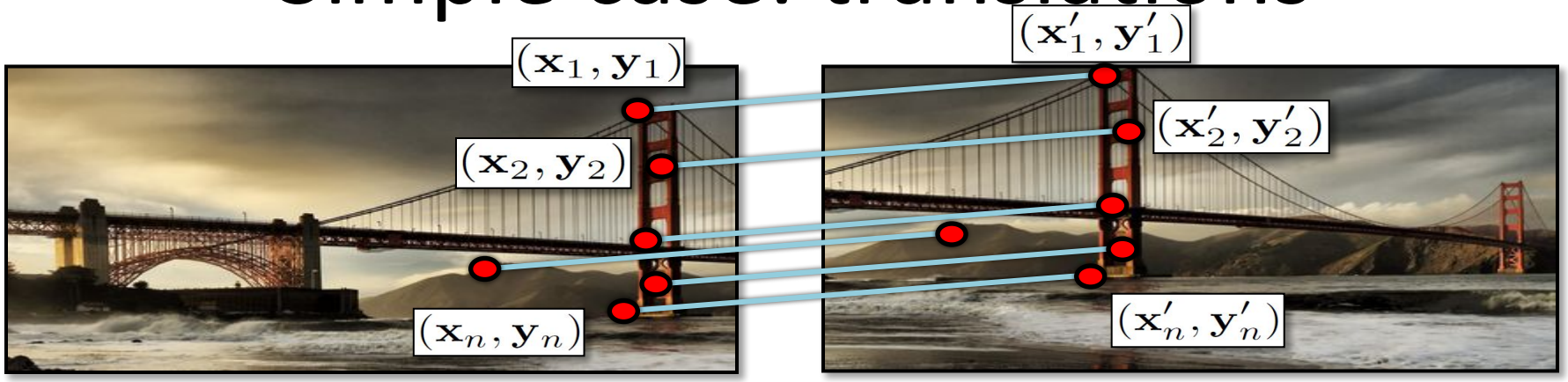


$$\mathbf{x}_i + \mathbf{x_t} = \mathbf{x}'_i$$

$$\mathbf{y}_i + \mathbf{y_t} = \mathbf{y}'_i$$

- System of linear equations
  - What are the knowns? Unknowns?

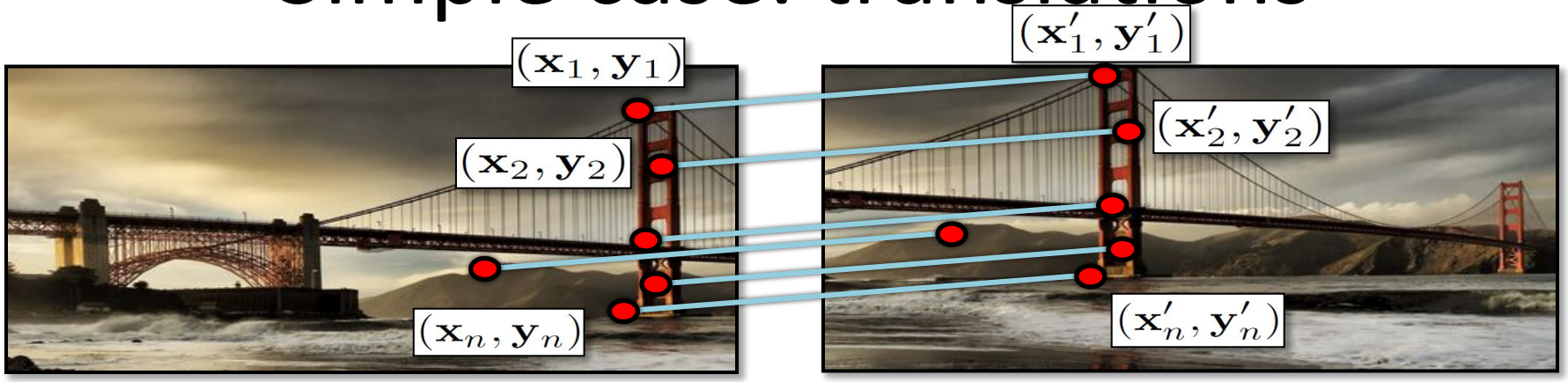
Simple case: translations



$$\mathbf{x}_i + \mathbf{x_t} = \mathbf{x}'_i$$
$$\mathbf{y}_i + \mathbf{y_t} = \mathbf{y}'_i$$

- System of linear equations
  - What are the knowns? Unknowns?
  - How many unknowns? How many equations (per match)?

Simple case: translations



$$\mathbf{x}_i + \mathbf{x_t} = \mathbf{x}'_i$$

$$\mathbf{y}_i + \mathbf{y_t} = \mathbf{y}'_i$$

- Problem: more equations than unknowns
  - "Overdetermined" system of equations
  - We will find the *least squares* solution

Least squares formulation

- For each point $(\mathbf{x}_i, \mathbf{y}_i)$

$$\mathbf{x}_i + \mathbf{x_t} = \mathbf{x}'_i$$
$$\mathbf{y}_i + \mathbf{y_t} = \mathbf{y}'_i$$

Least squares formulation

- For each point $(\mathbf{x}_i, \mathbf{y}_i)$

$$\begin{aligned} \mathbf{x}_i + \mathbf{x_t} &= \mathbf{x}'_i \\ \mathbf{y}_i + \mathbf{y_t} &= \mathbf{y}'_i \end{aligned}$$

- we define the *residuals* as

$$\begin{aligned} r_{\mathbf{x}_i}(\mathbf{x}_t) &= (\mathbf{x}_i + \mathbf{x}_t) - \mathbf{x}'_i \\ r_{\mathbf{y}_i}(\mathbf{y}_t) &= (\mathbf{y}_i + \mathbf{y}_t) - \mathbf{y}'_i \end{aligned}$$

Least squares formulation

- Goal: minimize sum of squared residuals

Least squares formulation

- Goal: minimize sum of squared residuals

$$C(\mathbf{x}_t, \mathbf{y}_t) = \sum_{i=1}^{n} \left(r_{\mathbf{x}_i}(\mathbf{x}_t)^2 + r_{\mathbf{y}_i}(\mathbf{y}_t)^2 \right)$$

- “Least squares” solution

Least squares formulation

- Goal: minimize sum of squared residuals

$$C(\mathbf{x}_t, \mathbf{y}_t) = \sum_{i=1}^{n} \left(r_{\mathbf{x}_i}(\mathbf{x}_t)^2 + r_{\mathbf{y}_i}(\mathbf{y}_t)^2 \right)$$

- "Least squares" solution

- For translations, is equal to mean displacement

Least squares

$$\mathbf{At} = \mathbf{b}$$

Least squares

$$\mathbf{At} = \mathbf{b}$$

- Find **t** that minimizes

$$||\mathbf{At} - \mathbf{b}||^2$$

Least squares

$$\mathbf{At} = \mathbf{b}$$

- Find **t** that minimizes

$$||\mathbf{At} - \mathbf{b}||^2$$

- To solve, form the *normal equations*

$$\mathbf{A}^{\mathrm{T}}\mathbf{At} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Least squares

$$\mathbf{At} = \mathbf{b}$$

- Find **t** that minimizes

$$||\mathbf{At} - \mathbf{b}||^2$$

- To solve, form the *normal equations*

$$\mathbf{A}^{\mathrm{T}}\mathbf{At} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

$$\mathbf{t} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Solving for translations

- Using least squares

Solving for translations

- Using least squares

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ \vdots & \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} x'_1 - x_1 \\ y'_1 - y_1 \\ x'_2 - x_2 \\ y'_2 - y_2 \\ \vdots \\ x'_n - x_n \\ y'_n - y_n \end{bmatrix}$$

$$\underset{2n \text{ x } 2}{\mathbf{A}} \quad \underset{2 \text{ x } 1}{\mathbf{t}} = \underset{2n \text{ x } 1}{\mathbf{b}}$$

Affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- How many unknowns?

Affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- How many unknowns?
- How many equations per match?

Affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- How many unknowns?
- How many equations per match?
- x´ = ax + by + c; y´ = dx + ey +f

Affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- How many unknowns?
- How many equations per match?
- x´ = ax + by + c; y´ = dx + ey +f
- How many matches do we need?

Affine transformations

- Residuals:

$$r_{x_i}(a,b,c,d,e,f) = (ax_i + by_i + c) - x'_i$$
$$r_{y_i}(a,b,c,d,e,f) = (dx_i + ey_i + f) - y'_i$$

Affine transformations

- Residuals:

$$r_{x_i}(a,b,c,d,e,f) = (ax_i + by_i + c) - x'_i$$
$$r_{y_i}(a,b,c,d,e,f) = (dx_i + ey_i + f) - y'_i$$

- Cost function:

$$C(a,b,c,d,e,f) = \sum_{i=1}^{n} \left(r_{x_i}(a,b,c,d,e,f)^2 + r_{y_i}(a,b,c,d,e,f)^2\right)$$

Affine transformations

- Matrix form

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ & & \vdots & & & \\ x_n & y_n & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_n \\ y'_n \end{bmatrix}$$

Affine transformations

- Matrix form

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ & & \vdots & & & \\ x_n & y_n & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_n \\ y'_n \end{bmatrix}$$

$$\underset{2n \text{ x } 6}{\mathbf{A}} \qquad \underset{6 \text{ x } 1}{\mathbf{t}} = \underset{2n \text{ x } 1}{\mathbf{b}}$$

Solving for homographies

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Solving for homographies

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Why is this now a variable and not just 1?

Solving for homographies

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Why is this now a variable and not just 1?

- A homography is a projective object, in that it has no scale. It is represented by the above matrix, up to scale.

Solving for homographies

$$\begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Why is this now a variable and not just 1?

- A homography is a projective object, in that it has no scale. It is represented by the above matrix, up to scale.
- One way of fixing the scale is to set one of the coordinates to 1, though that choice is arbitrary.

Solving for homographies

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Why is this now a variable and not just 1?

- A homography is a projective object, in that it has no scale. It is represented by the above matrix, up to scale.
- One way of fixing the scale is to set one of the coordinates to 1, though that choice is arbitrary.
- But that's what most people do and your assignment code does.

Solving for homographies

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

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Why the division?

Solving for homographies

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$$y_i'(h_{20}x_i + h_{21}y_i + h_{22}) = h_{10}x_i + h_{11}y_i + h_{12}$$

Solving for homographies

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\begin{aligned}
x_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{00}x_i + h_{01}y_i + h_{02} \\
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\end{aligned}
$$

$$
\begin{bmatrix}
x_i & y_i & 1 & 0 & 0 & 0 & -x_i'x_i & -x_i'y_i & -x_i' \\
0 & 0 & 0 & x_i & y_i & 1 & -y_i'x_i & -y_i'y_i & -y_i'
\end{bmatrix}
\begin{bmatrix}
h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0
\end{bmatrix}
$$

Direct Linear Transforms

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ & & & & & \vdots & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

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$$\underset{2n \times 9}{\mathbf{A}} \qquad \underset{9}{\mathbf{h}} \qquad \underset{2n}{\mathbf{0}}$$

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Defines a least squares problem: $\text{minimize } ||\mathbf{Ah} - \mathbf{0}||^2$

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- Solution: $\hat{\mathbf{h}}$ = eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue

Direct Linear Transforms

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Defines a least squares problem: $\text{minimize } ||\mathbf{Ah} - \mathbf{0}||^2$

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- Solution: $\hat{\mathbf{h}}$ = eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points

Matching features

Matching features

Matching features

Matching features

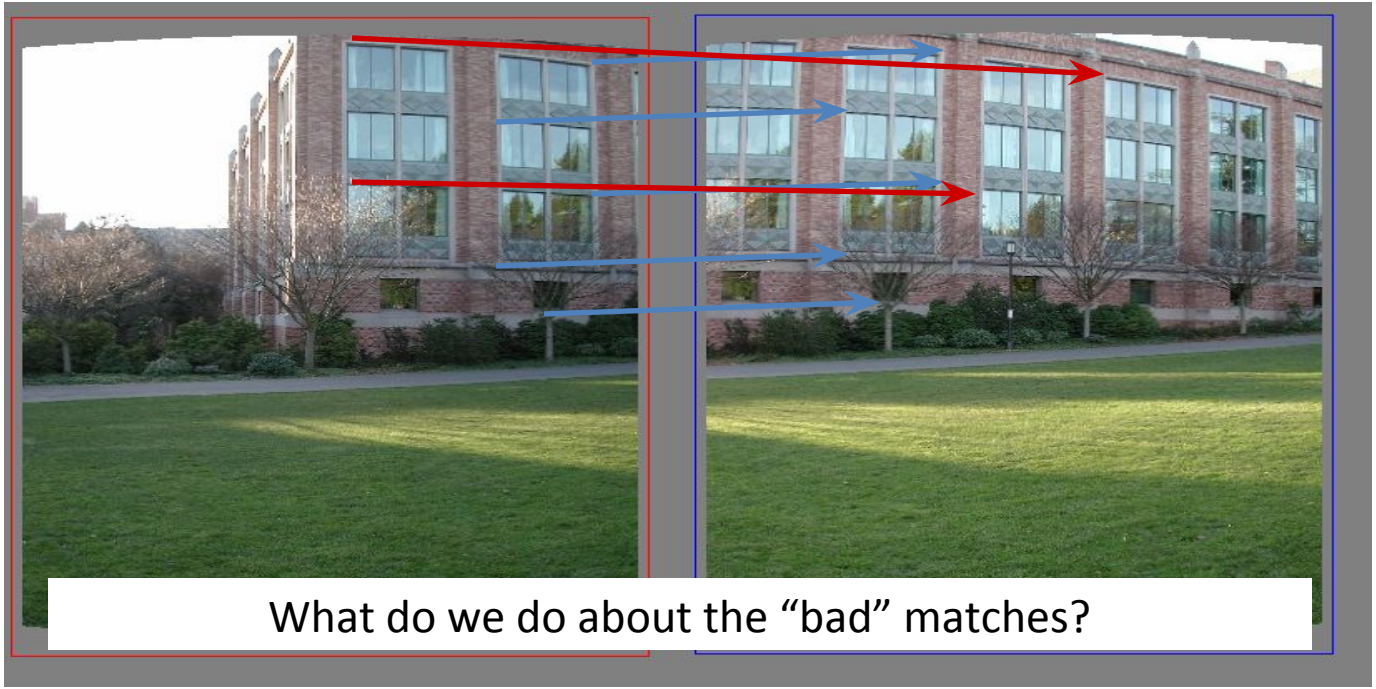
Matching features

Matching features

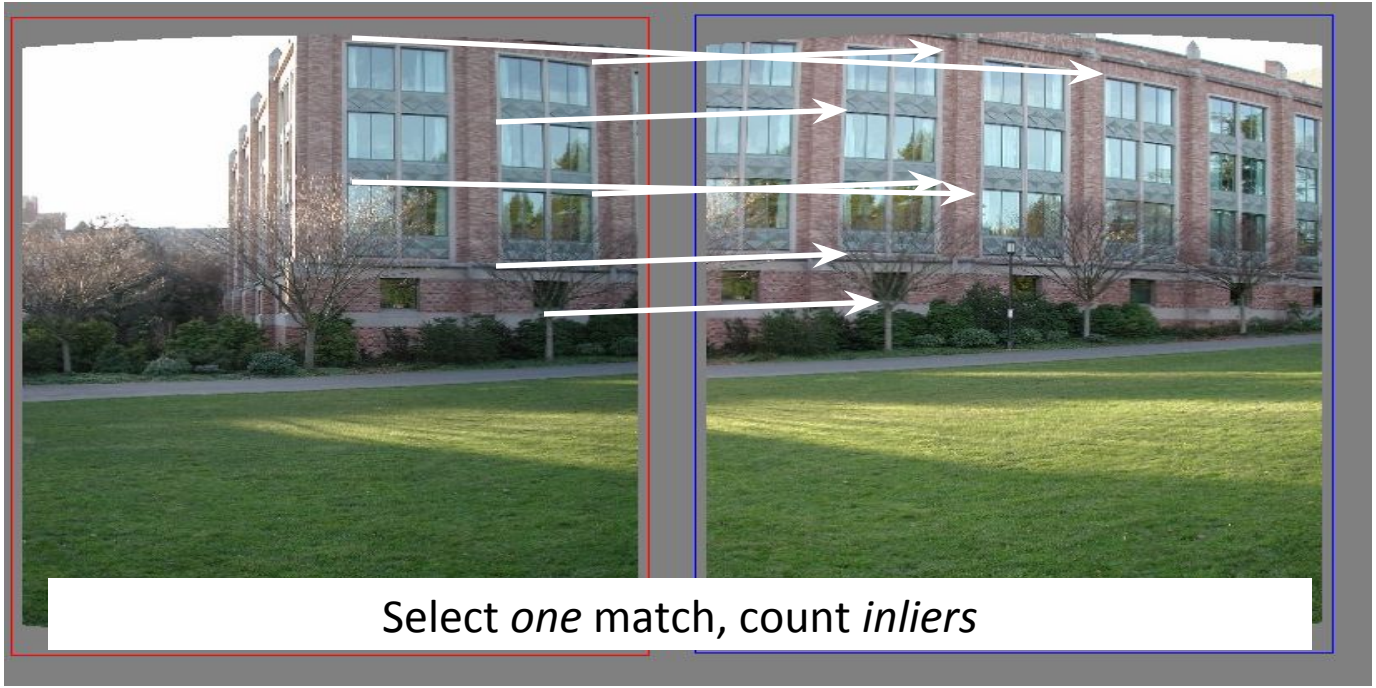
Matching features

Matching features

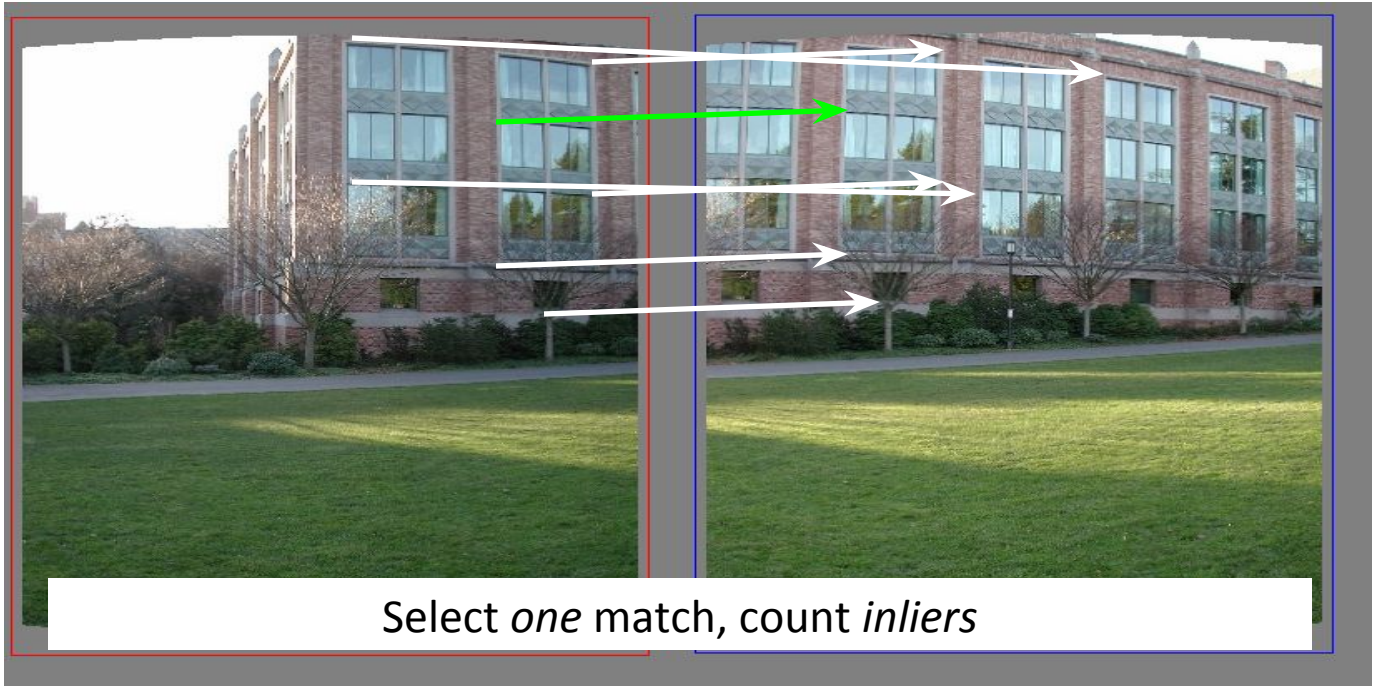
Matching features



RANdom SAmple Consensus



RANdom SAmple Consensus



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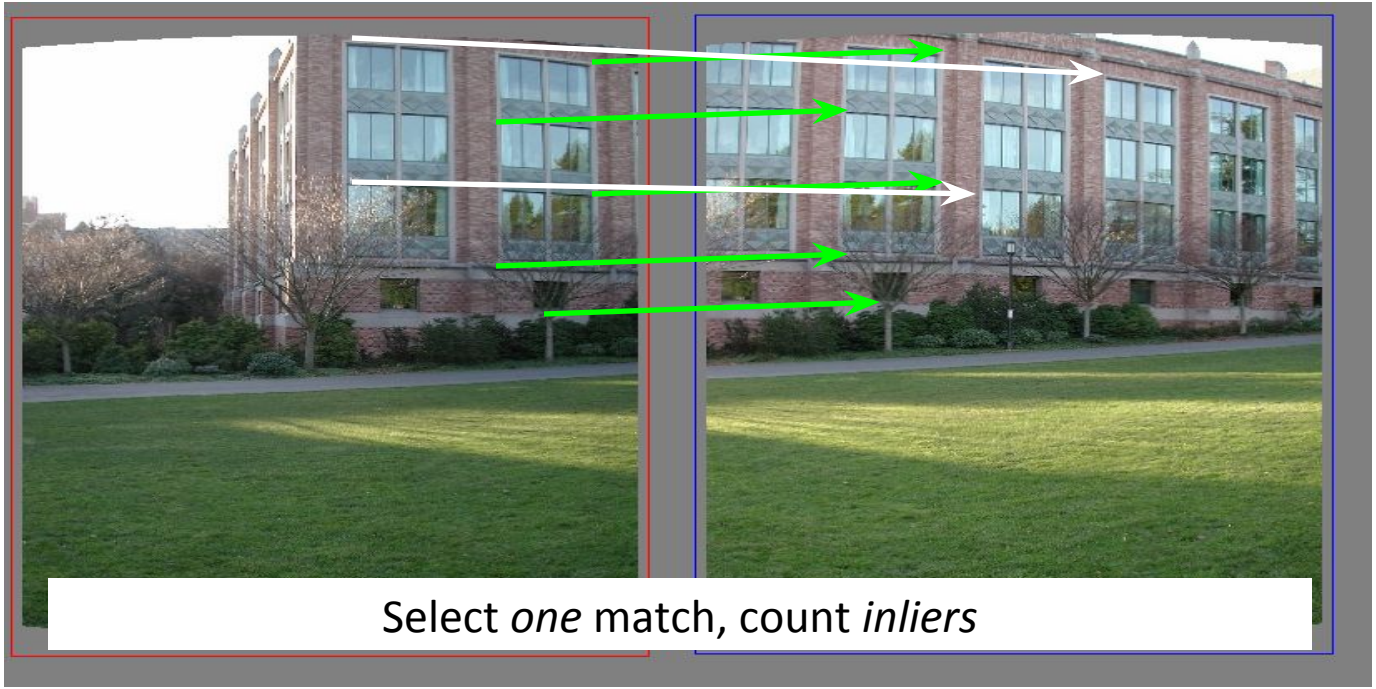

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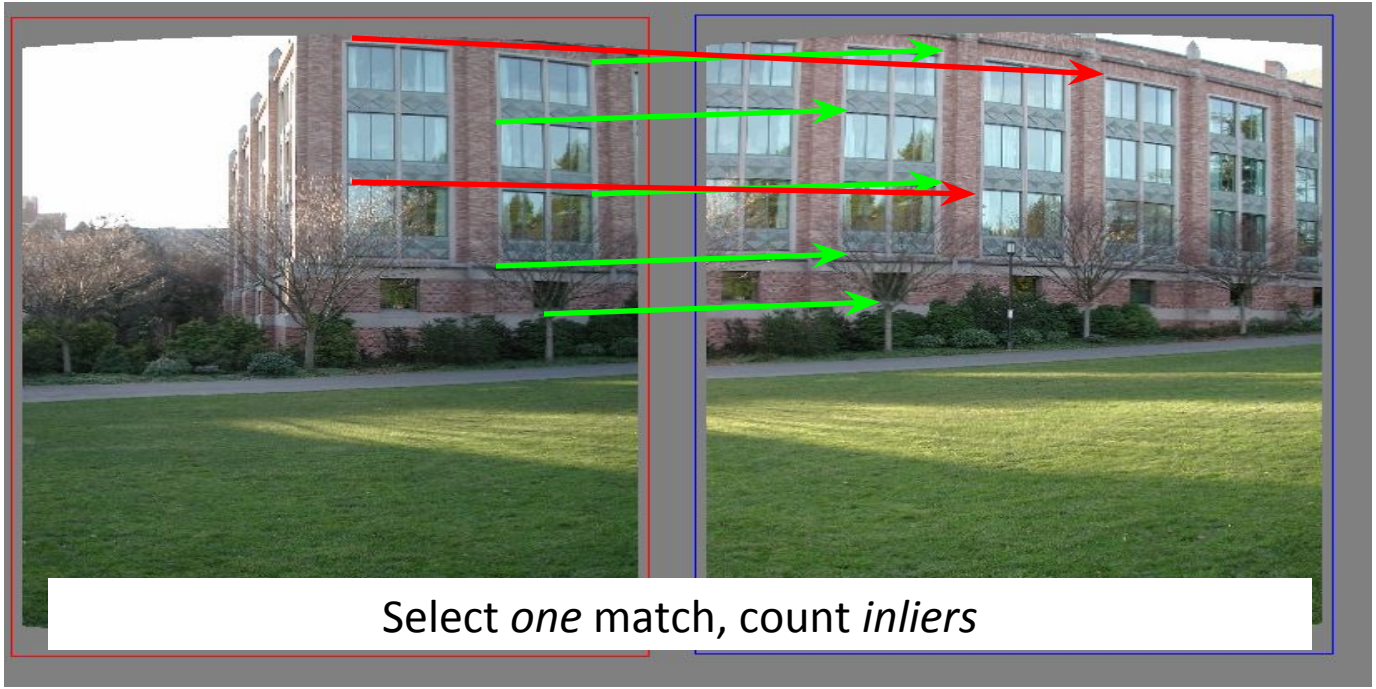

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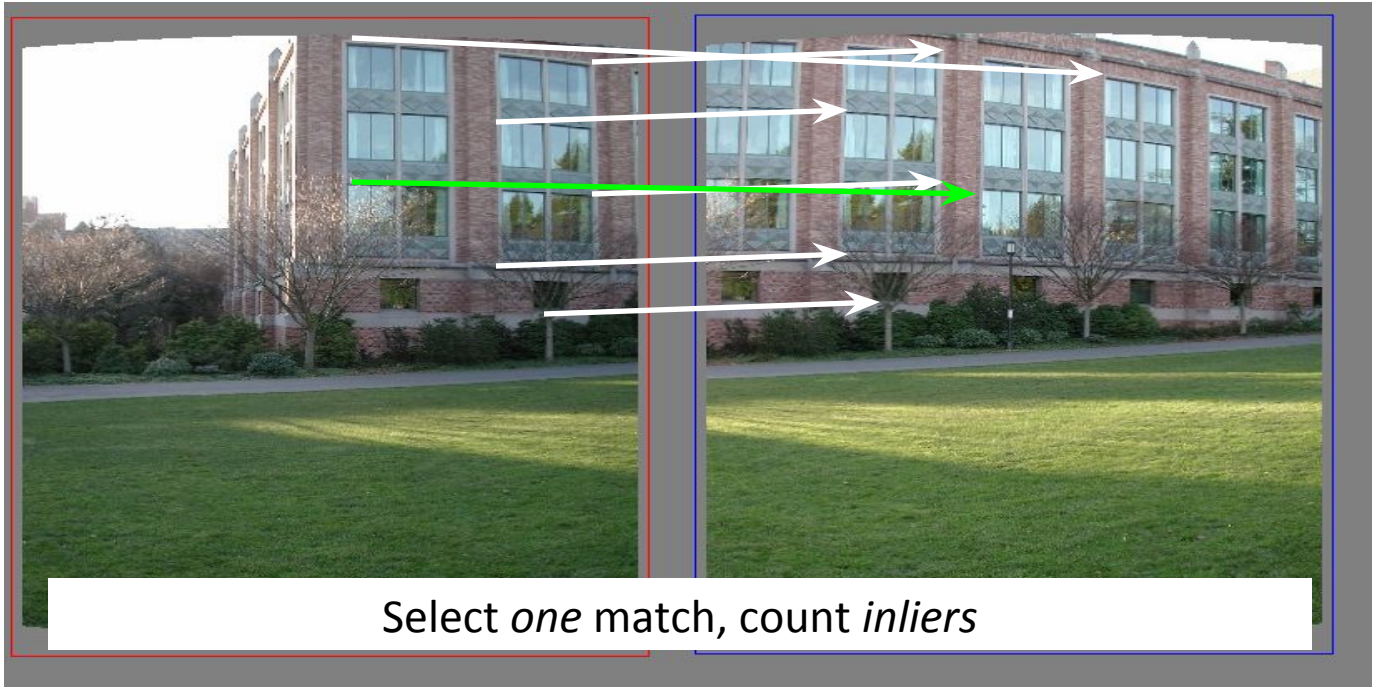
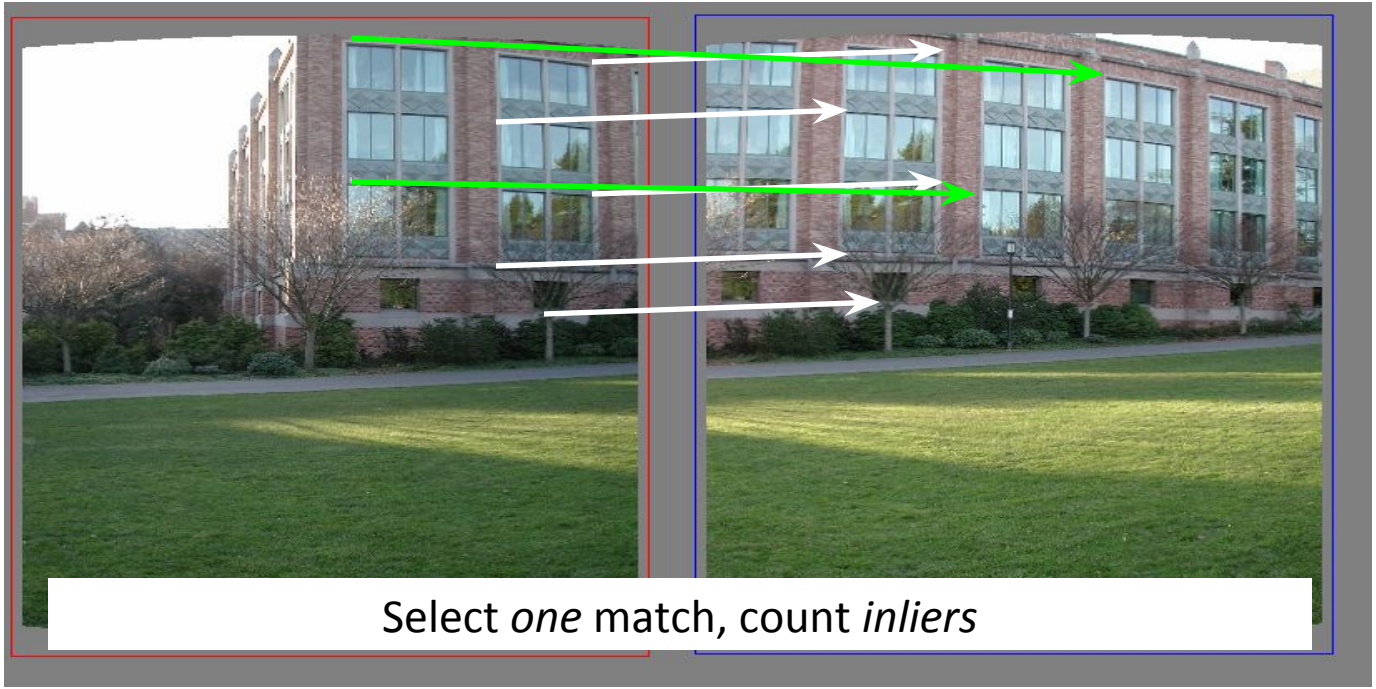
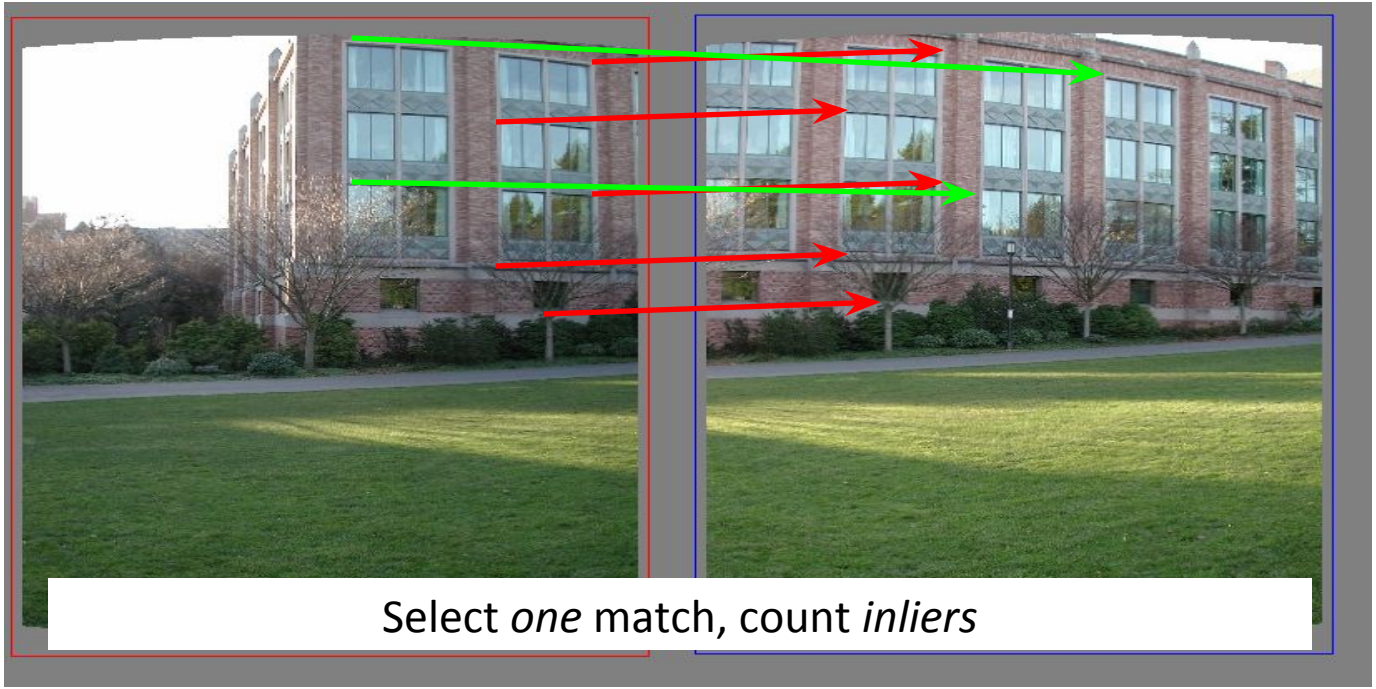


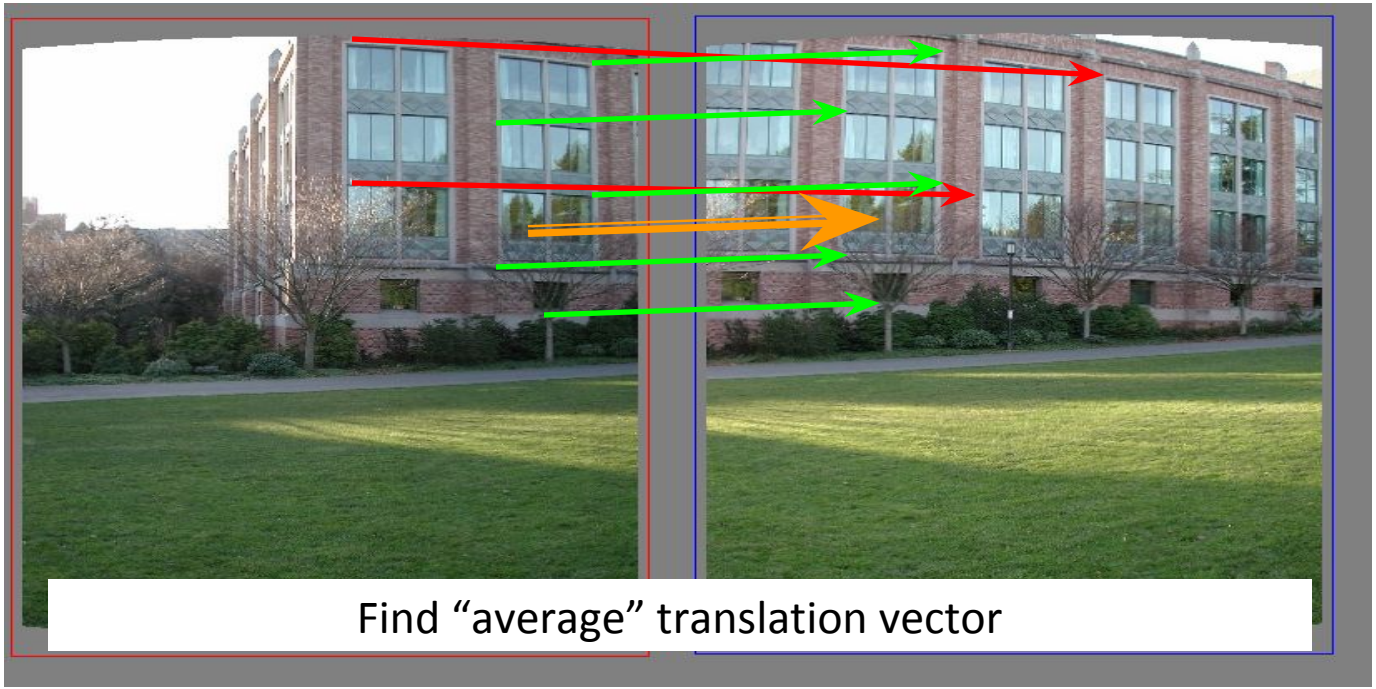
RANdom SAmple Consensus

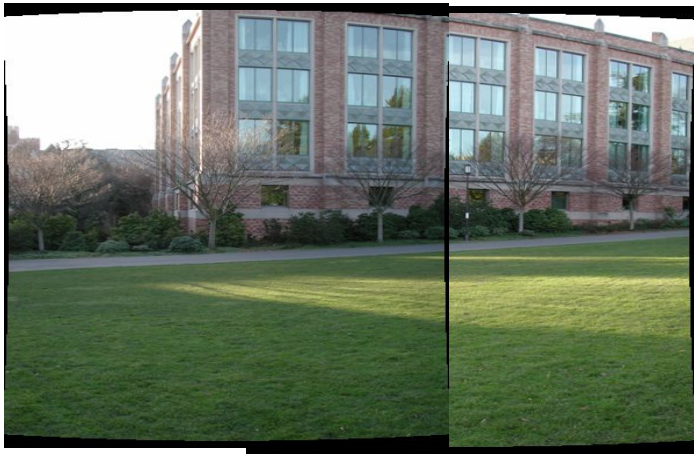


RAndom SAmple Consensus



Least squares fit (from inliers)





RANSAC for estimating homography

- RANSAC loop:

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1. Select four feature pairs (at random)

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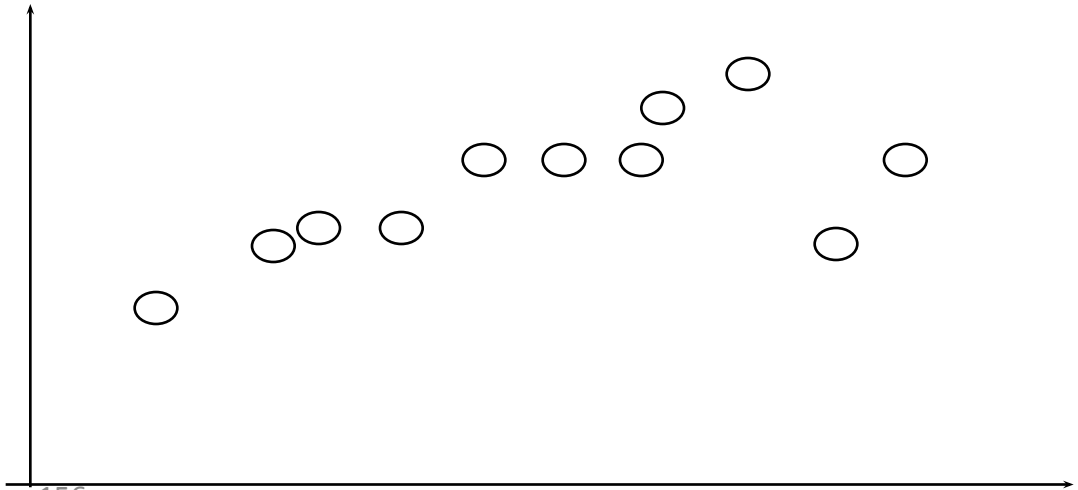
- Keep largest set of inliers
- Re-compute least-squares $\boldsymbol{H}$ estimate using all of the inliers

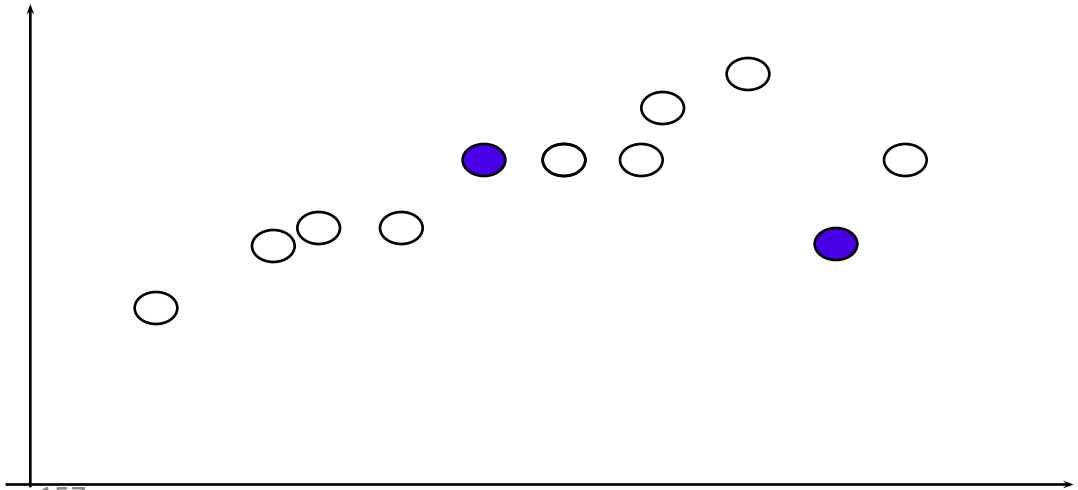
Simple example: fit a line

- Rather than homography H (8 numbers) fit y=ax+b (2 numbers a, b) to 2D pairs



Simple example: fit a line

- Pick 2 points



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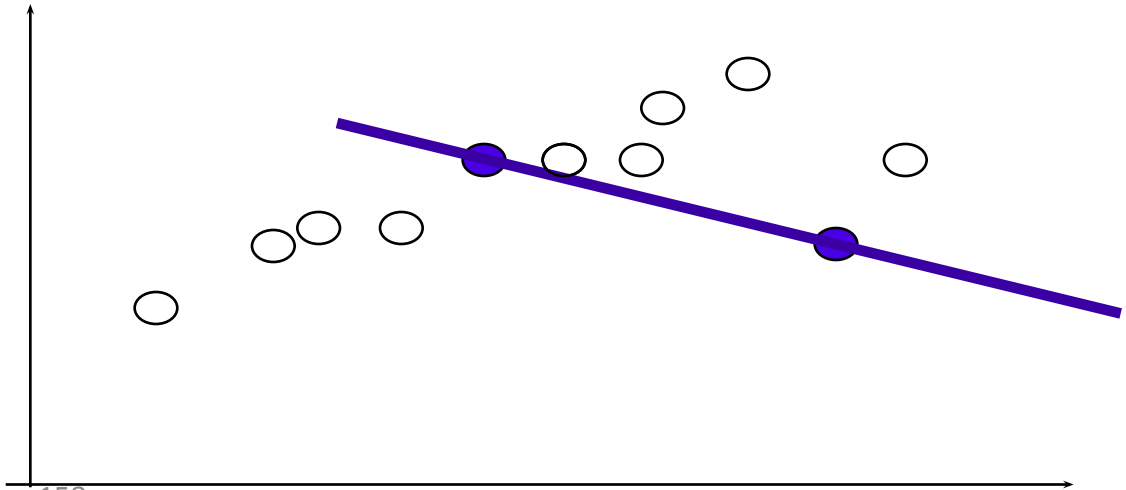
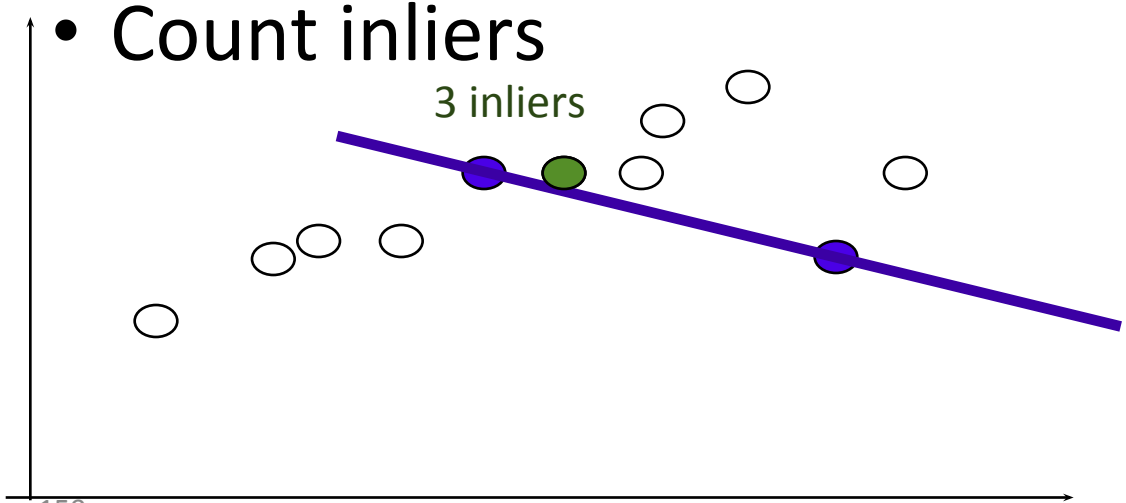


Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



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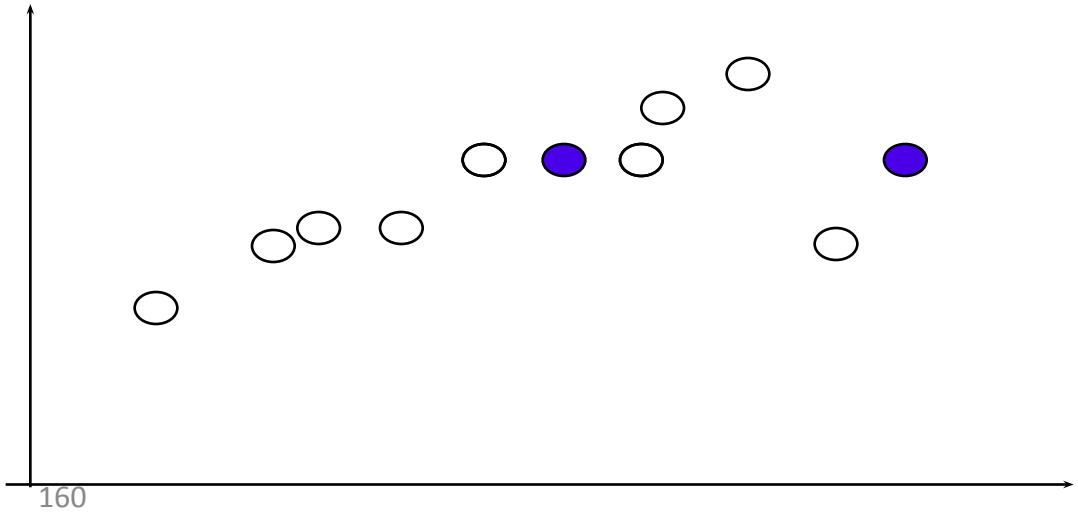

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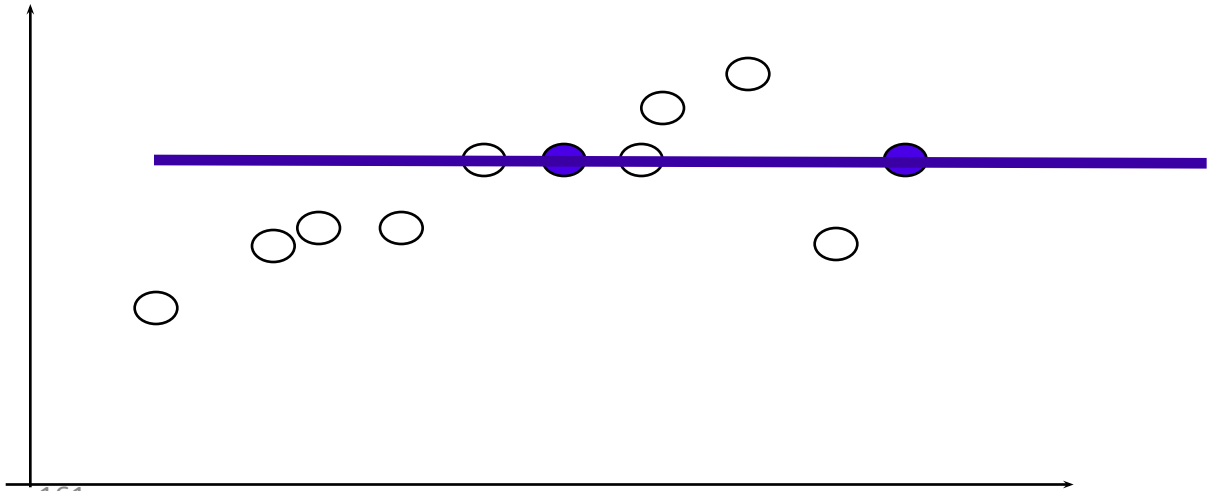

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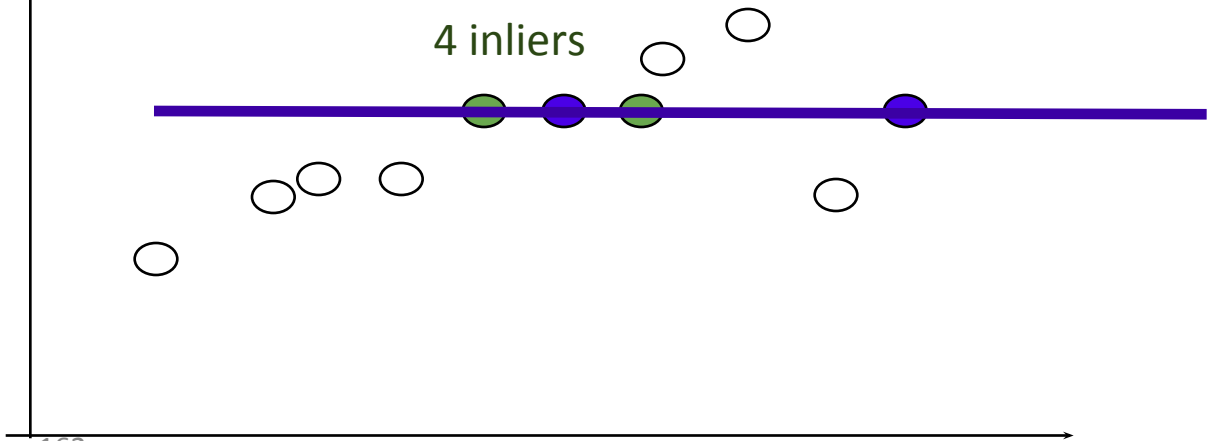

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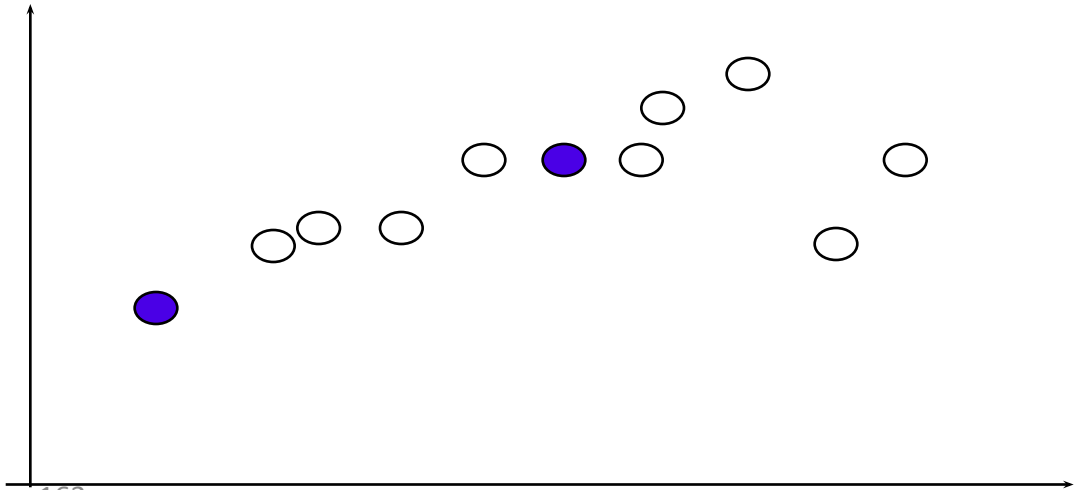

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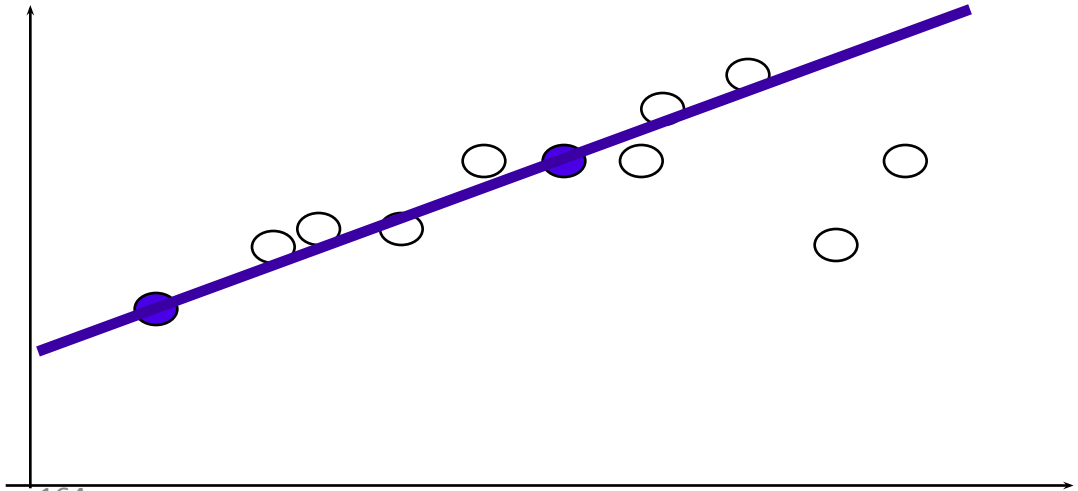

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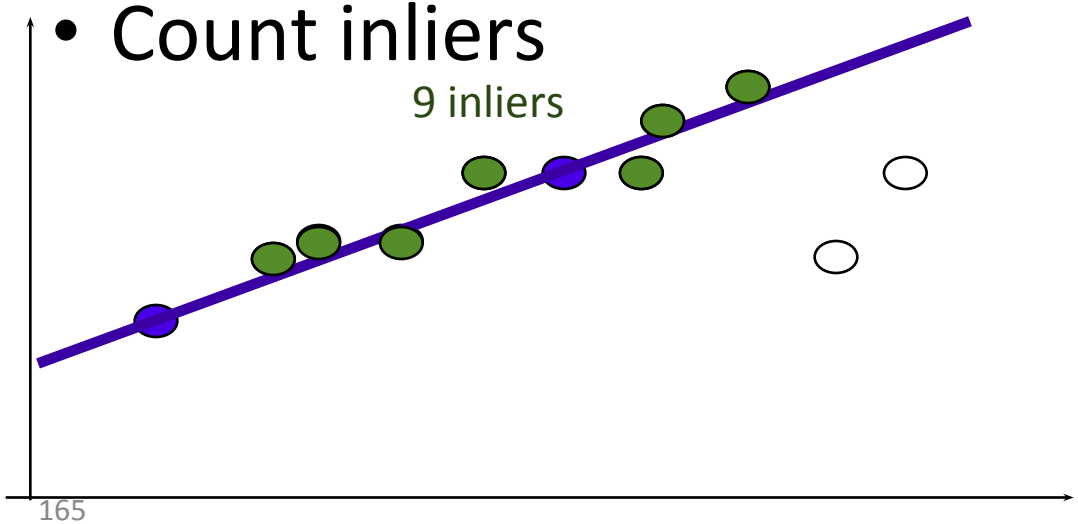

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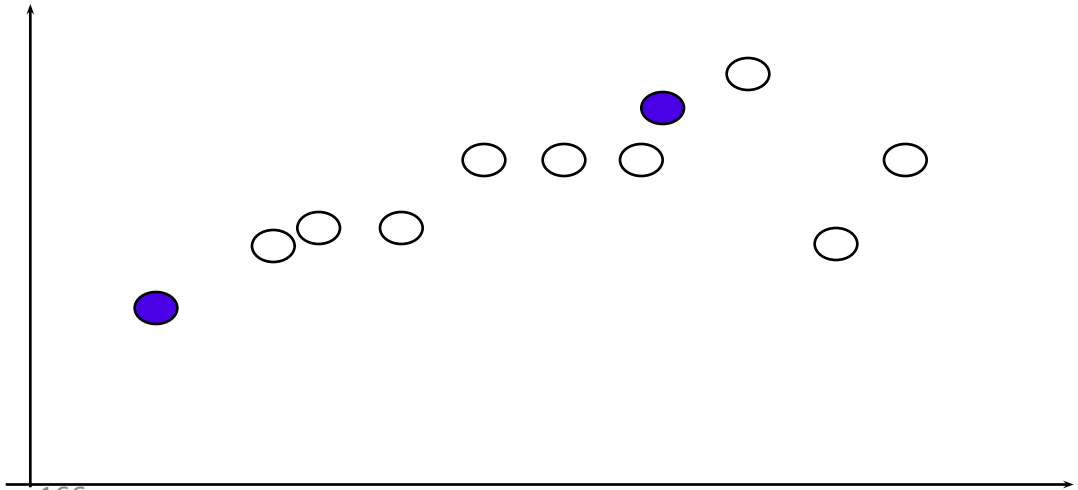

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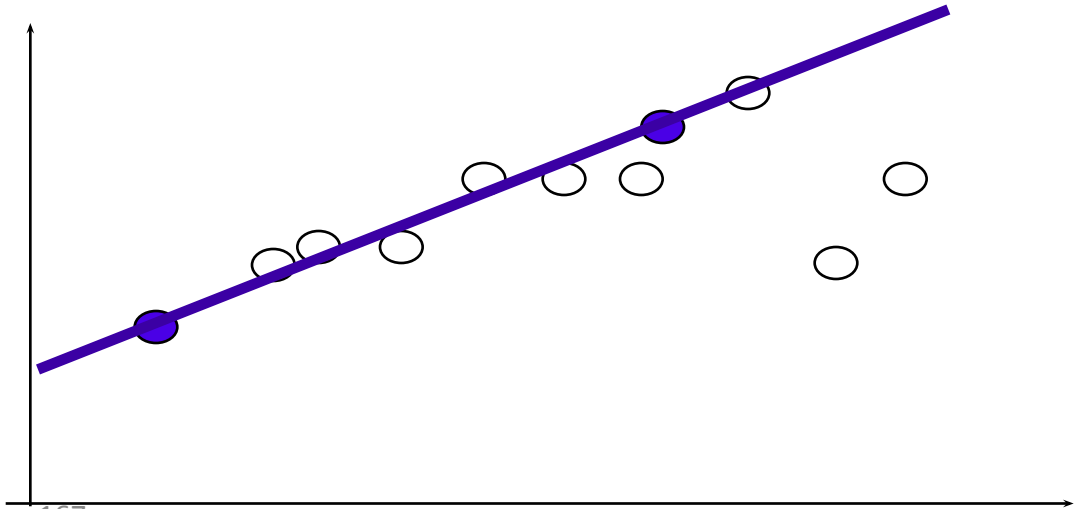

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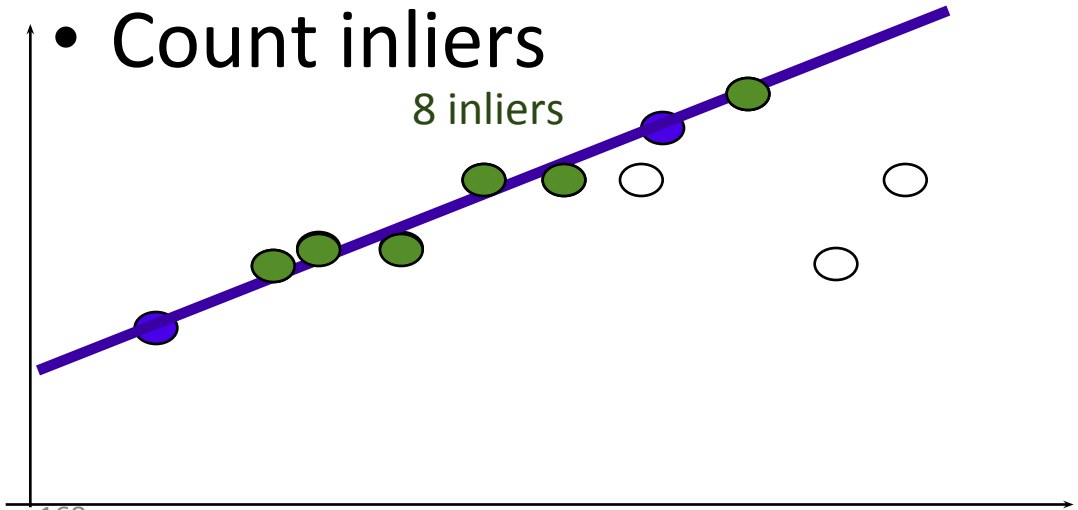
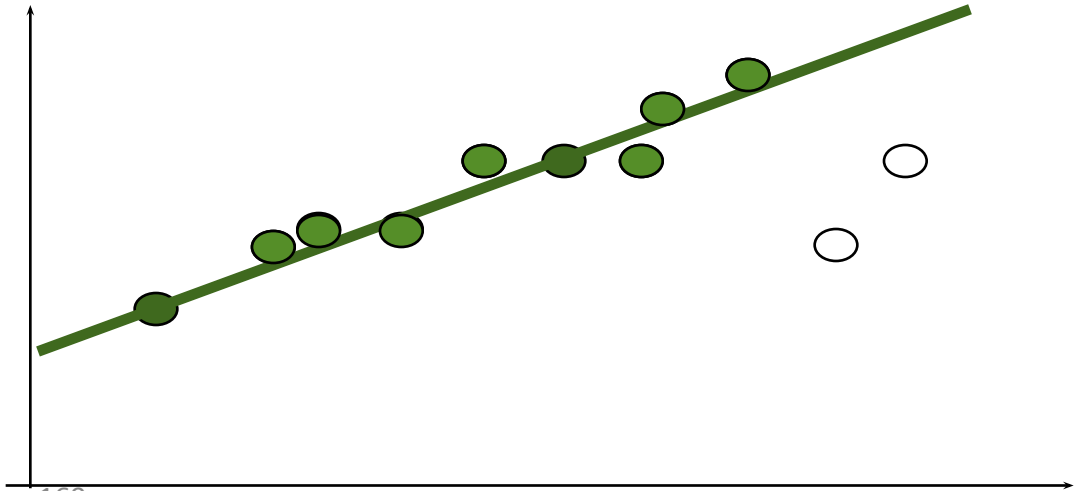


Simple example: fit a line

- Use biggest set of inliers
- Do least-square fit



Where are we?

- Basic Procedure

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  1. Take a sequence of images from the same position

     (Rotate the camera about its optical center)

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     (Rotate the camera about its optical center)
  2. Compute transformation between second image and first
  3. Shift the second image to overlap with the first
  4. Blend the two together to create a mosaic
  5. If there are more images, repeat

Review

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Plane perspective mosaics

– 8-parameter generalization of affine motion

  • works for pure rotation or planar surfaces

– Limitations:

  • local minima
  • slow convergence
  • difficult to control interactively

Direct Linear Transforms

- Why could we not solve for the homography in exactly the same way we did for the affine transform, ie.

$$\mathbf{t} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Answer from Sameer Agrawal

- For an affine transform, we have equations of the form $Ax_i + b = y_i$, solvable by linear regression.
- For the homography, the equation is of the form

  $H\tilde{x}_i \sim \tilde{y}_i$ (homogeneous coordinates)

and the ~means it holds only up to scale. The affine solution does not hold.

- To get rid of the scale ambiguity, we can break up the H into 3 row vectors and divide out the third coordinate to make it 1.

  $[h_1' \tilde{x}_i / h_3' \tilde{x}_i , h_2' \tilde{x}_i / h_3' \tilde{x}_i] = [y_{i1}, y_{i2}]$ for each i.

Continued

- Expanding these out leads to (for each i)

$$h_1' \tilde{x}_i - y_{i1} h_3' \tilde{x}_i = 0$$
$$h_2' \tilde{x}_i - y_{i2} h_3' \tilde{x}_i = 0$$

a system with no constant terms.

- The resultant system to be solved is of the form A h = 0

- Then this is solved with the eigenvector approach.