UNIVERSITÉ Concordia UNIVERSITY

FACULTY OF ENGINEERING AND COMPUTER SCIENCE

Department of Computer Science and Software Engineering

COMP498G/691G COMPUTER VISION

LECTURE 4
GEOMETRIC TRANSFORMATIONS

Administrative

- Lab tutorials
- Course is full
  - undergraduate
  - graduate
- Programming Assignment #1 deadline tonight
  - submission site open until 5am tomorrow
- Programming Assignment #2 is out

Review of last week's lectures

- Image Sampling
  - Downsampling, Upsampling, Gaussian Pyramid, Laplacian Pyramid
- Edge Detection
  - derivatives, LoG, DoG
- Questions

Today’s Lecture

- Geometric Transformations
    - Slides acknowledgment: L. Shapiro
- Questions

What are geometric transformations?

What are geometric transformations?

Why do we need them?

Translation

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

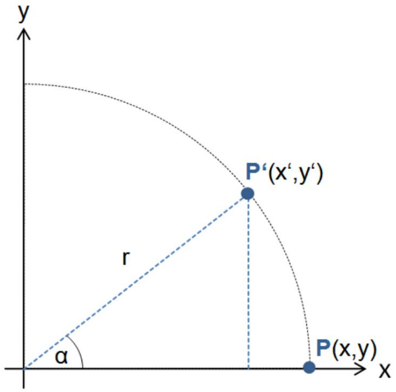
Translation

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Preserves: Orientation

Rotation (2D)

- Convention: positive angles rotates counterclockwise

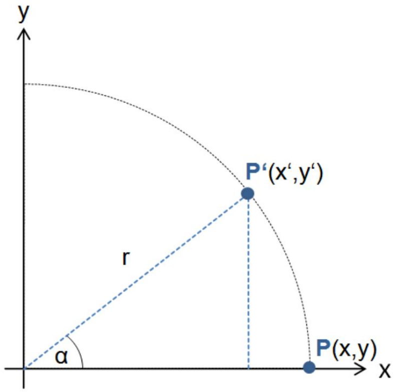


$$\cos\alpha = \frac{x'}{r} \quad \rightarrow \quad x' = r * \cos\alpha = x * \cos\alpha$$

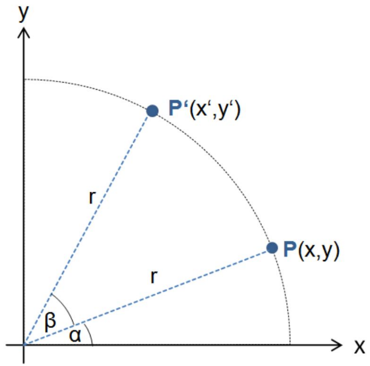
$$\sin\alpha = \frac{y'}{r} \quad \rightarrow \quad y' = r * \sin\alpha = x * \sin\alpha$$

Rotation (2D)

- Convention: positive angles rotates counterclockwise



$$\cos\alpha = \frac{x'}{r} \quad \rightarrow \quad x' = r * \cos\alpha = x * \cos\alpha$$
$$\sin\alpha = \frac{y'}{r} \quad \rightarrow \quad y' = r * \sin\alpha = x * \sin\alpha$$



$$x = r * \cos\alpha$$
$$y = r * \sin\alpha$$

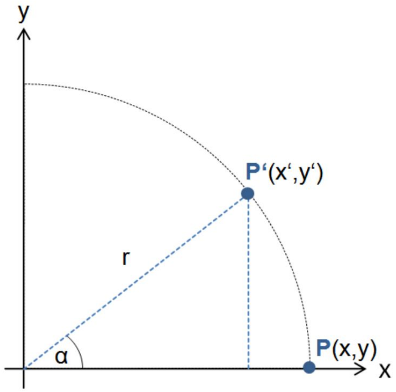
$$x' = r * \cos(\alpha + \beta)$$
$$y' = r * \sin(\alpha + \beta)$$

$$\begin{aligned} x' &= r * \cos(\alpha + \beta) \\ &= r * (\cos\alpha * \cos\beta - \sin\alpha * \sin\beta) \\ &= r * \cos\alpha\cos\beta - r * \sin\alpha\sin\beta \\ &= x * \cos\beta - y * \sin\beta \end{aligned}$$

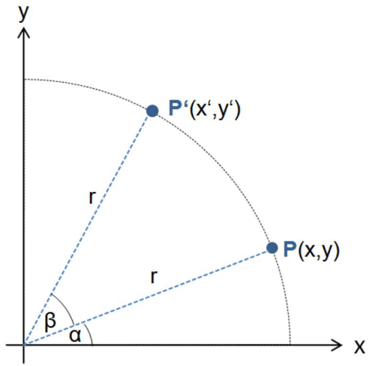
$$\begin{aligned} y' &= r * \sin(\alpha + \beta) \\ &= r * (\sin\alpha * \cos\beta + \cos\alpha * \sin\beta) \\ &= r * \sin\alpha\cos\beta - r * \cos\alpha\sin\beta \\ &= y * \cos\beta + x * \sin\beta \end{aligned}$$

Rotation (2D)

- Convention: positive angles rotates counterclockwise



$$\cos\alpha = \frac{x'}{r} \;\rightarrow\; x' = r * \cos\alpha = x * \cos\alpha$$
$$\sin\alpha = \frac{y'}{r} \;\rightarrow\; y' = r * \sin\alpha = x * \sin\alpha$$



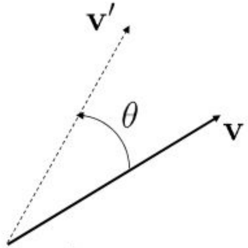
$$x = r * \cos\alpha$$
$$y = r * \sin\alpha$$

$$x' = r * \cos(\alpha+\beta)$$
$$y' = r * \sin(\alpha+\beta)$$

$$\begin{aligned}x' &= r * \cos(\alpha+\beta)\\ &= r * (\cos\alpha * \cos\beta - \sin\alpha * \sin\beta)\\ &= r * \cos\alpha\cos\beta - r * \sin\alpha\sin\beta\\ &= x * \cos\beta - y * \sin\beta\end{aligned}$$

$$\begin{aligned}y' &= r * \sin(\alpha+\beta)\\ &= r * (\sin\alpha * \cos\beta + \cos\alpha * \sin\beta)\\ &= r * \sin\alpha\cos\beta - r * \cos\alpha\sin\beta\\ &= y * \cos\beta + x * \sin\beta\end{aligned}$$

$$\mathbf{R}(\theta) = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

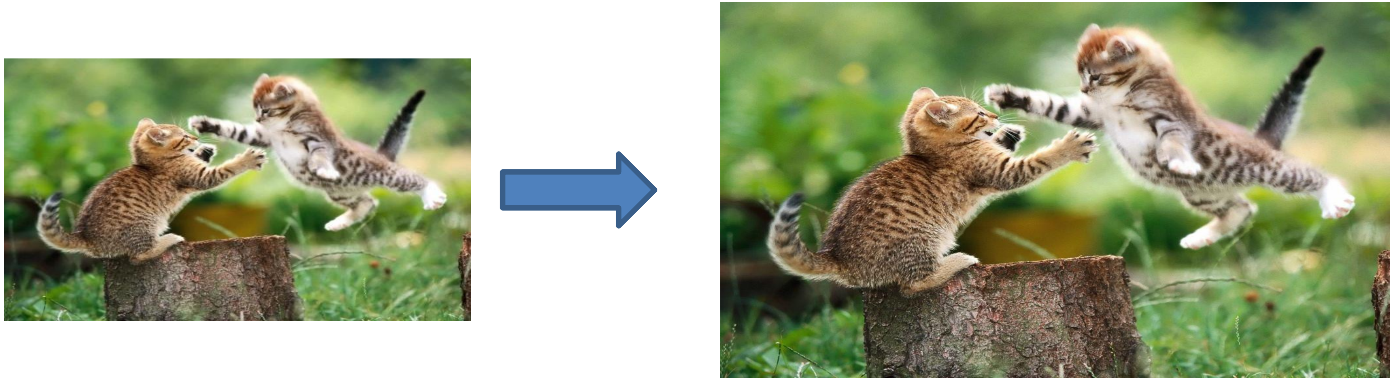


$$\mathbf{v}' = \mathbf{R}(\theta)\mathbf{v}$$

Translation and rotation

$$\begin{bmatrix} cos(\theta) & -sin(\theta) & t_x \\ sin(\theta) & cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

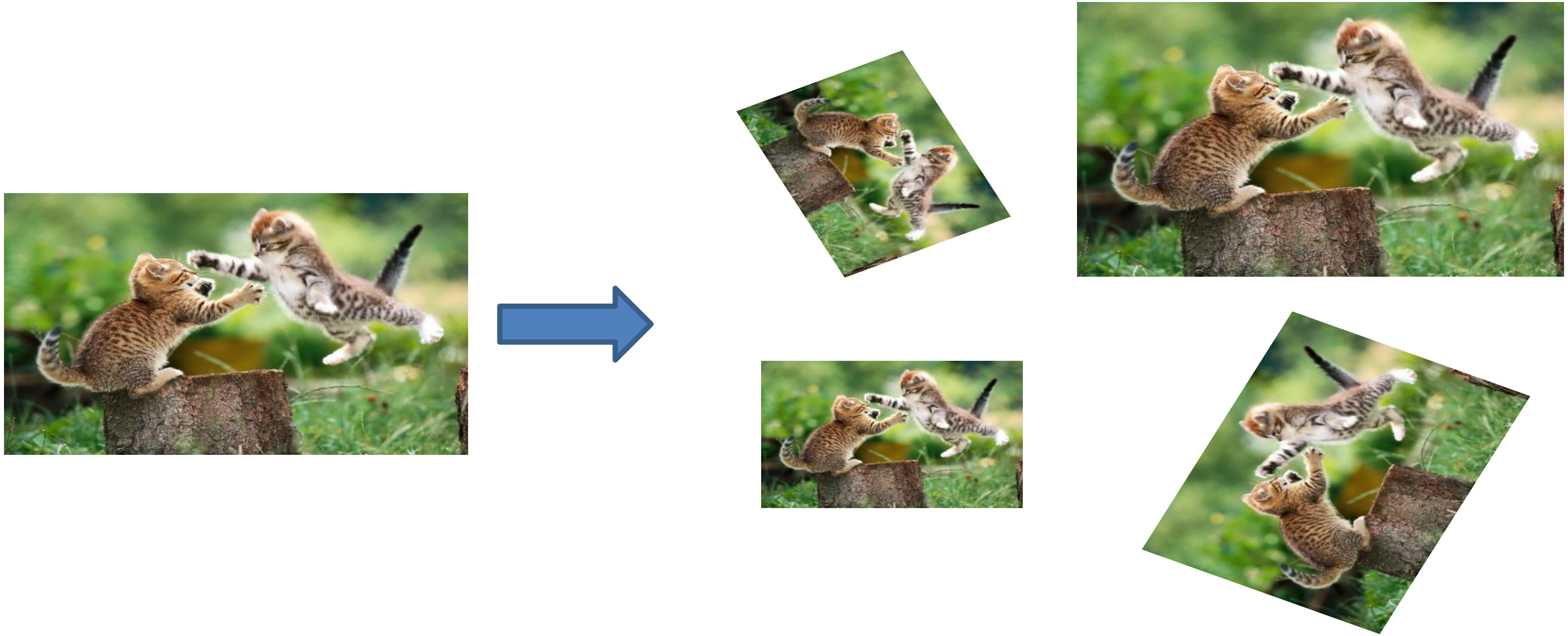


$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Scale

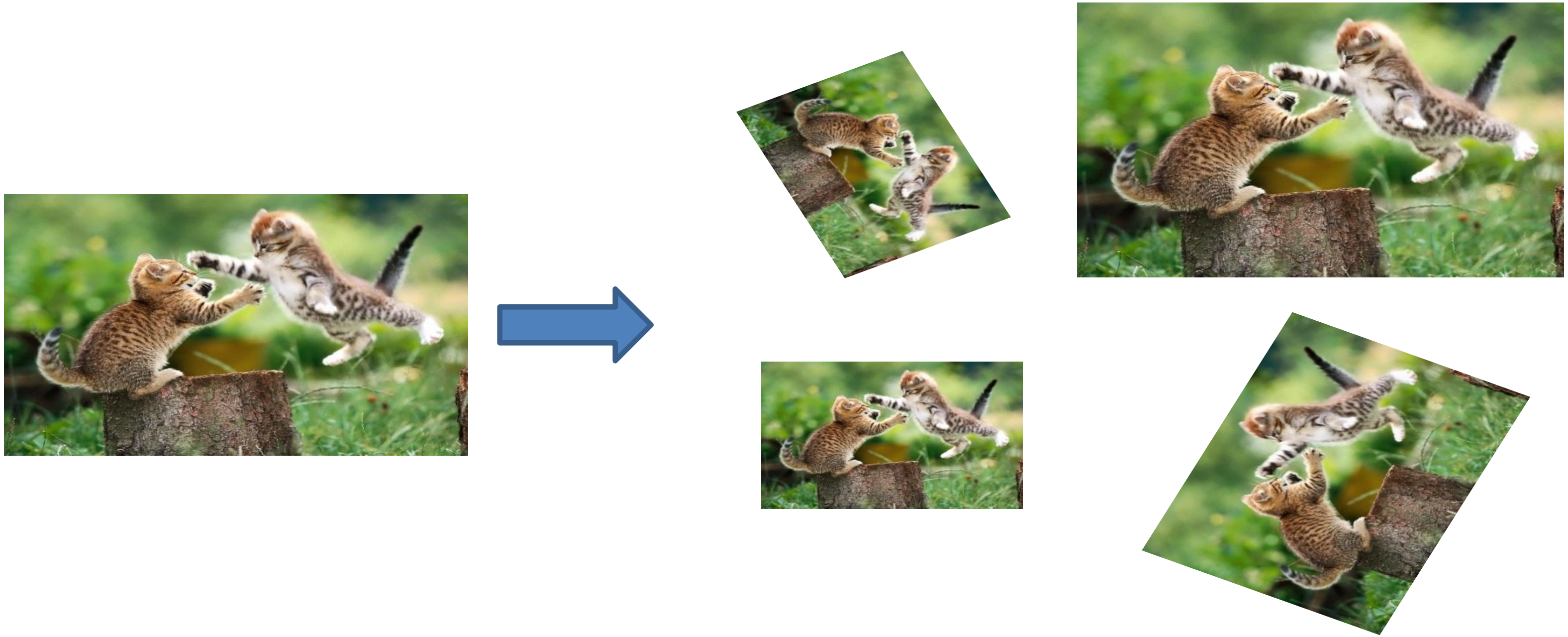
$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Similarity transformations



Similarity transform (4 DoF) = translation + rotation + scale

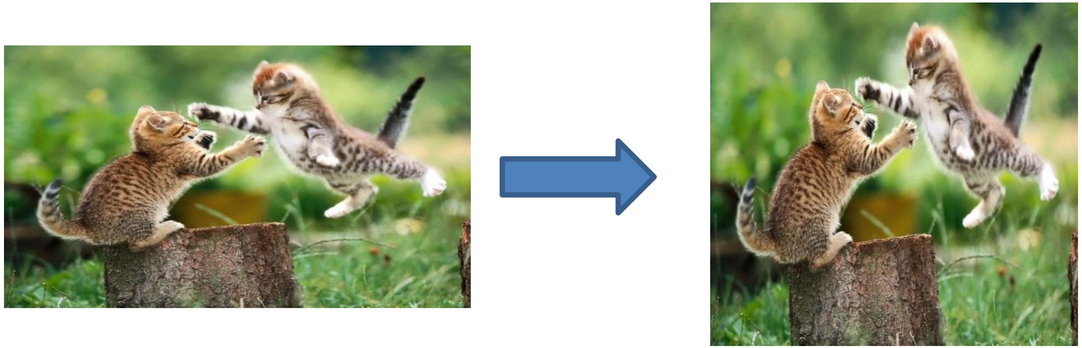
Similarity transformations



Similarity transform (4 DoF) = translation + rotation + scale

Preserves: Angles

$$\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

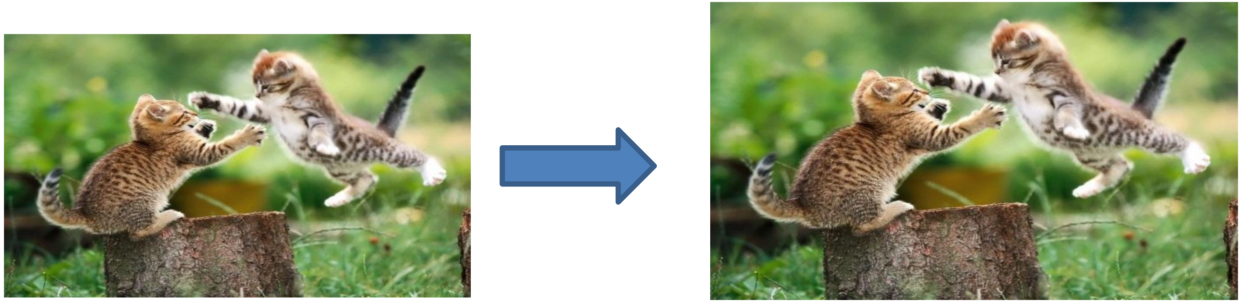


$$\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Aspect ratio

$$\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

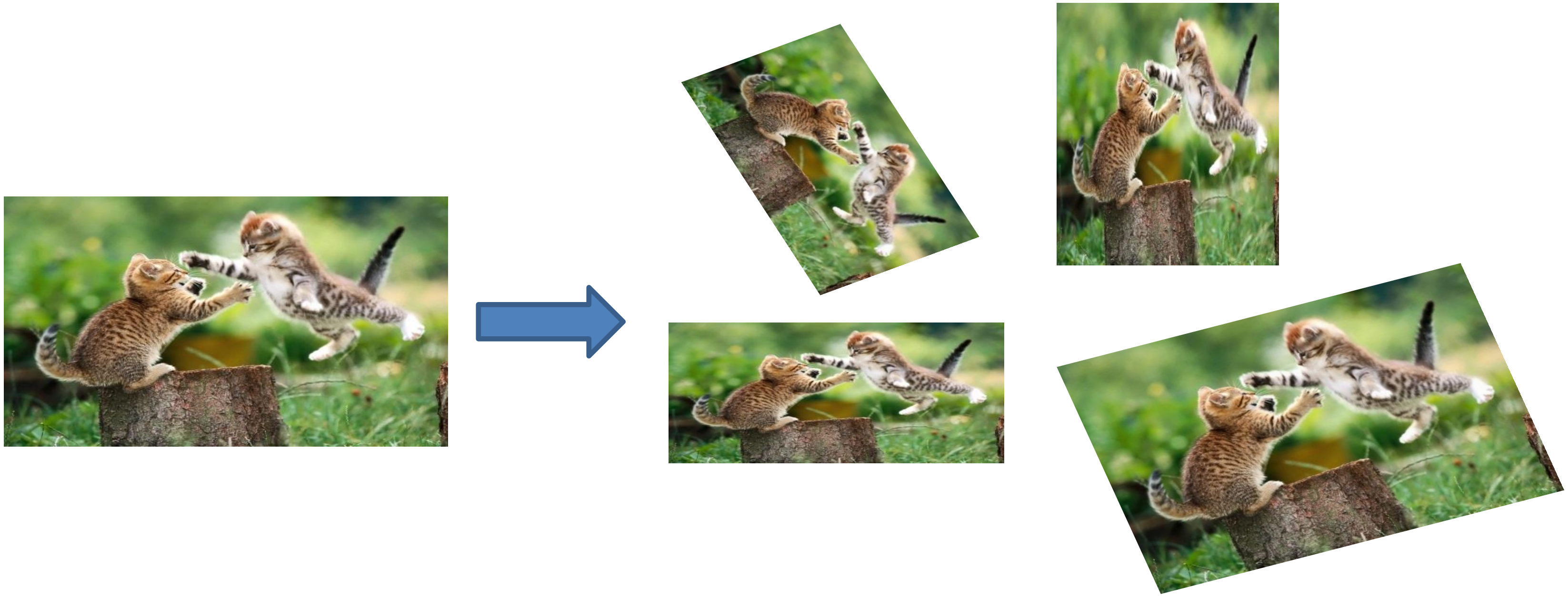


$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Shear

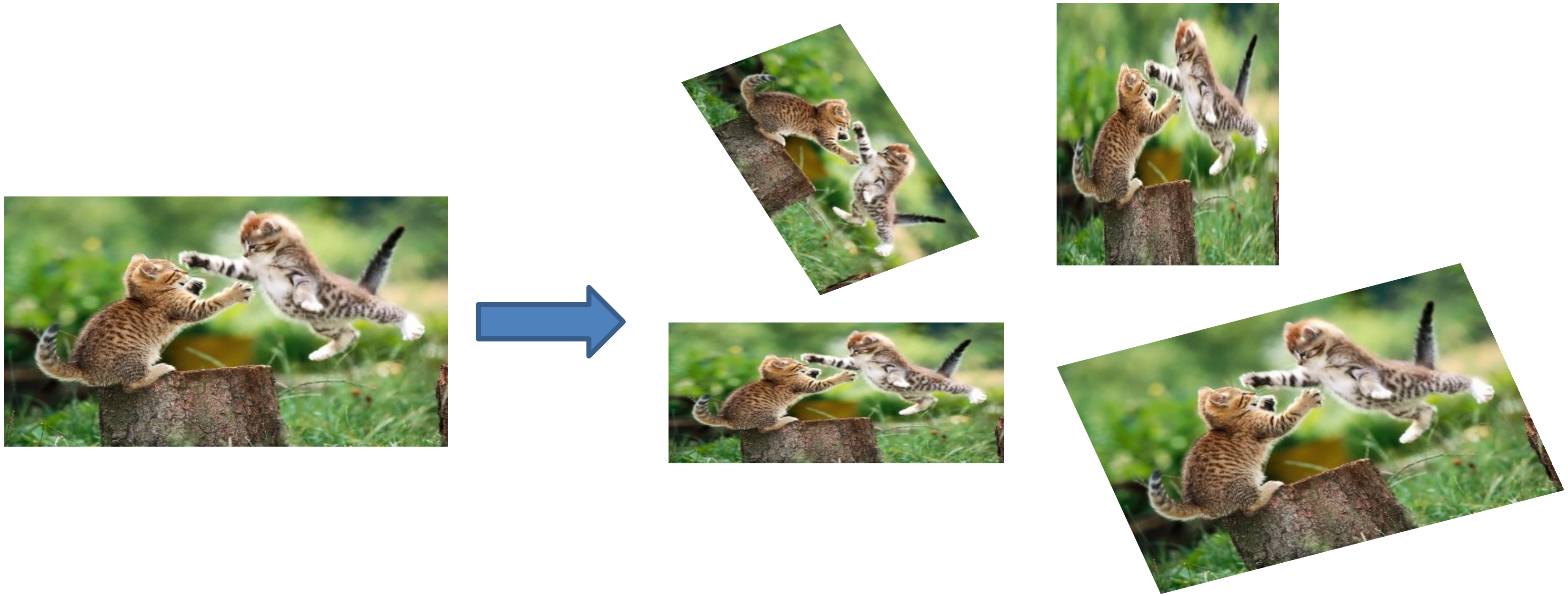
$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Affine transformations



Affine transform (6 DoF) = translation + rotation + scale + aspect ratio + shear

Affine transformations



Affine transform (6 DoF) = translation + rotation + scale + aspect ratio + shear

Preserves: Parallelism

What is missing?

Canaletto

Are there any other planar transformations?

General affine

We already used these

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

How do we compute projective transformations?

Homogeneous coordinates

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

One extra step:

$$x' = u/w$$
$$y' = v/w$$

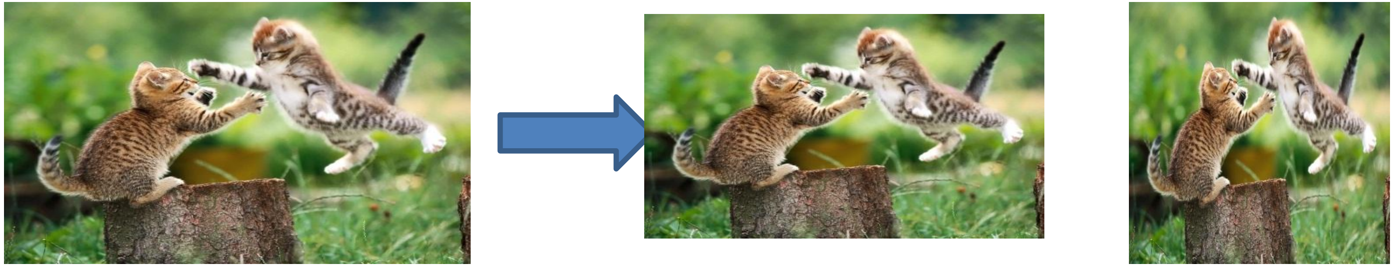
Projective transformations

a.k.a. Homographies

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$x' = u/w$$
$$y' = v/w$$

"keystone" distortions



Projective transformations

a.k.a. Homographies

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$x' = u/w$$
$$y' = v/w$$

"keystone" distortions

Preserves: Straight Lines

Finding the transformation

Translation	=	2 degrees of freedom
Similarity	=	4 degrees of freedom
Affine	=	6 degrees of freedom
Homography	=	8 degrees of freedom

How many corresponding points do we need to solve?

Finding the transformation

- How can we find the transformation between these images?
- How many corresponding points do we need to solve?

What can I use homographies for?

For one thing: Panoramas

Review

- Transformations
  - Similarity
  - Affine
  - Projective/Homographies
  - Homogeneous coordinates

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