Université Concordia University
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
Department of Computer Science and Software Engineering

COMP498G/691G COMPUTER VISION

LECTURE 3
IMAGE SAMPLING

Administrative

- Lab session starts tonight
  - Attend, follow the instructions, and ask questions

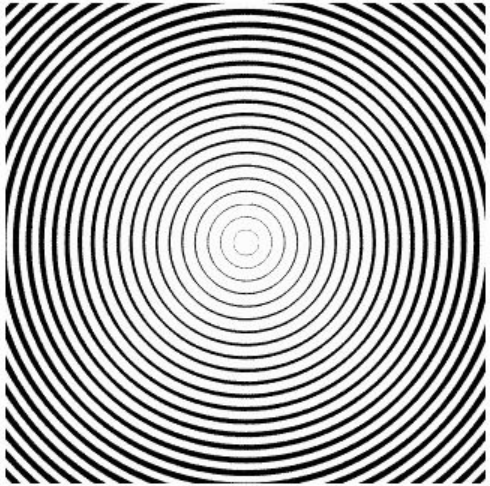
Review of last week's lectures

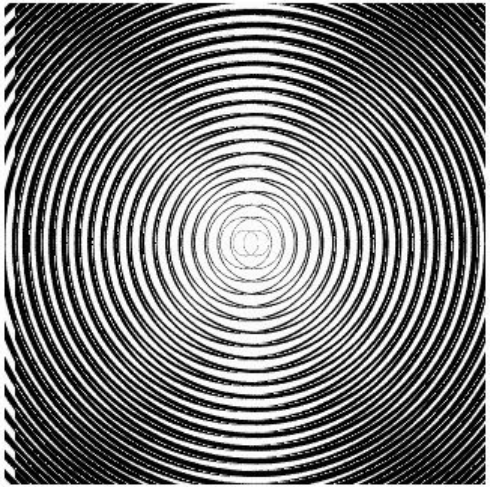
- Introduction to computer vision
- Image filtering
  - Images
    - Morph demo
    - GIMP demo
- Questions

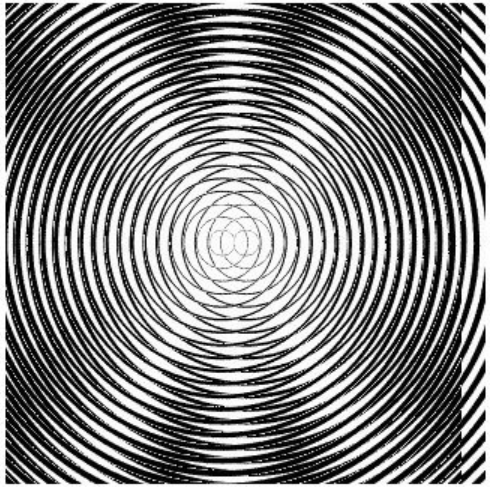
Today's Lecture

- Image Sampling
  - Slides acknowledgment: A. Farhadi, S. Seitz
- Questions

Image Sampling







Moire patterns

- http://www.sandlotscience.com/Moire/Circular_3_Moire.htm

Image Sampling

F() = 

F() = 

Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

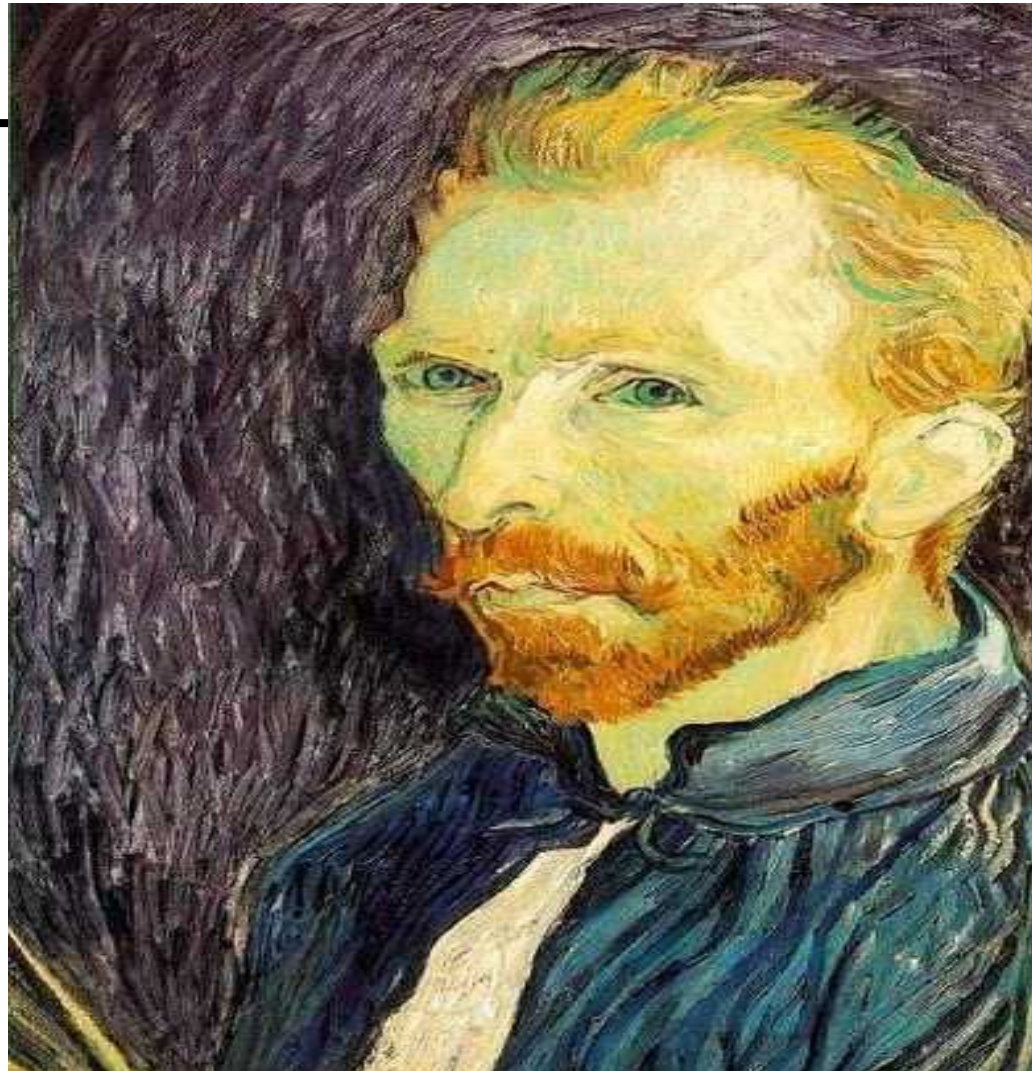
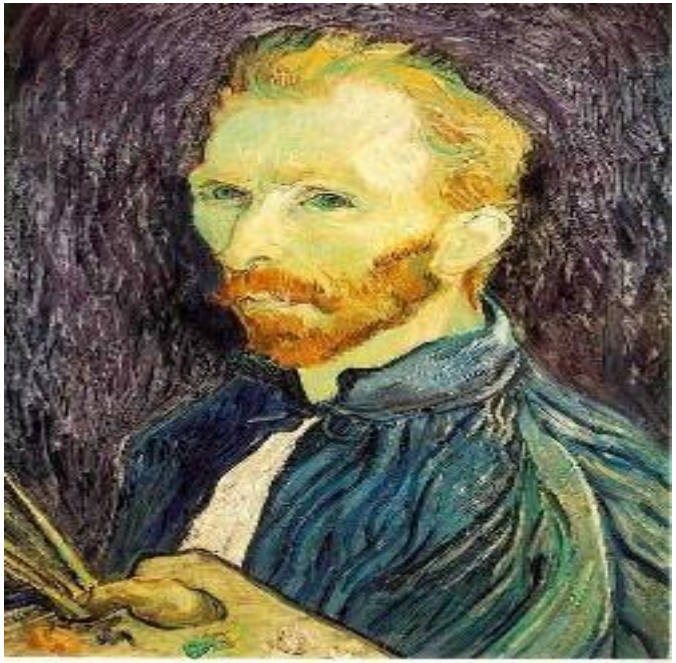
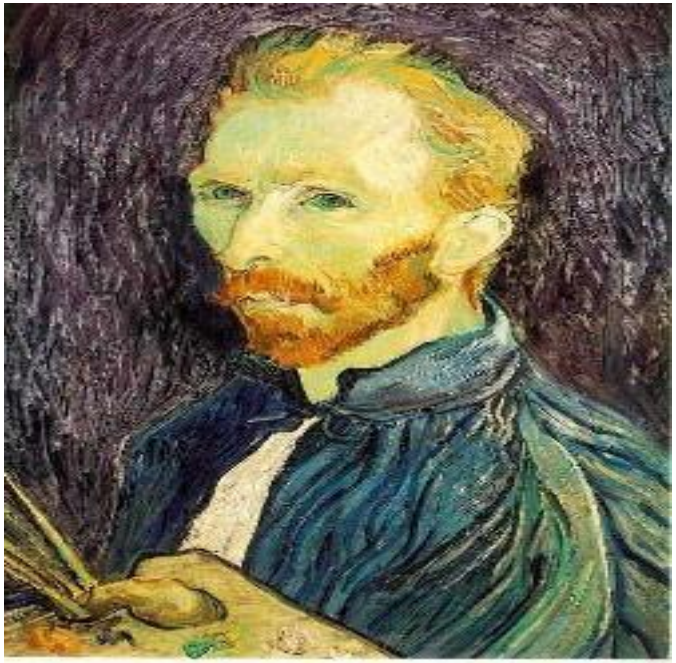


Image sub-sampling



Throw away every other row and column to create a 1/2 size image - called image sub-sampling

Image sub-sampling



1/4

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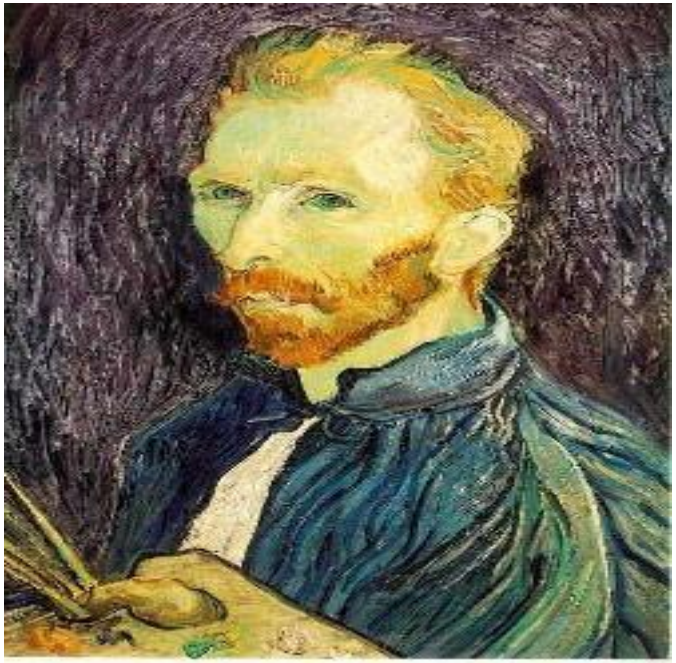
Image sub-sampling



1/4

1/8

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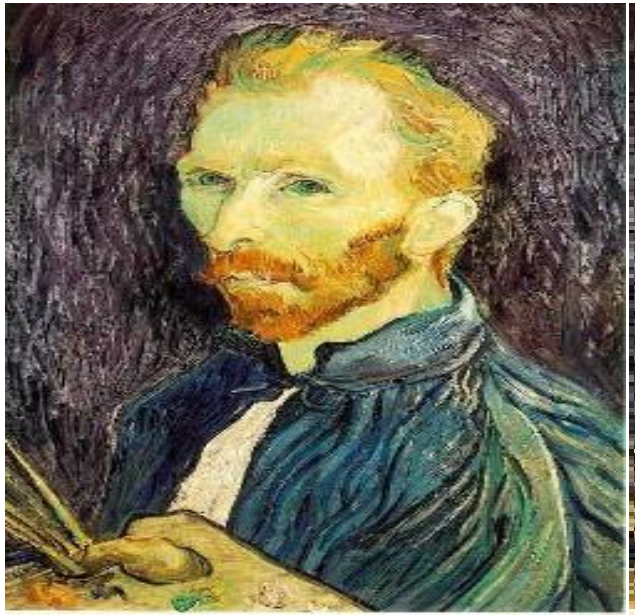
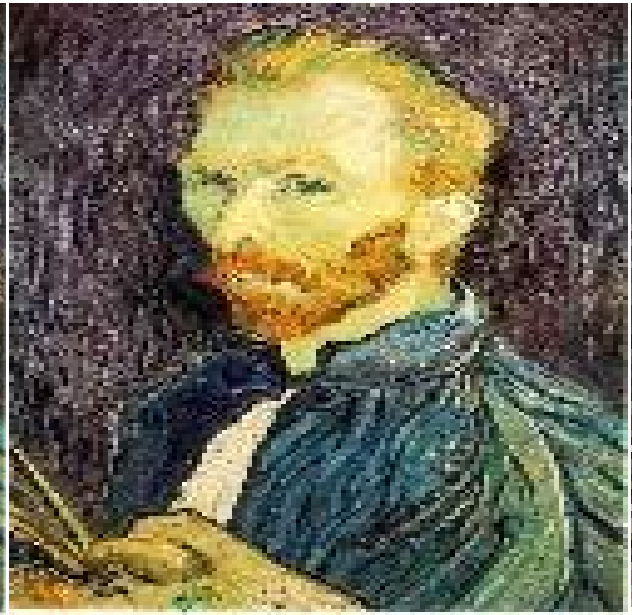
Image sub-sampling



1/2

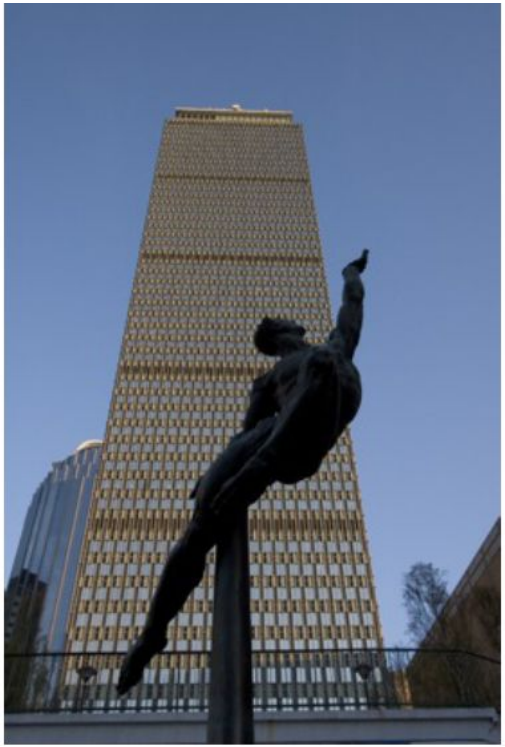


1/4 (2x zoom)

1/8 (4x zoom)

Why is this a bad effect?

Image sub-sampling



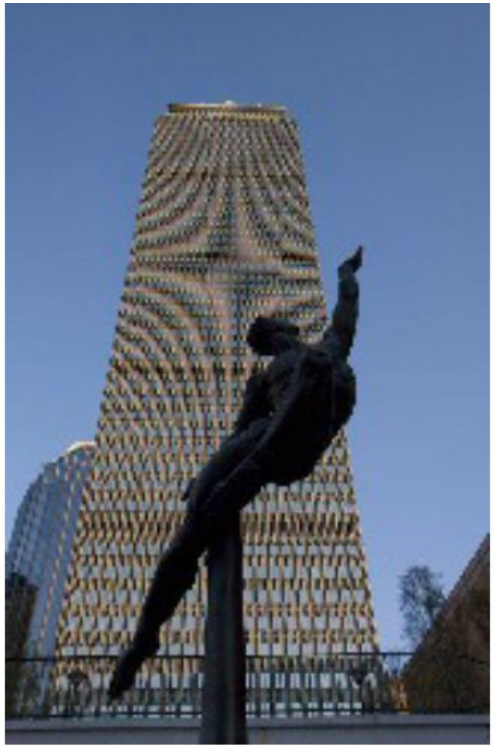
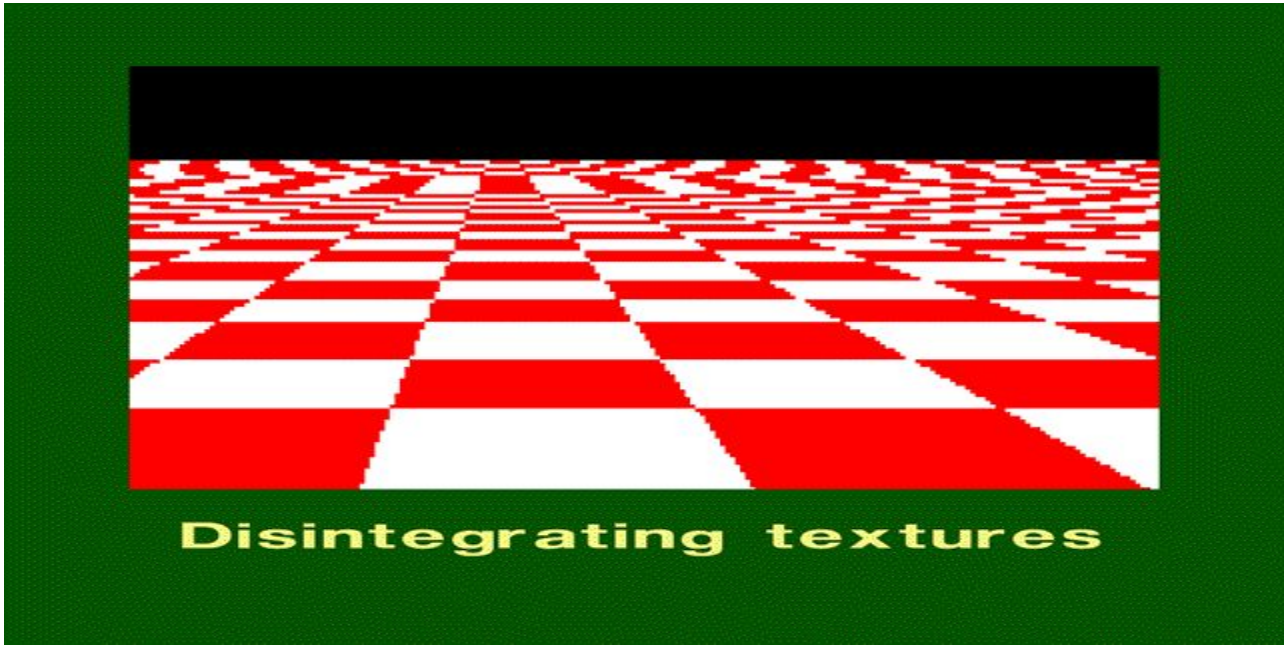


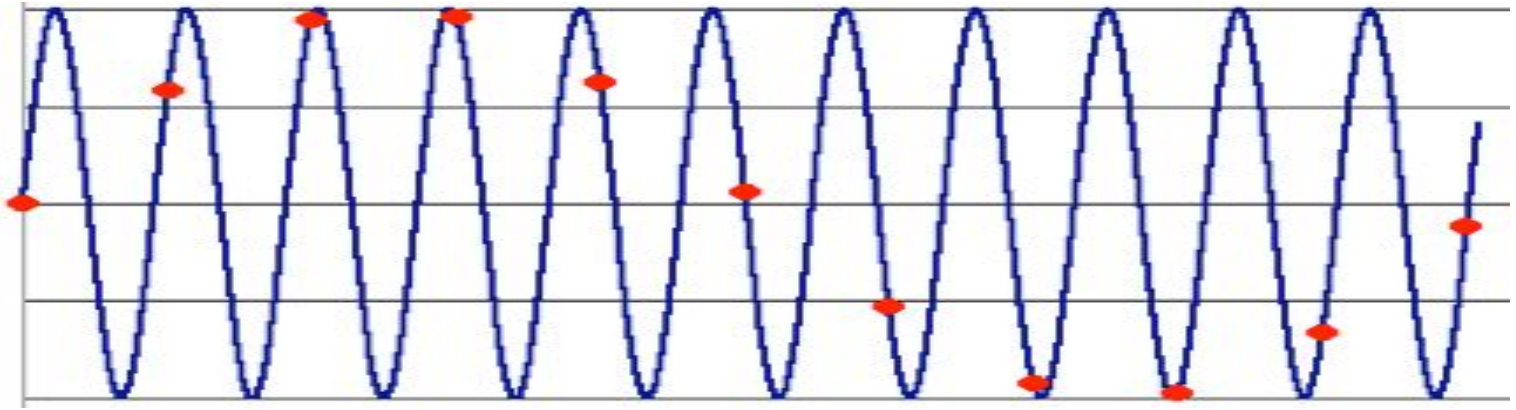
Image credit: F. Durand

Why is this a bad effect?

Even worse for synthetic images



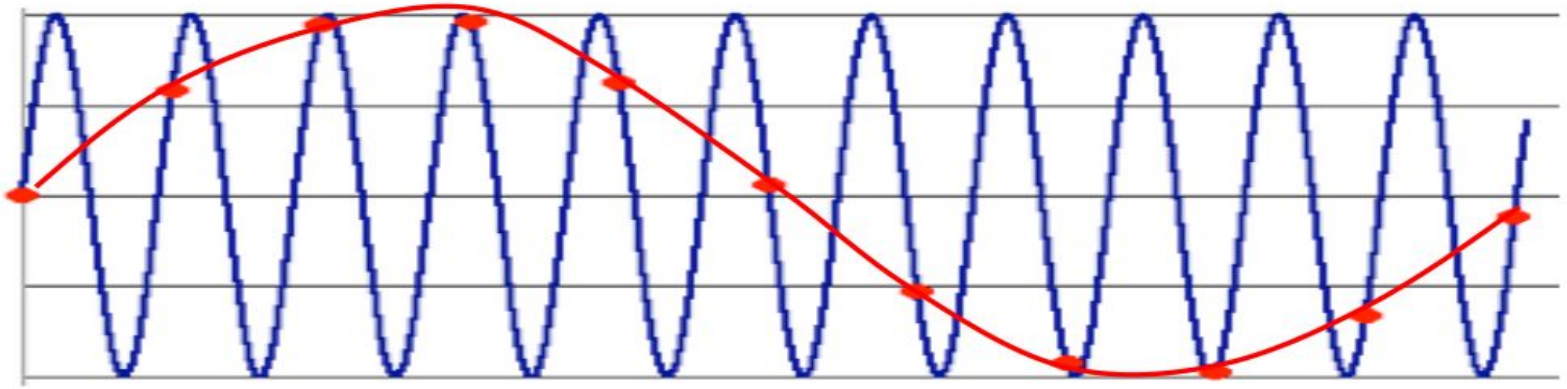
Down-sampling



Aliasing can arise when you sample a continuous signal or image

- occurs when your sampling rate is not high enough to capture the amount of detail in your image
- Can give you the wrong signal/image—an alias
- formally, the image contains structure at different scales
  - called “frequencies” in the Fourier domain
- the sampling rate must be high enough to capture the highest frequency in the image

Sampling and the Nyquist rate

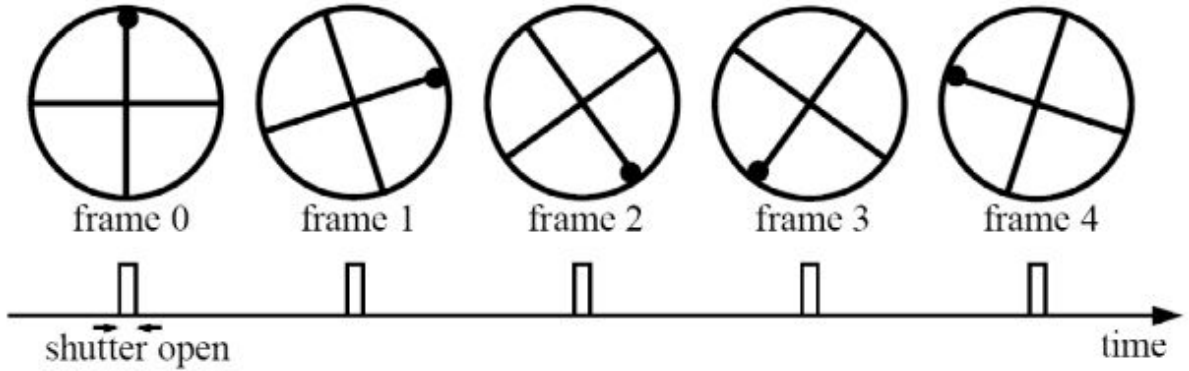


- To avoid aliasing:
  - sampling rate ≥ 2 * max frequency in the image
    - said another way: ≥ two samples per cycle
  - This minimum sampling rate is called the **Nyquist rate**

Wagon-wheel effect

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

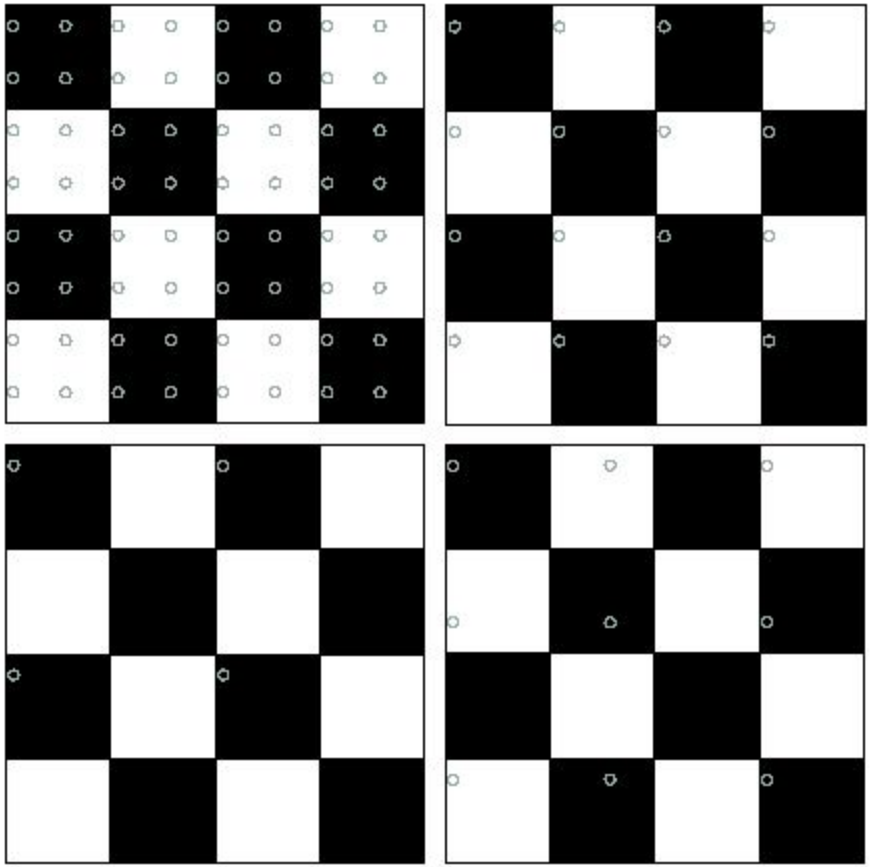


Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

http://www.michaelbach.de/ot/mot-wagonWheel/index.html

Source: L. Zhang

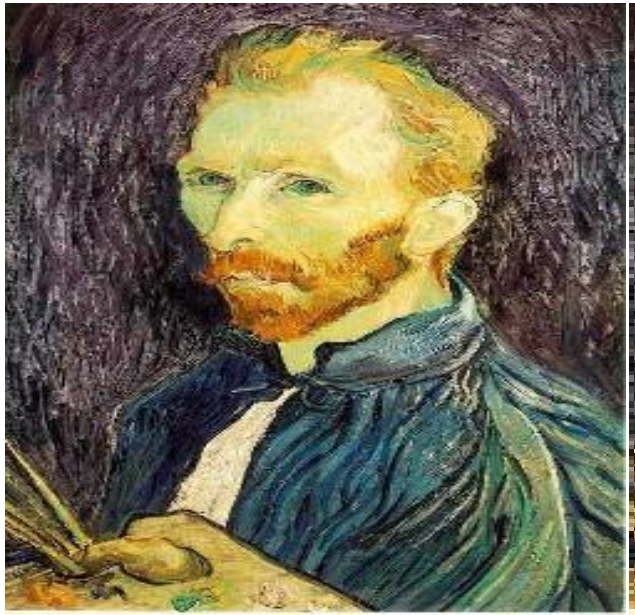
2D example



Good sampling

Bad sampling

How to fix this?

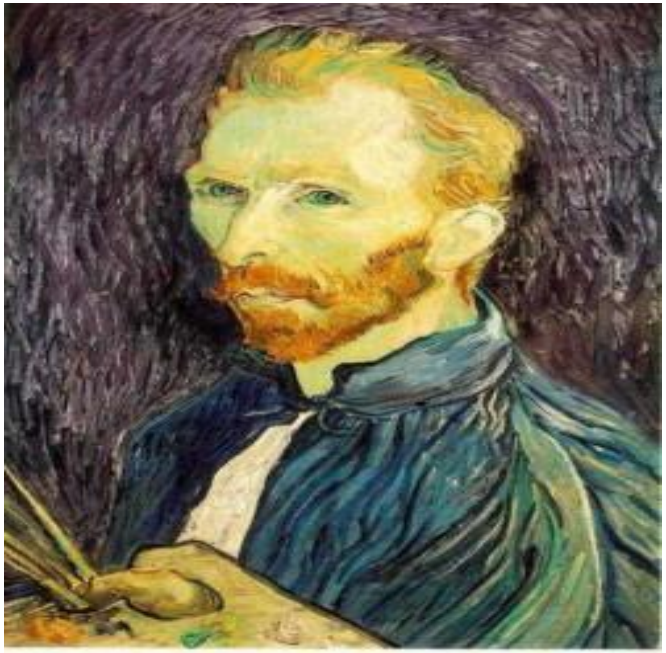


1/2

1/4 (2x zoom)

1/8 (4x zoom)

Subsampling with Gaussian pre-filtering

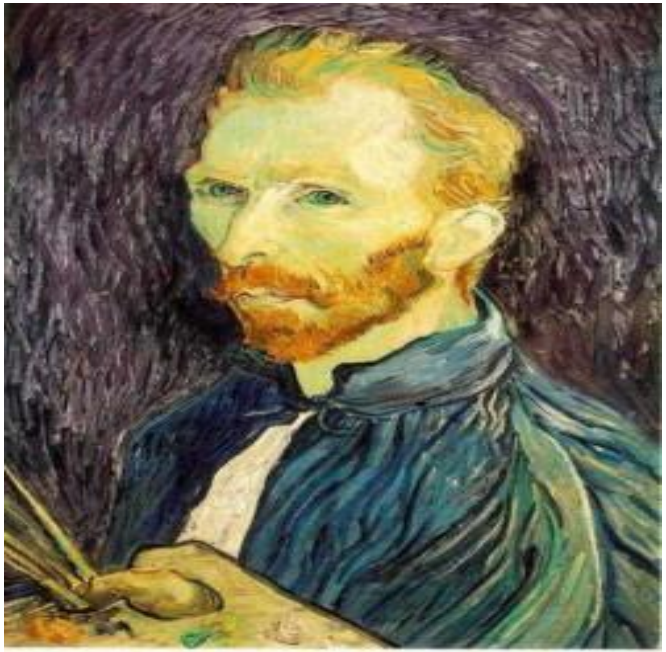


Gaussian 1/2

Solution: filter the image, then subsample

- Filter size should double for each ½ size reduction. Why?

Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4

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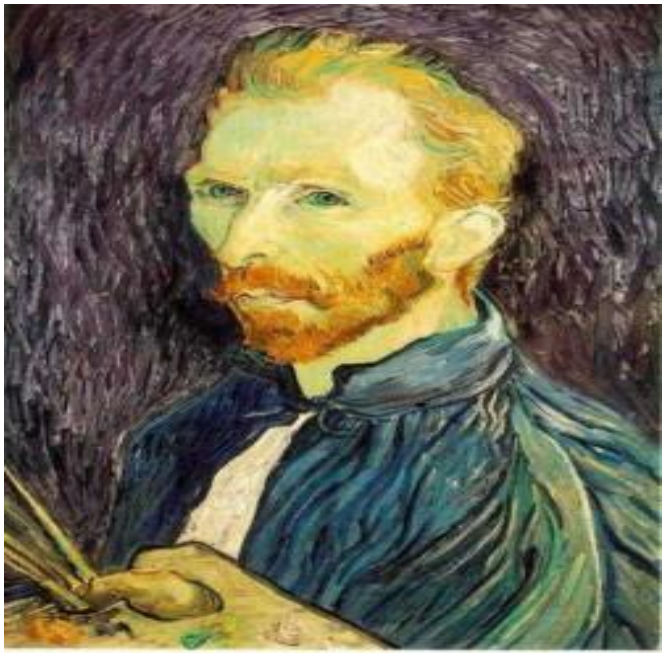

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G 1/4

G 1/8

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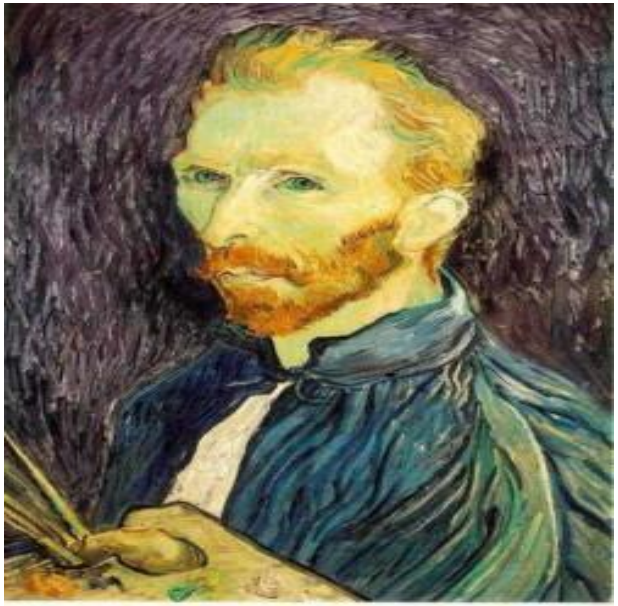
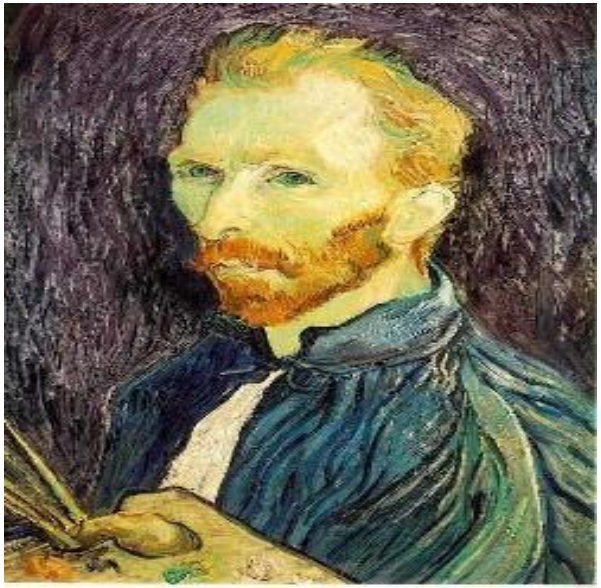


Gaussian 1/2 G 1/4 G 1/8

Solution: filter the image, then subsample

- Why does this work?

Compare with...

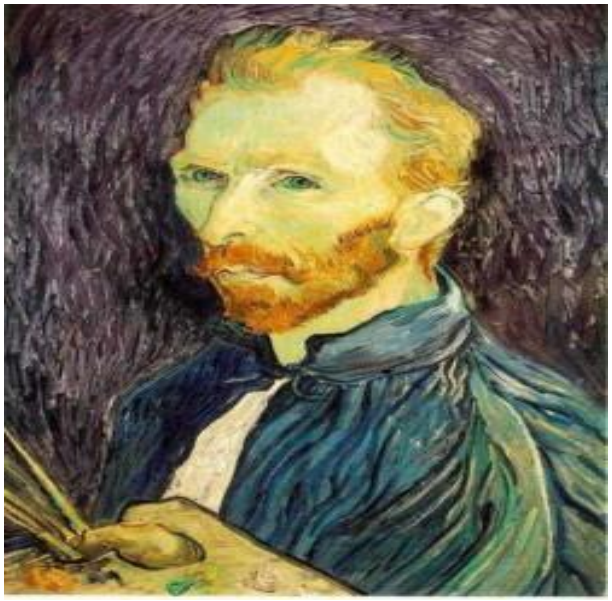


1/2

1/4 (2x zoom)

1/8 (4x zoom)

Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4

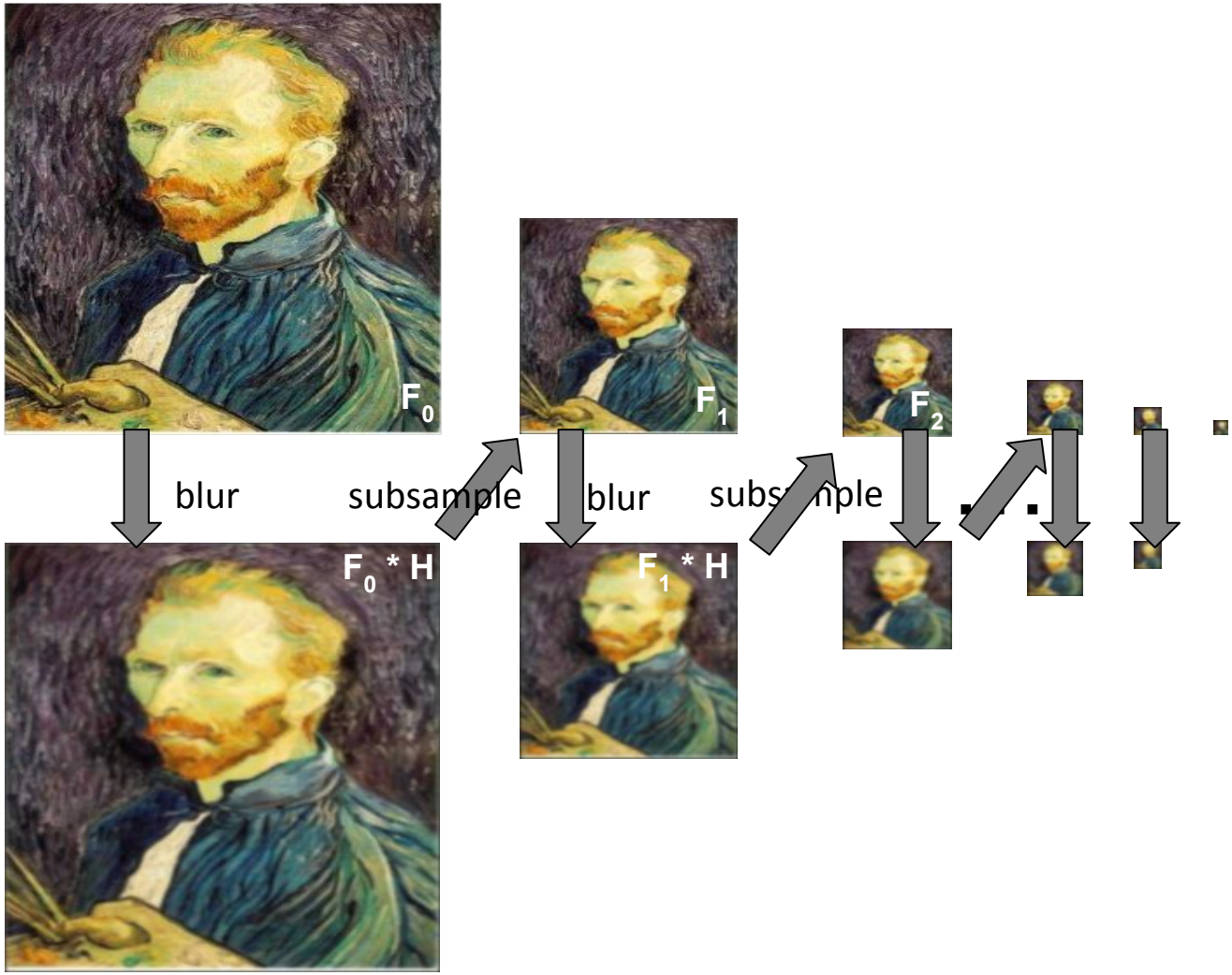
G 1/8

Solution: filter the image, then subsample

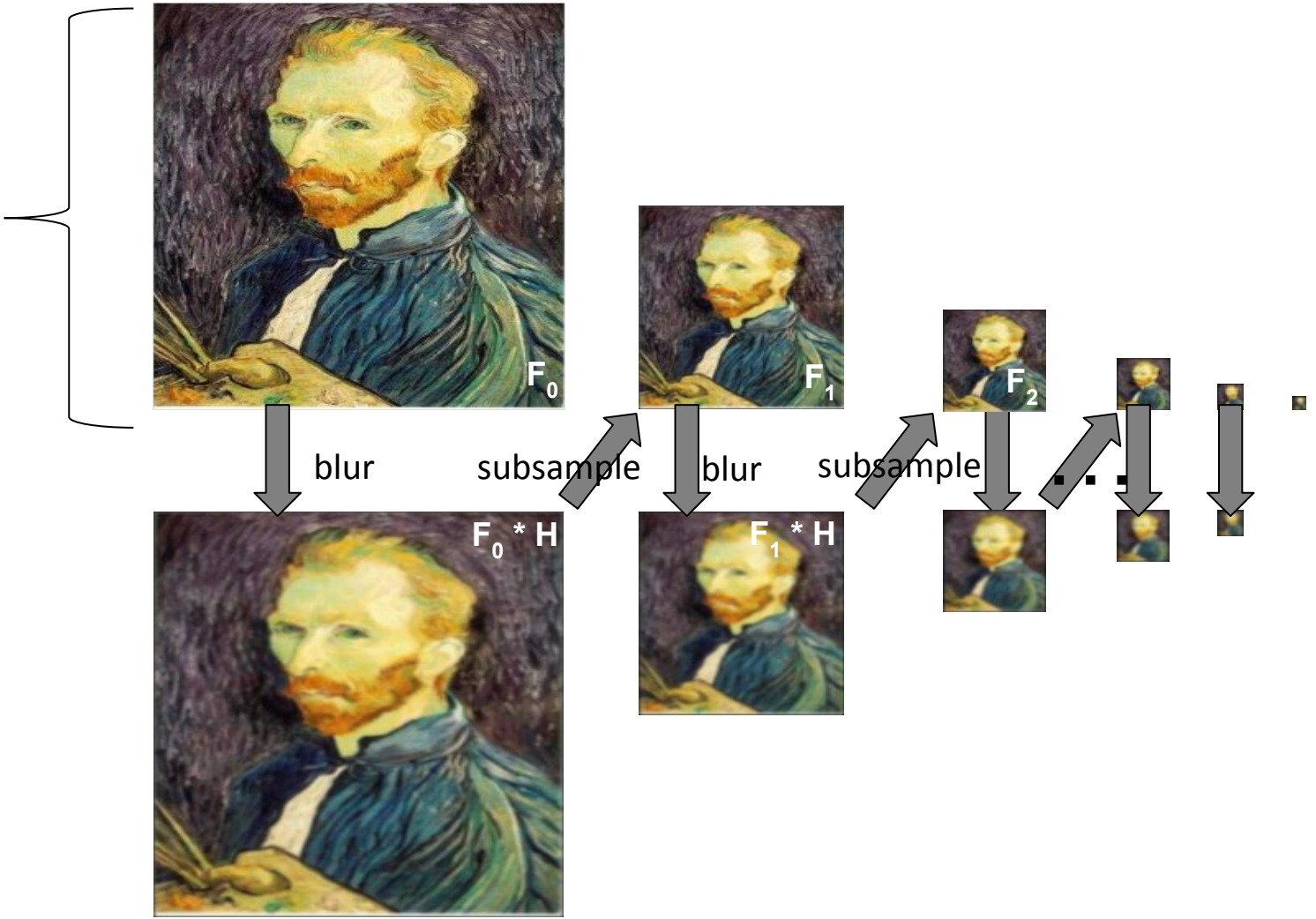
- Filter size should double for each ½ size reduction. Why?
- How can we speed this up?

Gaussian pre-filtering

- Solution: filter the image, *then* subsample



Gaussian pyramid



The Gaussian Pyramid

Low resolution

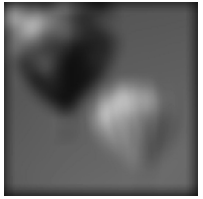
$$G_4 = (G_3 * gaussian) \downarrow 2$$

$$G_3 = (G_2 * gaussian) \downarrow 2$$

$$G_2 = (G_1 * gaussian) \downarrow 2$$

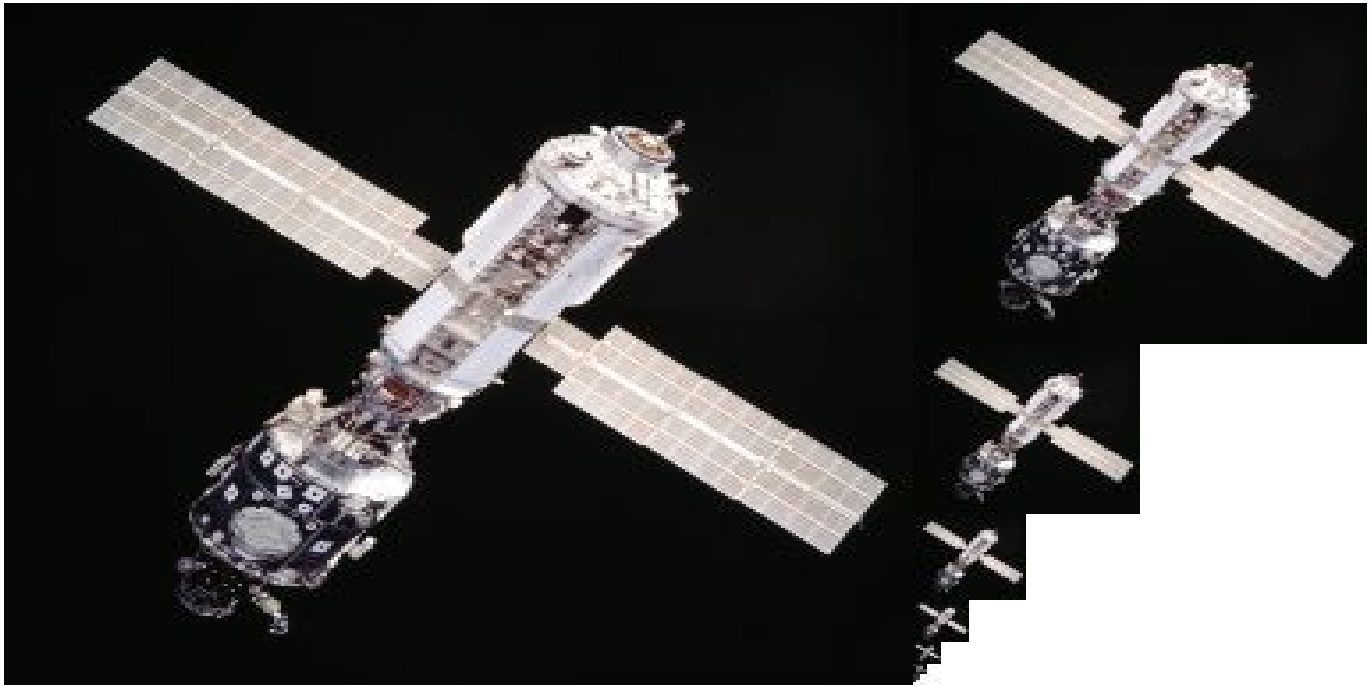
$$G_1 = (G_0 * gaussian) \downarrow 2$$

$$G_0 = \mathbf{Image}$$



High resolution

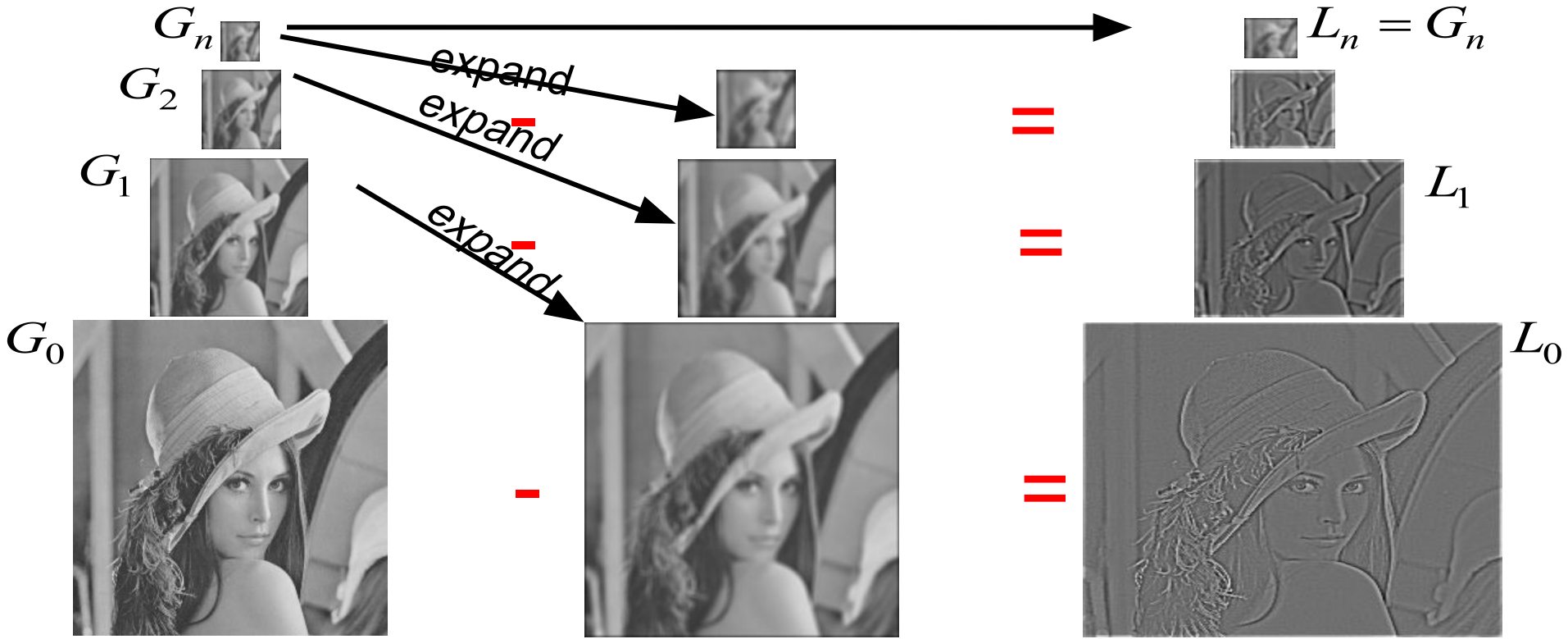
Gaussian Pyramid



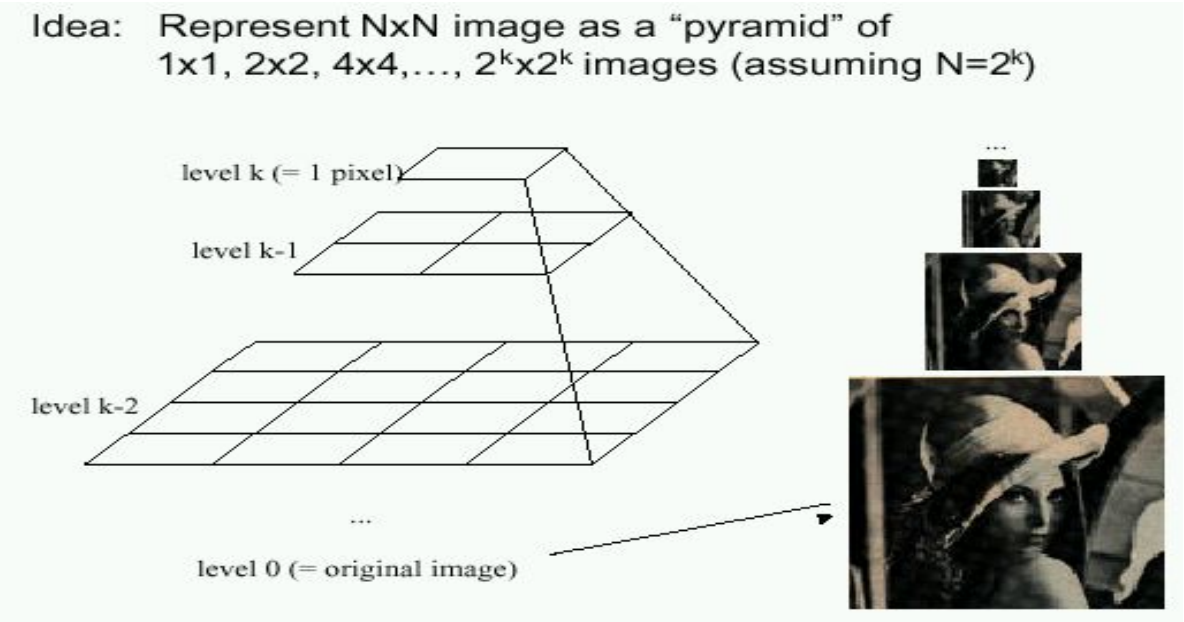
The Laplacian Pyramid

$$L_i = G_i - \text{expand}(G_{i+1})$$

Gaussian Pyramid $\quad G_i = L_i + \text{expand}(G_{i+1})$ Laplacian Pyramid



Some times we want many resolutions



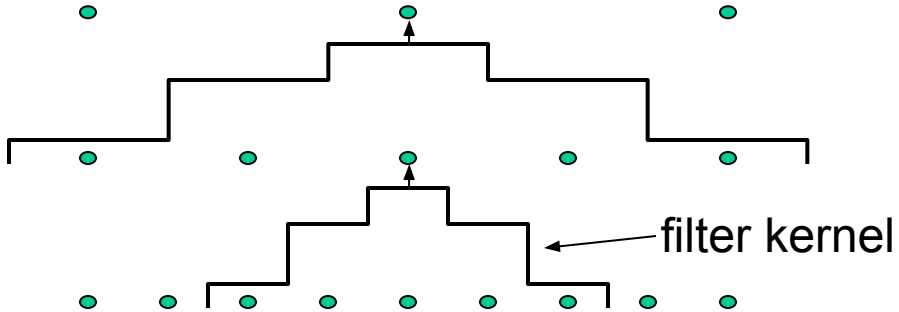
Known as a Gaussian Pyramid [Burt and Adelson, 1983]

- In computer graphics, a mip map [Williams, 1983]
- A precursor to wavelet transform

Gaussian Pyramids have all sorts of applications in computer vision

- We'll talk about these later in the course

Gaussian pyramid construction



Repeat

- Filter
- Subsample

Until minimum resolution reached

- can specify desired number of levels (e.g., 3-level pyramid)

Moire patterns in real-world images. Here are comparison images by Dave Etchells of Imaging Resource using the Canon D60 (with an antialias filter) and the Sigma SD-9 (which has no antialias filter). The bands below the fur in the image at right are the kinds of artifacts that appear in images when no antialias filter is used. Sigma chose to eliminate the filter to get more sharpness, but the resulting apparent detail may or may not reflect features in the image.

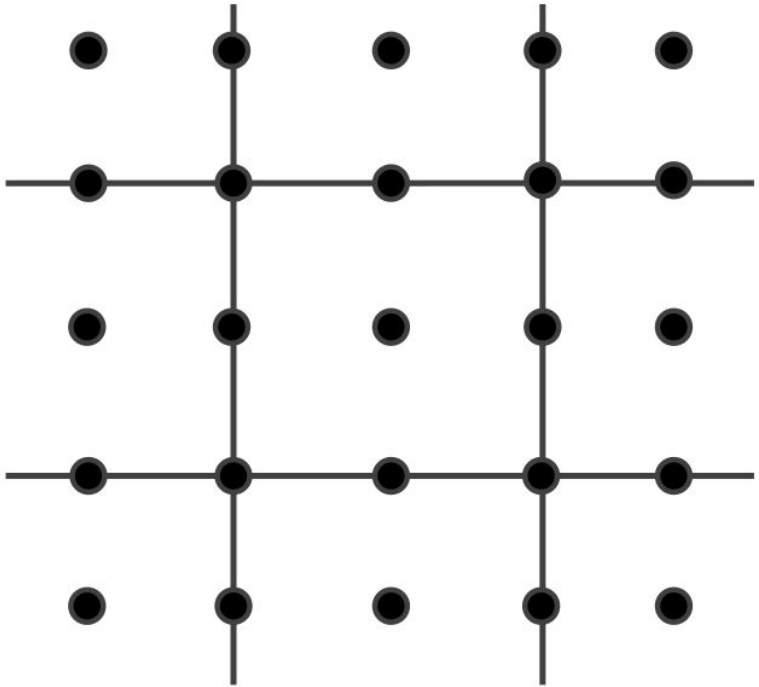
More examples

Up-sampling

How do we compute the values of pixels at fractional positions?

Up-sampling

How do we compute the values of pixels at fractional positions?



Up-sampling

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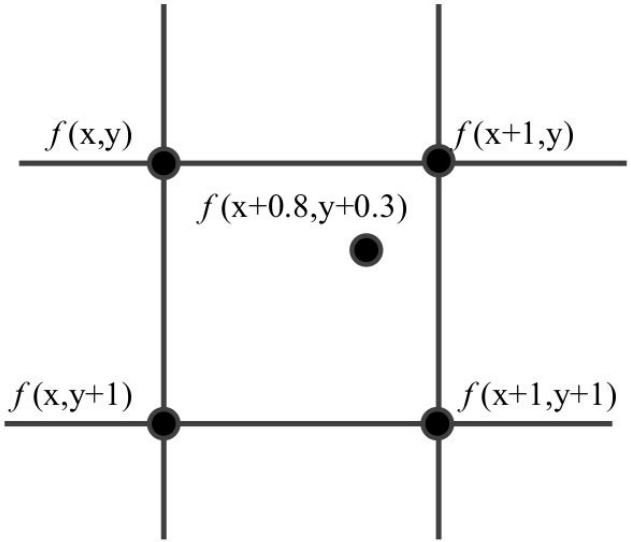
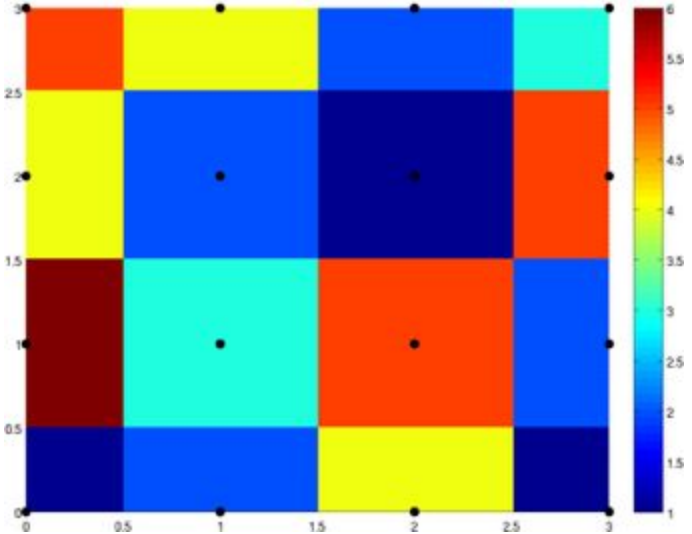


Bilinear sampling:

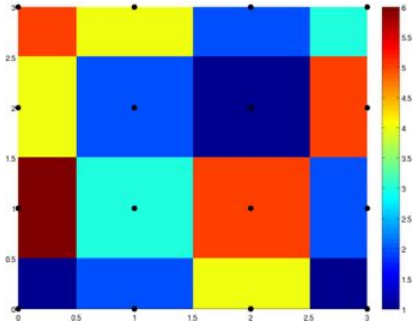
$$
\begin{aligned}
f(x + a, y + b) = \\
(1 - a)(1 - b) f(x, y) + \\
a(1 - b) f(x + 1, y) + \\
(1 - a)b f(x, y + 1) + \\
ab f(x + 1, y + 1)
\end{aligned}
$$

Bicubic sampling fits a higher order function using a larger area of support

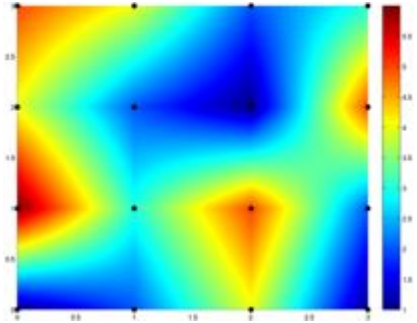
Up-sampling methods



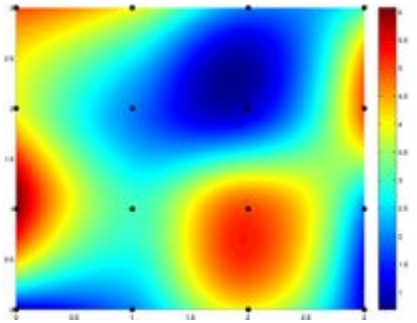
Up-sampling



nearest neighbour

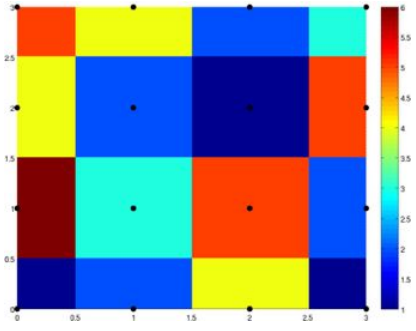


bilinear

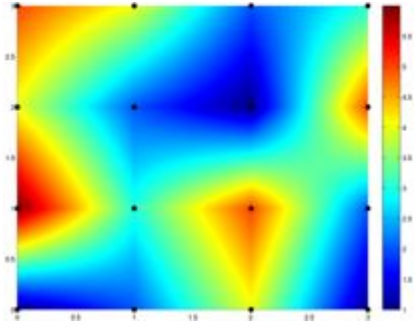


bicubic

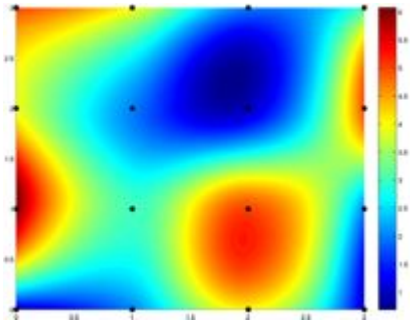
Up-sampling



nearest neighbour



bilinear



bicubic

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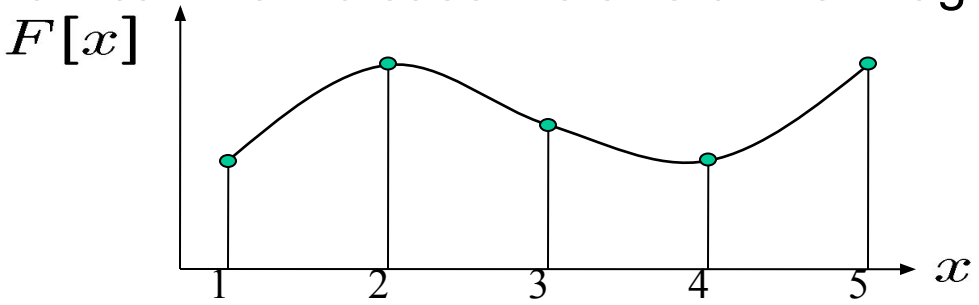
Image resampling

So far, we considered only power-of-two subsampling

- What about arbitrary scale reduction?
- How can we increase the size of the image?



d = 1 in this example

Recall how a digital image is formed

$$F[x, y] = \text{quantize}\{f(xd, yd)\}$$

- It is a discrete point-sampling of a continuous function
- If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale

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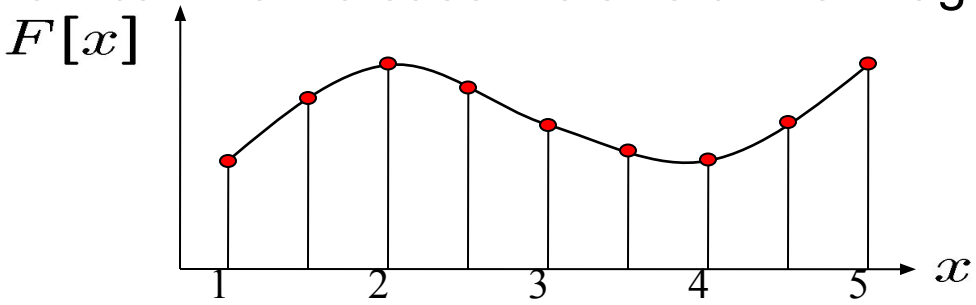

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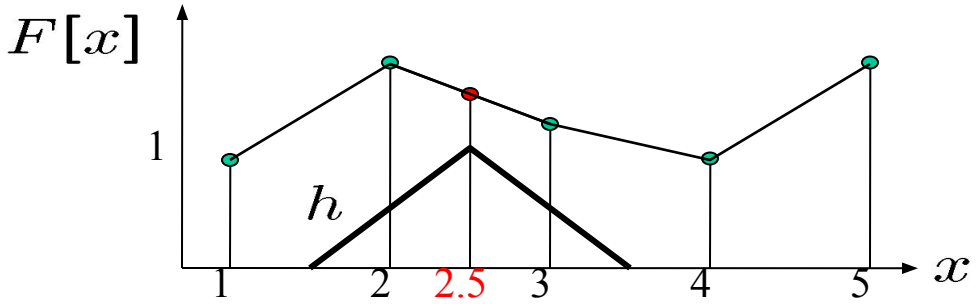
Image resampling

So what to do if we don't know f

- Answer: guess an approximation $\tilde{f}$
- Can be done in a principled way: filtering



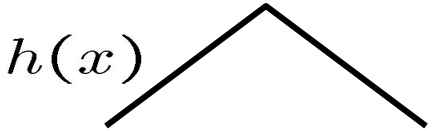
d = 1 in this example

Image reconstruction

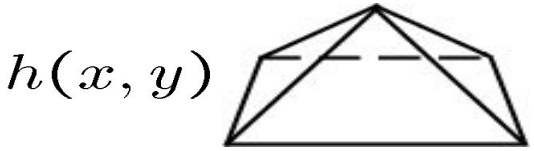
- Convert F to a continuous function
  $f_F(x) = F(\frac{x}{d})$ when $\frac{x}{d}$ is an integer, 0 otherwise
- Reconstruct by convolution: $\tilde{f} = h \otimes f_F$

Resampling filters

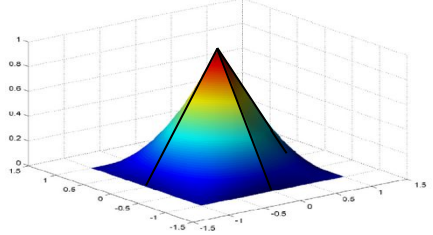
What does the 2D version of this hat function look like?



performs
linear interpolation



(tent function) performs
bilinear interpolation



Simpler implementation of bilinear interpolation

Better filters give better resampled images

- Bicubic is common choice
- fit 3rd degree polynomial surface to pixels in neighborhood
- can also be implemented by a convolution

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