Homework 2

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Question 1:

(a) Target variable: total evaluation comes from reviews

(b) $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$, where x_1 indicates the numeric score and x_2 indicates the frequency of occurrence of words that convey judgment like "bad", "good", and "doesn't work."

(c) I would change x_1 to a fraction (=numeric score/total score), instead of a numeric score.

(d) $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$, where x_1 indicates the percentage of the score, x_2 represents a rating that is simply good or bad(ex. good=1; bad=0), x_3 comes from the frequency of occurrence of some judgmental word like "bad", "good", and "doesn't work".

(e) I would use (b) as a predictor. From (b), we can see the good review rate and therefore know how customers like one product. It is meaningful for us to know the total number of reviews with the word "good".

Question 2:

(a) $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$,

(b) least square solution: $\beta = (A^T A)^{-1} A^T y = \begin{bmatrix} 0.75 \\ 2.5 \\ 3.5 \end{bmatrix}$

Minimum RSS = 0.25

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In [1]: import numpy as np

In [2]: A = [[1,0,0],[1,0,1],[1,1,0],[1,1,1]]
        y = [1,4,3,7]

In [3]: out = np.linalg.lstsq(A, y)

In [4]: beta = out[0]

In [5]: print(beta)

        [ 0.75   2.5    3.5 ]
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Question 3:

(a) $y_k = a_1 y_{k-1} + a_2 y_{k-2} + \cdots + a_M y_{k-M} + b_0 x_k + b_1 x_{k-1} + \cdots + b_N x_{k-N} + \varepsilon_k$

$$\beta = \begin{bmatrix} a_1 \\ \dots \\ a_M \\ b_0 \\ \dots \\ b_N \end{bmatrix}$$

There are M+N+1 unknown parameters.

(b)

$$y = \underbrace{\begin{bmatrix} 0 & 0 & \cdots & 0 & x_0 & 0 & 0 & \cdots & 0 \\ y_0 & 0 & \cdots & 0 & x_1 & x_0 & 0 & \cdots & 0 \\ y_1 & y_2 & \cdots & 0 & x_2 & x_1 & x_0 & \cdots & 0 \\ & & & & & & & & \\ y_{T-3} & y_{T-4} & \cdots & y_{T-M-2} & x_{T-2} & x_{T-3} & & \cdots & x_{T-N-2} \\ y_{T-2} & y_{T-3} & \cdots & y_{T-M-1} & x_{T-1} & x_{T-2} & & & x_{T-N-1} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \\ b_0 \\ b_1 \\ \vdots \\ b_N \end{bmatrix}}_{\beta} + \underbrace{\begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \vdots \\ \varepsilon_{T-1} \end{bmatrix}}_{\varepsilon}$$

(c)

$$\left(\frac{1}{T}\right) A^T A \approx$$

$$\begin{array}{c} \\ M \left\{ \begin{array}{c} \\ \\ \\ \\ \end{array} \right. \\ N+1 \left\{ \begin{array}{c} \\ \\ \\ \\ \end{array} \right. \end{array} \left[\begin{array}{ccccc|ccccc} \multicolumn{5}{c|}{\overbrace{\hspace{12em}}^{M}} & \multicolumn{5}{c}{\overbrace{\hspace{12em}}^{N+1}} \\ R_{yy}(0) & R_{yy}(1) & R_{yy}(2) & \cdots & R_{yy}(M-1) & R_{xy}(-1) & R_{xy}(0) & R_{xy}(1) & \cdots & R_{xy}(N-1) \\ R_{yy}(-1) & R_{yy}(0) & R_{yy}(1) & \cdots & R_{yy}(M-2) & R_{xy}(-2) & R_{xy}(1) & R_{xy}(0) & \cdots & R_{xy}(N-2) \\ & & & \vdots & & & & & \vdots & \\ R_{yy}(-M+1) & R_{yy}(-M+2) & R_{yy}(-M+3) & \cdots & R_{yy}(0) & R_{xy}(-M) & R_{xy}(-M+1) & R_{xy}(-M+2) & \cdots & R_{xy}(N-M) \\ \hline R_{xy}(-1) & R_{xy}(-2) & R_{xy}(-3) & \cdots & R_{xy}(-M) & R_{xx}(0) & R_{xx}(1) & R_{xx}(2) & \cdots & R_{xx}(N) \\ R_{xy}(0) & R_{xy}(-1) & R_{xy}(-2) & \cdots & R_{xy}(-M+1) & R_{xx}(-1) & R_{xx}(0) & R_{xx}(1) & \cdots & R_{xx}(N-1) \\ R_{xy}(+1) & R_{xy}(0) & R_{xy}(-1) & \cdots & R_{xy}(-M+2) & R_{xx}(-2) & R_{xx}(-1) & R_{xx}(0) & \cdots & R_{xx}(N-2) \\ & & & \vdots & & & & & \vdots & \\ R_{xy}(N-1) & R_{xy}(N-2) & R_{xy}(N-3) & \cdots & R_{xy}(N-M) & R_{xx}(-N) & R_{xx}(-N+1) & R_{xx}(-N+2) & \cdots & R_{xx}(0) \end{array} \right]$$

$$\frac{1}{T} A^T y \approx \begin{bmatrix} R_{yy}(1) \\ R_{yy}(2) \\ \vdots \\ R_{yy}(M) \\ R_{xy}(0) \\ R_{xy}(1) \\ \vdots \\ R_{xy}(N) \end{bmatrix}$$

Thus, $\frac{1}{T} A^T A$ and $\frac{1}{T} A^T y$ can be approximately computed from auto-correlation.

Question 4:

4. (a)

$$x_k = a_1 \cos(\Omega_1 k) + a_2 \cos(\Omega_2 k) + \cdots + a_\ell \cos(\Omega_\ell k) + b_1 \sin(\Omega_1 k) + b_2 \sin(\Omega_2 k) + \cdots + b_\ell \sin(\Omega_\ell k)$$

$k = 0, \dots, N-1$

$$A = \begin{bmatrix} \cos(0\cdot\Omega_1) & \cos(0\cdot\Omega_2) & \cos(0\cdot\Omega_3) \cdots & \cos(0\cdot\Omega_\ell) & \sin(0\cdot\Omega_1) & \sin(0\cdot\Omega_2) \cdots & \sin(0\cdot\Omega_\ell) \\ \cos(1\cdot\Omega_1) & \cos(1\cdot\Omega_2) & \cos(1\cdot\Omega_3) \cdots & \cos(1\cdot\Omega_\ell) & \sin(1\cdot\Omega_1) & \sin(1\cdot\Omega_2) \cdots & \sin(1\cdot\Omega_\ell) \\ \vdots & & & \vdots & & & \\ \cancel{\cos((N-1)\Omega_1)} & \cancel{\cos((N-1)\Omega_2)} & \cancel{\cos((N-1)\Omega_3)} \cdots & \cancel{\cos((N-1)\Omega_\ell)} & \cancel{\sin((N-1)\Omega_1)} & \cancel{\sin((N-1)\Omega_2)} \cdots & \cancel{\sin((N-1)\Omega_\ell)} \\ \cos((N-1)\Omega_1) & \cos((N-1)\Omega_2) & \cos((N-1)\Omega_3) \cdots & \cos((N-1)\Omega_\ell) & \sin((N-1)\Omega_1) & \sin((N-1)\Omega_2) \cdots & \sin((N-1)\Omega_\ell) \end{bmatrix}$$

$$\beta = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_\ell \\ b_1 \\ \vdots \\ b_\ell \end{bmatrix} \qquad x \approx A\cdot\beta = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix}$$

we can calculate a_ℓ and b_ℓ by computing the least-squares estimate for β,

$$\beta = (A^T A)^{-1} A^T y$$

b) ~~The problem~~ This is not a linear regression problem.