# COMP4680/COMP8650: Advanced Topics in SML

## Assignment #3: Convex Optimization and Duality

**Due:** 11:55pm on Sunday 2 September, 2018.
Submit as a single PDF file via Wattle.

All questions are of equal value.

- **Shortest path.** Consider a network with $n$ nodes. Let $c_{ij} \geq 0$ be the distance when directly going from node $i$ to node $j$ in the network. If there is no edge between $i$ and $j$ then we set $c_{ij}$ to $\infty$. We wish to find the shortest path between node 1 and node $n$. Let $x_{ij} \in \{0, 1\}$ represent whether the edge $(i, j)$ is in a path ($x_{ij} = 1$) or not ($x_{ij} = 0$). That is, if $x_{ij} = 1$ it means that the path passes through node $i$ and then node $j$. Note that not all possible assignments to $x_{ij}$ define valid paths. For example, $x_{ij} = x_{ji} = 1$ defines a loop and is therefore not a valid path.
  (a) Write down equation(s) to enforce that a path starts at node 1 and ends at node $n$. (You do not need to worry (yet) about enforcing that an assignment to all $x_{ij}$ represents a valid path.)
  (b) Write down equation(s) to enforce the constraint that a path can pass through a node a most once.
  (c) Write down equation(s) to enforce the constraint that the number of times a path goes into a node is the same as the number of times a path goes out of a node (except for nodes 1 and $n$).
  (d) Given a path defined by the $x_{ij}$, write down a function that computes the cost (i.e., length of the path).
  (e) Write down an optimization problem to find the shortest path between node 1 and node $n$.
  (f) Is your problem convex? If not, is there a way to make it convex?

The remaining questions are from the textbook *Boyd and Vandenberghe*, "Convex Optimization", 2004.

- **4.3**
- **4.8** (a), (c), (e)
- **4.9**
- **5.12**
- **5.31**