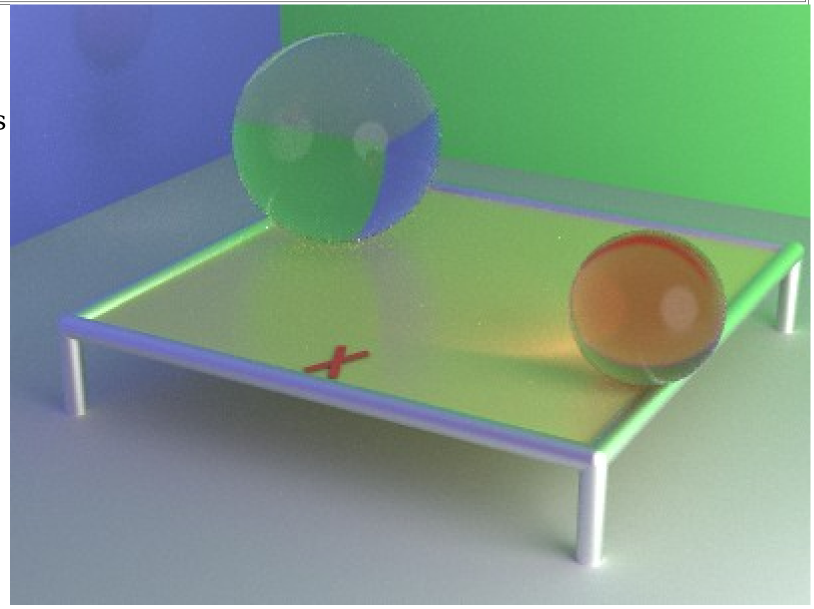
CS500 Project 4

Synopsis



Enhance the reflective micro-facet BRDF from Project 3 to include transmission across the micro-faceted surface into and through objects:

$$\frac{K_d}{\pi} + \frac{D(m)\, G(\omega_i, \omega_o, m)\, F(\omega_i \cdot m)}{4|\omega_i \cdot N||\omega_o \cdot N|}$$
$$+ \frac{D(m)\, G(\omega_i, \omega_o, m)\, (1 - F(\omega_i \cdot m))}{(\ldots)|\omega_i \cdot N||\omega_o \cdot N|}.$$

Include the following features:

- refraction (Snell's law),
- light absorption through colored medium (Beer's law), and
- Multiple Importance Sampling (MIS) weights.

Instructions

Objects may now allow light to pass through them, changing direction (refracting) whenever they cross a surface boundary. Most of the work for this project has to do with the three functions that provide the interface to the BRDF: SampleBrdf, PdfBrdf, and EvalScattering. In addition, your code must now be able to handle path rays that start on the boundary of an object, and proceed *through* the object to an intersection with the same object from the inside.

Changes to the BRDF:

Two new BRDF parameters are needed:

- An RGB value K_t which specifies the color of light after it transmits through one unit of a material (absorption calculated according to Beer's law),
- An index-of-refraction (a real number between 1 and about 1.6) which specifies how light refracts when crossing in/out of an object (refraction calculated according to Snell's law).
- The BRDF lines in the test scene now have four extra parameters, three for K_t and one for the IOR. The parser should be able to accept BRDF lines with either 7 or 11 parameters.
- It make no sense for both K_d and K_t to be non-zero for any material.

Index of Refraction

The following calculations also need η_i and η_o, the indices of refraction of the region though which ω_i and ω_o travel, respectively, as well as their ratio $\eta = \eta_i / \eta_o$. We make the assumption that surfaces with an index-of-refraction are surrounded by air (whose index-of-refraction is 1). So when a **path** enters or leaves an object, calculate η as follows:

- If $\omega_o \cdot N > 0$ (Path leaves an object, passing from inside to outside),

  $\eta_i = 1.0$

  use $\eta_o = \text{<BRDF's IOR>}$,

  $\eta = \eta_i / \eta_o$
- If $\omega_o \cdot N < 0$ (Path enters an object, passing from outside to inside),

  $\eta_i = \text{<BRDF's IOR>}$

  use $\eta_o = 1.0$.

  $\eta = \eta_i / \eta_o$

Sampling a BRDF: $\omega_i \Leftarrow \mathrm{SampleBrdf}(\omega_o, N)$

The sampling of the BRDF must now choose between three sampling strategies, **diffuse**, **reflection** and **transmission**. A reasonable set of probabilities is

$$p_d = \|K_d\|/s, \quad p_r = \|K_s\|/s, \quad p_t = \|K_t\|/s, \text{ respectively, where } s = \|K_d\| + \|K_s\| + \|K_t\|.$$

Choose **diffuse, reflection,** or **transmission** via a random number ξ:

if $\xi < p_d$: *choice*=**diffuse**

if $\xi < p_d + p_r$: *choice*=**reflection**

otherwise: *choice*=**transmission**

Sample a micro-facet around the normal N for some random numbers ξ_1, ξ_2:

If choice=**diffuse**

return $\omega_i = \mathit{SampleLobe}(N, \sqrt{\xi_1}, 2\pi\xi_2)$

If choice=**reflection,** sample a micro-facet normal in a narrow lobe around N, then reflect:

$m = \mathit{SampleLobe}(N, \cos\theta_m, 2\pi\xi_2)$

return $\omega_i = 2\,|\omega_o \cdot m|\, m - \omega_o$

where $\cos\theta_m$ depends on the $D()$ function used.

Phong BRDF: $\cos\theta_m = \xi_1^{\frac{1}{\alpha+1}}$

GGX BRDF: $\cos\theta_m = \cos\left(\tan^{-1}\left(\frac{\alpha_g\sqrt{\xi_1}}{\sqrt{1-\xi_1}}\right)\right)$

Beckman BRDF: $\cos\theta_m = \cos\left(\tan^{-1}\left(\sqrt{-\alpha_b^2\log(1-\xi_1)}\right)\right)$

If choice=**transmission,** sample a micro-facet normal in a narrow lobe around N, then refract:

$m = \ldots$ (Same as reflection's m, with the same three choices of BRDF.)

Test radicand $r = 1-\eta^2\left(1-(\omega_o\cdot m)^2\right)$.

If negative, declare **total internal reflection**, and

return the above reflection's ω_i,

else

return $\omega_i = \left(\eta\,(\omega_o\cdot m) - \mathit{sign}(\omega_o\cdot N)\sqrt{r}\right) m - \eta\,\omega_o$

(Note: The sign(x) function above is defined by **sign(x) = (x>=0) ? 1 : -1**)

PDF of the sample: $\text{PdfBrdf}(\omega_o, N, \omega_i)$

Given p_d , p_r , and p_t as the probabilities of *choice* made in SampleBRDF, calculate P_d , P_r , and P_t as the probabilities of the vector ω_i being calculated for the diffuse, reflection, and transmission directions respectively. Return the combined probability:

The **diffuse** probability calculation:

$$P_d = |\omega_i \cdot N| / \pi$$

The (specular) **reflection** probability calculation:

$$P_r = D(m)|m \cdot N| \frac{1}{4|\omega_i \cdot m|} \quad \text{where} \quad m = (\omega_o + \omega_i).normalized()$$

The **transmission** probability calculation:

$$m = -(\eta_o \omega_i + \eta_i \omega_o).normalized()$$

Test radicand $r = 1 - \eta^2 (1 - (\omega_o \cdot m)^2)$.

If negative, declare **total internal reflection,** and calculate the reflection term

$$P_t = D(m)|m \cdot N| \frac{1}{4|\omega_i \cdot m|} \quad \text{where} \quad m = (\omega_o + \omega_i).normalized()$$

else calculate the transmission term

$$P_t = D(m)|m \cdot N| \frac{\eta_o^2 |\omega_i \cdot m|}{(\eta_o(\omega_i \cdot m) + \eta_i(\omega_o \cdot m))^2}$$

Return $p_d P_d + p_r P_r + p_t P_t$

Eval a micro-facet BRDF: $\text{EvalScattering}(\omega_o, N, \omega_i)$

The BRDF is now the sum of the diffuse, specular, and transmission terms. Note: If a surface does not have one of those modes of light interaction, do not do the corresponding calculation, and do not include it in the returned sum. Use p_d , p_r , and p_t to make this determination.

Diffuse: $E_d = K_d / \pi$,

Specular: $E_r = \dfrac{D(m)\, G(\omega_i, \omega_o, m)\, F(\omega_i \cdot m)}{4|\omega_i \cdot N||\omega_o \cdot N|}$ where $m = (\omega_o + \omega_i).normalized()$

Transmission:

If doing Beer's law, calculate the attenuation factor $A(t)$ here, If not, then $A(t)=1$

$$m = -(\eta_o \omega_i + \eta_i \omega_o).normalized()$$

Test radicand $r = 1 - \eta^2 (1 - (\omega_o \cdot m)^2)$.

If negative, declare **total internal reflection,** and calculate the reflection term

$$E_t = A(t) \frac{D(m)\, G(\omega_i, \omega_o, m)\, F(\omega_i \cdot m)}{4|\omega_i \cdot N||\omega_o \cdot N|} \quad \text{where} \quad m = (\omega_o + \omega_i).normalized()$$

else calculate the transmission term

$$E_t = A(t) \frac{D(m)\, G(\omega_i, \omega_o, m)\, (1 - F(\omega_i \cdot m))}{|\omega_i \cdot N||\omega_o \cdot N|} \frac{|\omega_i \cdot m|\, |\omega_o \cdot m|\, \eta_o^2}{(\eta_o(\omega_i \cdot m) + \eta_i(\omega_o \cdot m))^2}$$

Return $|N \cdot \omega_i| \, (E_d + E_r + E_t)$

Beer's law:

In the above transmission cases (both TIR and not) the amount of light being transmitted through the object must be *attenuated* (i.e., multiplied by a factor $A(t)<1$) to account for absorption while passing through the object. Beer's law, in general, states that a light ray attenuates by a factor of $A(t)=e^{-ct}$ after traveling a distance t through the object, where c is a parameter controlling the amount of attenuation. An easy way to specify c is to let $A(1)=K_t$ be the color of unit light after traveling one unit through the object. Then $K_t=e^{-c}$ so let $c=-\log_e K_t$, and so light traveling through a distance of t is attenuated by a factor of $A(t)=e^{-t\log_e K_t}$.

In the choice=**transmission** case of EvalBRDF calculate the attenuation factor

If $\omega_o \cdot N<0$:

$A(t)=e^{t\log_e K_t}$ (Path has intersected the inside of its surface at a distance t along ray)

else

$A(t)=1$

and apply this attenuation factor $A(t)$ to both return cases (TIR or not TIR) of **transmission** section of EvalBRDF.

MIS weights

Any path which adds light to the final pixel total may adjust that light by a weight (the *multiple importance sampling* weight) which accounts for the probability which generated the path compared to the probability of any other ways that path could have been generated.

In both the "Explicit" and "Extend/Implicit" sections of the basic algorithm (from project 3), find the calculation of p, and add a new calculation of an extra probability q, and weight $w_{mis}=p^2/(p^2+q^2)$, and modify the summation "**C += ...**" line with that w_{mis} weight. In each case the new weight q is a probability calculation from the other case.

// Explicit light connection

L = SampleLight() // Randomly choose a light and a point on that light.

p = PdfLight(L)/GeometryFactor(P,L) // Probability of L, converted to angular measure

q = PdfBrdf (ω_o, N, ω_i) * RussianRoulette // Prob the explicit light could be chosen implicitly

$w_{mis}=p^2/(p^2+q^2)$

ω_i = direction from P toward L

I = Trace ray from P toward L // Sometimes called a shadow-ray

if p>0 and I exists and is the chosen point on the chosen light:

  f = EvalScattering(ω_o, N, ω_i)

  C += W * w_{mis} *f/p * EvalRadiance(L)

// Extend path

ω_i = SampleBrdf (ω_o, N) // Choose a sample direction from P

Q = Trace ray from P in direction ω_i into the scene

if Q is non-existent: **break**

f = EvalScattering(ω_o, N, ω_i)

p = PdfBrdf (ω_o, N, ω_i) * RussianRoulette

if p < ϵ : break // Avoid division by zero or nearly zero: $\epsilon=10^{-6}$

W *= f/p

// Implicit light connection

if Q is a light:

  q = PdfLight(Q)/GeometryFactor(P,Q) // Prob the implicit light could be chosen explicitly

  $w_{mis}=p^2/(p^2+q^2)$

  C += W * w_{mis} * Radiance(Q)

  break

Microfacet Models for Refraction through Rough Surfaces

https://faculty.digipen.edu/~gherron/references/References/BRDF/EGSR07-btdf.pdf

All microfacet BRDFs have this general form:

$$\frac{K_d}{\pi} + \frac{D(m)\, G(\omega_i, \omega_o, m)\, F(\omega_i \cdot m)}{4|\omega_i \cdot N||\omega_o \cdot N|}$$

Where

- $m = (\omega_o + \omega_i)/||\omega_o + \omega_i||$ is the half-vector.
- K_d is the diffuse reflection (albedo) of the surface
- K_s is the specular reflection in the $\omega_i = \omega_o = N$ direction,
- α_p, α_b, α_g are the surface roughness values for **Phong**, **Beckman** and **GGX** respectively.

The paper lists three common versions for **D** and **G: Phong**, **Beckman** and **GGX**.
The easiest is **Phong**, but recently, graphics has been trending toward **GGX**.

Characteristic factor:

The so called characteristic function in the **D** and **G** factors below is defined as

$$\chi^+(d) = \begin{cases} 1 & \text{if } d > 0 \\ 0 & \text{if } d \le 0 \end{cases}$$

and implemented with a simple "if" statement.

F factor

F is the Fresnel (reflection) is usually approximated by Schlick as

$$F(d) = K_s + (1 - K_s)(1 - |d|)^5$$

where K_s is the specular reflection color at $L = V = N = H$.

The exact formulation (if you are interested) is

$$F(L, H) = \frac{1}{2} \frac{(g-c)^2}{(g+c)^2} \left(1 + \frac{(c(g+c)-1)^2}{(c(g-c)+1)^2}\right)$$

where

$$g = \sqrt{\frac{\eta_t^2}{\eta_i^2} - 1 + c^2}$$

and

$$c = |L \cdot H|$$

and η_i and η_t are indices of refraction of the two materials.

D factor

D is the micro-facet distribution.

In the following, $\tan\theta_m = \sqrt{(1.0-(m\cdot N)^2)} / (m\cdot N)$ and $\tan\theta_v = \sqrt{(1.0-(v\cdot N)^2)} / (v\cdot N)$.

Phong: $$D_p(m) \ = \ \chi^+(m\cdot N) \ \frac{\alpha_p+2}{2\pi}(m\cdot N)^{\alpha_p}$$

(α_p : $1..\infty$; increasing means smoother surface)

Beckman: $$D_b(m) = \chi^+(m\cdot N) \ \frac{1}{\pi\,\alpha_b^2 (N\cdot m)^4} \ e^{\frac{-\tan^2\theta_m}{\alpha_b^2}}$$

(α_b : $0..1$; increasing means rougher surface)

similar to Phong for smooth surfaces using $\alpha_p = 2\alpha_b^{-2} - 2$

GGX: $$D_g(m) = \chi^+(m\cdot N) \ \frac{\alpha_g^2}{\pi (N\cdot m)^4 (\alpha_g^2 + \tan^2\theta_m)^2}$$

G factor:

The G term should be calculated via the smith method:

$$G(\omega_i, \omega_o, m) = G_1(\omega_i, m)\ G_1(\omega_o, m)$$

where $G_1(...)$:

Beckman:

$$G_1(v, m) = \chi^+\left(\frac{v\cdot m}{v\cdot N}\right) \begin{cases} \dfrac{3.535a + 2.181a^2}{1.0 + 2.276a + 2.577a^2} & \text{if } a < 1.6 \\ 1 & \text{otherwise} \end{cases}$$

where $a = 1/(\alpha_b \tan\theta_v)$, and $\tan\theta_v$ is defined above.

Phong:

Same G_1 as **Beckman**, but with $a = (\sqrt{\alpha_p/2+1}) / \tan\theta_v$.

GGX:

$$G_1(v, m) = \chi^+\left(\frac{v\cdot m}{v\cdot N}\right) \frac{2}{1 \ + \ \sqrt{1+\alpha_g^2 \tan^2\theta_v}}$$

Beware round off errors in calculating the G_1 function:

- The value of $(v\cdot N)$ may round up to greater than 1.0 (mathematically it shouldn't, but computationally it sometimes does). If so, return $G_1(...) = 1.0$.
- The calculation of $\tan\theta_v$ may be zero. If so, don't divide by it, instead return $G_1(...) = 1.0$.