Multi-robot Rendezvous in The Plane

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Multi-robot Rendezvous in The Plane

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Abstract — *In this paper, we discuss a multi-robot rendezvous problem. The aim is to design decentralized control laws for N robots moving in the horizontal plane in order to reach an unpredictably moving point at the same time. The rendezvous problem is modeled using the relative kinematics equations. Our control laws are derived using the integration of geometrical rules with the relative kinematics model. Two approaches are used, namely reference–robot and leader–follower. In the first approach the robots move independently from each other, but they depend on the motion of the reference trajectory. In the second approach the motion of each following robot depends only on its leader, and the motion of one leader depends on the motion of the reference trajectory. Each follower has one leader. For both approaches, two control laws are derived for each robot. Our strategy is illustrated using various simulation examples.*

Keywords: Multi-robot navigation, leader-follower, rendezvous.

1 Introduction

Multi-robot cooperation and coordination is an important topic in robotics. Different aspects of multi-robot systems have been intensively studied over the last decade. Multi-robot systems are important for various applications, for they can accomplish tasks that are difficult or time consuming for a single robot. Different methodologies have been suggested to achieve different cooperative tasks. Multi-robot systems can be controlled using centralized and decentralized strategies. These strategies use behavioral or model-based controls. Multi-robot formations based on behavior-based strategies are suggested in ([1, 2, 3]), where it is shown that behavior-based methods allow robots to navigate, avoid hazards, and keep formation at the same time. The problem of multi-robot formations control was also considered using model-based strategies. In ([4, 5]) geometric approaches are used to generate smooth trajectories for the robots in the formation. Geometric methods are robust in general. Robots formation using graph theory was also studied, where decentralized control laws are used to control the robots in a leader-follower approach. Note that graph theoretic approaches suffer from the expensive computational cost which appears mainly in large formations. Cooperative hunting behavior for a group of mobile robots was addressed in [7]. The method suggested by the author is model-based, where a formation vector is defined for each robot, and the formation is controlled through this formation vector. The same strategy is used for robot formation in [8]. Navigation strategies for multiple autonomous robots are addressed in ([9, 10, 11]), where a line of sight communication strategy is used in [11]. Cooperative multi-robot tracking and observation of a moving target was discussed in ([12, 13, 14]). Various sensors ranging from directional sensors to visual sensors are used.

In this paper, we discuss a multi-robot navigation problem, where the aim is to control a group of robots in order to reach an unpredictably moving point at the same time. We call this problem a rendezvous problem. This terminology is different from the terminology used in [15], where the word rendezvous is used for an optimal control problem for minimum control energy needed to reach a target. Our strategy is model-based, and is based on the integration of the kinematics equations with geometric rules. For robots navigation, we suggest the use of the deviated pursuit, which is a closed loop control ([16, 17]). Two approaches called reference–robot and leader–follower are developed. Our method is illustrated using an extensive simulation.

2 Statement of the problem

Given N wheeled mobile robots moving in the horizontal plane, the aim is to derive control laws that allow the robots to reach a moving point at the same time. The motion of the moving point is not a priori known to the robots. Otherwise, the solution of the problem becomes quite simple. In this paper, the moving point is assumed to be a simple robot, called the reference robot. We have the following assumptions:

1. The path traveled by the reference robot is continuous.

2. The robots have a communication system or a sensory system that allows them to determine the position of the reference robot. In the leader–follower approach, each following robot is assumed to be able to measure the position of its leader. This information can be obtained using a communication system as well.

3. All robots are faster than the reference robot. In the leader-follower approach, it is assumed that a following robot is faster than its leader.

In the next section, we discuss the geometry and we derive a set of kinematics equations in order to provide a mathematical model of the rendezvous problem.

3 Robots modeling, kinematics, and geometry

The N holonomic wheeled mobile robots are moving in the Cartesian frame of reference according to the following kinematics equations

$$\begin{aligned} \dot{x}_i &= v_i \cos\theta_i \\ \dot{y}_i &= v_i \sin\theta_i \\ i &= 1, ..., N \end{aligned} \tag{1}$$

where (x_i, y_i) represents of the position of robot R_i; v_i is its linear velocity and θ_i is its orientation angle. The reference robot moves in the Cartesian frame of reference under the following equations

$$\begin{aligned} \dot{x}_r &= v_r \cos\theta_r \\ \dot{y}_r &= v_r \sin\theta_r \end{aligned} \tag{2}$$

where (x_r, y_r) represents of the position of reference robot denoted by R_r, v_r is its linear velocity and θ_r is its orientation angle. The motion of the reference robot is not a priori known to the other robots. Our aim is to design a control strategy which allows the robots to reach the reference robot at the same time. Our solution is based on the integration of geometric rules with the kinematics equations. The navigation problem is modeled using the kinematics equations in polar coordinates. Consider the following change of variable

$$\begin{aligned} x &= r\cos\lambda \\ y &= r\sin\lambda \end{aligned} \tag{3}$$

where r and λ are the radial and angular variables of the polar representation, respectively. By taking the time derivative of r and λ, and using systems (3) we get

$$\begin{aligned} \dot{r}_i &= v_i \cos(\theta_i - \lambda_i) \\ r_i\dot{\lambda}_i &= v_i \sin(\theta_i - \lambda_i) \\ i &= 1, ..., N \end{aligned}$$

For the reference robot, we have

$$\begin{aligned} \dot{r}_r &= v_r \cos(\theta_r - \lambda_r) \\ r_r\dot{\lambda}_r &= v_r \sin(\theta_r - \lambda_r) \end{aligned}$$

Figures 1 and 2 show important geometric variables. In figure 1, we have the following variables

(a) The imaginary straight line that starts at robot R_i $(i = 1, ..., N)$ and is directed towards the reference robot is called the line of sight between R_i and R_r. This line is denoted by $\vec{L}_{ri}$.

(b) The angle λ_{ri} $(i = 1, ..., N)$ is the angle between the reference line (parallel to the x-axis) and the line of sight $\vec{L}_{ri}$. This angle is called the angle of the line of sight between R_i and R_r.

In figure 2, we have

(c) The imaginary straight line that starts at robot R_i and is directed towards robot R_j, $(i \neq j)$ is called the line of sight between R_i and R_j. This line is denoted by $\vec{L}_{ji}$.

(d) The angle λ_{ji} is the angle between the reference line and the line of sight $\vec{L}_{ji}$. This angle is called the angle of the line of sight between R_i and R_j.

The difference between $\left(\vec{L}_{ri}, \lambda_{ri}\right)$ and $\left(\vec{L}_{ji}, \lambda_{ji}\right)$ is that $\left(\vec{L}_{ri}, \lambda_{ri}\right)$ relates each robot in the workspace with the reference robot while $\left(\vec{L}_{ji}, \lambda_{ji}\right)$ relates two different robots in the context of leader-follower. Thus, the relationship between robots R_j and R_i is a leader-follower relationship. For example in figure 2, R_k is the leader of both R_1 and R_2. Consider the relative velocity vector between robot R_i and the reference robot given by

$$\vec{v}_{ri} = \vec{v}_r - \vec{v}_i \tag{4}$$

$\vec{v}_{ri}$ is decomposed into two components along and across the line of sight $\vec{L}_{ri}$. We have

$$\vec{v}_{ri} = v_{ri}^{\parallel}\vec{u}_{\parallel} - v_{ri}^{\perp}\vec{u}_{\perp} \tag{5}$$

where $\vec{u}_{\parallel}$ and $\vec{u}_{\perp}$ are the unit vectors along and across the line of sight $\vec{L}_{ri}$. The values of $v_{ri}^{\parallel}$ and $v_{ri}^{\perp}$ are given by

$$\begin{aligned} v_{ri}^{\parallel} &= \dot{r}_{ri} = v_r \cos(\theta_r - \lambda_{ri}) - v_i \cos(\theta_i - \lambda_{ri}) \\ v_{ri}^{\perp} &= r_{ri}\dot{\lambda}_{ri} = v_r \sin(\theta_r - \lambda_{ri}) - v_i \sin(\theta_i - \lambda_{ri}) \\ i &= 1, ..., N \end{aligned} \tag{6}$$

System (6) models the motion of the reference robot seen by robot R_i. The first equation gives the relative range and the second equation gives the rate of turn of the reference robot with respect to R_i. $(i = 1, ..., N)$. In a similar way, the relative velocity between two different robots is given by

$$\vec{v}_{ji} = \vec{v}_j - \vec{v}_i \tag{7}$$

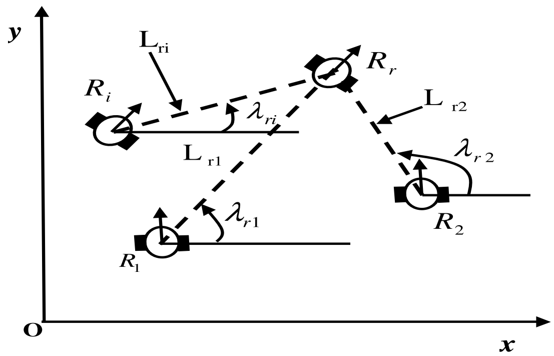

Figure 1. Geometry of rendezvous problem, reference–robot approach

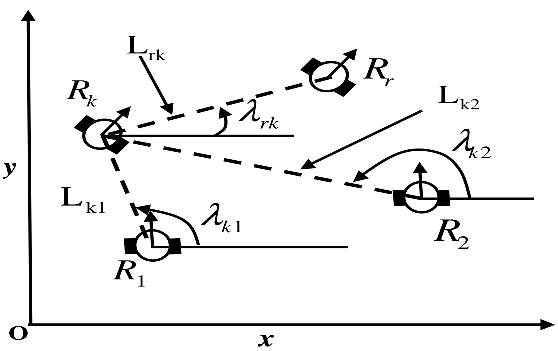

Figure 2. Geometry of rendezvous problem, leader-follower approach

Using the same reasoning, we get the following relative model

$$\begin{array}{l} v_{ji}^{\parallel} = \dot{r}_{ji} = v_j \cos\left(\theta_j - \lambda_{ji}\right) - v_i \cos\left(\theta_i - \lambda_{ji}\right) \\ v_{ji}^{\perp} = r_{ji}\dot{\lambda}_{ji} = v_j \sin\left(\theta_j - \lambda_{ji}\right) - v_i \sin\left(\theta_i - \lambda_{ji}\right) \\ i \neq j \end{array} \quad (8)$$

Systems (6) and (8) represent a set of differential equations, which model the rendezvous problem. As we mentioned previously, we use two different approaches, reference–robot and leader–follower. In both cases, the control laws are decentralized.

3.1 Reference–robot approach

This approach uses the kinematics model given in (6). Note that (6) is a set of N systems of two differential equations. In this approach, the trajectory of each robot is a direct function of the trajectory of the reference robot, and it is completely independent from the trajectory of the other robots.

3.2 Leader–follower approach

This approach uses the kinematics model given in (8). The trajectory of one robot (say R_k) depends on the trajectory of the reference robot. The trajectory of the other robots is generated based on the a leader-follower approach.

4 Decentralized deviated pursuit control laws

Our aim in this section is to derive control laws for the robots to accomplish the task of rendezvous in the absence of obstacles. We use the deviated pursuit control law in a decentralized fashion. The deviated pursuit is a closed loop control law ([16, 17]) based on geometric rules. This law is well known for its robustness under uncertainties. The control strategy is discussed as follows.

4.1 Reference–robot

The aim here is to make the angular velocity of the robots equal to the rate of turn of the line of sight angle between each robot and the reference robot. The orientation angle of robot R_i is a function of the angle of the line of sight between R_i and R_r as follows

$$\theta_i = \lambda_{ri} - \gamma_{0i} \quad (9)$$

where γ_{0i} is a constant deviation angle. Recall that λ_{ri} depends on the positions of R_i and R_r in the Cartesian frame of reference.

In order to reach the moving point (represented as a reference robot) by robot R_i, the deviation angle of robot R_i must satisfy

$$\gamma_{0i} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \quad (10)$$

The detailed proof is beyond the scope of this paper. But we provide a sketch of the proof. The proof can be achieved as follows

1. Write the range rate equation under the velocity pursuit control law, which gives
$$\dot{r}_{ri} = v_r \cos\left(\theta_r - \lambda_{ri}\right) - v_i \cos\left(\gamma_{0i}\right) \quad (11)$$
2. It is possible to rigorously prove that under the deviated pursuit law, we have
$$\lambda_{ri} \to \theta_r - \sin^{-1}\left(\frac{v_i \sin\gamma_{0i}}{v_r}\right) \quad (12)$$
3. Thus, the rate range becomes
$$\dot{r}_{ri} = v_r \cos\left(\sin^{-1}\left(\frac{v_i \sin\gamma_{0i}}{v_r}\right)\right) - v_i \cos\left(\gamma_{0i}\right) \quad (13)$$
Equation (13) shows that $\dot{r}_{ri} < 0$, when $v_i > v_r$ and $\gamma_{0i} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

4.2 Leader–follower

In this case, robot R_k tracks the reference robot using the deviated pursuit control law. The other robots are controlled in a leader-follower fashion. The control law for R_k is given by

$$\theta_k = \lambda_{rk} - \gamma_{0k} \tag{14}$$

When robot R_i tracks robot R_j, the control input for its orientation angle becomes

$$\theta_i = \lambda_{ji} - \gamma_{0i} \tag{15}$$

In this case robot R_j is the leader and robot R_i is the follower. Each follower has only one leader however a leader may have more than one follower. Note that robot R_k must be among the leader robots in equation (15). The proof that every following robot moving under the velocity pursuit reaches its leader robot and thus, the reference robot is quite similar to the previous case. Equation (10) must be satisfied in this case also. The leader-follower approach presents a local strategy. However, the reference–robot approach is simpler in general.

4.3 Speed Command

Control laws given in equations (10) and (15) guarantee that the robots reach the moving point at a given time. In the second stage, the aim is to control the robots in the linear velocity in order to reach the reference point at the same time. Our strategy is based on the range rate equation in the kinematics model. The range rate under the velocity pursuit control law is as follows

$$\dot{r}_{ri} = v_r \cos(\theta_r - \lambda_{ri}) - v_i \cos(\gamma_{0i}) \tag{16}$$

Let t_f be the final time and $r_{0i} = r_{ri}(t_0)$ the initial distance between the reference robot and robot R_i. We write the range rate as follows

$$\dot{r}_{ri} = -K_i(t) \tag{17}$$

where K_i is always positive. Therefore, the relative range is a decreasing function. The solution for this equation is given by

$$r_{ri}(t) = -\int_{t_0}^{t_f} K_i dt + r_{0i} \tag{18}$$

Robot R_i reaches the reference robot at time t_f when $r_{ri}(t_f) = 0$, and thus

$$\int_{t_0}^{t_f} K_i dt = r_{0i} \tag{19}$$

Function $K_i(t)$ is a slope representing the decreasing rate of the relative range. The goal is to choose K_i by taking into account the initial and the instantaneous values of the relative range. The control law for the relative velocity is given by

$$v_i = \frac{K_i(t) + v_r \cos(\theta_r - \lambda_{ri})}{\cos(\gamma_{0i})} \tag{20}$$

There exist various possibilities to choose $K_i(t)$. Details will be given in future work. In our simulation K_i is proportional to the relative range.

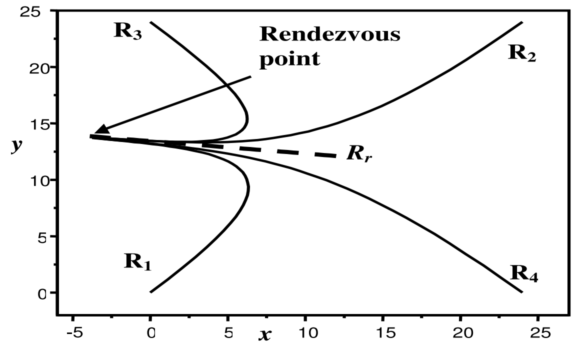

Figure 3. Rendezvous in the plane using the reference–robot approach. The reference robot (path in dashed line) is moving with constant orientation angle

5 Simulation

Here, we illustrate our control strategies using simulation examples. We consider 4 examples to illustrate both approaches. We use the deviated pursuit control law with $\alpha_{0i} = 0$ to control the orientation angle of the robots. The robots are modeled as geometric points.

Example 1: This example uses the reference–robot strategy. The initial positions of the robots are $R_1(0,0)$, $R_2(24,24)$, $R_3(0,24)$, $R_4(24,0)$. The reference robot is moving with a constant speed $v_r = 4m/s$, and constant orientation angle. The paths traveled by the robots are shown in figure 3. The velocity profile of the robots is shown in figure 4. Clearly, the velocity of R_i tracks the velocity of the reference robot. The velocity of R_i vary according to the distance r_{ri}. For this reason, R_2 and R_4 move faster than R_1 and R_3.

Example 2:

This example also uses the reference–robot strategy. The initial positions of the robots are $R_1(0,0)$, $R_2(24,0)$, $R_3(0,24)$, $R_4(12,0)$. The reference robot is moving with time varying orientation angle. The paths traveled by the robots are shown in figure 5. The time variation of the orientation angles for R_i and R_r is shown in figure 6, where it can be seen that $\theta_i \to \theta_r$.

Example 3: This example uses the leader-follower strategy. The initial positions of the robots are $R_1(0,0)$,

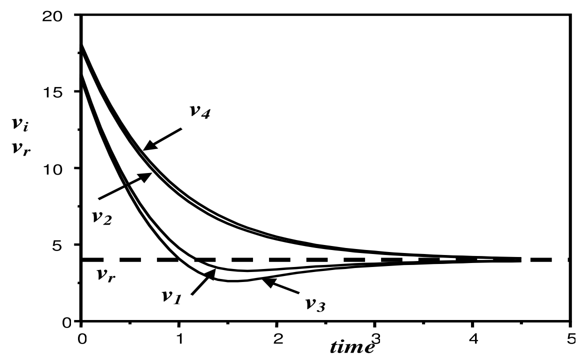

Figure 4. Velocity profile of the robots for example 1. Clearly v_i tracks v_r

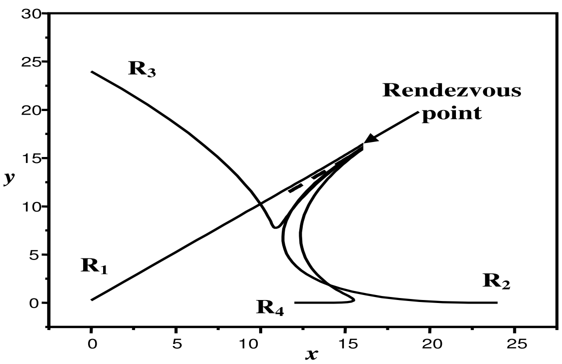

Figure 7. Rendezvous in the plane using the leader-follower approach. The leader-follower configuration is $R_r - R_1$, $R_1 - R_2$, $R_2 - R_3$, and $R_2 - R_4$

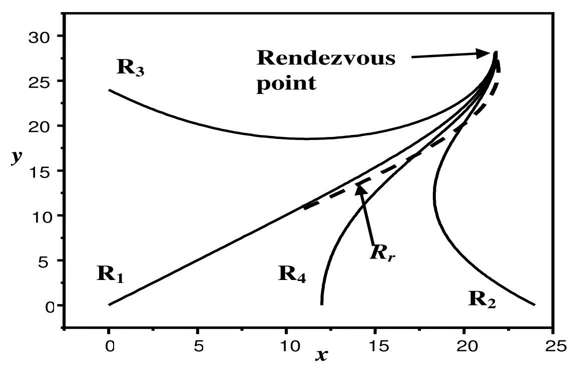

Figure 5. Rendezvous in the plane using the reference–robot approach. The reference robot (path in dashed line) is moving with time-varying orientation angle

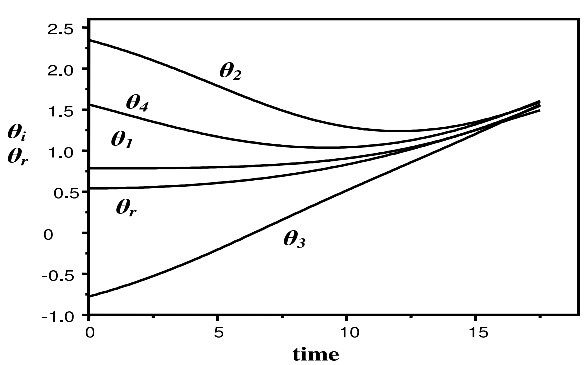

Figure 6. Orientation angles of the robots for the scenario of example 2. Clearly θ_i tracks θ_r.

$R_2\,(24, 0)$, $R_3\,(0, 24)$, $R_4\,(12, 0)$. The reference robot performs the same type of motion as the previous example. The relationship leader–follower is as follows: $R_r - R_1$, $R_1 - R_2$, $R_2 - R_3$, and $R_2 - R_4$. The paths traveled by the robots are shown in figure 7. Unlike the previous examples, the velocity of the each robot tracks the velocity of its leader robot. In this case, we have $v_1 \rightarrow v_r$, $v_2 \rightarrow v_1$, $v_3 \rightarrow v_2$, $v_4 \rightarrow v_2$. As a result, we have $v_i \rightarrow v_r$, $(i = 1, .., 4)$. In a similar way, we have for the orientation angle $\theta_1 \rightarrow \theta_r$, $\theta_2 \rightarrow \theta_1$, $\theta_3 \rightarrow \theta_2$, $\theta_4 \rightarrow \theta_2$, and thus, $\theta_i \rightarrow \theta_r$, $(i = 1, .., 4)$.

Example 4: This example is similar to example 3, but with a different configuration leader–follower. The aim is to show that different paths are obtained for different leader-follower configurations. The relationship leader-follower is as follows: $R_r - R_4$, $R_4 - R_1$, $R_1 - R_2$, and $R_1 - R_3$. The paths traveled by the robots are shown in figure 8, which are quite different from the paths obtained in the previous example. In this example, we have $\theta_4 \rightarrow \theta_r$, $\theta_1 \rightarrow \theta_4$, $\theta_3 \rightarrow \theta_1$, $\theta_2 \rightarrow \theta_1$, and thus all $\theta_i \rightarrow \theta_r$.

6 Conclusion

In this paper, we presented control strategies for multi-robot rendezvous in the plane, where the robots are controlled to reach an unpredictably moving point at the same time. We model the problem using the relative kinematics equations. The control law, which consists of the velocity pursuit, is derived based on this model. Two different approaches are used, namely, the reference–robot and the leader-follower. In the first approach the angular velocity of robot R_i is equal to the rate of turn of the line of sight angle between the reference robot and robot R_i. In the second approach the angular velocity of the follower robot is equal to the rate of turn of the line of sight angle between the leader and the follower. In both cases the control laws are decen-

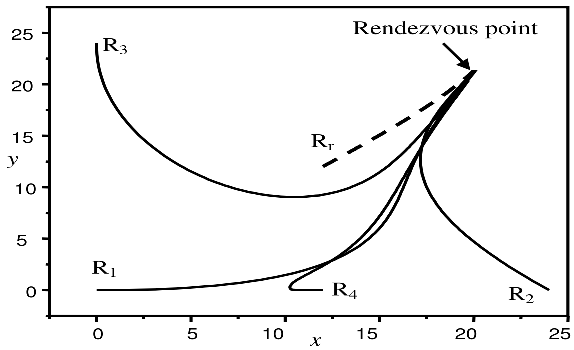

Figure 8. Rendezvous in the plane using the leader-follower approach. The leader-follower configuration is $R_r - R_4$, $R_4 - R_1$, $R_1 - R_2$, and $R_1 - R_3$

tralized. An extensive simulation is carried out to illustrate the method.

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