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Recovering Clipped OFDM Symbols with Bayesian Inference

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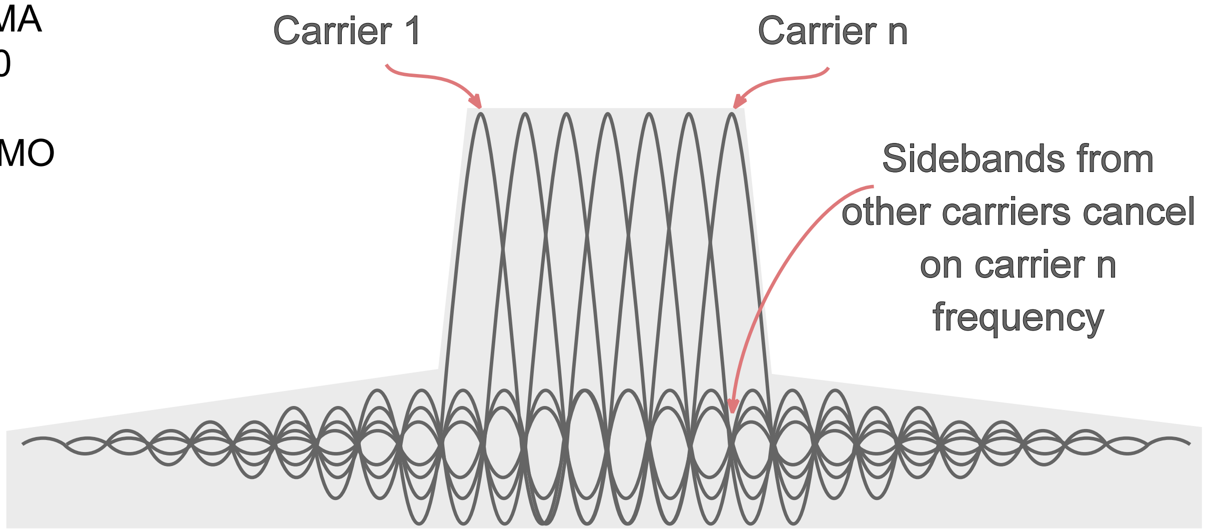
OFDM System

- Orthogonal Frequency-Division Multiplexing (OFDM) is a method of encoding digital data on multiple orthogonal carrier frequencies.
- OFDM was first introduced in 1966 (Bell Lab)
- OFDM is one of the basic idea of 4G LTE.

1G -> Analog System
2G -> TDMA, CDMA
3G -> CDMA-2000
4G -> OFDM
5G -> Massive MIMO



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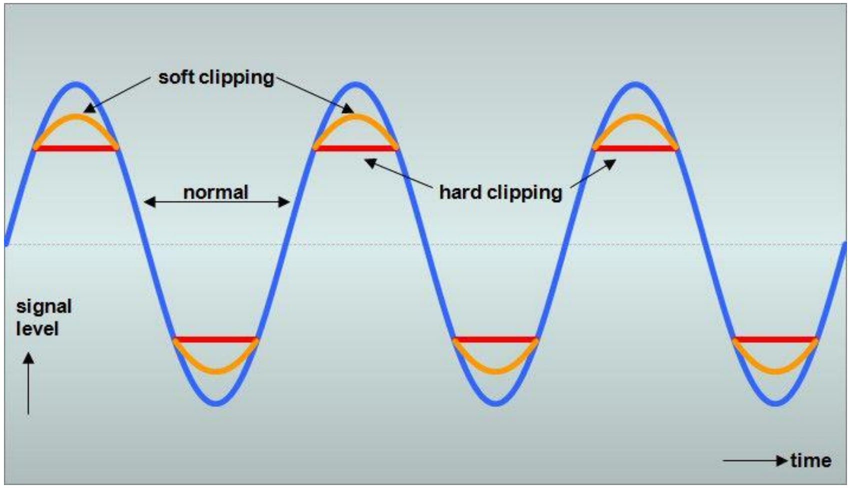
Clipping Distortion

- Frequency domain signal is drawn from a finite set (Constellation)

$$X_l \in \mathcal{S}, \quad |\mathcal{S}| = Q$$

- Time domain signal may have high peak power

$$x_n = \frac{1}{\sqrt{N}} \sum_{l=1}^{N} X_l e^{2\pi j \frac{ln}{N}} \qquad \tilde{x}_n = \begin{cases} x_n & \text{, if } |x_n| \leq A \\ A e^{j\mathrm{Arg}(x_n)} & \text{, if } |x_n| > A \end{cases}$$



Input-BackOff:

$$(IBO)_{dB} = 10 \log_{10}\left(\frac{A^2}{\mathbf{E}\left[x_n^2\right]}\right)$$

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Channel

- We assume time invariant frequency selective channel

$$y_n = \sum_{m=0}^{M-1} h_m \tilde{x}_{n-m} + w_n$$

- Also assume perfect sync. and equalization

$$\bar{Y}_l = \frac{Y_l}{H_l} = \tilde{X}_l + \frac{W_l}{H_l}$$

$$\bar{y}_n = \tilde{x}_n + \epsilon_n \quad , \sigma_\epsilon^2 = \frac{\sigma_w^2}{N} \sum_{l=1}^{N} \frac{1}{|H_l|^2}$$

MAP Approach

- Set hyperparameter z_n

$$\bar{y}_n = \tilde{x}_n + \epsilon_n = x_n(1 - z_n) + Ae^{j\mathrm{Arg}(x_n)} z_n + \epsilon_n, \quad z_n \in \{0, 1\}$$

- Prior

$$\Pr(z_n) = \begin{cases} 1/2 & , z_n = 0 \\ 1/2 & , z_n = 1 \end{cases}$$

$$\Pr(X_l = S_u) = \frac{1}{Q}$$

$$\Pr(\sigma_\epsilon^2) = \mathcal{IG}(a_0, b_0)$$

MAP Approach

- Likelihood

$$\Pr(\bar{\mathbf{y}}|\mathbf{x},\mathbf{z},\sigma_\epsilon^2) = \prod_{n=1}^{N} \Pr(\bar{y}_n|x_n, z_n, \sigma_\epsilon^2)$$
$$= \prod_{n:z_n=0} \mathcal{N}(\bar{y}_n|x_n,\sigma_\epsilon^2) \times \prod_{n:z_n=1} \mathcal{N}(\bar{y}_n|Ae^{j\phi_{x_n}},\sigma_\epsilon^2)$$

- Directly maximizing the full posterior w.r.t all variables would be hard to implement because the parameter space is too large

Iterative MAP Approach

- We introduce iterative/stochastic algorithm, update each variable separately.
- This approach can be interpreted as an extension of EM strategy.

Algorithm 1: Iterative/Stochastic MAP Estimation

Result: transmitted signal $\hat{\mathbf{x}}$, hyperparameter $\hat{\mathbf{z}}$, receiver noise power $\hat{\sigma}_\epsilon^2$

Compute $\bar{\mathbf{y}}$ at the receiver;

Start point $\mathbf{x}^{(0)}$, $\mathbf{z}^{(0)}$, $\sigma_\epsilon^{(0)}$ drawn from the given prior;

$k \leftarrow 1$;

do

  Drawn $\mathbf{z}^{(k)} \sim \Pr(\mathbf{z} | \mathbf{x}^{(k-1)}, \sigma_\epsilon^{(k-1)}, \bar{\mathbf{y}})$;

  Update: $\mathbf{x}^{(k)} = \arg\max_{\mathbf{x}} \Pr(\mathbf{x} | \mathbf{z}^{(k)}, \sigma_\epsilon^{(k-1)}, \bar{\mathbf{y}})$;

  Drawn $(\sigma_\epsilon^2)^{(k)} \sim \Pr(\sigma_\epsilon^2 | \mathbf{z}^{(k)}, \mathbf{x}^{(k)}, \bar{\mathbf{y}})$;

  $k \leftarrow k + 1$;

while $k \leq K_{max}$;

return $(\hat{\mathbf{x}}, \hat{\mathbf{z}}, \hat{\sigma}_\epsilon^2) \leftarrow (\mathbf{x}^{(k)}, \mathbf{z}^{(k)}, (\sigma_\epsilon^2)^{(k)})$

Iterative MAP Approach (Algorithm)

- Detail conditional posterior

- z_n

$$p = \frac{\Pr(z_n = 1|\bar{y}_n, x_n, \sigma_\epsilon^2)}{\Pr(z_n = 0|\bar{y}_n, x_n, \sigma_\epsilon^2) + \Pr(z_n = 1|\bar{y}_n, x_n, \sigma_\epsilon^2)}$$
$$= \text{Sigmoid}\left(\frac{1}{\sigma_\epsilon^2}\left[\left|\bar{y}_n - Ae^{j\phi_{x_n}}\right|^2 - |\bar{y}_n - x_n|^2\right]\right)$$

- X_l

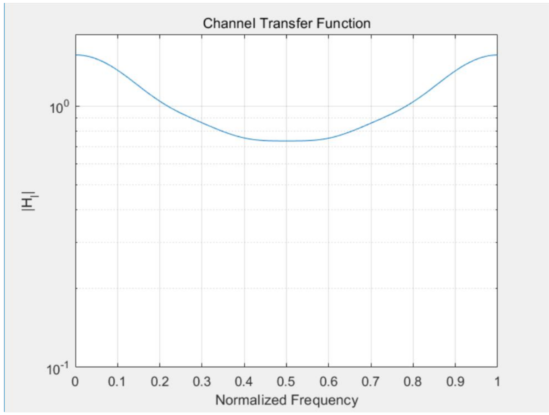
$$\hat{X}_l = \arg\max_{S_u} \Pr(X_l = S_u|\mathbf{X}_{-l}, \bar{\mathbf{y}}, \mathbf{z}, \sigma_\epsilon^2)$$
$$= \arg\max_{S_u} \sum_{n:z_n=0} |\bar{y}_n - x_{n,X_l=S_u}|^2 + \sum_{n:z_n=1} \left|\bar{y}_n - Ae^{j\phi_{x_{n,X_l=S_u}}}\right|^2$$

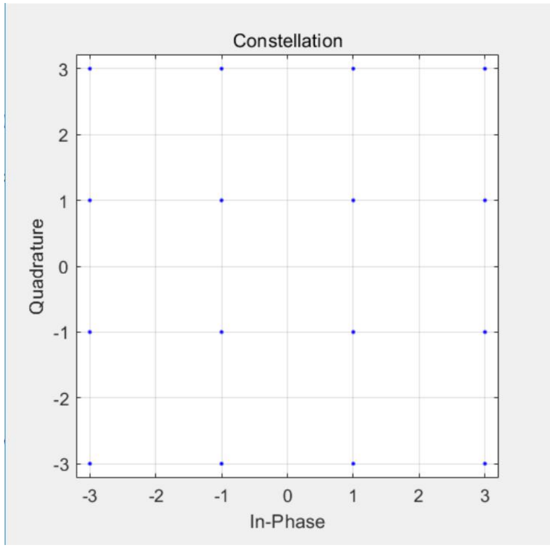
- σ_ε

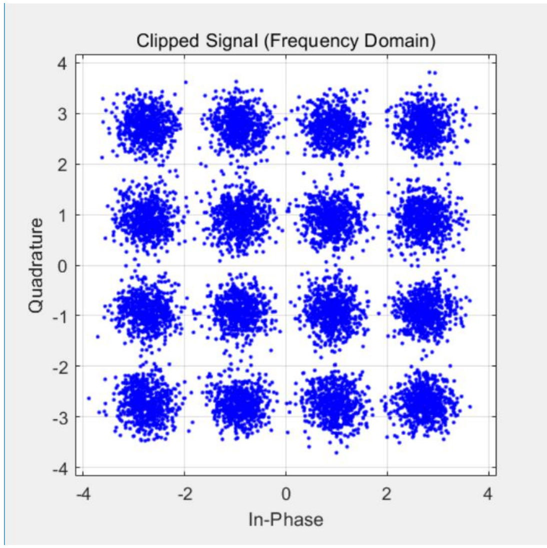
$$\Pr(\sigma_\epsilon^2|\bar{\mathbf{y}}, \mathbf{x}, \mathbf{z}) \propto \Pr(\bar{\mathbf{y}}|\mathbf{x}, \mathbf{z}, \sigma_\epsilon^2)\Pr(\sigma_\epsilon^2)$$
$$\propto \mathcal{IG}(a_N, b_N)$$
$$a_N = a_0 + \frac{N}{2}$$
$$b_N = b_0 + \frac{1}{2}\sum_{n=1}^{N}\left(\bar{y}_n - x_n(1 - z_n) - Ae^{j\phi_{x_n}} z_n\right)^2$$

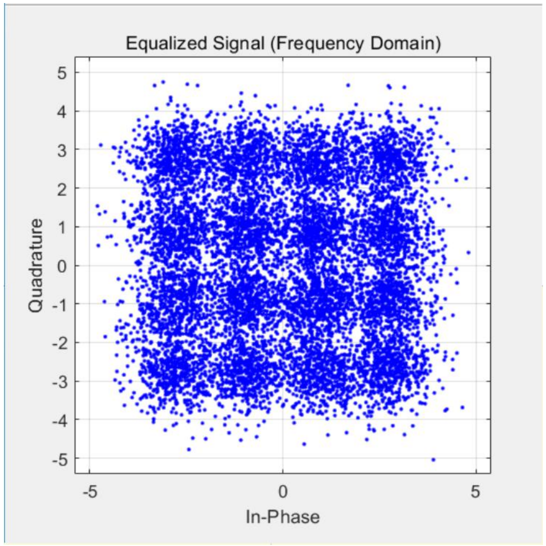
Implementation

- Simple convolution channel
- 16QAM Constellation



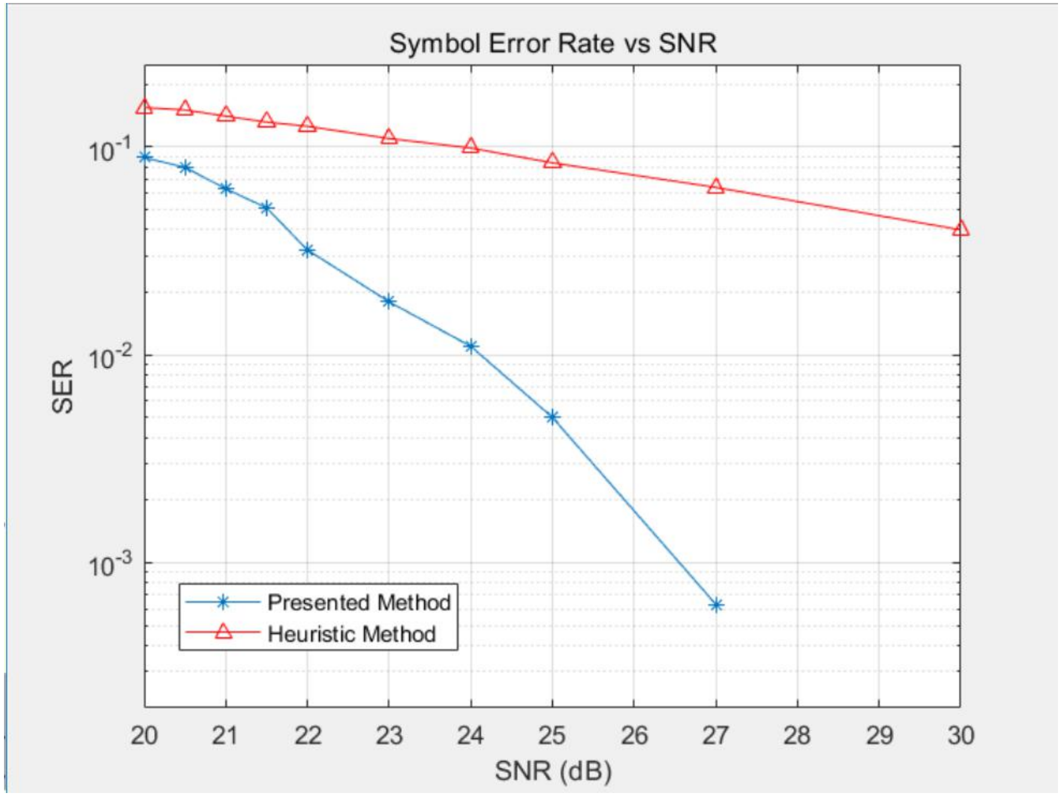






Implementation

- $N = 100000$, IBO = 2.93dB, $K_{Max} = 10$
- 20 trails
- Time Complexity: $O(KQN^2)$



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