CFA

LEVEL Ⅰ

Quantitative Methods

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一级数量—假设检验

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Hypothesis Testing

- Four steps of hypothesis testing
- Stating the decision rule
- Type I and type II errors
- Test-Statistic

Four steps of hypothesis testing

- Step 1: Stating the hypothesis
- Step 2: Selecting and calculating the appropriate test statistic
- Step 3: Specify the level of significance (α)
- Step 4: Stating the decision rule regarding the hypothesis

Hypothesis Testing

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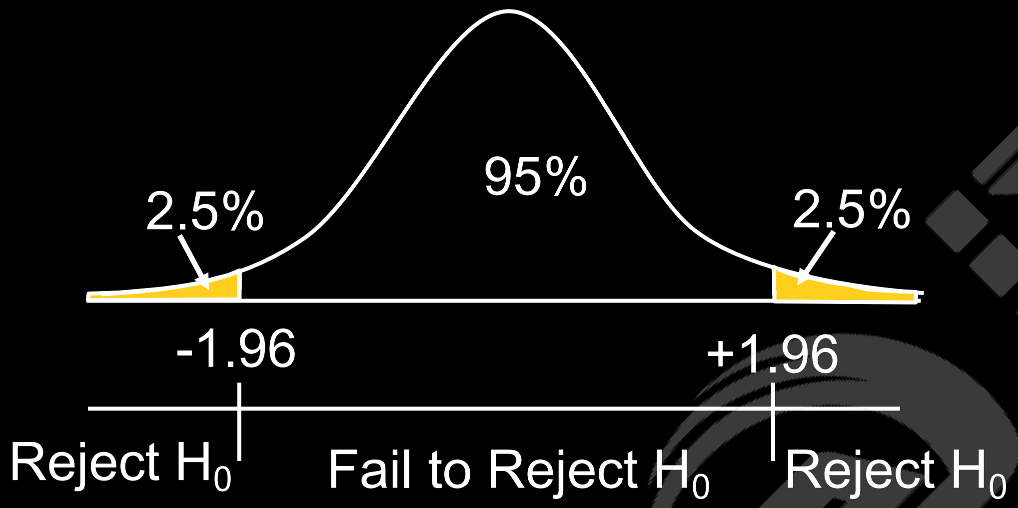
Stating the decision rule

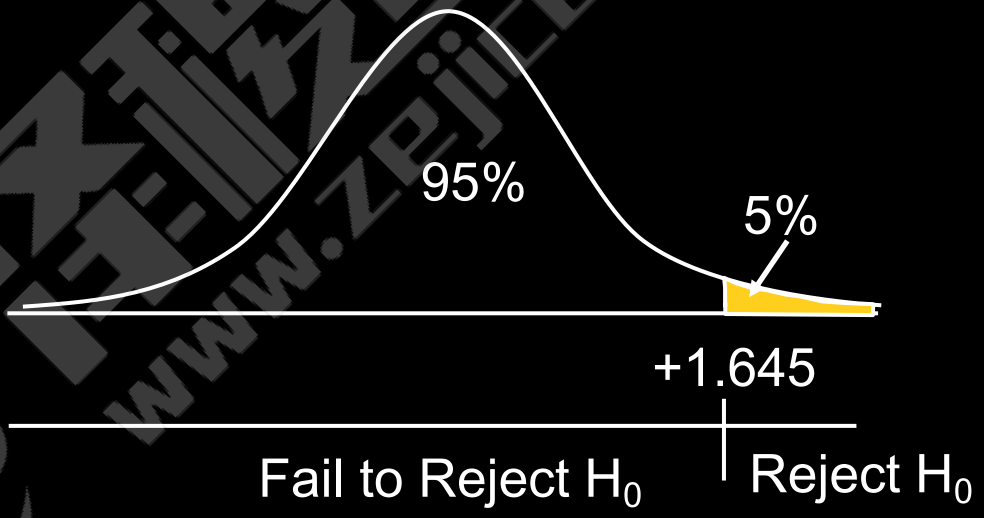
- Null hypothesis(原假设 H_0) & Alternative hypothesis(备择假设 H_a)
  - ✓ Two-tailed test
    - H_0: $\mu=0$ H_a: $\mu\neq0$
  - ✓ One-tailed test
    - H_0: $\mu\geq0$ H_a: $\mu<0$
    - H_0: $\mu\leq0$ H_a: $\mu>0$

Stating the decision rule

- Reject H_0 if |test statistic|>critical value
  - ✓ μ is significantly different from μ_0
- Fail to reject H_0 if |test statistic|<critical value
  - ✓ μ is not significantly different from μ_0
  - ✓ We can never say "accept" H_0

Stating the decision rule

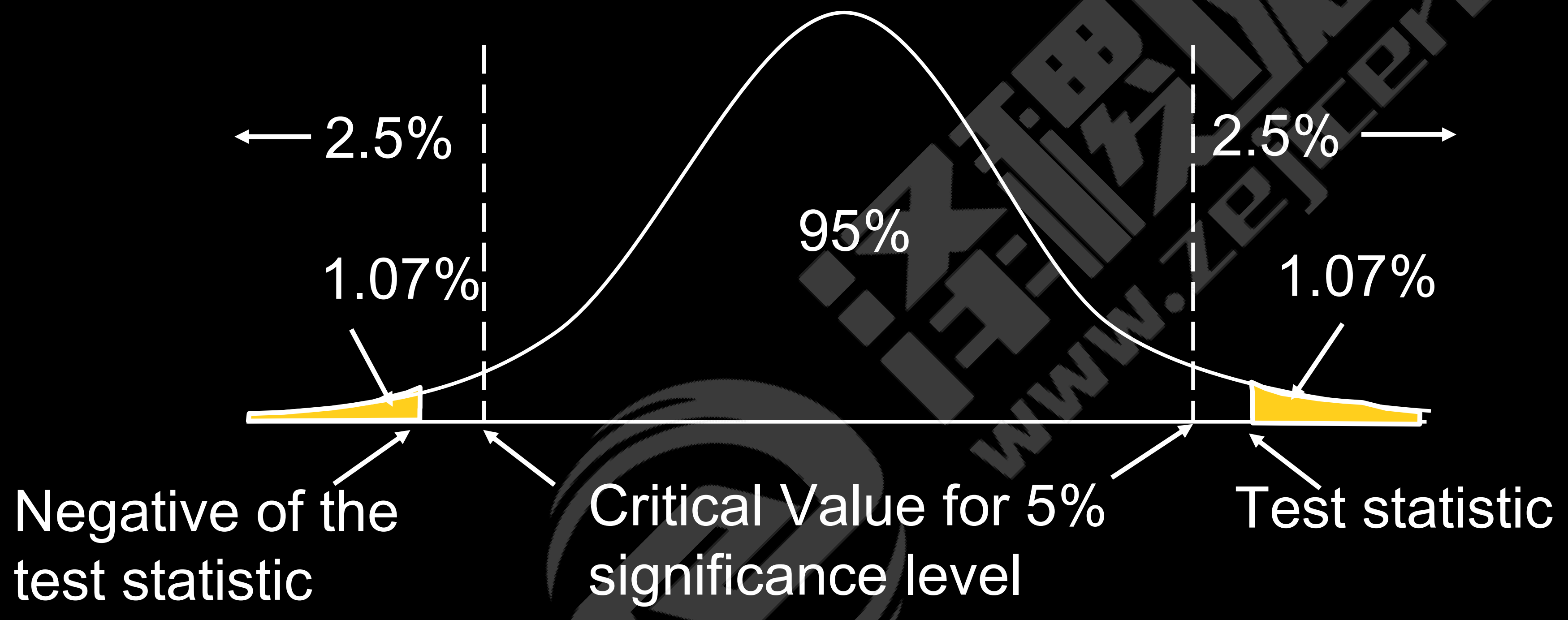




P-value

- The p-value is the smallest level of significance at which the null hypothesis can be rejected
  - ✓ If P-value<alpha, we reject null hypothesis

P-value


2.5%
2.5%
95%
1.07%
1.07%
Negative of the test statistic
Critical Value for 5% significance level
Test statistic

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Type I and type II errors

- Type I error: reject the null hypothesis when it’s actually true
- Type II error: fail to reject the null hypothesis when it’s actually false
- Significance level (α): the probability of making a Type I error
  - ✓ Significance level =P (Type I error)
- Power of a test (检验势): the probability of correctly rejecting the null hypothesis when it is false
  - ✓ Power of a test=1−P(Type II error)

Type I and type II errors

Decision	True Condition	
	H_0 is true	H_0 is false
Do not reject H_0	Correct decision	Incorrect decision Type Ⅱ error
Reject H_0	Incorrect decision Type Ⅰ error Significance level, α, =P(Type Ⅰ error)	Correct decision Power of the test =1-P(Type Ⅱ error)

Example

Which of the following statements about hypothesis testing is least accurate?

A. The higher the significance level, the higher the power of the test

B. If the alternative hypothesis is H_a: $\mu > \mu_0$, a two-tailed test is appropriate

C. A Type II error is failing to reject a false null hypothesis

Hypothesis Testing

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Test-Statistic

- Tests concerning a single mean
  - ✓ $H_0: \mu = \mu_0$
  - ✓ $z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$ $t_{n-1} = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$

When sampling from a:	Test statistic	
	Small sample($n<30$)	Large sample($n\geq30$)
正态分布，总体方差已知	z-statistic	z-statistic
正态分布，总体方差未知	t-statistic	t-statistic /z-statistic
非正态分布，总体方差已知	Not available	z-statistic
非正态分布，总体方差未知	Not available	t-statistic /z-statistic

Test-Statistic

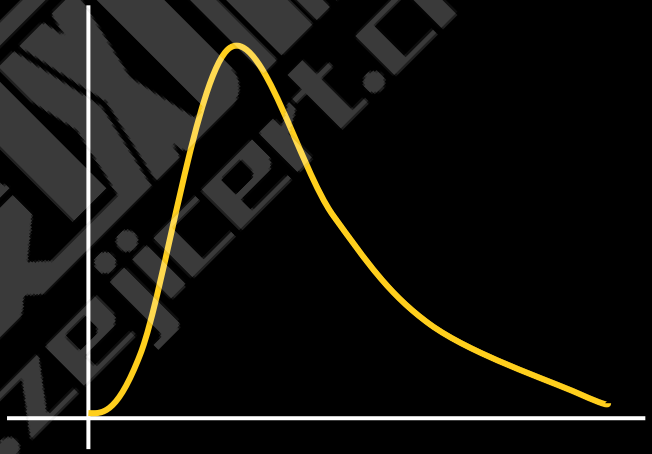
- Tests concerning the mean of the differences between paired observations
  - ✓ When two samples are dependent
  - ✓ H_0: $\mu_d=0$ H_a: $\mu_d\neq0$
  - ✓ t-test
    - $t=\frac{\bar{d}-0}{s_{\bar{d}}}$

Test-Statistic

- Tests concerning the differences in means
  - ✓ When two samples are independent
  - ✓ H_0: $\mu_1=\mu_2$
  - ✓ t-test
    - 方差相等，未知 ($\sigma_1^2=\sigma_1^2$)
      - $t_{n_1+n_2-2}=\dfrac{\bar{x}_1-\bar{x}_2}{s_w\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)^{1/2}}$；$S_w^2=\dfrac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$
    - 方差不相等，未知 ($\sigma_1^2\neq\sigma_1^2$)
      - $t=\dfrac{\bar{x}_1-\bar{x}_2}{\left(\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}\right)^{1/2}}$

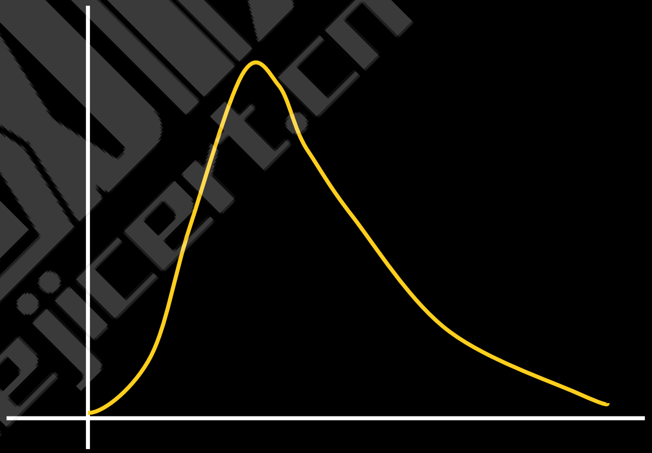
Test-Statistic

- Tests concerning a single variance
  - ✓ Chi-Square test
    - H_0: $\sigma^2 = \sigma_0^2$
    - The chi-square test (χ^2-test)
    - Test-Statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$, df=n-1
      - – n=Sample size s^2=Sample variance;
      - – σ_0^2=Hypothesized value for the population variance;
      - – df=Degree of freedom.



Test-Statistic

- Tests concerning two variance
  - ✓ F-test
    - H_0: $\sigma_1^2=\sigma_2^2$
    - Test-Statistic: $F=\dfrac{s_1^2}{s_2^2}$, $df_1=n_1-1$; $df_2=n_2-1$
    - Always put the larger variance in the numerator($s_1^2>s_2^2$).
    - 不管是单尾检验还是双尾检验，拒绝域总是在右尾。



Test-Statistic

Test type	Assumptions	H_0	Test-statistic	Critical value
Mean hypothesis testing	Normally distributed population, known population variance	$\mu=0$	$z=\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$	N(0,1)
	Normally distributed population, unknown population variance	$\mu=0$	$t=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$	t(n-1)
	Independent populations, unknown population variances assumed equal	$\mu_1-\mu_2=0$	t	$t(n_1+n_2-2)$
	Independent populations, unknown population variances not assumed equal	$\mu_1-\mu_2=0$	t	t

Test-Statistic

Test type	Assumptions	H_0	Test-statistic	Critical value
Variance hypothesis testing	Normally distributed population	$\sigma^2 = \sigma_0^2$	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2(n-1)$
	Two independent normally distributed populations	$\sigma_1^2 = \sigma_2^2$	$F = \frac{s_1^2}{s_2^2}$	$F(n_1-1, n_2-1)$

Example 1

Roberts believes that the mean price of houses in the area is greater than $145,000. A random sample of 36 houses in the area has a mean price of $149,750. The population standard deviation is $24,000, and Roberts wants to conduct a hypothesis test at a 1% level of significance.(The critical value of the z-statistic is 2.33)

The appropriate alternative hypothesis is?

The value of the calculated test statistic is closest to?

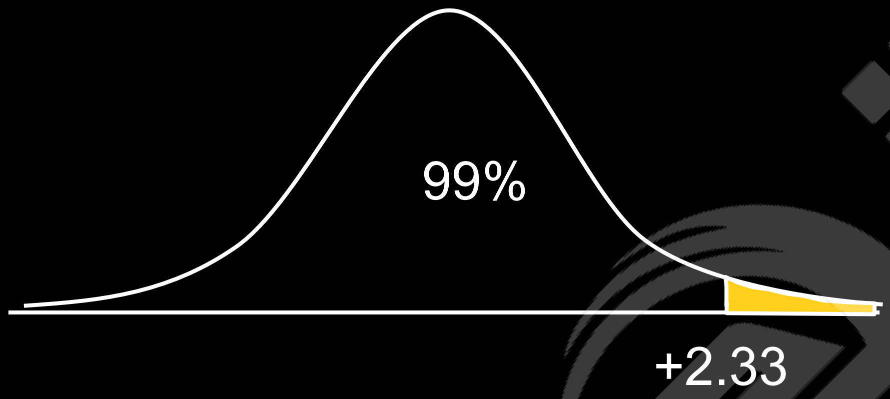
Two-tailed test or One-tailed test?

Should Roberts reject or not reject the null hypothesis?

Example 1

H_0: $\mu \le 145,000$
H_a: $\mu > 145,000$

$$z = \frac{149,750 - 145,000}{24,000/\sqrt{36}} = 1.1875$$



Roberts should not reject the null hypothesis

Example 2

An analyst is conducting a hypothesis test to determine if the mean time spent on investment research is different from three hours per day. The test is performed at the 5% level of significance and uses a random sample of 64 portfolio managers, where the mean time spent on research is found to be 2.5 hours. The sample standard deviation is 1.5 hours.

At a 5% level of significance, Analyst should reject or not reject the null hypothesis?

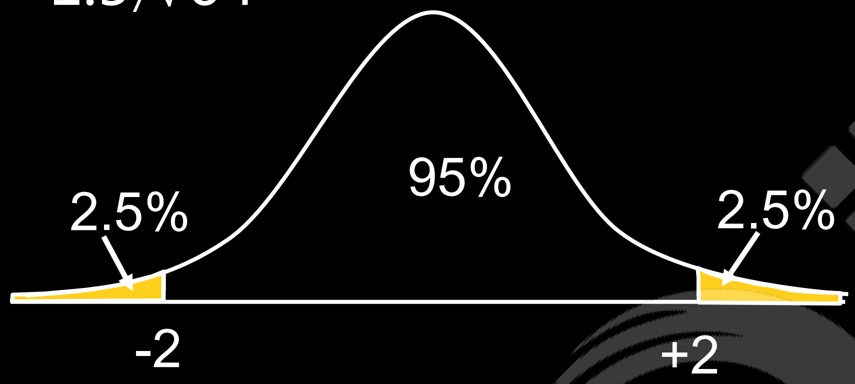
The 95% confidence interval for the population mean is?

R12: Hypothesis Testing

H_0: $\mu=3$

H_a: $\mu\neq3$

$$t=\frac{2.5-3}{1.5/\sqrt{64}}=-2.6667$$



Analyst should reject the null hypothesis

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