# Boson Sampling Simulation Software

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## 1 Transition Amplitude

In the model of boson sampling, the amplitude of transforming state $|t_1,\, t_2,\, \ldots,\, t_m\rangle$ to state $|s_1, s_2, \ldots, s_m\rangle$ is

$$\langle s_1 \ldots s_m | U | t_1 \ldots t_m \rangle \;=\; \frac{G(U, s, t)}{\sqrt{(s_1! \cdots s_m!)(t_1! \cdots t_m!)}} \tag{1}$$

where

$$\begin{aligned} G(U, s, t) &= \frac{v_s^2}{d^m} \sum_{\{z\}} (\bar{z}_1^{s_1} \bar{z}_2^{s_2} \cdots \bar{z}_m^{s_m}) \Pi_{k=1}^m (w_{k,1} z_1 + \ldots + w_{k,m} z_m)^{t_k} \\ &= \mathbb{E}(v_s^2 (\bar{z}_1^{s_1} \bar{z}_2^{s_2} \cdots \bar{z}_m^{s_m}) \Pi_{k=1}^m (w_{k,1} z_1 + \ldots + w_{k,m} z_m)^{t_k}) \\ &\equiv \mathbb{E}(\mathrm{mGenGly}(z)) \end{aligned}$$

$$v_s \equiv \sqrt{(s_1!/s_1^{s_1})(s_2!/s_2^{s_2}) \cdots (s_m!/s_m^{s_m})} \tag{2}$$

$$z_i \in \{\sqrt{s_i} w^0, \sqrt{s_i} w^1, \ldots, \sqrt{s_i} w^{n-1}\} \tag{3}$$

$$w = e^{2\pi i/n} \tag{4}$$

## 2 Gurvits's Algorithm

Glynn's algorithm is an exact deterministic classical algorithm, while Gurvits's algorithm is an approximate sampling algorithm. In the following we analysized the success probability of Gurvits's algorithm.

From the Chernoff bound for complex variables, We have the following result. Let $X_i = \frac{\mathrm{mGenGly}(z_i)}{\prod_{k=1}^m (s_k! t_k!)^{1/2}}$, $X = \sum_{i=1}^T X_i$, $\mu \equiv \mathbb{E}(X_i) = \langle s_1 \ldots s_m | U | t_1 \ldots t_m \rangle$, $\;b \equiv \min\,\{v_s / v_t\;,\; v_t / v_s\}$, then $|X_i| \leqslant v_s / v_t$ . Considering the conjugate of $X_i$ , we conclude $|X_i| \leqslant b$.

$$\boldsymbol{P}\left(\frac{\left|\frac{\boldsymbol{X}}{\boldsymbol{T}} - \boldsymbol{\mu}\right|}{|\boldsymbol{\mu}|} \geqslant \boldsymbol{\varepsilon}\right) \leqslant \boldsymbol{4e^{\frac{-T\,|\mu|^2\,\varepsilon^2}{4b^2}}} \tag{5}$$

To conclude, we have estimated $\langle s_1 \ldots s_m | U | t_1 \ldots t_m \rangle$ using $T = O(1/\varepsilon^2)$ samples from $\frac{\mathrm{mGenGly}(z_i)}{\prod_{k=1}^m (s_k! t_k!)^{1/2}}$ within additive error $\pm \varepsilon \times \min(v_s/v_t, v_t/v_s)$, and have a high probabilty $\left(1 - 4e^{\frac{-T\,|\mu|^2\,\varepsilon^2}{4b^2}}\right)$ success. For each mGenGly($z$)we need $mn$ operations, thus the polynomian-time sampling algorithm scales as $O(mn/\varepsilon^2)$.

Note that in order to success in a high probability, we also require the permanent of unitary, i.e. $|\mu|$ not too small.

## 3 The Classical Simulation Software

In this simulation software we simulated boson sampling from 6 photons up to 10 photons. The number of modes are supposed to be $O(n^2)$, however in here for the purpose of demonstrating the efficiency of Gurvits's algorithm, we simplify the simulation by constraining $m = n$. In all simulation, **the initial state and final state are both set to be standard state**, i.e. there are exactly one photon in each mode. **We calculated the transition amplitude using both Glynn's algorithm and Gurvits's algorithm.**

The time efficiency for both Glynn and Gurvits are analyzed in all cases from 6 photons to 10 photons. Due to the capability of classical hardware, however, the successful probability for Gurvits's algorithm is analyzed only in the case with only 6 photons.

As seen from the plot for time efficiency, the time required for Glynn's algorithm grows exponetially, while the time for Gurvits's method grows much slower. This result is consistent with the result in [1].

In the case of 6 photons, the results for success probability vs fidelity is plotted in file "6photons.pdf". To compute the success probability, Gurvits's method was run by 1000 times. The dashed line stands for the theoretical prediction from Chernoff bound. From this theoretical bound, to achieve fidelity 90%, success probability 97%, the number of samples should be $T = 5 \times 10^6$. However, in practice, the actual samples we need in Gurvits are much less than we expected. In this simulation, even if the number of samples is only T=$5 \times 10^2$, the accuracy of Gurvits's algorithm goes beyond our expectation. When fidelity is set as 95%, the success probability is 100%.When fidelity is set as 96%, the success probability is still 97%.

[1] Neville, Alex, et al. "No imminent quantum supremacy by boson sampling." *arXiv preprint arXiv:1705.00686* (2017).