Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2227819

A Powerful Trading Strategy Model Brings More Returns

Summary

In the stock exchange market, where risks and opportunities coexist, savers invest to increase their capital. However, the choice of trading strategy is very important for investors. Trading strategies include how to carry out **investment portfolios**, grasp investment trends, and maximize returns under a certain level of risk. With the continuous development of the field of machine learning, compared with quantitative econometric models and other technologies, machine learning can dig out deeply hidden information in huge data to assist financial practitioners or investors in decision-making [2–4].

In this paper, we build an **econometric model** and a **deep reinforcement learning model** to simulate the trading strategy.

For question 1, we hold an initial capital of $1,000 to maximize returns by modeling a trading strategy on a given dataset (dates when Bitcoin and Gold are open for trading over a period of time and the closing prices of the trading days).After studying Markowitz's optimal investment theory, we established our econometric model by combining two strategies, the optimal risky asset portfolio and the optimal investment portfolio.We divide the model results into three categories: **low-risk returns, balanced returns, and high-risk returns**. At the low-risk return point, the trading strategy tends to increase the investment weight of gold, and finally the return as of the trading day is **$2140.163**, with an annualized return of about **0.077 (107.7%)**; at the balance point, the trading strategy automatically balances the return according to the market And risk, the final investment weights of Bitcoin and gold are **45.46%** and **54.54%** respectively, the return as of the final trading day is **$15576.597**, and the annualized return is about **0.531 (153.1%)**; at the high-risk return point, the trading strategy tends to invest The riskier bitcoin, as of the last trading day, has a return of **$29532.632**, an annualized return of about **0.781 (178.1%)**.

For question 2, we demonstrate that our model provides the best trading strategy by analyzing economic theory and comparing actual market returns.

For question 3, we analyze the sensitivity of investment weight portfolio to annual related of return, investment weight portfolio to risk, and rate of return to risk. We found that the investment weight of Bitcoin has a positive correlation with the rate of return and risk, and the investment weight of gold has a negative correlation with the return and risk. It can be seen that Bitcoin is a high-risk and high-yield commodity while gold is a low-risk and low-risk commodity with **less sensitive**.

Keywords: Optimal Portfolio Theory,Deep Reinforcement Learning,Convex Optimization

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1 Introduction

1.1 Problem Background

As the financial and economic markets are rapidly changing and the impact is huge, investors need to have a keen eye and understand the investment portfolio strategy, grasp the trend, and maximize returns under a certain risk level. Among them, the choice of investment portfolio strategy can greatly affect the return of investors. Portfolio strategy refers to continuously allocating a certain amount of funds to multiple financial products and continuously adjusting the investment weights of different financial products to achieve high returns while controlling risks. The investment theory based on the mean-variance model, first proposed by Markowitz in 1952 [1], has been proved to be effective and has been widely used in asset allocation and portfolio management.

In recent years, with the rapid development of artificial intelligence technology, "intelligence" has entered our lives. After Alphago, developed by Deep Mind, defeated a group of world-class Go players, people were amazed at the rapid development of intelligent systems. It is precisely because artificial intelligence can capture tiny details that are deeply hidden in a large amount of data, people try to design intelligent systems to assist investors in making financial investment decisions.

1.2 Clarifications and Restatements

In this problem, we analyze the trade data of gold and bitcoin to build a trading strategy model, which includes the open trading day and the closing price (USD) from the London Bullion Market Association and NASDAQ.

We will solve the following problems based on the given data:

1. By analyzing the given data, get the expected return, risk (standard deviation) and the correlation coefficient between gold and bitcoin.

2. Establish econometric models, implement effective trading strategies, and assist traders in maximizing returns on their portfolios.

3. From a theoretical point of view, the feasibility of the econometric trading strategy model and the optimality of results are analyzed and demonstrated.

4. By controlling the difference of the given original assets and transaction frequency, using the same trading strategy, observe and analyze the sensitivity of different amounts of initial capital and transaction frequency to the final return.

5. Generate a deep reinforcement learning model based on artificial intelligence technology, learn trading strategies heuristically through its policy-value, and finally train it to become a keen investment portfolio decision-making model.

1.3 Our Work

- Analyze the given data , add a transaction status attribute according to the trading open day, and then describe the fluctuation of the closing price between the current trading day and the previous trading day.

- Construct an econometric model based on Markowitz's optimal portfolio theory to analyze investment weights in Bitcoin and Gold.

- An investment decision-making model is established based on deep reinforcement learning, and an agent is trained to independently judge trading strategies through its value strategy network.

- From the perspectives of economics, artificial intelligence technology and real data comparison, it is confirmed that our trading strategy is the optimal trading strategy.

- Analyze how trading costs affect strategies and structures by restricting or encouraging traders to execute trading strategies.

2 Reasonable Assumptions

Assumptions about the data provided.

1. Trading opening dates for gold and bitcoin provided by the London Bullion Market Association and NASDAQ are accurate.

2. The London Bullion Market Association and NASDAQ provide the exact prices of gold and bitcoin on open days of trading that do not lead to deviations in model predictions.

3. Assume that the opening price of the day after the current trading day is the closing price of the current trading day.

Assumptions about the behavior of trading strařegy.

1. Traders can predict future prices based on the price prediction model established by all prices before the current trading date, and use this to conduct trading strategies.

2. Assuming that inflation and other conditions are not taken into account, the cash held to purchase treasury bonds (risk-free return) can be redeemed at any time to purchase gold and bitcoin.

3 Abbreviations and Symbols

Before we begin analyzing the problems, it is necessary to clarify the abbreviations and symbols that we will be using in our discussion. These are shown below in Table 1:

Table 1: Abbreviations and Symbols

Symbol/Abbreviation	Description
α_i	The commission for each transaction (purchase or sale)
ω_i	Weight (investment ratio) of risky asset n in the portfolio of risky assets
$E(r_i)$	Expected rate of return
r_f	Expected rate of return on a risk-free asset
σ_i	Risk of Risky Assets (Standard Deviation)
ρ	Correlation coefficient between risky asset 1 and risky asset 2
$E(R)$	The mathematical expectation of the past return of the asset
W	The weight vector assigned to each asset

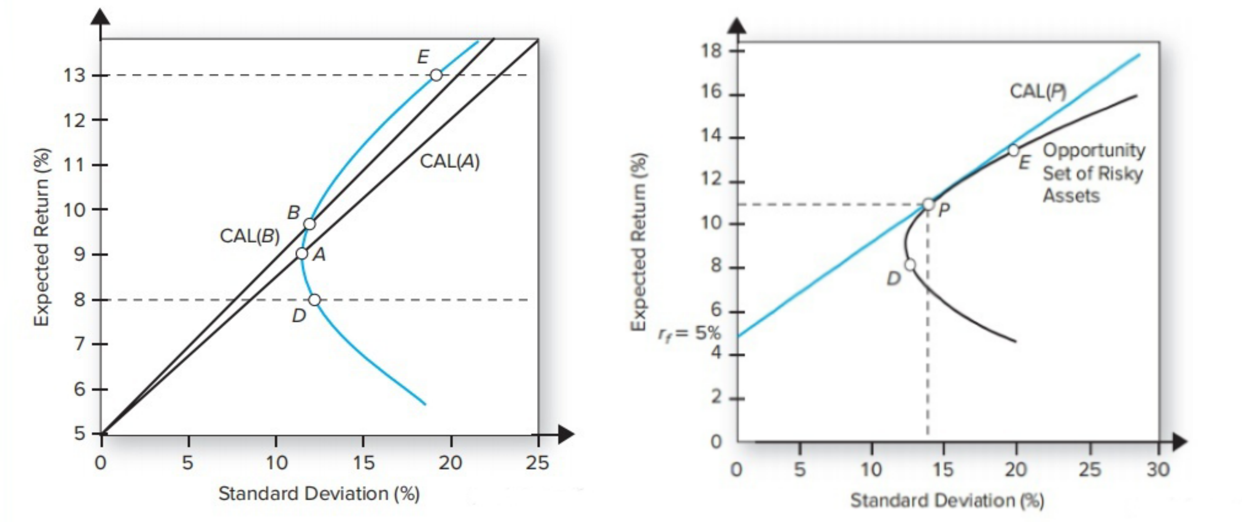
4 Constructing the best trading strategy

4.1 Econometric Models

Markowitz proposed a mean-variance model that influenced **modern portfolio theory**(MPT) in 1952, and relevant practitioners have confirmed the feasibility and reliability of the theory in continuous research and practice. Many economists have further modified this model, such as the **Capital Asset Pricing Model (CAPM)** [5] model proposed by Sharpe and the **Arbitrage pricing theory (APT)** [6] model proposed by Ross.

Low risk has a good chance of producing low reward, but high risk usually means high reward. MPT assumes that investors are risk-neutral, which means that if there are two portfolios with the same return, investors tend to prefer the less risky portfolio. However, if high returns exist, investors will likely choose a high-risk portfolio. "Diversity" is an important factor affecting modern investment portfolios. Modern Portfolio Theory states that for a particular stock, it is not enough if we only focus on its risk and return. Investors can reduce portfolio risk by investing in multiple stocks, diversifying risk, and profiting from investment diversification.

We will build an econometric model based on the mean-variance model, the optimal risky asset portfolio, and the optimal risky investment portfolio to simulate trading and calculate returns.



(a) Portfolio Allocation Line (b) Highest slope of Capital Allocation Line (CAL)

Figure 1: Optimal risk portfolio

4.1.1 Optimal risky asset portfolio

Portfolio Opportunity Set (Figure 1 left)[1]: Two risky assets, forming the new risky asset.Point A is the first new risky asset formed by the combination of two risky assets under a certain weight, and then combined with the risk-free asset, the first new risky asset at this time accounts for 100% of the weight.Point B is the same as point A, where the weight of the new risky asset is 100%.

Optimal risky portfolio: The blue line in the Figure 1 (right) is the **Capital Allocation Line (CAL)**. With the introduction of risk-free lending, CAL allocates a certain amount of money between risky assets and specific combinations of risk-free assets, a line that describes the relationship between expected return and risk for all possible new combinations.

Sharpe ratio: Sharpe proposed a simplification of Markowitz's model in 1966 [5], which decomposes the return on investment into two major parts: the additional return of the investment product itself and the external risk of the market. The model provides a practically calculable framework in which investors can allocate assets based on their value weights.Here we use the Sharpe ratio to measure the portfolio that maximizes expected return, the formula is as follows:

$$SharpeRatio = \frac{E(r_n) - r_f}{\sigma_n} \tag{1}$$

In the above formula, $E(r_n)$: the expected annualized rate of return of the portfolio, r_f: the annualized risk-free rate, σ_n: the standard deviation of the annualized rate of return of the portfolio. Compared with the risk-free return of investment assets, the ratio of the excess return obtained to the

[1]Bodie Z, Merton R C, Cl D L. Financial Economics[M]. China Renmin University Press, 2009.

risk assumed is the excess return per unit risk. Obviously, the greater the ratio, the greater the cost performance of the asset.

Since SharpRatio is connected by the risk-free asset point (0, r_f) and one point in the range of all assets in the market, the curve connected by these two points is the CAL mentioned earlier, and its straight line equation for:

$$E(R) = \frac{E(r_n) - r_f}{\sigma_n}\sigma(R) + r_f \tag{2}$$

Among equation (2): $(\sigma(R), r_f)$ is any point on the CAL straight line. It can be seen that when adding risk-free asset allocation, the asset portfolio is the best on CAL, and there is a point P that is the market equilibrium point (market portfolio point). According to Sharpe's hypothesis: Investors in the whole market have the same expected return or the same expected risk, so that the market value ratio of risk assets in the whole market will be exactly the allocation ratio of this market portfolio point, which will ultimately ensure market stability.

4.1.2 Optimal portfolio

Optimal portfolio: An optimal portfolio is a form of investment portfolio in which an investor can achieve maximum returns in a selected portfolio of possible investments. Generally speaking, it refers to the selection of investment products and forms that can produce positive effects in the process of gathering various securities in the securities, stock and fund markets according to the risks, returns and future development trends of each security. The uniqueness of the optimal combination is determined by the convexity of the efficient set and the concaveness of the indifference curve.

The purpose of a portfolio is to diversify risk. The investment portfolio can be regarded as a multi-level investment portfolio, which can be comprehensively considered according to your own needs and actual conditions. It has the dual consideration of safety and effectiveness, effectiveness and flexibility, flexibility and effectiveness.

Asset return and asset risk as defined by Markowitz:

The return of an asset $E(R)$ is: the mathematical expectation of the past return of the asset.

$$E(R) = \sum_{i}^{n} \omega_i E(R_i) \tag{3}$$

The risk of an asset $\sigma(r)$ is: the mathematical standard deviation (variance) of the asset's past returns.

$$\sigma_N = \sqrt{W^T \Sigma W} \tag{4}$$

In the above formula, $W = (w_1, w_2, ..., w_n)^T$ is the weight vector assigned to each asset, the sum of its components is 1, $E(R_i)$ is the expected return of the i th asset, and the calculation method is the

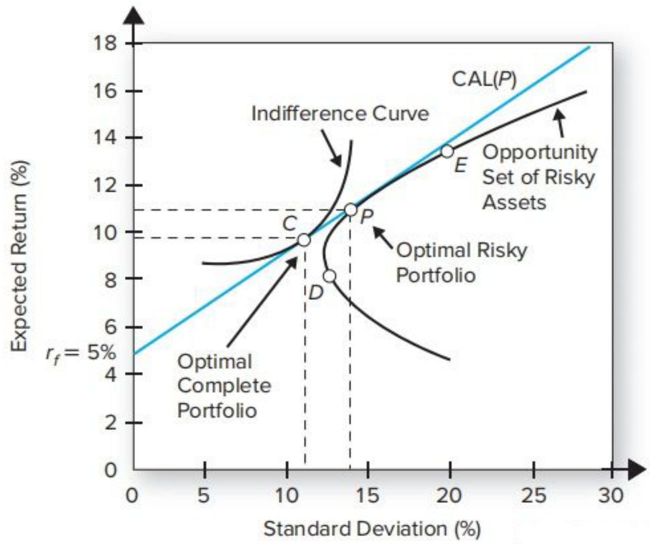


Figure 2: Optional portfolio

past period of the ith asset time average returns. n is the total number of assets and Σ is the covariance matrix of n assets.

4.1.3 Econometric model solving

First, we analyzed the daily fluctuations of gold and bitcoin in the given data as shown in Figure 3, where the yellow line represents the daily price fluctuation of gold, and the blue line represents the daily price fluctuation of bitcoin. Among them, it can be clearly observed that the price of gold has remained basically stable while the price of Bitcoin has oscillated very clearly.

After that, we implemented the following formulas by computer to solve the yield and risk of gold and bitcoin.

The following formula can be obtained from the optimal combination of risky assets:

$$E(r_N) = \omega E(r_1) + (1-\omega)E(r_2) \tag{5}$$

$$\sigma_N = \sqrt{W^T \Sigma W} = \sqrt{\omega^2\sigma_1^2 + (1-\omega)^2\sigma_2^2 + 2\rho\sigma_1\sigma_2\omega(1-\omega)} \tag{6}$$

Assuming that the straight line and the curve intersect at point P like Figure 1 (right), then:

$$k = \frac{E(r_n) - r_f}{\sigma_n} = \frac{\omega E(r_1) + (1-\omega)E(r_2) - r_f}{\sqrt{\omega^2\sigma_1^2 + (1-\omega)^2\sigma_2^2 + 2\rho\sigma_1\sigma_2\omega(1-\omega)}} \tag{7}$$

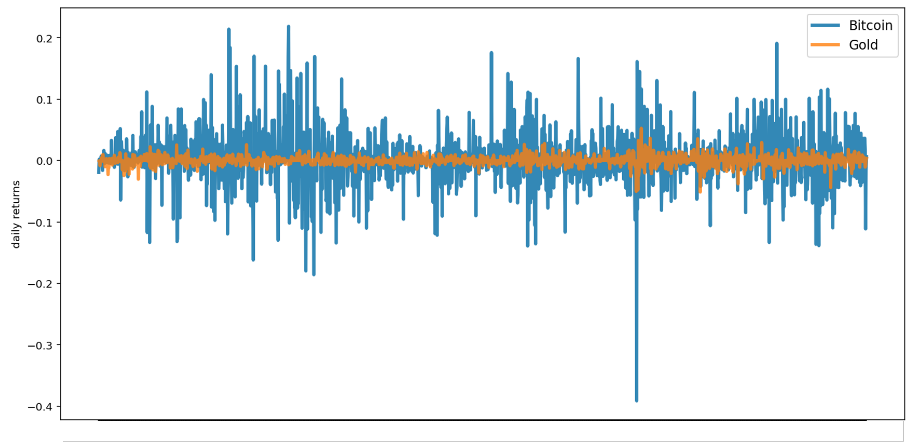


Figure 3: Daily volatility

This ratio describes the excess return you receive for the extra volatility you are exposed to holding riskier assets.Our goal is to maximize the value of k, that is:

$$k = k_{max} = \frac{E(r_n) - r_f}{\sigma_n} \tag{8}$$

We find ω by taking the derivative of k ($\frac{dk}{d\omega}$) with respect to ω such that the derivative is zero:

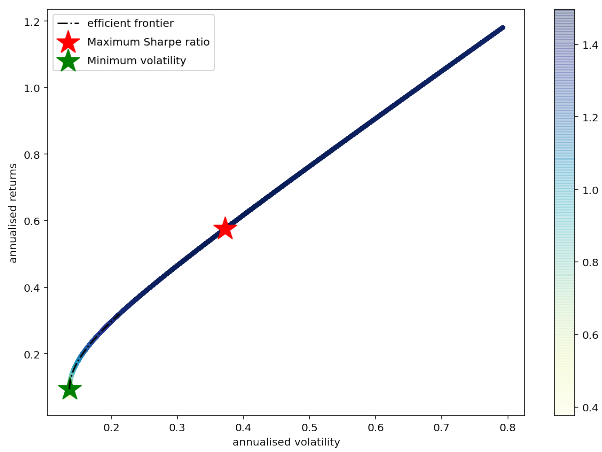
$$\omega = \frac{[E(r_1) - r_f]\sigma_2^2 - [E(r_2) - r_f]\rho\sigma_1\sigma_2}{[E(r_1) - r_f]\sigma_2^2 - [E(r_2) - r_f]\sigma_1^2 - [E(r_1) - r_f + E(r_2) - r_f]\rho\sigma_1\sigma_2} \tag{9}$$

4.1.4 Model's results and analyze

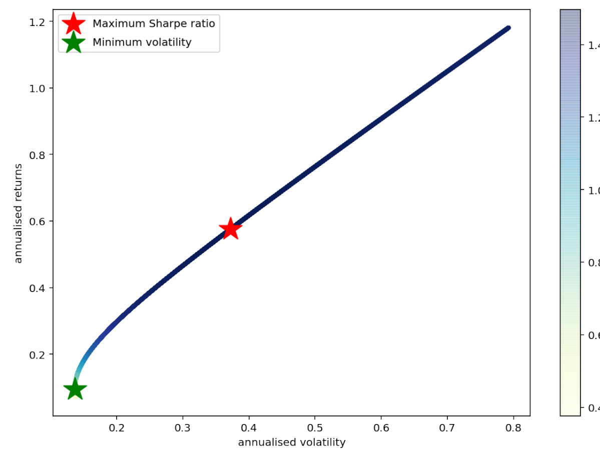
From the Figure 3, we find that the investment risk of Bitcoin is relatively large, and the investment risk of gold is relatively small.How to balance a risky investment product and a less risky investment product directly affects our expected rate of return, so we initialize the investment weights of different products randomly, and then simulate the trading strategy according to the randomly generated weights.

We randomly simulated 100,000 different investment weight distributions, and the results are shown in Figure 4(a). Find the portfolio with the highest Sharpe ratio among them and mark the portfolio with the highest Sharpe ratio with a red star. Also find the portfolio with the least volatility and mark it with a green star. Finally, plot the Sharpe ratios of all randomly generated portfolios with the color map. The bluer the color, the higher the Sharpe ratio.

Through the efficient frontier theory, we calculate the optimal SharpeRation investment weight and the lowest risk investment weight according to the formula in section 4.1.3, as shown in Figure 4(b).



(a) Calculated Portfolio Optimization based on Efficient Frontier



(b) Simulated Portfolio Optimization based on Efficient Frontier

Figure 4: Optimal risk portfolio

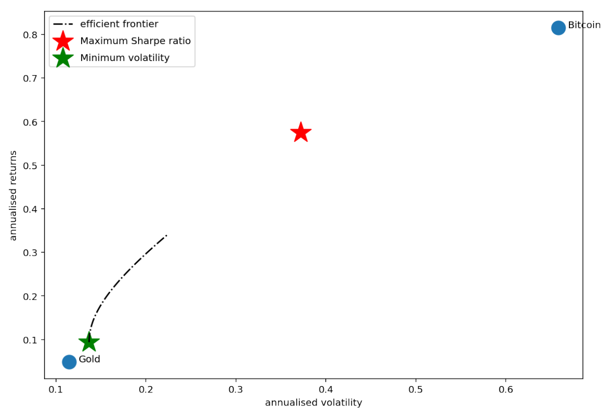


Figure 5: Portfolio Optimization with Bitcoin and Gold

As can be seen from the Figure 5, the lowest risk is gold, which is about 0.11. But in the optimal portfolio, the lowest risk can reach 0.14, and it has a higher yield than gold. If we can accept the higher risk of gold, we can get a yield of 0.58 under the optimal portfolio.

In the results of our weight simulation, we ordered the simulation results from low to high indexed by the average annual rate of return as follows:

Table 2: Partial yield results

Combination	Returns	Volatility	Bitcoin weight	Gold weight
16937	0.048132	0.113996	0.000002	0.999998
49995	0.260497	0.204508	0.279189	0.720811
49996	0.525982	0.416401	0.628211	0.371789
4	0.690287	0.554967	0.844217	0.155783
15892	0.808740	0.655931	0.999941	0.000059

From the results in the Table 2, we can observe that the size of the rate of return increases with the increase of the Bitcoin investment weight, and it can be analyzed that Bitcoin and the size of the rate of return are positively correlated. From the Figure 3, it can also be judged that the greater the investment weight of Bitcoin, the greater the return, because the daily price fluctuations of Bitcoin are accompanied by greater risks relative to gold , and higher risks are often accompanied by greater returns. On the contrary, when the weight of investment in gold is too large, the rate of return is too small, and it can be analyzed that there is a negative correlation between gold and the size of the rate of return. Although the yield of gold is not high, investing in gold with low risk can almost be regarded as a stable income. We finally measure the balance between return and risk by using Sharperatio.

Table 3: Maximum Balanced Profit and Minimum Risk

Maximum Sharperatio Allocation	Return	Risk	Bitcoin	Gold
	0.58	0.37	45.46	54.54
Minimun Volatility Allocation	Return	Risk	Bitcoin	Gold
	0.09	0.14	2.11	97.89

According to the investment proportion obtained by the econometric model, we used the computer to simulate traders from 9/11/2016 to 9/10/2021 and used this weight to simulate the transaction with an initial capital of 1,000 US dollars. The final results are as follows:

Table 4: Return

	Returns	Annualized rate of return
Low risk	2140.163	0.077
Balance benefits and risk	15576.597	0.531
High risk	29532.632	0.781

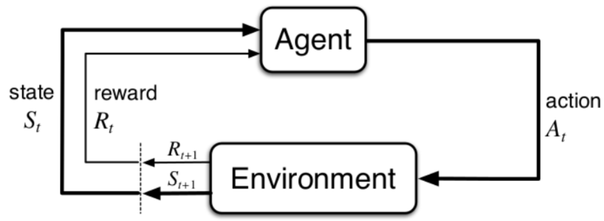


Figure 6: Reinforcement Learning

4.2 Deep Reinforcement Learning Models

Deep reinforcement learning is an artificial intelligence method similar to human thinking that has both deep learning perception ability and reinforcement learning decision-making ability.Deep reinforcement learning models that combine perception and decision-making capabilities can solve the perception problem of complex systems to a certain extent.

A common deep reinforcement learning model is shown in Figure 6, where Agent represents an smart object. Its role in the model is similar to the role of humans in the learning process, by taking certain actions, and these actions will have an impact (positive or negative) on the environment in which the agent is currently located, and the impact of the environment is finally fed back to the intelligence. The specific performance is that if the behavior of the agent has a positive impact on the environment, the model will reward the agent (but generally the reward is small), and when the behavior of the agent has a negative impact on the environment, the model will reward the agent. Punish the agent with a relatively large amount of force, and let the agent learn how to maximize the reward when the model converges through the reward and punishment mechanism.

4.2.1 Data arrange

The given data contains only the closing prices of bitcoin and gold for the trading open day from 9/11/2016 to 9/10/2021. We make certain assumptions based on the given data and calculate the open trading day's price from the data Closing price change, historical transaction volume, historical transaction value, and Bitcoin and gold transaction status and other attributes.

Table 5: Data arrange

Feature Name	Feature Description	Feature Explanation
Date	Trading open date	Format: MM-DD-YYYY
Open	Opening price today	Precision: 4 decimal places; Unit: USD
Close	Closing price today	Precision: 4 decimal places; Unit: USD
Volume	The number of transactions	Unit: share
Amount	Clinch price	Precision: 4 decimal places; Unit: USD
Tradestatus	Trading status	1: Normal trading 0: Trading suspension
pctChg	Change (percentage)	Precision: 6 decimal places

The agent needs to observe the environment to generate action, which in this case is the data of Bitcoin and Gold. In order for the network to converge during training, when the observed state data is input, it must be normalized to transform the various data into the interval $[-1, 1]$.

4.2.2 Model design

Action: Assuming that the transaction has three operations: buy, sell and hold, define the action as an array of length 2.

- $action[0]$ is the operation type
- $action[1]$ is the percentage of buy or sell

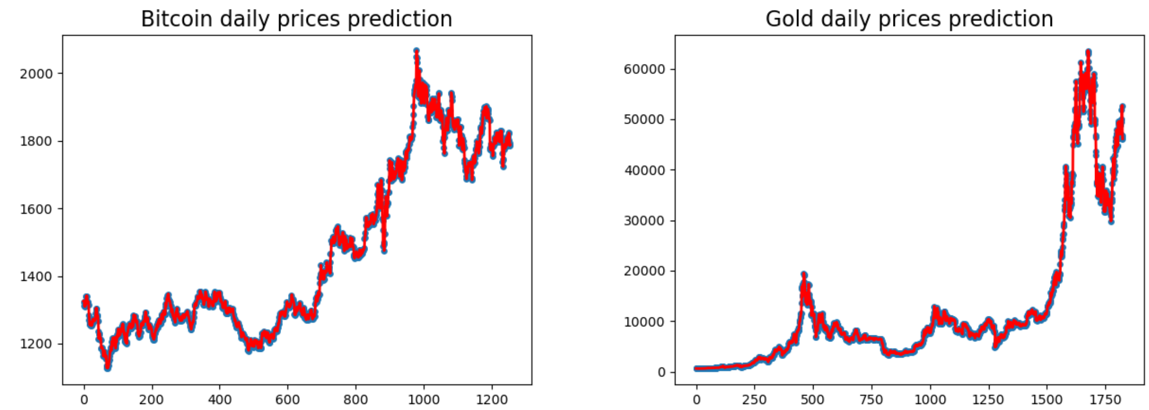
Reward: How to design the reward function is a crucial link in reinforcement learning. In the current Bitcoin and Gold trading environment, we are concerned with the profitability of the model, so we use the current net profit as the reward function.

$$CurrentFunds + ValueofBitcoinandGold - InitialPrincipal - TransactionCost = Profit \quad (10)$$

In order to make the model learn the trading strategy faster, we use a larger penalty for the model when the profit is negative.

Policy gradient: Since the value of the output of the action in the model is continuous, the **Proximal Policy Optimization Algorithms** [7] based on the policy gradient is adopted and has good performance (especially for continuous control problems).

Price prediction: We preliminarily determined that price prediction is a regression problem, so we used machine learning methods such as **Linear regression, LogisticRegression, BaggingRegressor, RandomForestRegressor, and Neural Network** for price prediction.



(a) Bitcoin (b) Gold

Figure 7: Price prediction

Table 6: Price prediction error

	Bitcoin	
	Meansquareerror	*R2Score*
LogisticRegression	0.08378	-0.68
AdaBoostRegressor	0.08324	-0.66
BaggingRegressor	0.08305	-0.67
GradientBoostingRegressor	0.08246	-0.65
RandomForestRegressor	0.08274	-0.66
Neural Network	**0.00016**	
	Gold	
	Meansquareerror	*R2Score*
LogisticRegression	0.19496	-1.75
AdaBoostRegressor	0.19128	-1.77
BaggingRegressor	0.19180	-1.74
GradientBoostingRegressor	0.19128	-1.80
RandomForestRegressor	0.19177	-1.76
Neural Network	**0.00021**	

From the Figure 7 and the Table 6, it is found that the neural network can predict the price curve very well, because when the neural network has enough neurons, it can approximate any function, so we use the neural network to predict the price.

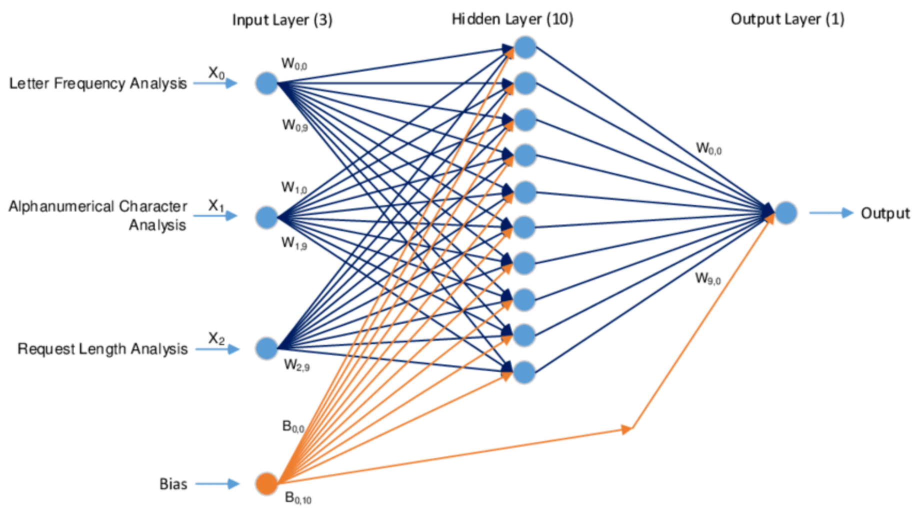


Figure 8: NeuralNetwok

5 Why is the best trading strategy

We try to analyze why the model is the optimal trading strategy from three perspectives: economic theory, artificial intelligence technology, and actual market research.

Artificial Intelligence: Since the proposal of AlexNet in 2012, deep learning(like Figure 8) has re-entered people's field of vision. After nearly 10 years of rapid development, deep learning has derived many sub-directions, such as **Convolutional neural networks**, **Recurrent Neural Networks**, **Graph Neural Networks**, and **Reinforcement learning**.The neural network is designed to imitate human neurons. With the deepening of the network layer and the complexity of the network structure, the neural network model is no longer interpretable, and researchers can only roughly perceive the direction of network architecture tuning. But it cannot be explained by rigorous proof that his improved performance shows excellent performance.

From a certain point of view, the neural network can approximate any function by increasing the number of neurons and deepening the number of layers of the neural network, but it is impossible to estimate what kind of network architecture and how many neurons are required for each function. You can only get it by trying.

Our model did not achieve particularly good results with our deep reinforcement model. Our model learned a certain trading strategy but the model did not converge. After our analysis, we believe that our model is unreasonable in the setting of some parameters, and the gradient does not converge. On the other hand, the model with less data given by the question cannot be really trained.

Economic Theory: For the asset allocation problem proposed by Markowitz, it makes the following assumptions about this problem:

- The return of an asset is expressed by the mean value of returns, and the risk of an asset is expressed as the standard deviation of returns.

- Replace the expected future return of the asset with the average value of the past return of the asset, which means that the return is stable.
- The standard deviation of the asset's past returns is used to replace the expected risk of the asset in the future, and the risk is considered to be stable.
- When making portfolio allocation, only consider benefits and risks.
- Investors are risk averse.

Through the above assumptions, the asset allocation problem becomes a mathematical convex optimization problem.

Through 54, the most cost-effective problem in asset allocation can be transformed into three models:

First:

$$\min_{w} \sqrt{W^T \Sigma W}$$
$$s.t. \quad \begin{cases} \sum_i^m w_i E(R_i) = R_0 \\ \sum_i^m w_i = 1 \\ w_i \geqslant 0 \end{cases}$$

Among them: there is a constraint of $w_i \geqslant 0$ that does not allow shorting, and removing this constraint is to allow shorting. R_0 is the given portfolio return.

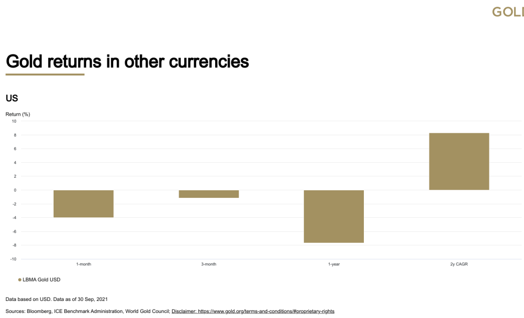
Second:

$$\max_{w} \sum_i^m w_i E(R_i)$$
$$s.t. \quad \begin{cases} \sqrt{W^T \Sigma W} = \sigma_0 \\ \sum_i^m w_i = 1 \\ w_i \geqslant 0 \end{cases}$$

Among them: there is a constraint of $w_i \geqslant 0$ that does not allow shorting, and removing this constraint is to allow shorting. σ_0 is the given portfolio return.

Third:

$$\max_{W} = \max_{W} (\sum_i^m w_i E(R_i) - \frac{1}{2} A W^T \Sigma W)$$
$$s.t. \quad \begin{cases} \sum_i^m w_i = 1 \\ w_i \geqslant 0 \end{cases}$$



(a) Gold reference return

Bitcoin Monthly Returns(%)

Type: Monthly Symbol: BTC

Time	January	February	March	April	May	June	July	August	September	October	November	December
2022	-16.68%	1.64%										
2021	14.51%	36.78%	29.84%	-1.98%	-35.31%	-5.95%	18.19%	13.8%	-7.03%	39.93%	-7.11%	-18.9%
2020	29.95%	-8.6%	-24.92%	34.26%	9.51%	-3.18%	24.03%	2.83%	-7.51%	27.7%	42.95%	46.92%
2019	-8.58%	11.14%	7.05%	34.36%	52.38%	26.67%	-6.59%	-4.6%	-13.38%	10.17%	-17.27%	-5.15%
2018	-25.41%	0.47%	-32.85%	33.43%	-18.99%	-14.62%	20.96%	-9.27%	-5.58%	-3.83%	-36.57%	-5.15%
2017	-0.04%	23.07%	-9.05%	32.71%	52.71%	10.45%	17.92%	65.32%	-7.44%	47.81%	53.48%	38.89%
2016	-14.83%	20.08%	-5.35%	7.27%	18.78%	27.14%	-7.67%	-7.49%	6.04%	14.71%	5.42%	30.8%
2015	-33.05%	18.43%	-4.38%	-3.46%	-3.17%	15.19%	8.2%	-18.67%	2.35%	33.49%	19.27%	13.83%
2014	10.03%	-31.03%	-17.25%	-1.6%	39.46%	2.2%	-9.69%	-17.55%	-19.01%	-12.95%	12.82%	-15.11%
2013	44.05%	61.77%	172.76%	50.01%	-8.56%	-29.89%	9.6%	30.42%	-1.76%	60.79%	449.35%	-34.81%

(b) Bitcoin reference return

Figure 9: Returns survey

Among them: there is a constraint of $w_i \geqslant 0$ that does not allow shorting, and removing this constraint is to allow shorting. σ_0 is the given portfolio return.

The third form is different from the first two, which are either maximizing return for a given risk, or minimizing risk for a given return. The third solution is the problem of maximizing the utility by given a utility function. When the utility function of the whole market is the same, what is obtained is the asset allocation at the market equilibrium point.It should be particularly pointed out that this model has a disadvantage. It is sensitive to the estimates of returns and risks. If the returns and risks change, the weights allocated by it will change greatly.

Actual market research: We googled the average or annual returns for Bitcoin[2] and Gold[3] from 9/11/2016 to 9/10/2021.

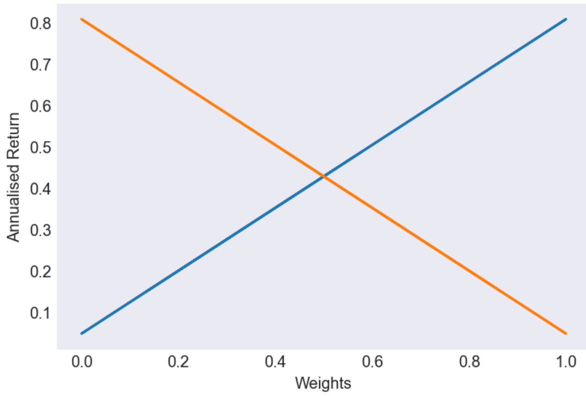
We searched the real bitcoin and gold returns through Google, and compared our returns to show that our trading strategy is the optimal trading strategy.

6 Conclusions

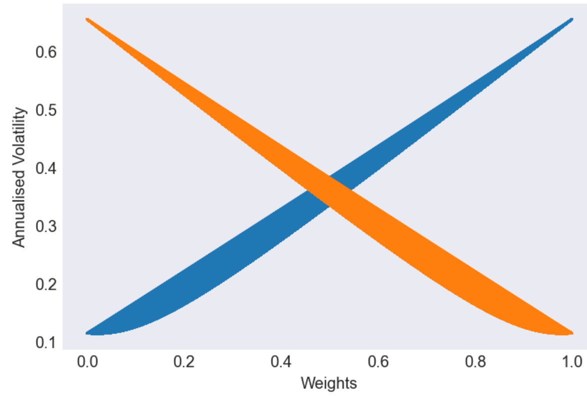
From the known information of the topic, we have established two models, an econometric model (based on Markowitzs best investment theory), and a deep reinforcement learning model to simulate investment and trading strategies. In a deep reinforcement learning model, our model can predict the price trend of bitcoin and gold respectively with very small errors (mse about 0.00016 and 0.00021), and the model has learned a simple trading strategy but has not yet learned how to maximize returns. For the econometric model, we calculate the weight of the portfolio before each transaction and then assign the weight to Bitcoin and gold respectively, and according to the market balance risk, the annual rate of return is about 0.077 when the risk is low. The maximum point of Sharpe ratio (and balancing risk and return) can achieve an annual rate of return of about 0.531, and if the high risk is acceptable, the

[2]https://www.coinglass.com/today

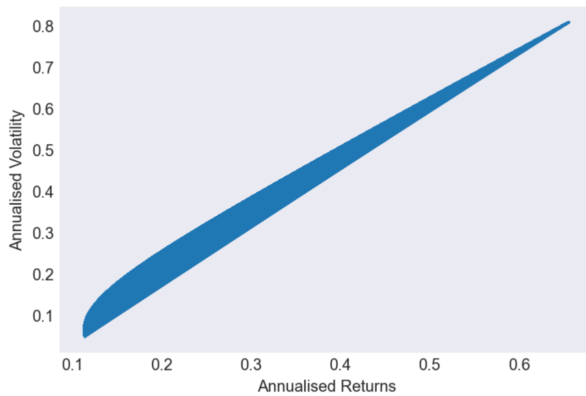
[3]https://www.gold.org/goldhub/data/gold-returns



(a) Annualised Return



(b) Anuualised volatility



(c) Returns with volatility

Figure 10: Sensitivity analysis

annual rate of return can reach 0.781. After rigorous assumptions and strict optimization, we believe that our trading strategy is the most efficient and excellent.

7 Sensitivity analysis

In this part, we do a sensitivity analysis on the different investment weights to the annual rate of return and the annual risk, and we also do a sensitivity analysis on the relationship between the annual rate of return and the annual risk.

In the Figure 10(a), the orange line represents gold, the blue line represents Bitcoin, the horizontal axis is their weight, and the vertical axis is their annual rate of return. We found that with the increasing investment weight of Bitcoin, the annual rate of return will increase, and the two show a positive correlation. On the contrary, with the increasing investment weight of gold, the annual rate of return will become lower and lower. negatively correlated relationship. According to the analysis of the

Figure 3, we can conclude that since the daily closing price of Bitcoin rises and falls more sharply than that of gold, investing in Bitcoin can achieve higher returns but higher risks, while investing in gold is a kind of stability choice of income.

In the Figure 10(b), gold is represented by an orange line, and Bitcoin is represented by a blue line, with their weights on the horizontal axis and their annual risk on the vertical axis. We found that as the investment weight of Bitcoin increases, the risk in the coming year will also increase, and the two show a positive correlation. On the contrary, as the investment weight of gold increases, the annual risk becomes lower and lower show a negative relationship. Since there may be different investment weights but the risk is the same, a wider blade-like shape appears in the figure. It can be seen from the figure that investing in gold in large quantities is a less risky investment method.

In the Figure 10(c), we find that with the growth of the annual rate of return on the horizontal axis, the risk is also increasing, and the relationship between the two is roughly linear. And there is a truth in the world that "high risk often means high reward".

Therefore, the investment weight is insensitive to the annual rate of return and annual risk, and the annual rate of return is also insensitive to the annual risk.

8 Possible Model Update in Future

We built two models to simulate the trading strategies of Bitcoin and Gold, and the two models respectively calculated the weights of the portfolio to carry out the trading strategies.

1. **The Economic Model:**Model is based on closing prices for Bitcoin and Gold trading from 9/11/2016 to 9/10/2021. In order to make the model more accurate, we will re-optimize the model and use longer data to train the model to get better performance. At the same time, we will continue to study related econometric models to find a more realistic model to calculate investment weights.

2. **The Deep Reinforcement Model:**The model is built based on the reprocessed data, but since the deep reinforcement learning model requires a large amount of data for training, we will collect more training data in the future to train a model with better performance. At the same time, we will also study the model architecture and build a Deep reinforcement learning models that can adapt to small samples.

9 Strengths and weaknesses

9.1 Strengths

- **Applies widely.**

- **Improve the quality of the trading straregy.**

- **This analysis method can be extended to excellent theories: efficient frontier theory, Sharpe capital asset pricing model, multi-factor distribution model and so on.**

9.2 Weaknesses

- **Models are built on a large number of assumptions and have limitations.**

- **In reality, the return and risk of most assets are not stable, and this determination is the biggest problem with this model. Past returns and risks cannot represent expectations.**

- **The benefits and risks defined in the model are too restrictive. For example, the actual risk is not necessarily the standard deviation, since upward volatility is not a risk.**

10 Memorandum

TO: Trader

FROM:MCM Team 2227819

DATE: February 21, 2022

SUBJECT: Build a trading strategy model that maximizes returns

The investment portfolio problem refers to the decision-making process of continuously allocating a certain amount of funds to multiple financial products. By continuously adjusting the investment weights of each financial product, it can achieve high returns while diversifying investment risks. Although Markowitz proposed Portfolio Theory in 1952, the theory is based on certain assumptions in order to be used and does not apply to all market situations.

OUR APPROACH

Our trading strategy model is based on optimal investment risk and optimal portfolio.

- We use the optimal portfolio strategy to calculate the weight of investing in different products, and then buy and sell according to the weight on different open trading days until the deadline, during which we hope to maximize our returns, that is, solve the convex optimization problem.
- After the solution is completed, we finally get a market equilibrium point, that is, the Sharpe rate point, by balancing risks and returns.

OUR Results

The model results are divided into three parts: low-risk return, balanced return, and high-risk return. Traders can choose trading strategies according to their appetite for risk. We prefer to choose balanced income, because balanced income balances the risk of the market to a certain extent and reduces the occurrence of losses.

Future Outlook

The deep reinforcement learning model is an important model in the investment field now and in the future. In the future, we will continue to train the deep reinforcement learning model to simulate trading strategies. We will use more valid data, use better-performing policy gradients, and mine the hidden information in the data by training the agents in the model to help us maximize our benefits.

Yours sincerely,
Team 2227819

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Appendices

Appendix A First appendix

Here are simulation programmes we used in our model as follow.

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import quandl
import scipy.optimize as sco

def neg_sharpe_ratio(weights, mean_returns, cov_matrix, risk_free_rate):
   p_var, p_ret = portfolio_annualised_performance(weights, mean_returns, cov_matrix)
   return -(p_ret - risk_free_rate) / p_var

def max_sharpe_ratio(mean_returns, cov_matrix, risk_free_rate):
   num_assets = len(mean_returns)
   args = (mean_returns, cov_matrix, risk_free_rate)
   constraints = ({'type': 'eq', 'fun': lambda x: np.sum(x) - 1})
   bound = (0.0,1.0)
   bounds = tuple(bound for asset in range(num_assets))
   result = sco.minimize(neg_sharpe_ratio, num_assets*[1./num_assets,], args=args,
                    method='SLSQP', bounds=bounds, constraints=constraints)
   return result

def min_variance(mean_returns, cov_matrix):
   num_assets = len(mean_returns)
   args = (mean_returns, cov_matrix)
   constraints = ({'type': 'eq', 'fun': lambda x: np.sum(x) - 1})
   bound = (0.0,1.0)
   bounds = tuple(bound for asset in range(num_assets))
   result = sco.minimize(portfolio_volatility, num_assets*[1./num_assets,], args=args,
                    method='SLSQP', bounds=bounds, constraints=constraints)

   return result

def efficient_frontier(mean_returns, cov_matrix, returns_range):
   efficients = []
```

```python
    for ret in returns_range:
        efficients.append(efficient_return(mean_returns, cov_matrix, ret))
    return efficients
```

Appendix B Second appendix

some more text

```python
import pickle
import pandas as pd
from stable_baselines.common.policies import MlpPolicy
from stable_baselines.common.vec_env import DummyVecEnv
from stable_baselines import PPO2
from rlenv.StockTradingEnv0 import StockTradingEnv

import matplotlib.pyplot as plt
import matplotlib.font_manager as fm

def stock_trade(stock_file):
    day_profits = []
    df = pd.read_csv(stock_file)
    df = df.sort_values('date')

    # The algorithms require a vectorized environment to run
    env = DummyVecEnv([lambda: StockTradingEnv(df)])

    model = PPO2(MlpPolicy, env, verbose=0, tensorboard_log='./log')
    model.learn(total_timesteps=int(1e4))

    df_test = pd.read_csv(stock_file.replace('train', 'test'))

    env = DummyVecEnv([lambda: StockTradingEnv(df_test)])
    obs = env.reset()
    for i in range(len(df_test) - 1):
        action, _states = model.predict(obs)
        obs, rewards, done, info = env.step(action)
        profit = env.render()
        day_profits.append(profit)
        if done:
            break
    return day_profits
```