

Granular Computing: Pursuing New Frontiers of Artificial Intelligence

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Agenda

Defining intelligence and AI

AI: a retrospective view

Information granules as constructs of abstraction

Formal frameworks of Granular Computing

Information granules of higher types and higher orders

Knowledge representation with information granules

Conclusions



Intelligence and AI

Ability to interact with the world (speech, vision, motion...)

Ability to model the world and reason about it

Ability to learn and adapt

To build systems that exhibit intelligent behavior

To understand intelligence in order to model



AI: retrospective view

1943- 1956: artificial neural networks

1956: The Dartmouth conference

1952-1969: checkers players, neural networks

1966-1974: reality (awareness limitations)

1969-1979: knowledge-based systems (expert systems)

1980-1988: industrial applications; commercial expert systems, diagnostic systems

1990-2000: computer chess, humanoid robots, entertainment robots, speech systems, vision systems, data analysis (summarization)

Top 10 AI technologies for 2018

- Deep learning
- Capsule networks
- Deep reinforcement learning
- Generative adversarial learning
- Lean and augmented data learning
- Probabilistic programming
- Hybrid learning models
- Automated machine learning
- Digital twin
- Explainable AI

Price Waterhouse, 2018

AI: symbolic and sub-symbolic perspective

Symbols → symbolic processing
knowledge representation,
processing,
programming

Sub-symbolic → numeric processing

Sub symbolic perspective: early (neural networks) and
recent (deep learning) periods of AI

Information granularity

Information granules as intuitively appealing constructs playing a pivotal role in human cognitive and decision-making activities.

complex phenomena perceived by organizing existing knowledge along with available experimental evidence.

Knowledge structured in a form of some meaningful, *semantically* sound entities, which are central to all ensuing processes of describing the world, reasoning about the environment and support decision-making activities.



Information granules

Information granules: entities composed of elements being drawn together on a basis of

similarity,

functional closeness,

temporal resemblance

spatial neighborhood, etc.

and subsequently regarded as a single semantically meaningful unit used in processing.

Information granules – examples (1)

Image processing.

In spite of the continuous progress in the area, a human being assumes a dominant and very much uncontested position when it comes to understanding and interpreting images.

We do not focus our attention on individual pixels and process them as such but group them together into semantically meaningful constructs – familiar objects we deal with in everyday life.

Such objects involve regions that consist of pixels or categories of pixels drawn together because of their proximity in the image, similar texture, color, etc.

This remarkable and unchallenged ability of humans dwells on our effortless ability to construct **information granules**, manipulate them and arrive at sound conclusions.

Information granules – examples (2)

Processing and interpretation of time series.

From our perspective we can describe time series in a semi-qualitative manner by pointing at specific regions of such signals.

One distinguishes some segments of temporal signals and interpret their combinations.

E.g., in stock market, one analyzes numerous time series by looking at existing amplitudes, trends, patterns, and relationships among them.



Information granules – examples (3)

Granulation of time

Time is another important and omnipresent variable that is subjected to granulation.

We use seconds, minutes, days, months, and years.

Depending upon a specific problem we have in mind who the user is, the size of **information granules** (time intervals) could vary quite significantly.



Information granules – examples (4)

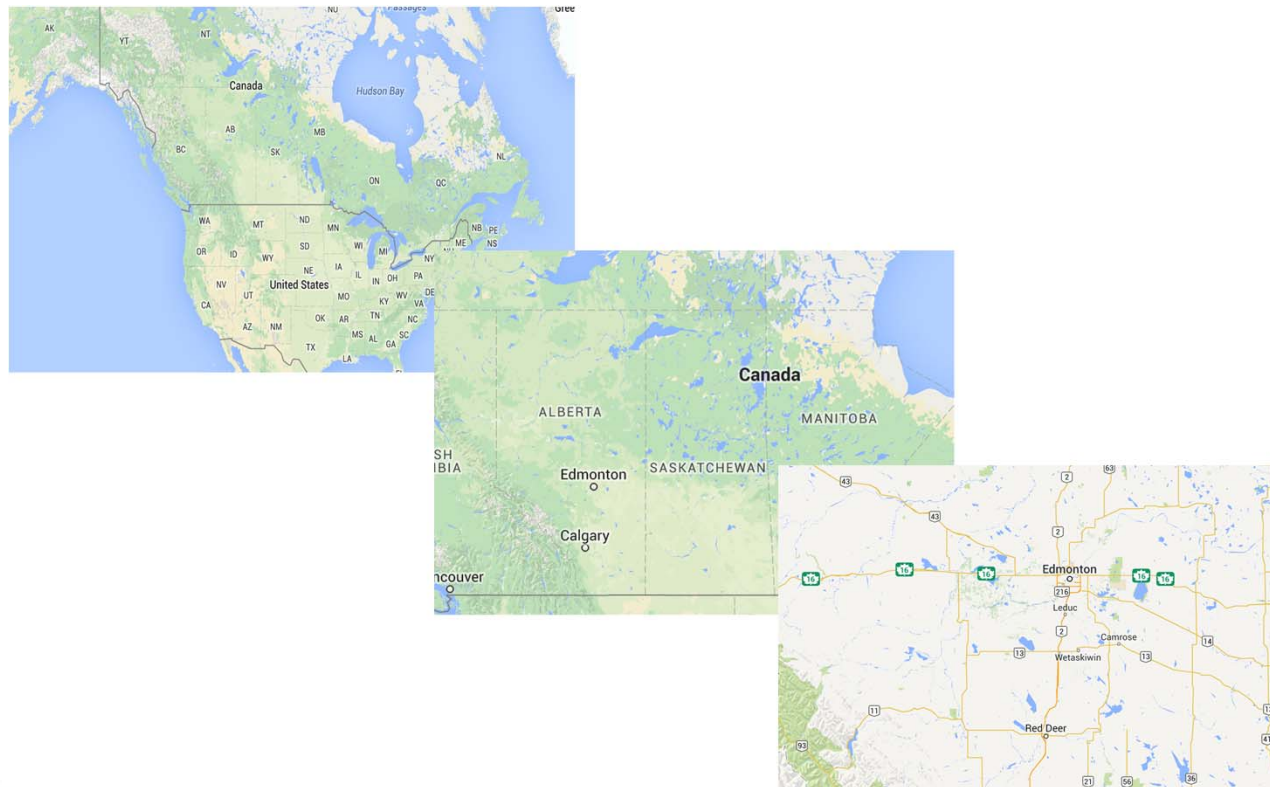
Design of software systems

We develop software artifacts by admitting a modular structure of an overall architecture of the designed system.

Each module is a result of identifying essential functional closeness of some components of the overall system.

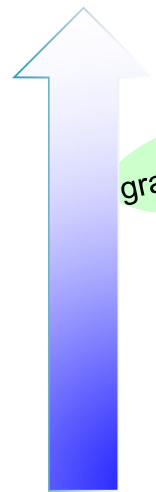
Modularity ([granularity](#)) is a holy grail of the systematic software design supporting a production of high quality software products.

Information granularity



Abstraction levels

**INFORMATION
GRANULES**



granularity of variable

spatial granularity

temporal granularity

long traffic delay close to university level around 9AM



Quality of findings:
Generality, specificity, relevance, stability

Traffic delay of 12 min at 116St at 9:07

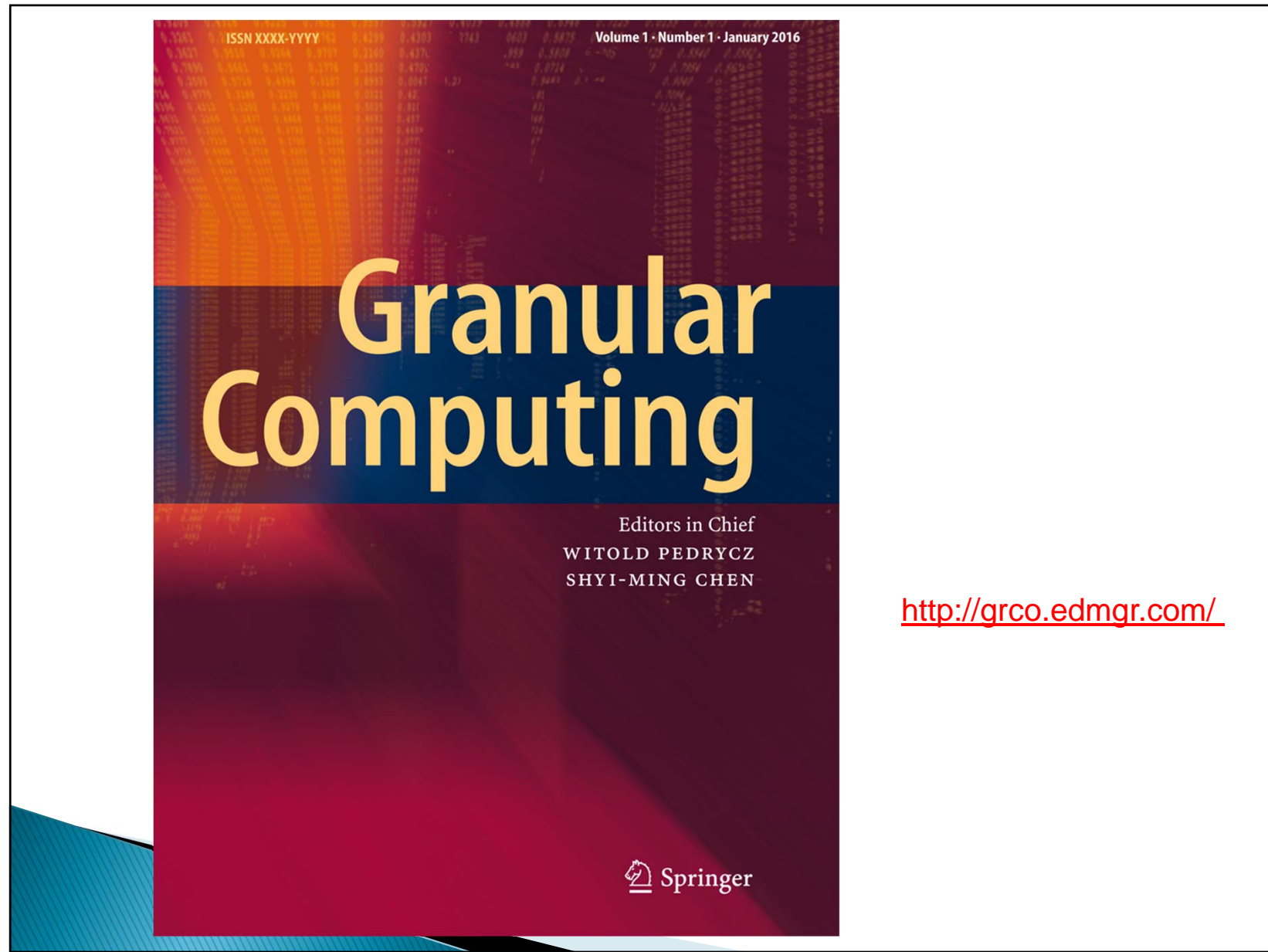
DATA

Information granularity

Our ability to conceptualize the world at different granularities and to switch among these granularities is fundamental to our intelligence and flexibility.

It enables us to map the complexities of the world around us into simple theories that are computationally tractable to reason in.

J. R. Hobbs, Proc. IJCAI, 1985



<http://grco.edmgr.com/>

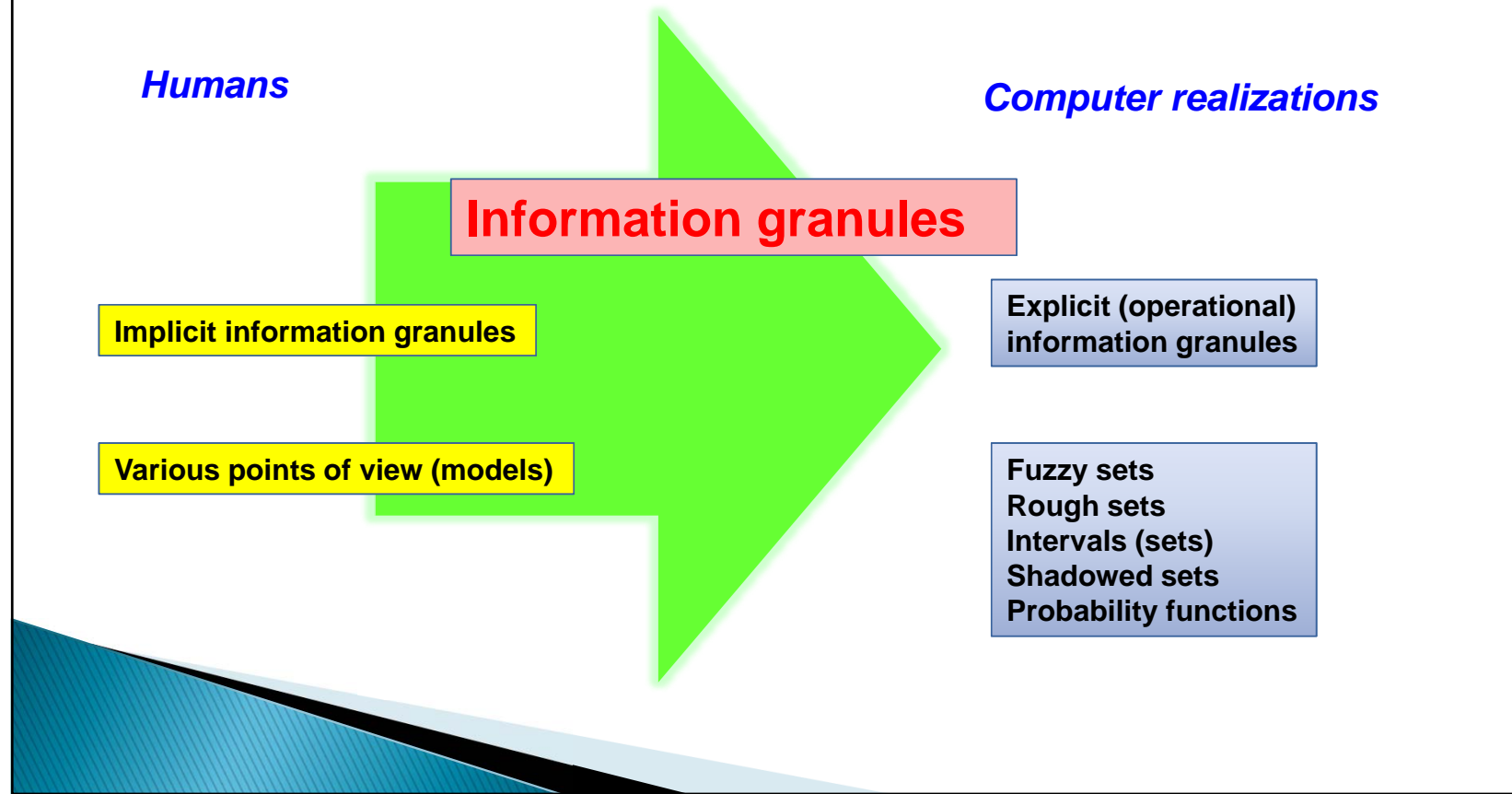
Information granules: key features

Information granules as generic mechanisms of abstraction

Customized, user-centric and business-centric approach to problem description and problem solving

Processing at the level of information granules optimized with respect to the specificity of the problem

Information granules: from their conceptualization to realization



Key formalisms of information granules

Formal settings

Sets and their basic concepts

Interval analysis and interval calculus

Fuzzy sets

Rough sets

Intuitionistic sets

Shadowed sets



Set and interval analysis

Sets are fundamental notions of mathematics, science, and engineering

Dichotomy as the underlying fundamental notion

- belongingness
- exclusion



Interval analysis

Numeric intervals as generic information granules

$$A = [a, b] \quad B = [c, d]$$

Processing concerns the use of

- set theoretic operations

- algebraic operations

- mappings

Interval analysis

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Cl. III — Vol. IV, No. 5, 1956

MATHEMATICS

Calculus of Approximations

by
M. WARMUS

Presented by H. STEINHAUS on February 6, 1956

This paper presents a theory which lays down the foundations for numerical computations and makes it possible to formulate properly many numerical problems.

By the approximate number $[a, A]$ we shall indicate the interval $[a, A]$, i. e. the set of all real numbers x that satisfy the inequality $a \leq x \leq A$.

The approximate number $[B - b, B + b]$ can also be denoted by B . Thus, the approximate number $[a, A]$ can be expressed in the form $\frac{1}{2}(A + a)$. We shall omit initial zeros in the upper part, if they lie to the left of the last digit of the lower one. For example, we shall write 3.1416⁰² instead of 3.1416⁰⁰⁰⁰⁰².

We say that the approximate numbers a and β are equal and we write $a = \beta$ if, and only if, they are two identical intervals. Hence, we have $[a, A] = [b, B]$ if, and only if, $a = b$ and $A = B$, and similarly $A = B$ if, and only if, $A = B$ and $a = b$.

We say that the approximate number β approximates the approximate number a and we write $a \Rightarrow \beta$ or $\beta \Leftarrow a$, if, and only if, the interval β includes the interval a . Thus, we have $[a, A] \Rightarrow [b, B]$ if, and only if, $a \geq b$ and $A \leq B$, and similarly $A \Rightarrow B$ if, and only if, $b - a \geq |B - A|$. It is easy to prove that the approximations $a \Rightarrow \beta$ and $\beta \Rightarrow \gamma$ imply $a \Rightarrow \gamma$.

The relation $a \Rightarrow \beta$ is a partial ordering of the set of all approximate numbers.

In practical computations it is convenient to use the following two rules:

the rounding-off rule: $\frac{a}{A} + c \Rightarrow \frac{a+c}{A}$;
the extending rule: $\frac{a}{A} \Rightarrow \frac{b}{A}$ if $b \geq a$.

Bulletin III PAN

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Fuzzy sets – a departure from dichotomy

Conceptually and algorithmically, fuzzy sets arise as one of the most fundamental and influential notions in science and engineering.

The notion of a fuzzy set is highly intuitive and transparent

Fuzzy set captures a way in which a real world is being perceived and described in our everyday activities.

Description of objects whose belongingness to a given category (concept) is a matter of degree



Fuzzy sets – examples

High temperature

Safe speed

Low humidity

Medium inflation

Low approximation error



From two-valued to three-valued logic

- Lukasiewicz (~1920)

true (0)

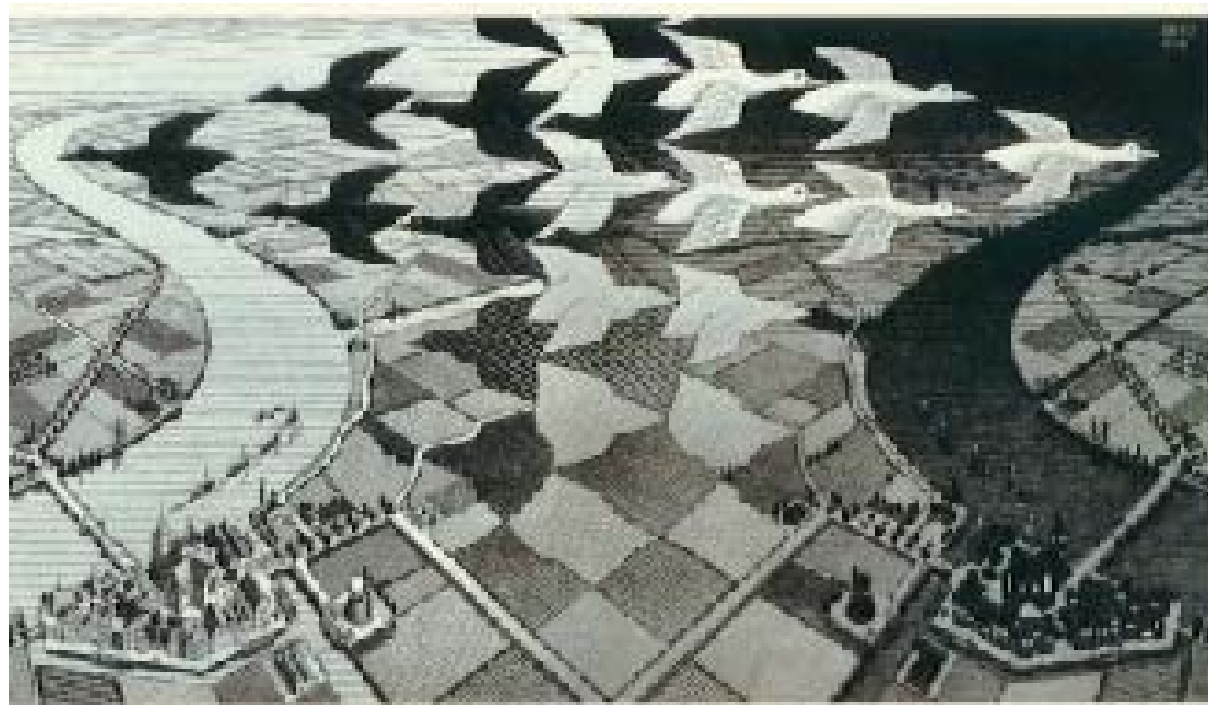
false (1)

don't know ($1/2$)

- Three valued logic and databases (concept of null)



Sets versus Fuzzy Sets



M.C. Escher

Dichotomy, two-valued logic and real world

true-false

yes-no

black –white



The underlying principle of *excluded middle*

“... the law of excluded middle is true when precise symbols are employed, but it is not true when symbols are vague, as, in fact, all symbols are.”

B. Russell (1923)

Fuzzy sets – definition

Fuzzy set A defined in universe of discourse X in terms of its *membership function*

$$A: X \rightarrow [0,1]$$

$A(x)$ – degree of membership

$A(x) = 1$ full membership

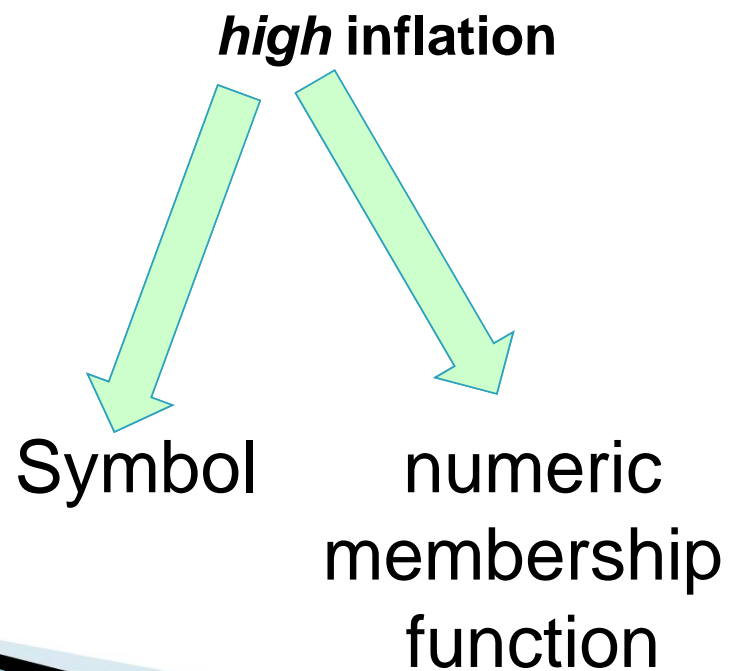
$A(x) = 0$ exclusion

$A(x)$ in-between 0 and 1 partial membership

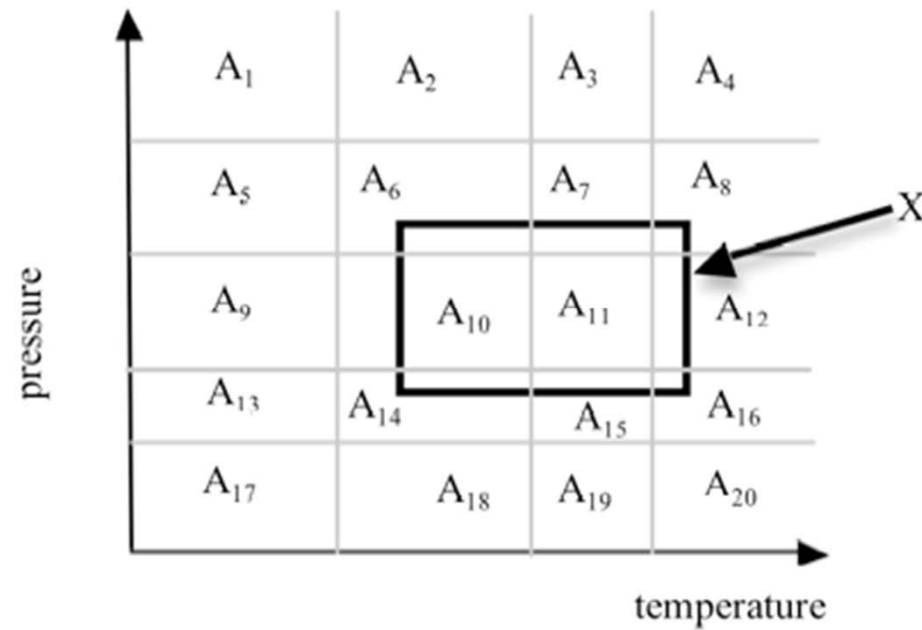


Information granules: duality

Symbolic and sub-symbolic view of granule

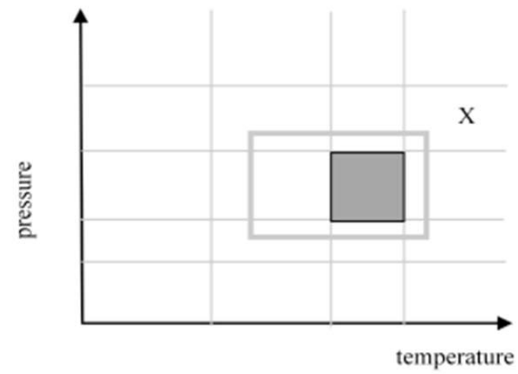
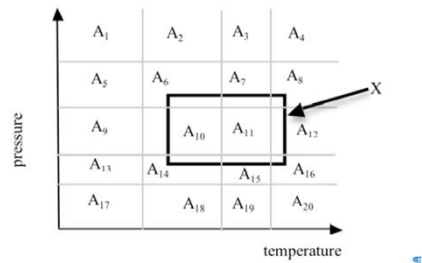


Rough sets

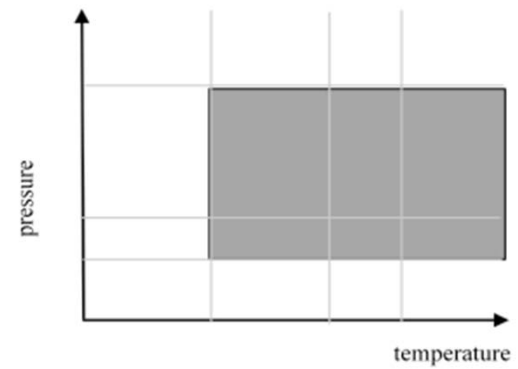


Vocabulary consisting of A_1, A_2, \dots, A_{20}
Describe X

Rough sets- lower and upper bound

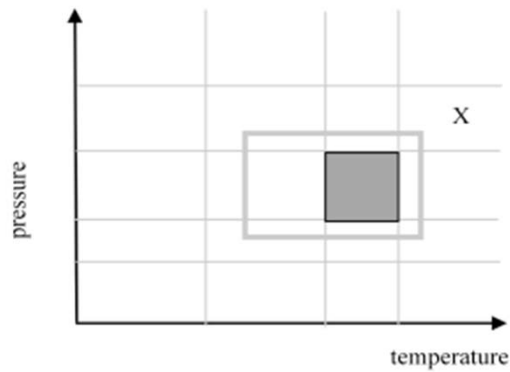


Lower bound



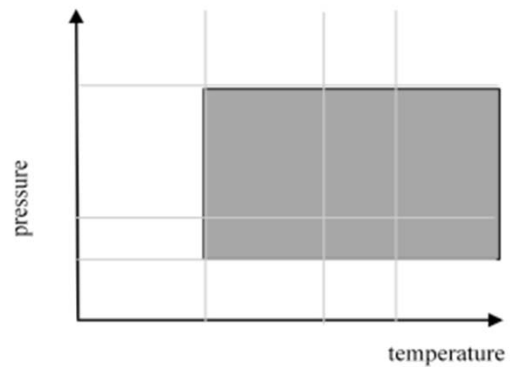
Upper bound

Example - details



$$X_+ = \{A_i \mid A_i \cap X \neq \emptyset\}$$

Lower bound $X_- = \{A_{11}\}$



$$X_- = \{A_i \mid A_i \subset X\}$$

Upper bound X_+

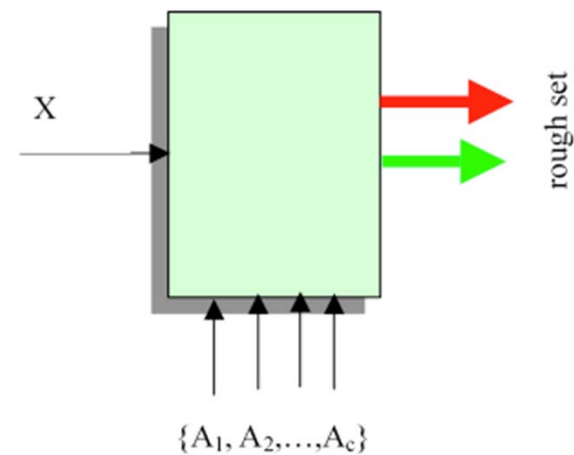
$$|X_+ = \{A_6, A_7, A_8, A_{10}, A_{11}, A_{12}, A_{14}, A_{15}, A_{16}\}.$$

Rough set – definition and interpretation

For given $\{A_i\}$ $\langle X_-, X_+ \rangle$

$$X_+ = \{A_i \mid \underline{A_i} \cap X \neq \emptyset\}$$

$$X_- = \{A_i \mid \underline{A_i} \subset X\}$$



Intuitionistic fuzzy sets (Atanassov)

Consideration given to degrees of membership and non-membership

Inclusion of *hesitation* component:

$$\text{membership} + \text{non-membership} \leq 1$$

$$\text{membership} + \text{non-membership} + \text{hesitancy level} = 1$$

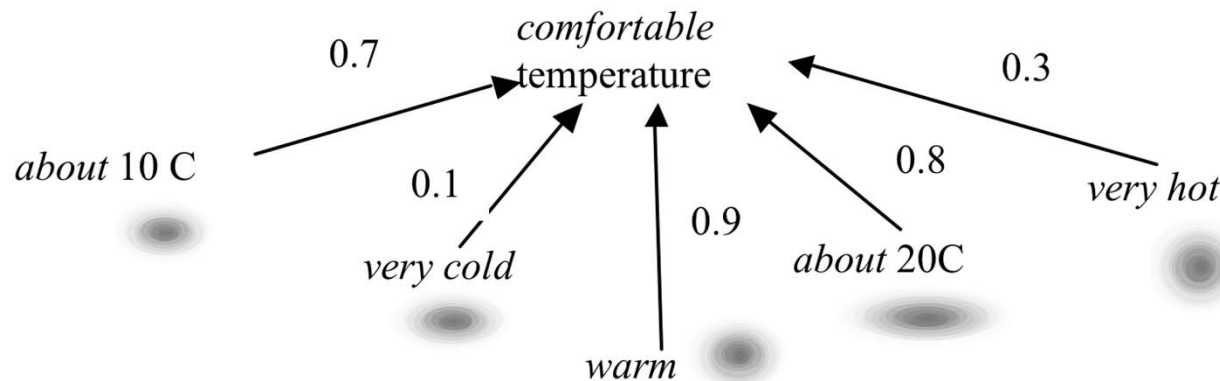
$$A = \langle A^+, A^- \rangle \quad A(x) = \langle A^+(x), A^-(x) \rangle \quad A^+(x) + A^-(x) \leq 1$$

Information granules of
higher type, higher order,
and hybrid information granules

Fuzzy sets of higher order

Fuzzy sets defined in a universe of discourse whose elements are **fuzzy sets** rather than individual elements

comfortable temperature – fuzzy set of second order

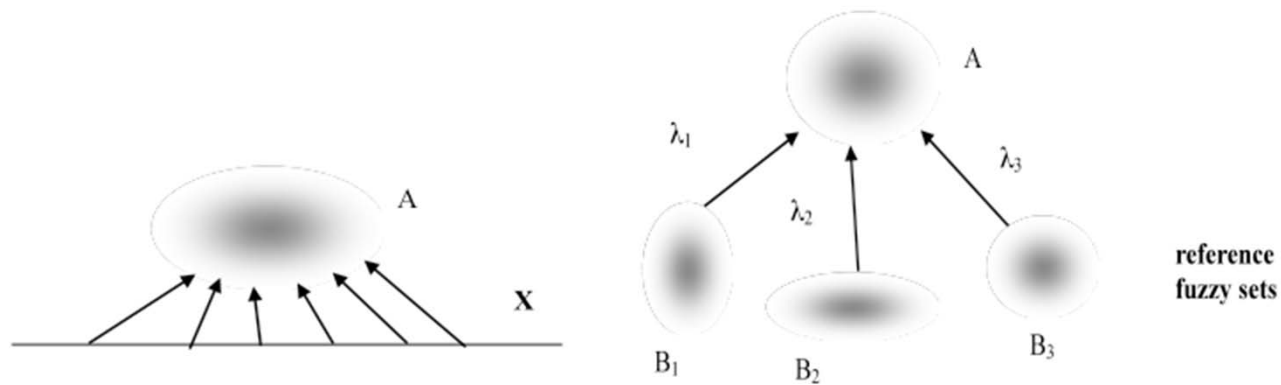


fuzzy sets as
elements of the universe

Fuzzy sets of higher order and fuzzy sets

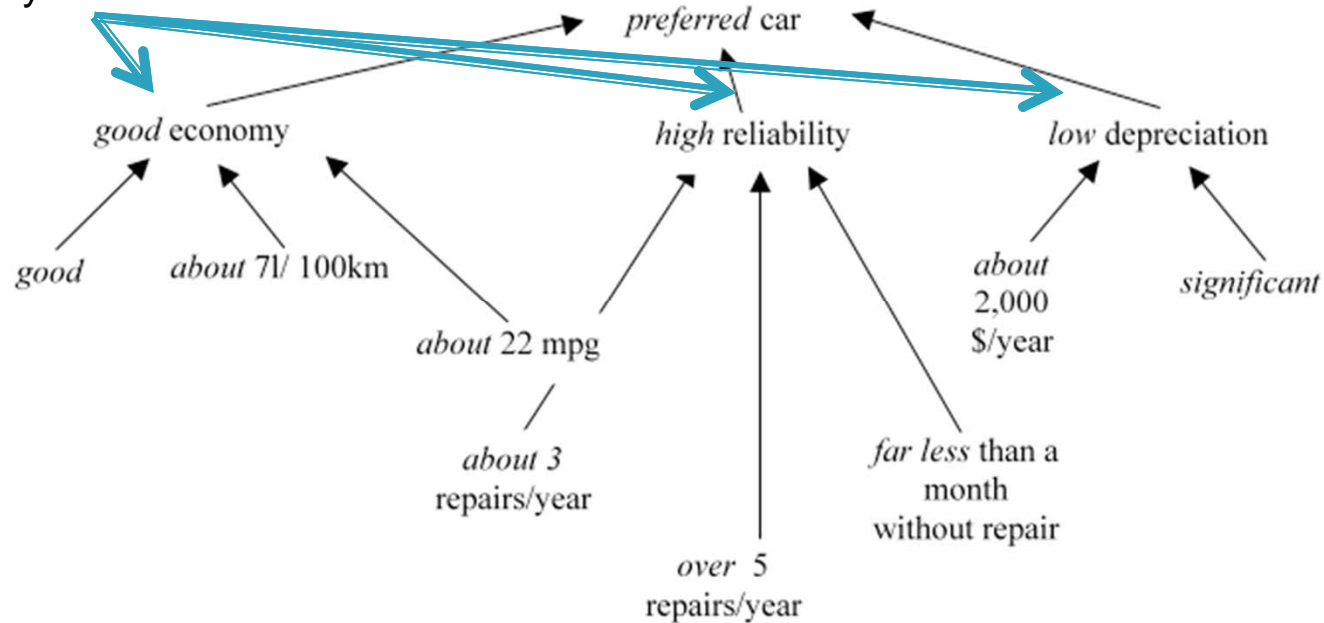
A nature of the universe of discourse

fuzzy set of second order



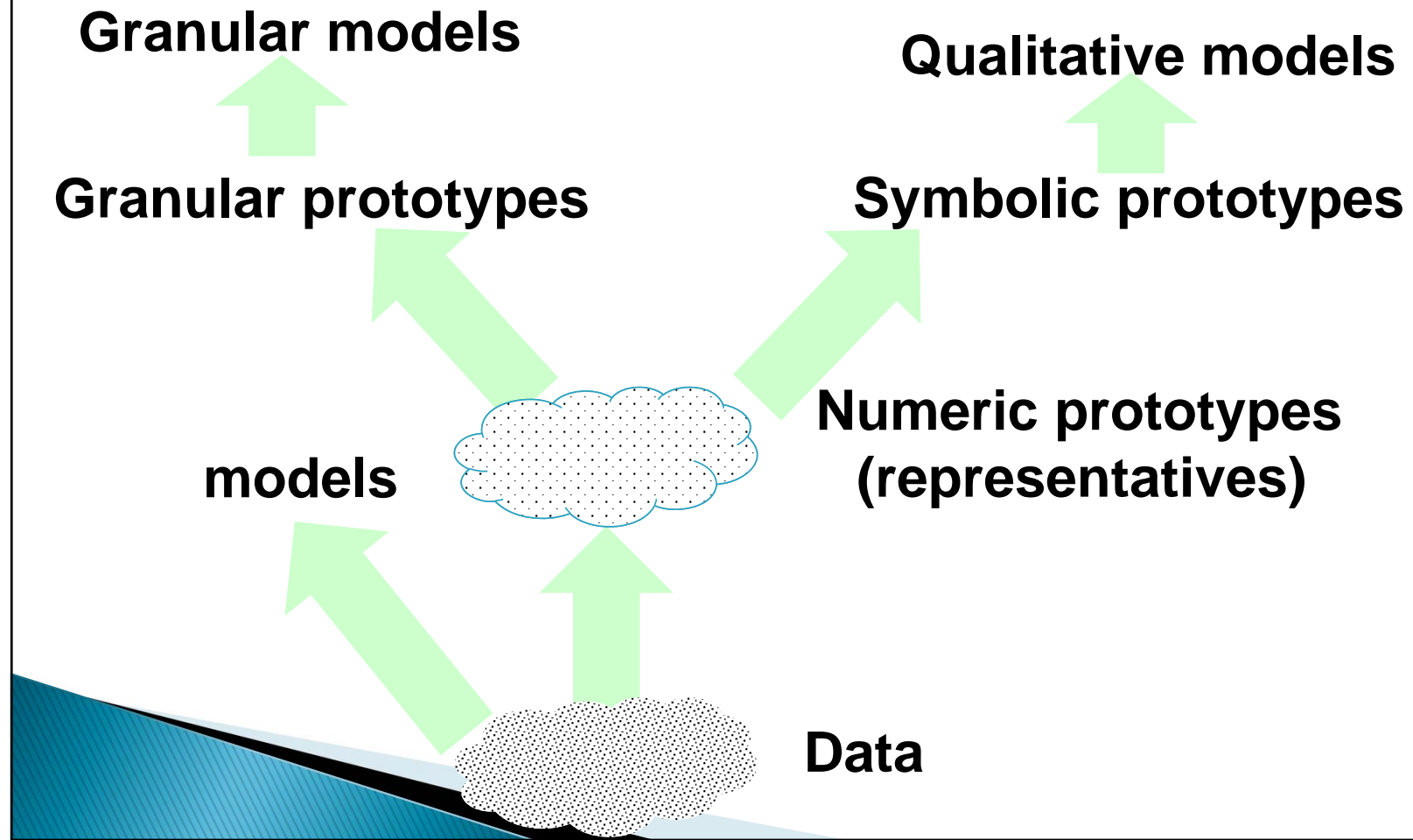
Fuzzy sets of higher order- a hierarchical construct

fuzzy set of order 2



A hierarchical definition of fuzzy set – a number of descriptors expressed as fuzzy sets
Implicit nature of concepts

Data, Granular Data and Granular Modeling: A Landscape



Clustering

Formation of information granules on a basis of experimental data

Abstraction mechanisms; especially visible with respect to the representation of large number of data: N data; c clusters, $c \ll N$ (abstraction in data space);

abstraction in feature space as another option

Unsupervised learning

Plethora of approaches

Fuzzy C – Means (FCM): Generic algorithm

Data in \mathbf{R}^n : $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N$

$$Q = \sum_{i=1}^c \sum_{k=1}^N u_{ik}^m \| \mathbf{x}_k - \mathbf{v}_i \|^2$$

Minimize Q with respect to partition matrix $U = [u_{ik}]$ and prototypes $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_c$

Iterative algorithm (Euclidean distance considered)

$$u_{ik} = \frac{1}{\sum_{j=1}^c \left(\frac{\| \mathbf{x}_k - \mathbf{v}_i \|}{\| \mathbf{x}_k - \mathbf{v}_j \|} \right)^{2/(m-1)}}$$

$$\mathbf{v}_i = \frac{\sum_{k=1}^N u_{ik}^m \mathbf{x}_k}{\sum_{k=1}^N u_{ik}^m}$$

Information granules: generative and discriminative aspects

Generative model

Description of data through a collection of information granules

A_1, A_2, \dots, A_c

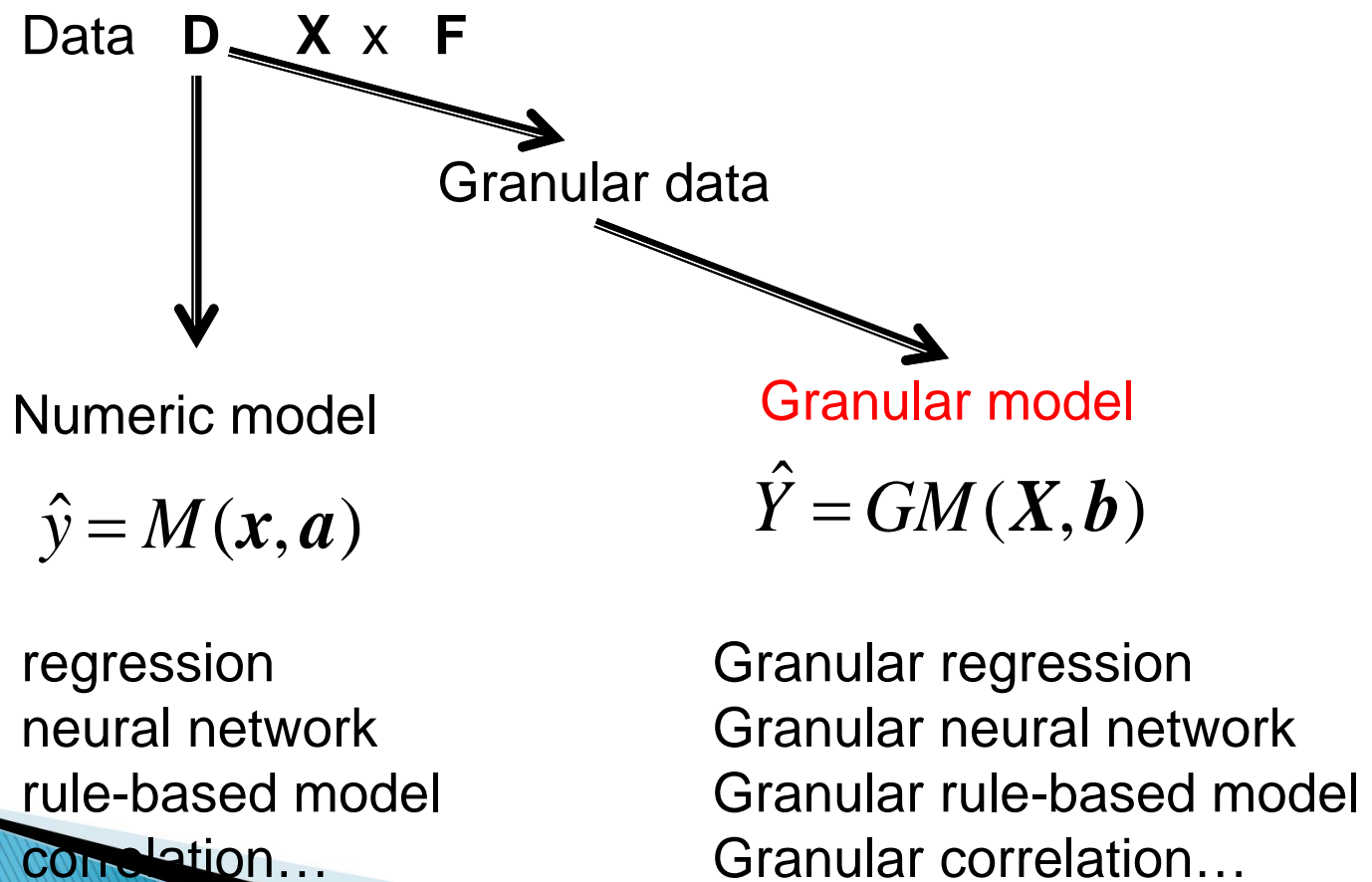
$$X \xrightarrow{A_1, A_2, \dots, A_c} D = G(X)$$

Discriminative model

Classification or prediction models realized at the level of information granules (granular blueprint)

$$D: \quad \omega = f(D) \quad y = g(D)$$

From information granules to modeling dependencies (1)



From information granules to modeling dependencies (2)

Data **D** **X** **x** **F**

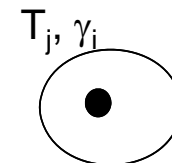
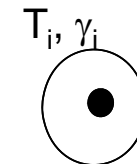
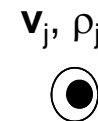
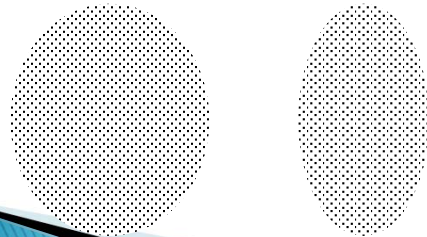
Granular data

Numeric model

$$\hat{y} = M(x, a)$$

Granular model

$$\hat{Y} = GM(X, b)$$



Conclusions

Roles of information granules in AI:

realization of abstraction

Formation of concepts and hierarchies of concepts

Delivery ways of knowledge representation

Facilitation of strategy problem solving (conceptual and computational aspects)

Emergence of Granular AI

