Hw1

April 11, 2024

1 Q1

In this problem, we looking for number of ways to combine u's and r's. So, we just need to choose 3 steps out of total 7 steps to be up, and the remaining steps will be right.

$$\begin{aligned} C(7,3) &= \binom{7}{3} \\ &= \frac{7!}{(7-3)! \times 3!} \\ &= \frac{7!}{4! \times 3!} \\ &= 35 \end{aligned}$$

Thus, there are total **35** paths from A to B.

2 Q2

2.1 Q2.1

In this problem, order doesn't matter. So, looking for combinations. Since,

$$P(A) = \frac{\text{Number of outcomes favorable to event A}}{\text{Total number of outcomes}}$$

The probability of k devices being occupied is

$$P(k) = \frac{\text{Numbers of combination of choosing k devices}}{\text{Total number of combinations in all cases}}$$

So the total number of combinations:

$$Total = \binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6} = 1 + 6 + 15 + 20 + 15 + 6 + 1 = 64$$

The number of devices being used is r = 2 or 3 or 4 or 5.

Thus,

$$P(k) = \frac{\binom{6}{k}}{\text{Total}}$$

When $k = 2$,

$$P(2) = \frac{15}{64}$$

When $k = 3$,

$$P(3) = \frac{5}{16}$$

When $k = 4$,

$$P(4) = \frac{15}{64}$$

When $k = 5$,

$$P(5) = \frac{3}{32}$$

Thus, for $2 \leq k \leq 5$,

$$P(k) = P(2) + P(3) + P(4) + P(5) = \frac{15}{64} + \frac{5}{16} + \frac{15}{64} + \frac{3}{32} = \frac{7}{8}$$

2.2 Q2.2

For "At least one buffer is occupied", which equals to 1 - P(0). (Since they are complementary)

So, when $k = 0$, none device is occupied,

$$P(0) = \frac{\binom{6}{0}}{64} = \frac{1}{64}$$

Thus, the probability of "At least one buffer is occupied" is (for $1 \leq k \leq 6$)

$$P(\text{At least one buffer is occupied}) = 1 - P(0) = P(k) = 1 - \frac{1}{64} = \frac{63}{64}$$

3 Q3

3.0.1 Q3.1

So, in this case, each song should be distinct and songs' order matters. The probability that the first Cake song heard is the 5th song played:

$$P(\text{the first Cake song heard is the 5th song played}) = \frac{90 \times 89 \times 88 \times 87 \times 10}{100 \times 99 \times 98 \times 97 \times 96} = 0.067878 = 6.79\%$$

3.0.2 Q3.2

[1]:
```
set.seed(123)
total_songs = 100
shuffle_times = 100000
prob = numeric(shuffle_times)
bingo = 0
for(k in 1:shuffle_times){
```

```r
    shuffled_songs = sample(1:100, total_songs, replace = FALSE)
    # lets say cake's song in range of [1,10]
    if((1 <= shuffled_songs[5] && shuffled_songs[5] <= 10) && shuffled_songs[1]␣
↪> 10 && shuffled_songs[2] > 10 && shuffled_songs[3] > 10 &&␣
↪shuffled_songs[4] > 10){
        bingo = bingo + 1  # record if it meets the condition
    }
    prob[k] = bingo / k
}
overall_prob = bingo / shuffle_times
print(overall_prob)
plot(prob, ylab="Probability", xlab="Number of shuffle times (k)")
```

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[1] 0.06831
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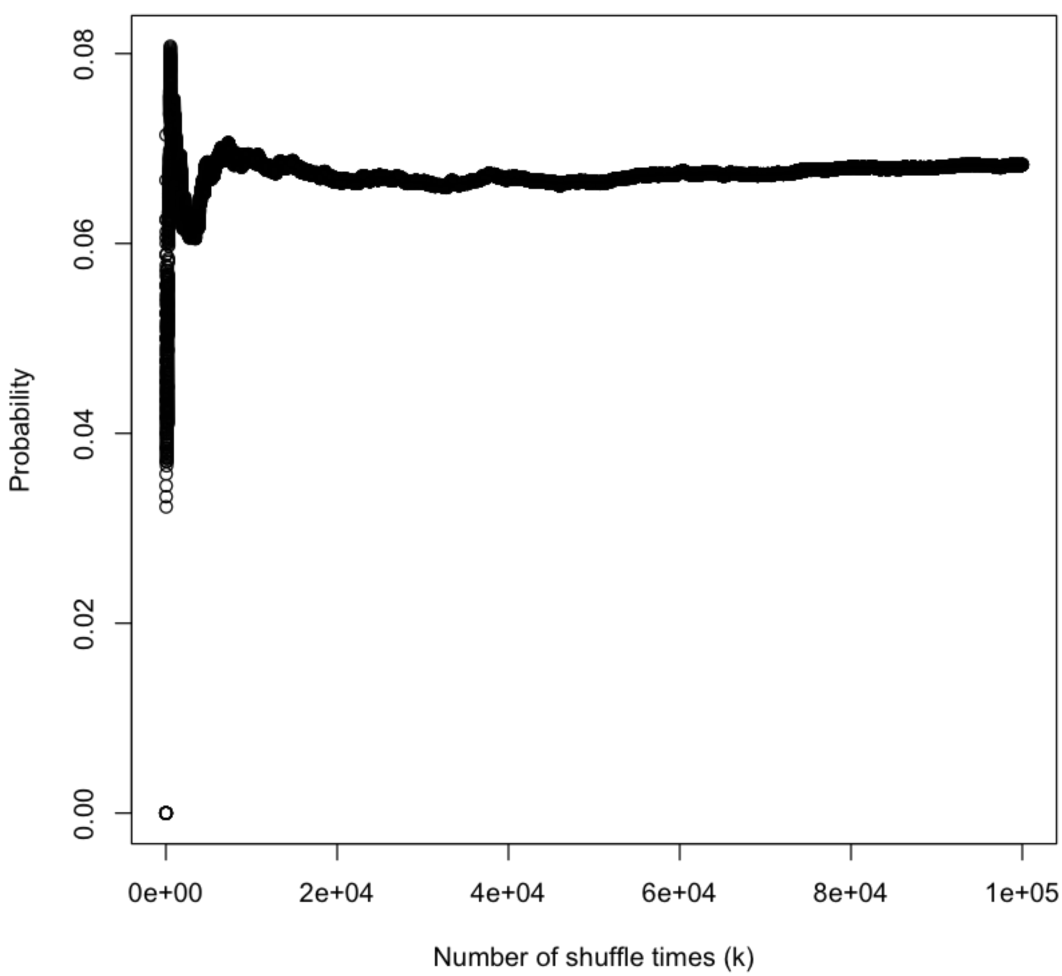

4 Q4

In this question, “how many different ways to arrange the songs” = “how many combinations of songs in all the drivers.”

Thus, in the first driver, choosing 20 songs from total 100 songs:

$$C1 = \binom{100}{20}$$

The second drive, choosing 30 songs from(100 - 20 =) 80 songs:

$$C2 = \binom{80}{30}$$

The third drive, choosing 40 songs from(80 - 30 =) 50 songs:

$$C3 = \binom{50}{40}$$

The second drive, choosing 10 songs from(50 - 40 =) 10 songs:

$$C4 = \binom{10}{10}$$

Therefore, the total ways to arrange songs is:

$$\begin{aligned} C &= \binom{100}{20} \times \binom{80}{30} \times \binom{50}{40} \times \binom{10}{10} \\ &= \frac{100!}{20! \times 80!} \times \frac{80!}{30! \times 50!} \times \frac{50!}{40! \times 10!} \times \frac{10!}{0! \times 10!} \\ &= \frac{100!}{20! \times 30! \times 40! \times 10!} \\ &\approx 4.88 \times 10^{52} \end{aligned}$$

5 Q5

Let’s denote,

$P(A)$ = the probability that program will access the first location = 0.5

$P(B)$ = the probability that program will access the second location = 0.4

$P(A \cap B)$ = the probability that program will access both locations = 0.3

Thus, the probability that program will access neither locations = $P(\overline{A \cup B})$ = 1 - $P(A \cup B)$

Since $P(A \cup B) = P(A) + P(B)$ - $P(A \cap B)$ = 0.5 + 0.4 - 0.3 = 0.6

Thus, the probability that program will access neither locations = $P(\overline{A \cup B})$ = 1 - $P(A \cup B)$ = 1 - 0.6 = 0.4

6 Q6

John is more probable to be a computer scientist. (Scenario A is more probable than Scenario B).

Scenario A with only condition is that John is a Computer scientist, while Scenario B has two conditions: Computer scientist and environmental activist.

So, we can consider that scenario A is a set of computer scientist with or with out any other positions.

Thus, scenario B will be the subset of A.

Scenario B is less probable than scenario A.

[ ]:

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