Hw3

April 26, 2024

1

Let: Event A = the probability that an email is spam = 80%

Thus the probability that an email is non-spma = 20%

Event B = the probability that an email contains the phrase

$$P(B) = 80\% \times 10\% + 20\% \times 1\%$$

The probability that a spam email contain the phrase "Large inheritance" = 10%

$$P(B|A) = 10\%$$

Thus, the probability P that a new email is received containing the phrase "Large inheritance" is spam is:

$$\begin{aligned} P = P(A|B) &= \frac{P(A) \times P(B|A)}{P(B)} \\ &= \frac{80\% \times 10\%}{80\% \times 10\% + 20\% \times 1\%} = 97.6\% \end{aligned}$$

2

The probability that **correctly** receiving 0 is 0.94

The probability that **Not correctly receiving** 0 is 1-0.94 = 0.06

The probability that **correctly** receiving 1 is 0.91

The probability that **Not correctly receiving** 1 is 1-0.91 = 0.09

The probability that transmit 0 is 0.45, then the probability that transmit 1 is 0.55 1. Probability that a 1 is received. (It could be correct or an error.)

$$\begin{aligned} P_1 &= 0.55 \times 0.91 + 0.45 \times 0.06 \\ &= 0.5275 \end{aligned}$$

2. Probability that a 0 is received. (It could be correct or an error.)

$$\begin{aligned} P_0 &= 0.45 \times 0.94 + 0.55 \times 0.09 \\ &= 0.4725 \end{aligned}$$

3. Probability that a 1 was transmitted given that a 1 was received.

$$\begin{aligned} P &= \frac{0.55 \times 0.91}{P_1} \\ &= \frac{0.55 \times 0.91}{0.5275} \\ &= 0.9488 \end{aligned}$$

4. Probability that a 0 was transmitted given that a 0 was received.

$$\begin{aligned} P &= \frac{0.45 \times 0.94}{P_0} \\ &= \frac{0.45 \times 0.94}{0.4725} \\ &= 0.8868 \end{aligned}$$

5. Probability of an error. (This means either a transmitted 0 was received as a 1, or a transmitted 1 was received as a 0)

$$\begin{aligned} P &= 0.45 \times 0.06 + 0.55 \times 0.09 \\ &= 0.0765 \end{aligned}$$

3

1. P(X = 1) is the probability that two sensors are active in all three time slots:

$$\begin{aligned} P(X=1) &= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \\ &= \frac{1}{64} \end{aligned}$$

2. X belongs to Bernoulli distribution:

$$X \sim \text{Bern}(\frac{1}{64})$$

.

3. The value of E(X):

$$\begin{aligned} E(X) &= 1 \times P + 0 \times (1-P) \\ &= \frac{1}{64} \end{aligned}$$

4. The value of Var(X):

$$\begin{aligned} Var(X) &= P \times (1-P) \\ &= \frac{1}{64} \times (1 - \frac{1}{64}) \\ &= \frac{1}{64} \times \frac{64}{64} \\ &= \frac{63}{4096} \end{aligned}$$

4

1. The probability that a patient recovers from a rare blood disease is **0.4**

   Thus, P = 0.4

   Total 10 people, n = 10.

Thus,

$$\begin{aligned} P(X == K) &= \binom{10}{k} \times 0.4^k \times (1-0.4)^{n-k} \\ &= \binom{10}{k} \times 0.4^k \times (0.6)^{n-k} \end{aligned}$$

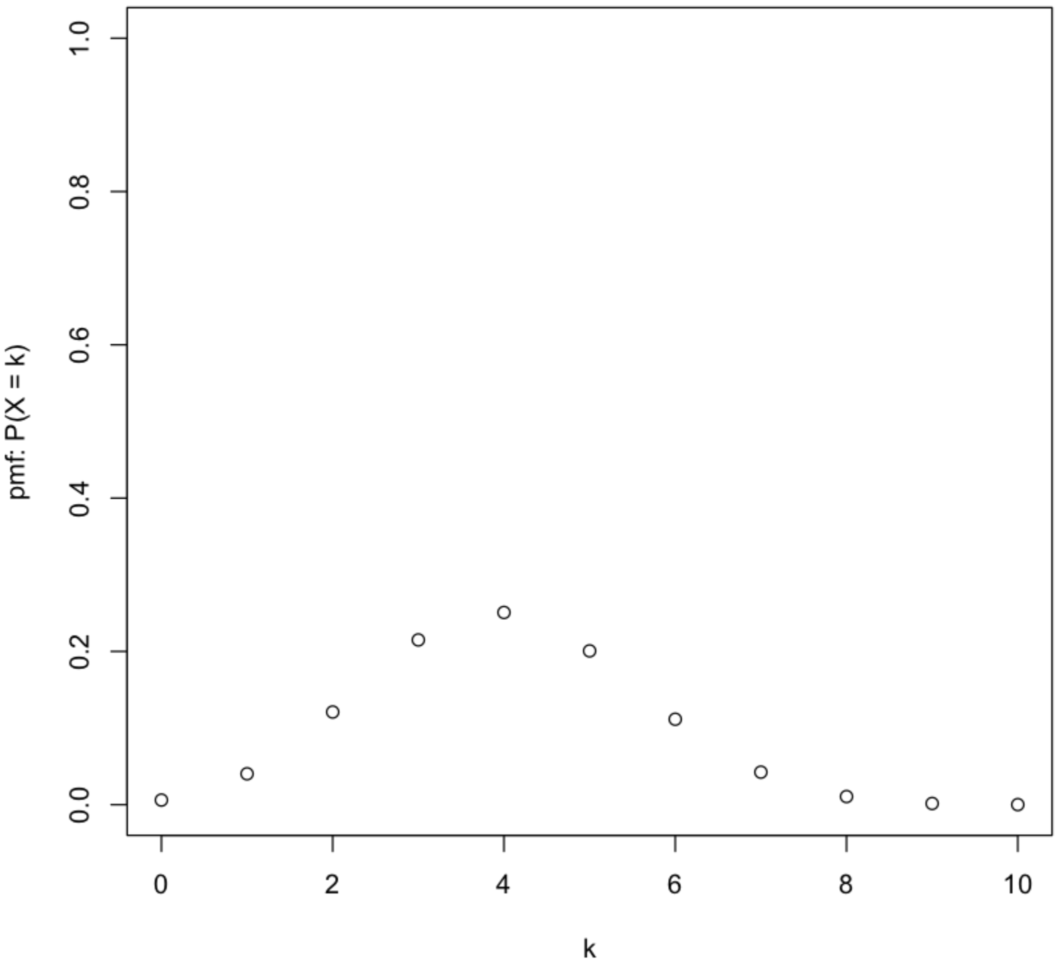
2. X belongs to Binomial distribution.

$$X \sim \text{Binom}(10, 0.4)$$

   .

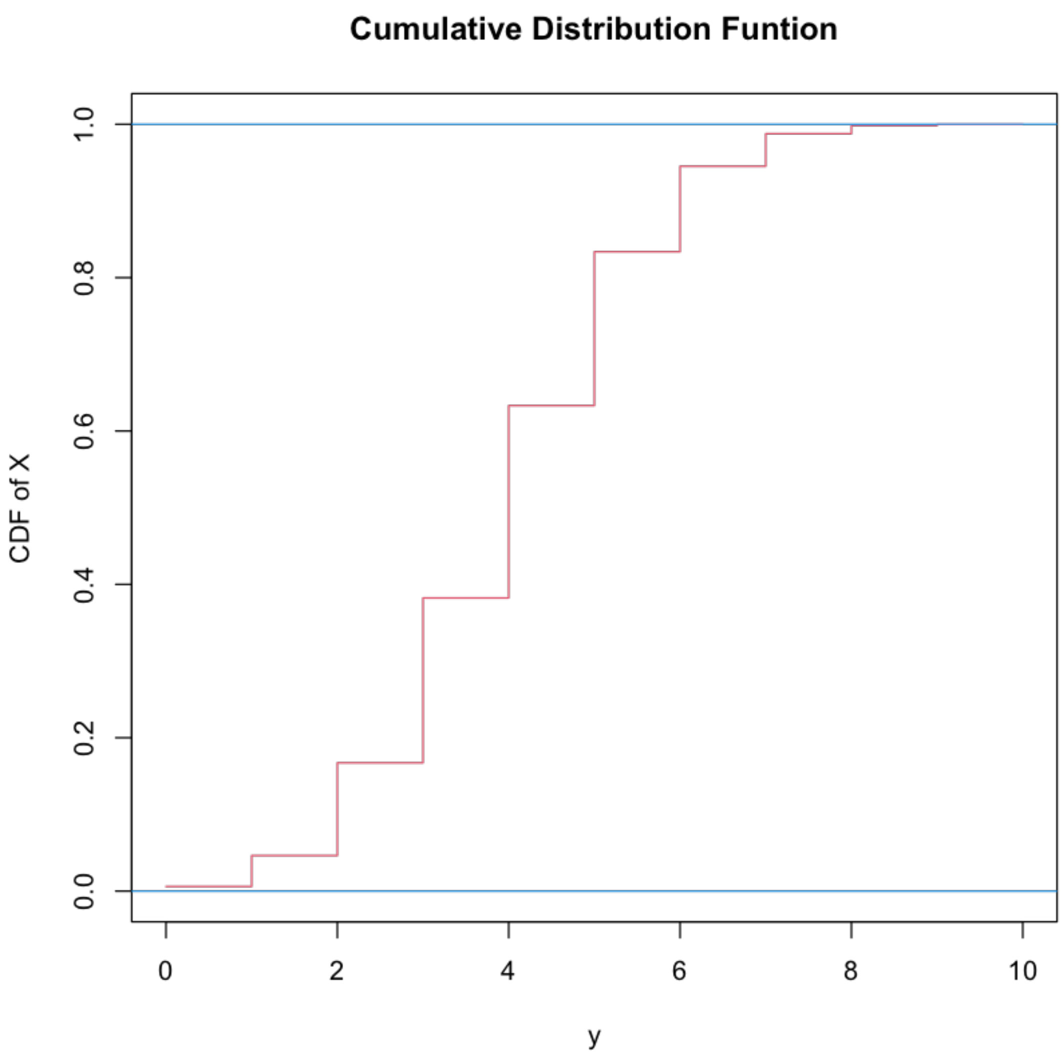
3. Plot the probability mass function (pmf) of X.

[11]:
```
n = 10
p = 0.4
y = dbinom(0:10, n,p)
plot(x = 0:10, ylim= (0:1.0), y,  xlab = "k", ylab = "pmf: P(X = k)")
```



4. Plot the cumulative distribution function (cdf) of X.

[16]:
```
pmf = dbinom(0:10, 10,0.4)
cdf = cumsum(pmf)    # note the use of the R function "cumsum"
#### Writing a general PMF ####
plot(x=(0:10), cdf, type = 's', ylim = c(0,1), ylab="CDF of X",col=2,xlab="y",
 main="Cumulative Distribution Funtion");abline(h=0:1,col=4)
```



5. What is the probability that at least 8 survive, i.e., P {X   8}

[37]:
```
n = 10
# the probability that can recover is 0.4
p = 0.4
# the probability that cannot recover is 1-p
# The probability that less than 2 people die = The probability that at least 8␣
↪survive
k = 2
print("The probability that at least 8 survive:")
print (pbinom(k, n, 1-p))
```

```
[1] "The probability that at least 8 survive:"
```

[1] 0.01229455

6. What is the probability that 3 to 8 survive, i.e., P {3 X 8}

[45]:
```
n = 10
# the probability that can recover is 0.4
p = 0.4
print("The probability that at least 3 to 8 survive:")
sum = 0
for(k in 3:8){
sum = sum + dbinom(k , n, p)
    }
print(sum)
```

```
[1] "The probability that at least 3 to 8 survive:"
[1] 0.8310325
```

5

1. Assume the that the loop execute k times and failure at k+1 times:

$$P(X = k + 1) \quad = \quad q^k \times p$$

2. The expected number of times the loop will be executed:

$$E(X) = \frac{p}{q}$$

3. The expected number of times the loop will be executed, when repeat S until B which mean exit loop until B is true:

$$E(X) = \frac{1}{p}$$

[ ]: