Hw2

April 19, 2024

1

1.1

Total sample $S = [GreenGreen, GreenBrown, BrownBrown, BrownGreen]$

The probability that both have green eyes:

$$P(\text{both have green eyes}) = \frac{1}{4}$$

1.2

Total sample that base on the condition **the older of the two has green eyes** is $S = [GreenGreen, GreenBrown]$

Thus, the probability that both have greens eyes conditioned on the older of the two has green eyes is:

$$P = \frac{1}{2}$$

2

2.1

Since discarding those that don't work. The probability that the first key failure to open the door is:

$$\frac{n-1}{n}$$

The probability that the second key failure to open the door is:

$$\frac{n-2}{n-1}$$

The probability that the kth key works is:

$$\frac{1}{n-k+1}$$

Thus, the overall probability that the kth key works is:

$$P_k = \frac{n-1}{n} \times \frac{n-2}{n-1} \times ...... \times \frac{1}{n-k+1} = \frac{1}{n}$$

2.2

If doesn't discard the not working tried keys, So similarly, the probability that the first key faliure to open the door is:

$$\frac{n-1}{n}$$

The probability that the second key opens the door is:

$$\frac{n-1}{n}$$

Thus, the kth keys that opens the door is:

$$\frac{1}{n}$$

Thus, the overall probability that the kth key works is:

$$\begin{aligned} P_k &= \frac{n-1}{n} \times \frac{n-1}{n} \times ...... \times \frac{1}{n} \\ &= (\frac{n-1}{n})^{k-1} \times \frac{1}{n} \end{aligned}$$

3

The total combinations of 10 red requests out of total 20 requests:

$$\begin{aligned} C(20,10) &= \binom{20}{10} \\ &= \frac{20!}{10! \times 10!} \\ &= 184756 \end{aligned}$$

Then, each of the rest of 10 requests could be green or black, which has total situations:

$$S_1 = 2^{10}$$

Thus, the overall total request situations for each request could be red or green or black:

$$S_0 = 3^{20}$$

So, the probability that receiving 10 red among of first 20 requests is:

$$\begin{aligned} P &= \frac{\binom{20}{10} \times 2^{10}}{3^{20}} \\ &= \frac{184756 \times 2^{10}}{3^{20}} \\ &= 0.0543 \end{aligned}$$

4

So the all combinations of 13 cards a player would get is:

$$\begin{aligned} C_(total) &= \binom{52}{13} \\ &= \frac{52!}{13! \times 39!} \\ &= 6.35 \times 10^{11} \end{aligned}$$

The combination that all 13 cards are heart is **1**

Thus, the probability that one player get all 13 hearts is:

$$\begin{aligned} P &= \frac{1}{C_(total)} \\ &= \frac{1}{6.35 \times 10^{11}} \\ &= 1.57 \times 10^{-12} \end{aligned}$$

5

5.1

Since each sensor is independent to each other.

Thus is time slot i, the probability that both sensors are active is:

$$P_{(A_i)} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

5.2

The probability that both sensors are active during two time slot:

$$\begin{aligned} P(A_i \cap A_j) &= P(A_1) \times P(A_2) + P(A_1) \times P(A_3) + P(A_2) \times P(A_3) \\ &= \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} \\ &= \frac{3}{16} \end{aligned}$$

5.3

The probability that all senors are active during all three time slot is:

$$\begin{aligned} P(A_1 \cap A_2 \cap A_3) &= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \\ &= \frac{1}{64} \end{aligned}$$

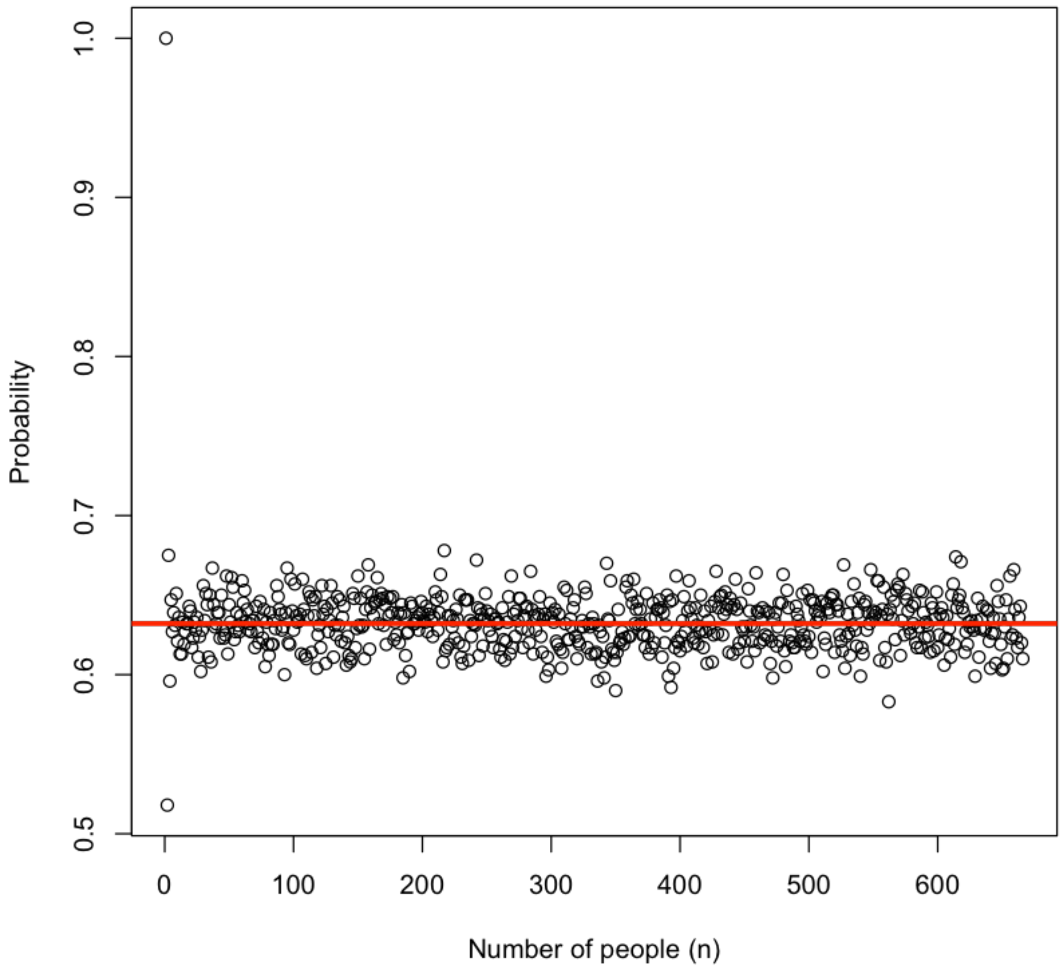
5.4

The probability that both sensors are active during at least one time slot is:

$$\begin{aligned} P(A_1 \cup A_2 \cup A_3) &= 1 - P(\text{None time slot that both sensor are active}) \\ &= 1 - (\frac{3}{4})^3 \\ &= \frac{37}{64} \end{aligned}$$

6

[1]:
```r
set.seed(123)
n = 666
experiment = 1000
prob = numeric(n)
for (num in 1:n){
    bingo = 0
    for (k in 1:experiment)
        {
            shuffled_hat = sample(1:num, num, replace = FALSE)
            for (i in 1:num)
                {
                    if (shuffled_hat[i] == i)
                        {
                            bingo = bingo + 1
                            break;
                        }
                }
        }
        prob[num] = bingo/experiment
}
plot(prob, ylab="Probability", xlab="Number of people (n)")
abline(h = 1 - 1/exp(1) , col = "red", lwd = 3)
```


Probability
1.0
0.9
0.8
0.7
0.6
0.5
0
100
200
300
400
500
600
Number of people (n)

[ ]:

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[ ]:

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