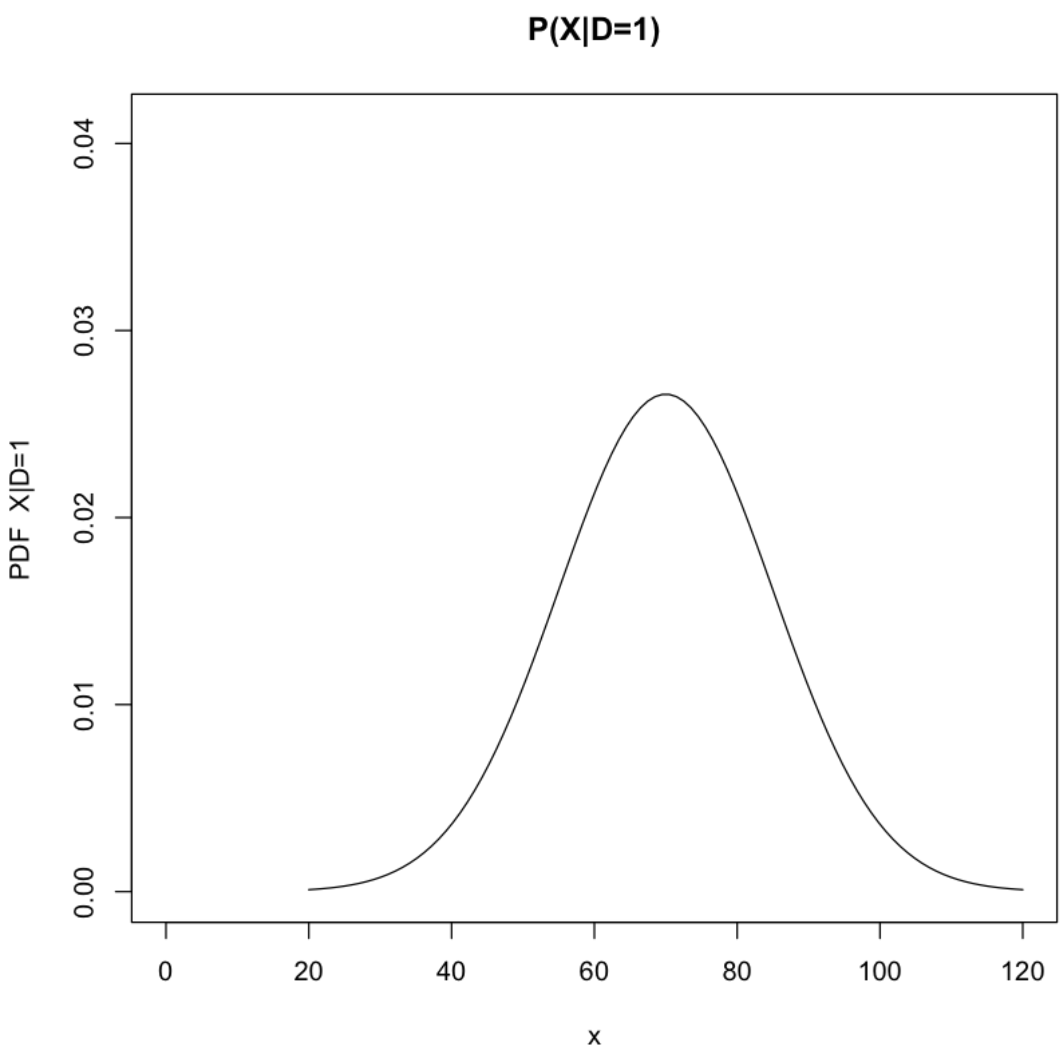
Hw6

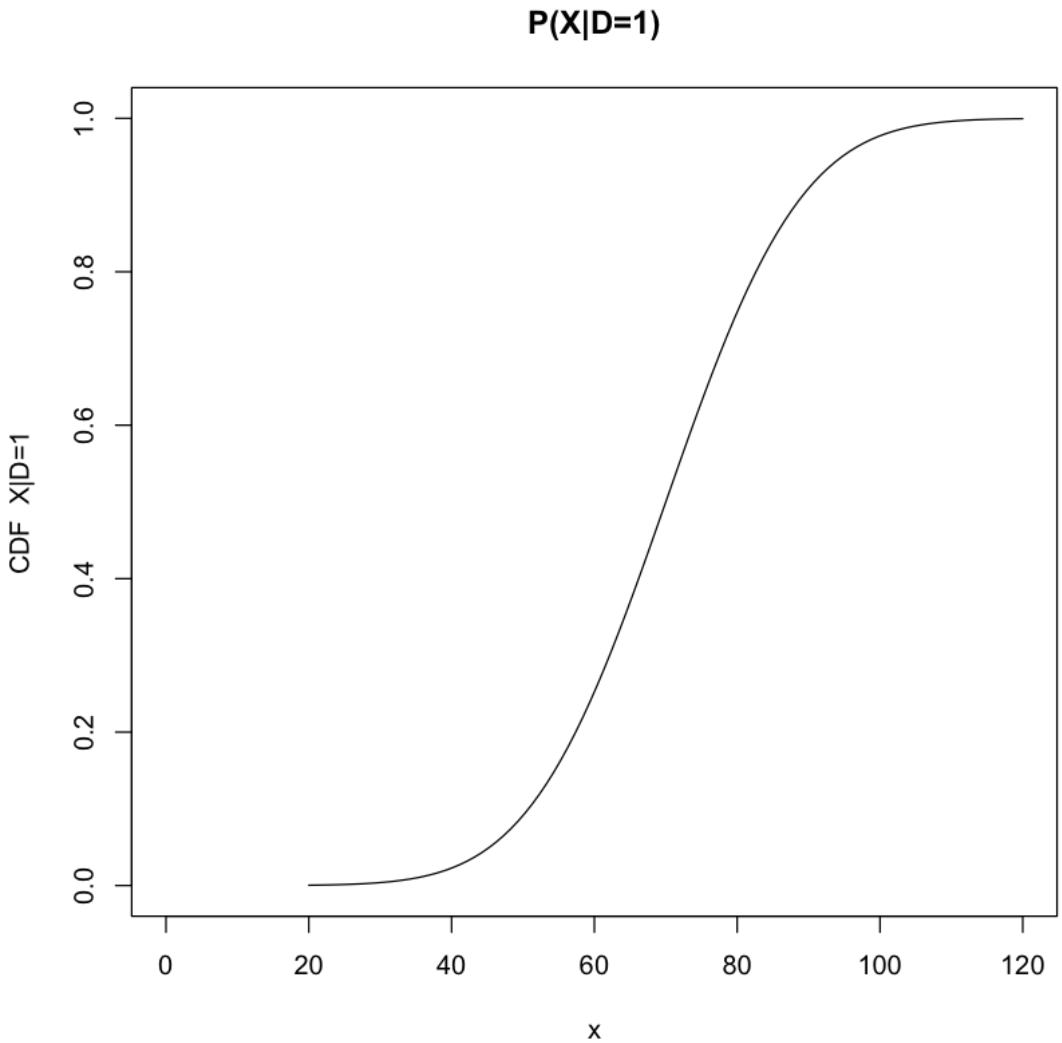
May 31, 2024

1

[1]:
```
x = seq(20, 120, length = 100)
hx = dnorm(x, 70, 15)
plot(x, hx, xlim = c(0,120), ylim = c(0, 0.041), type='l', ylab="PDF  X|D=1",␣
↪xlab="x", main="P(X|D=1)")
```



[5]:
```
x = seq(20, 120, length = 100)
hx = pnorm(x, 70, 15) # CDF
plot(x, hx, xlim = c(0,120), ylim = c(0, 1), type='l', ylab="CDF  X|D=1",␣
↪xlab="x", main="P(X|D=1)")
```



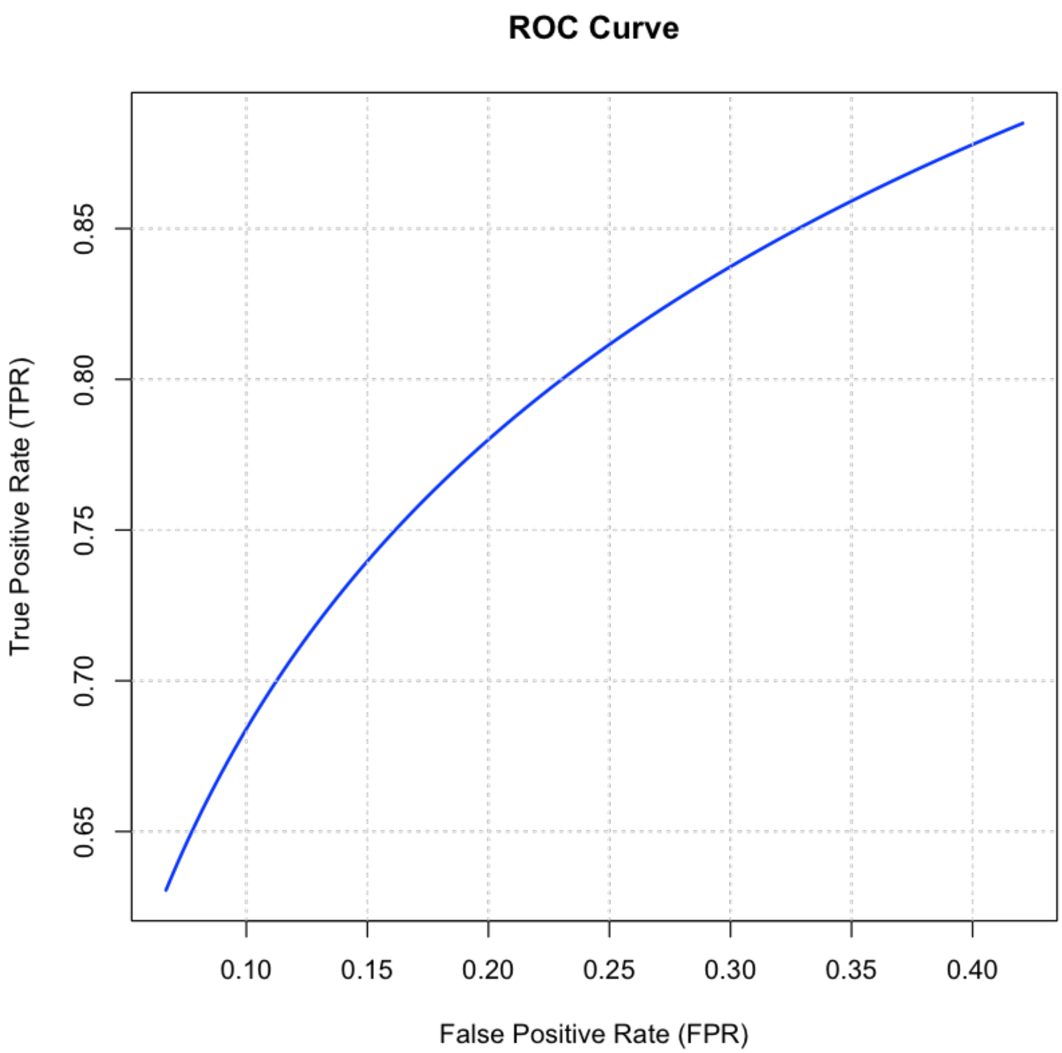
[10]:
```
# Define the range for x*
x_star_range <- seq(52, 65, length.out = 100)

# Mean and standard deviation for the distributions
mean_D0 <- 50
```

```r
std_D0 <- 10
mean_D1 <- 70
std_D1 <- 15

# Calculate TPR and FPR for each value of x*
TPR <- 1 - pnorm(x_star_range, mean = mean_D1, sd = std_D1)
FPR <- 1 - pnorm(x_star_range, mean = mean_D0, sd = std_D0)

# Plot the ROC curve
plot(FPR, TPR, type = 'l', col = 'blue', lwd = 2, xlab = 'False Positive Rate␣
  ↪(FPR)', ylab = 'True Positive Rate (TPR)', main = 'ROC Curve')
abline(0, 1, col = 'red', lty = 2) # Add diagonal line representing random␣
  ↪guessing
grid() # Add grid
```



3. **False Positive Rate** $= P(T=1|D=0)$

False Negative Rate $= P(T=0|D=1)$

If they are equally bad:

$$\begin{aligned} P(T=0|D=1) &= P(T=1|D=0) \\ \frac{x^*-70}{15} &= 1-\frac{x^*-50}{10} \\ x &= 58 \end{aligned}$$

2

1. The specifity θ :

$$\begin{aligned} \theta &= \frac{TP}{TN+FP} \\ &= \frac{100}{100+10} \\ &= 0.909 \end{aligned}$$

2. The sensitivity η:

$$\begin{aligned} \eta &= \frac{TP}{TP+FN} \\ &= \frac{15}{15+10} \\ &= 0.6 \end{aligned}$$

3. The prevalence π:

$$\begin{aligned} \pi &= \frac{TP+FN}{TP+FP+FN+TN} \\ &= \frac{15+10}{15+10+10+100} \\ &= 0.185 \end{aligned}$$

3

1. This is a 2-state Markov chain with the transition matrix given by

$$P = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

2. The that probability of finding the Markov Chain in state 0 is $P_0 = 80\%$

   The that probability of finding the Markov Chain in state 1 is $P_1 = 20\%$

Thus,

$$\pi P = (0.8, 0.2) \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

This gives us two equations:

$$P(X=0) = (1-\alpha)0.8 + \beta 0.2 = 0.8$$

$$P(X=1) = \alpha 0.8 + (1-\beta)0.2 = 0.2$$

Thus,

$$\beta = 4\alpha$$

3. Since $\pi_r = \pi_0 P^r$

   Thus, $\pi_5 = \pi_0 P^5$

$$\pi_5 = \pi_0 P^5 = (0.8, 0.2)(\frac{1}{\alpha+\beta} \begin{bmatrix} \beta & \alpha \\ \beta & \alpha \end{bmatrix} + \frac{(1-\alpha-\beta)^5}{\alpha+\beta} \begin{bmatrix} \alpha & -\alpha \\ -\beta & \beta \end{bmatrix})$$

4. P^r converges geometrically as $r \to \infty$ to

$$P^\infty = \frac{1}{\alpha+\beta} \begin{bmatrix} \beta & \alpha \\ \beta & \alpha \end{bmatrix}$$

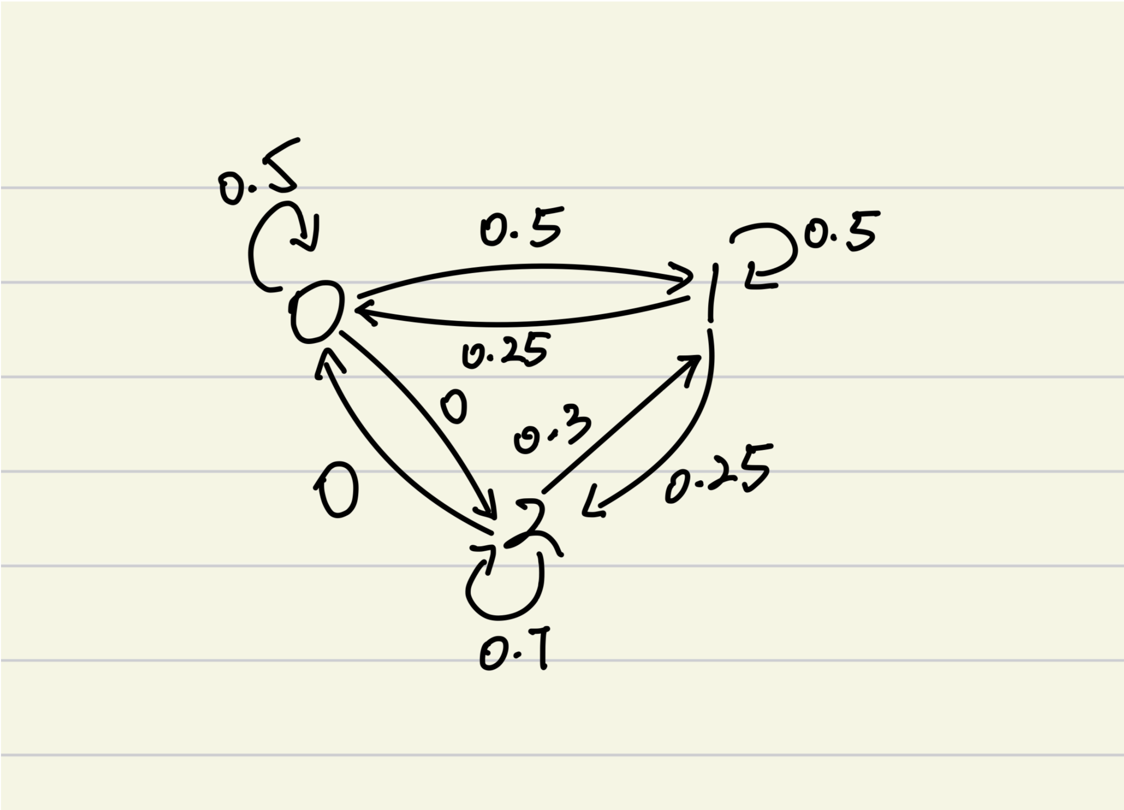
Thus, for $\pi_\infty = \pi_0 P^\infty$

$$\pi_0 P^\infty = [0.8, 0.2] \left(\frac{1}{\alpha+\beta} \begin{bmatrix} \beta & \alpha \\ \beta & \alpha \end{bmatrix} \right) = \left[\frac{\beta}{\alpha+\beta}, \frac{\alpha}{\alpha+\beta} \right]$$

4

1.

$$\begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.3 & 0.7 \end{bmatrix}$$



2.

3. The probability the sensor is awake is 0.4285714

[4]:
```
m = matrix(0, nrow=3,ncol=3)  #define a vector
m[1,] = c(0.5,0.5,0)          #specify the row entries
m[2,] = c(0.25,0.5,0.25)
m[3,] = c(0,0.3,0.7)
print(m)

e = eigen(t(m))               #solve for the eigenvalues and eigenvectors of
 ↪the transpose matrix
print(e)                      #Note, the leading eigenvalue is 1, and all the
 ↪rest are smaller

pi = e$vectors[,1]/sum(e$vectors[,1])  #Extract the corresponding eigenvector
 ↪and normalize it
print(pi)
```

```
     [,1] [,2] [,3]
[1,] 0.50  0.5 0.00
[2,] 0.25  0.5 0.25
[3,] 0.00  0.3 0.70
eigen() decomposition
```

```
$values
[1] 1.00000000 0.61925824 0.08074176

$vectors
          [,1]       [,2]       [,3]
[1,] 0.3585686  0.5415949  0.4838878
[2,] 0.7171372  0.2583586 -0.8114959
[3,] 0.5976143 -0.7999536  0.3276080

[1] 0.2142857 0.4285714 0.3571429
```

```
[ ]:
```