



UNIVERSITÉ DE NANTES



UMR CNRS 6597

UNSUPERVISED HANDWRITTEN GRAPHICAL SYMBOL LEARNING

-Using Minimum Description Length Principle on
Relational Graph

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KDIR 2011

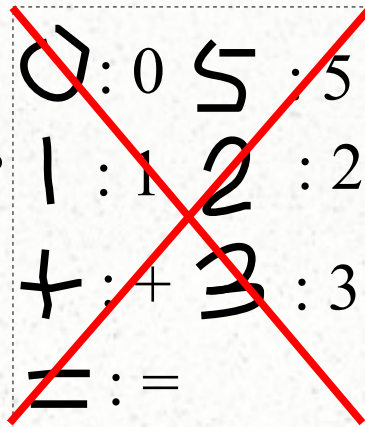
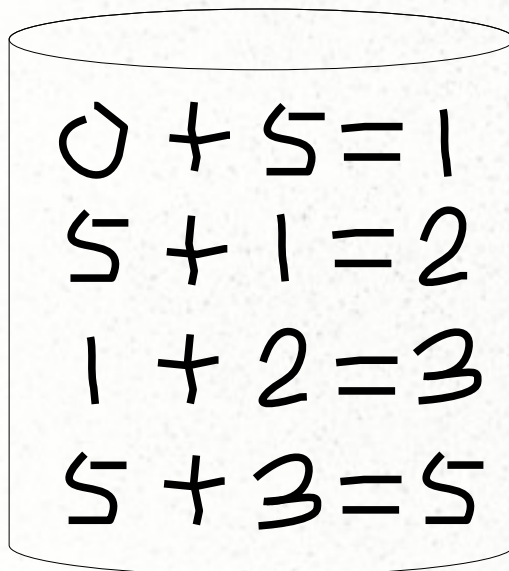
www.projet-depart.org

Outline

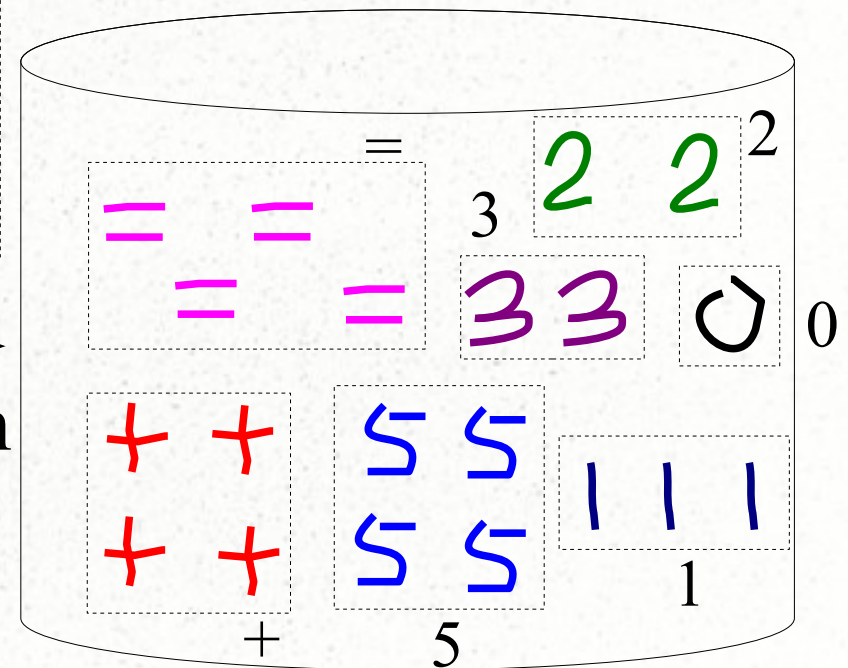
- 1. Background
- 2. Unsupervised Handwritten Graphical Symbol Learning
 - Quantization of strokes
 - Relational Graph Construction Between Strokes
 - Discover Symbols (Sub-graphs)
- 3. Experiment
- 4. Conclusion

Traditional Recognition (Background)

Unlabeled
handwritten symbols

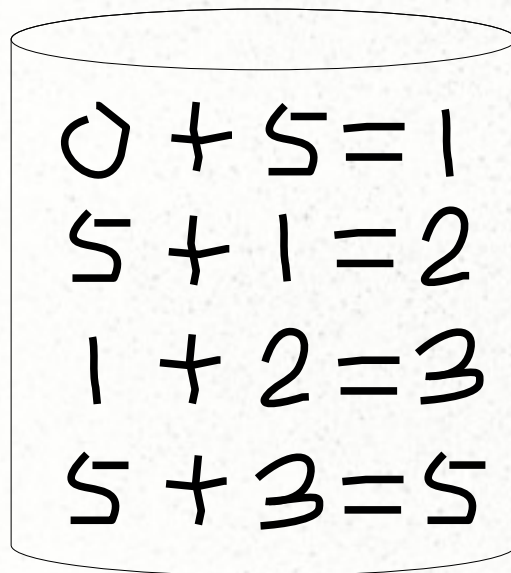


Recognition

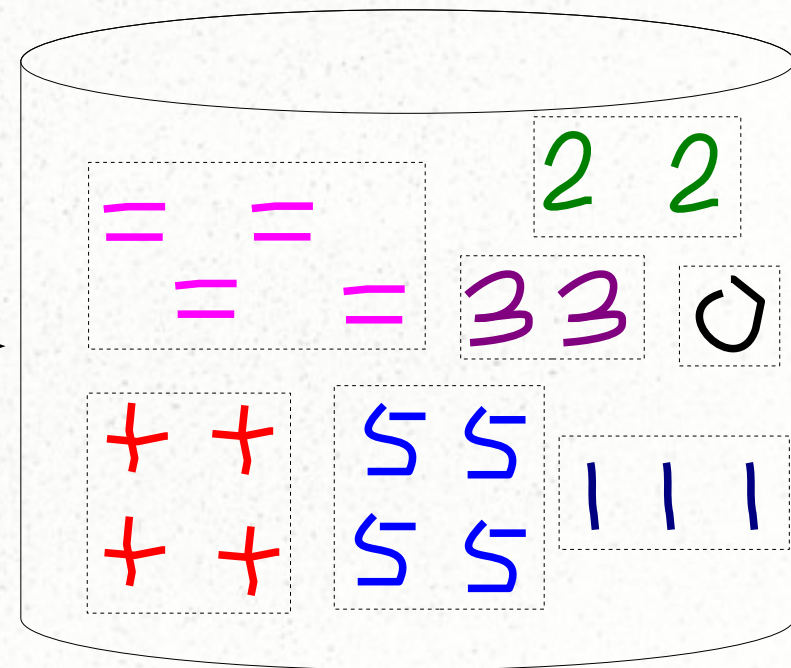


Annotation (Application)

Unlabeled
handwritten symbols



Symbol
Extraction →

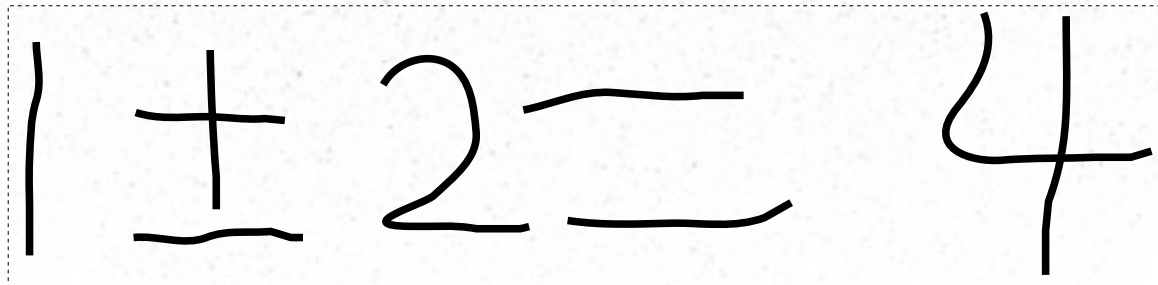


40 symbols
have to be labeled

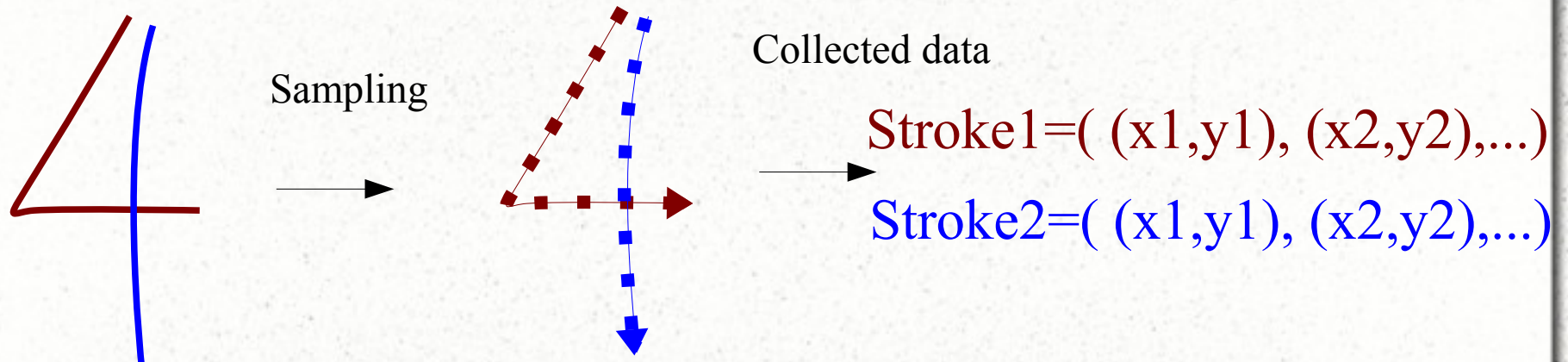
7 symbols (sets)
have to be labeled

UNSUPERVISED HANDWRITTEN GRAPHICAL SYMBOL LEARNING

As an example, we use mathematical expressions as an **unknown** graphical language.

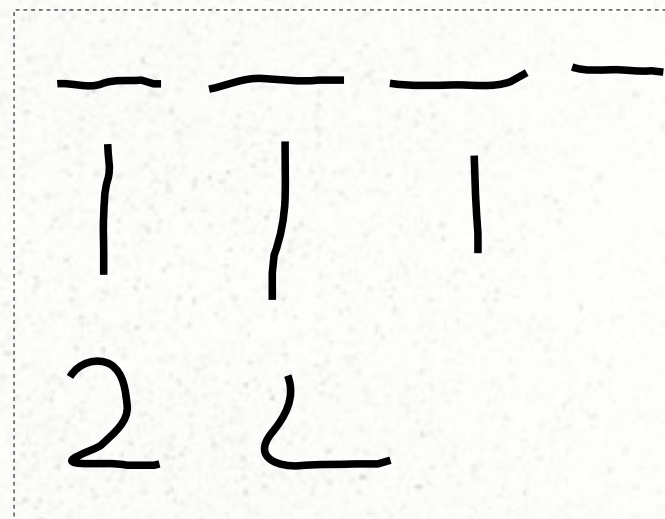


Online handwriting



Unsupervised symbol learning

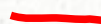
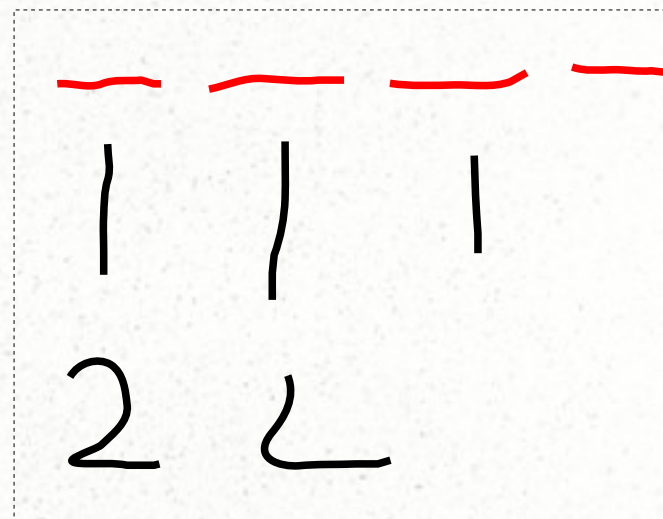
1 ± 2 = 4 $\xrightarrow[\text{are strokes}]{\text{The base elements}}$



We call the frequent strokes as
the **Grapheme**.

Unsupervised symbol learning

1 ± 2 = 4 $\xrightarrow{\text{The base elements are strokes}}$



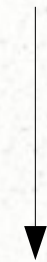
The horizontal stroke
repeats 4 times.

We call the frequent strokes as
the **Grapheme**.

Grapheme

1 ± 2 = 4

From a part of symbol "plus".



Where does
the horizontal stroke
come from?

1 ± 2 = 4

From a symbol "equal".



1 ± 2 = 4

From a symbol "minus".

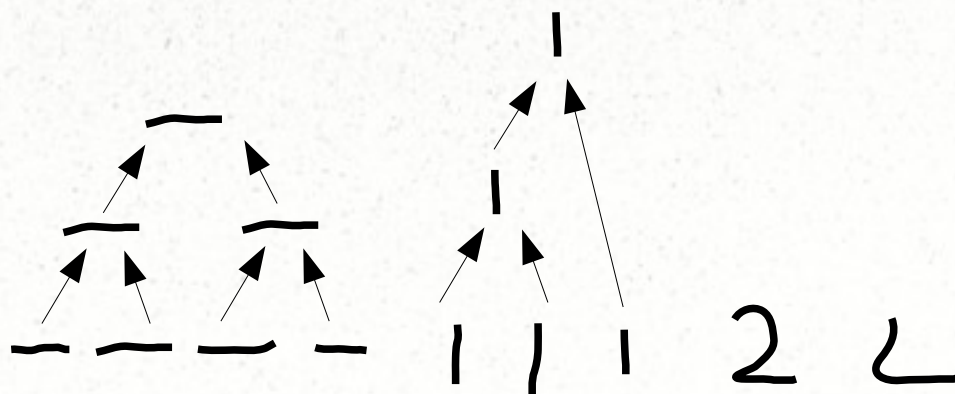
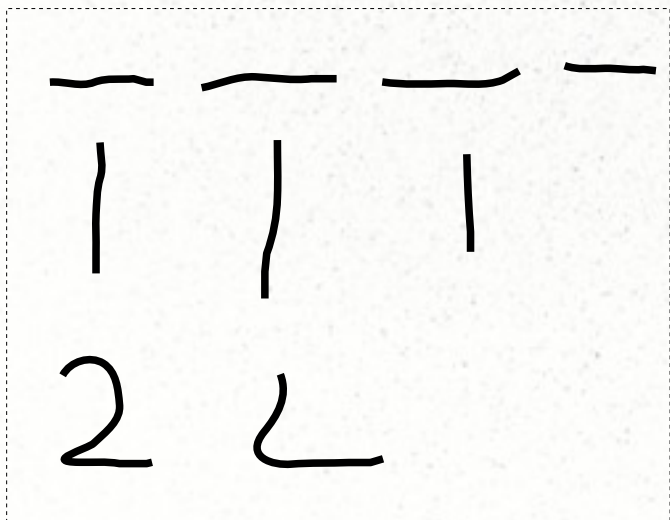
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- 1. Background
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 - **Quantization of strokes**
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Hierarchical clustering

1 ± 2 = 4

↓
Strokes

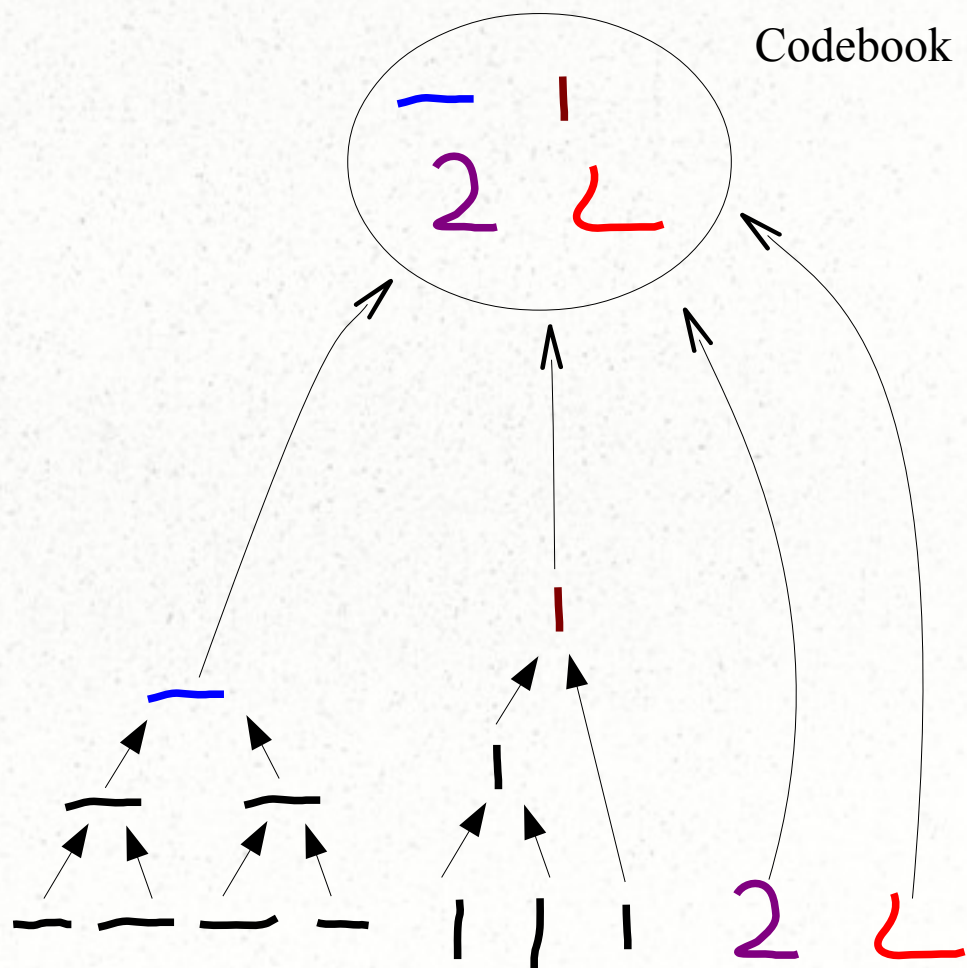
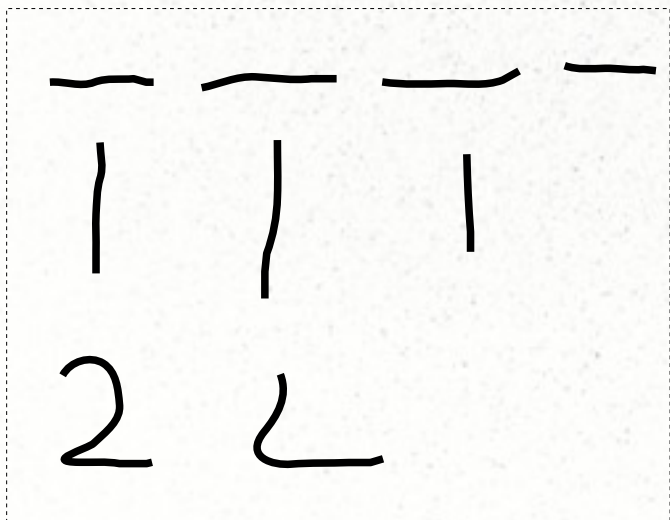


Dynamic Time Warping Distance

Hierarchical clustering

1 ± 2 = 4

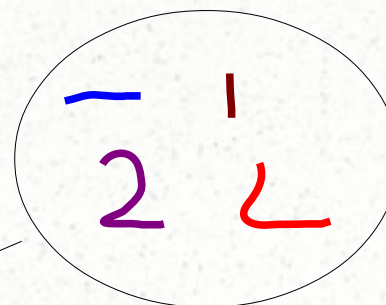
↓
Strokes



Dynamic Time Warping Distance

Quantization of strokes

1 ± 2 = 4



Codebook

1. Quantization of strokes

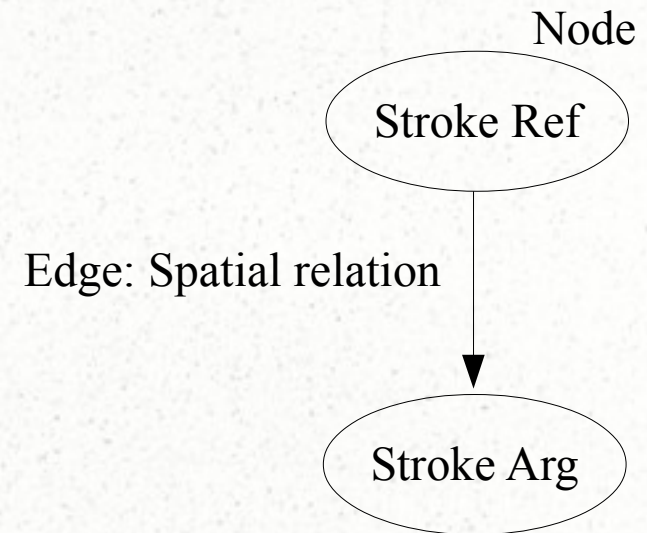
1 ± 2 = 4

Outline

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Relational Graph Construction

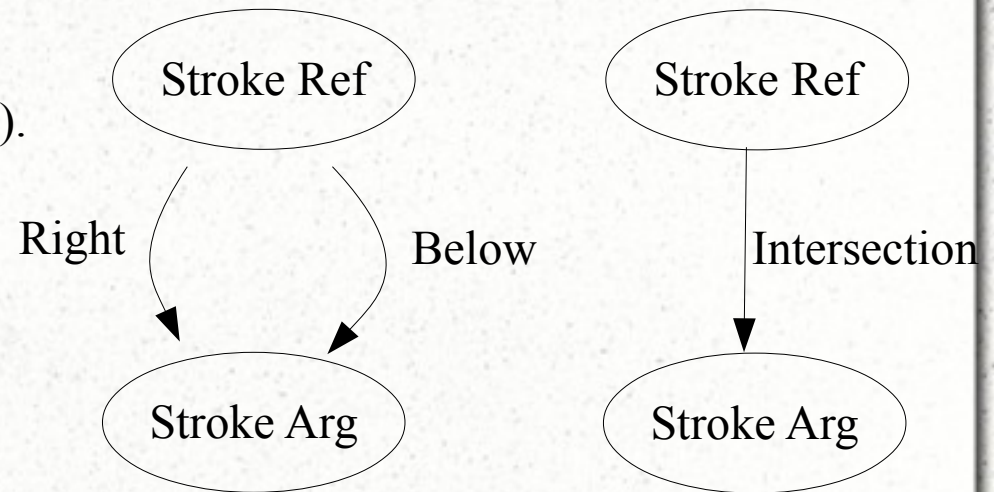
Spatial relation: from a reference stroke to an argument stroke.



Relational Graph Construction

Spatial relation: from a reference stroke to an argument stroke.

We predefine three spatial relations:
right (R), below (B), and intersection (I).



n_{str} : the number of strokes (number of nodes).

n_r : the number of different spatial relations from a reference stroke to an argument stroke.

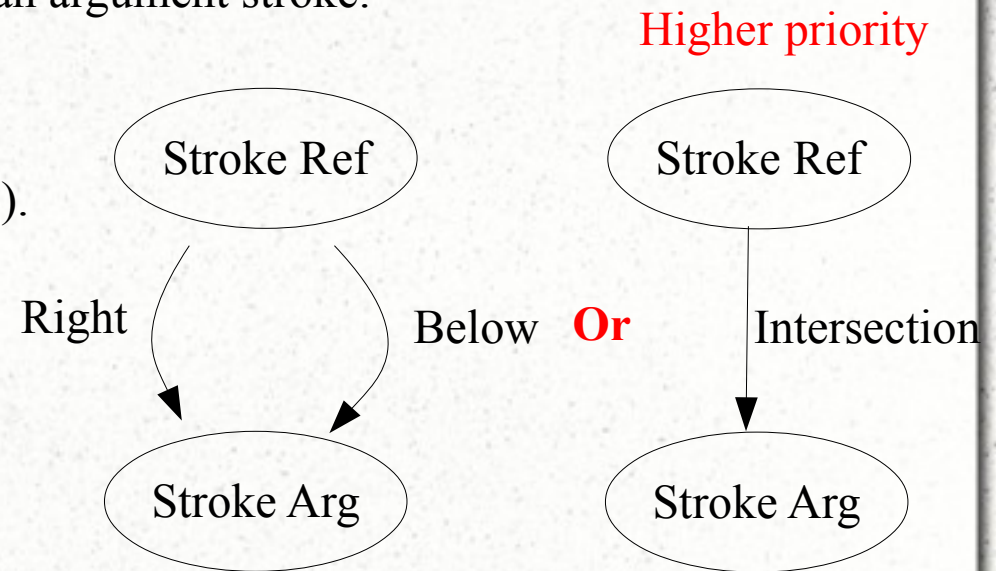
$$n_r = 3$$

Relational Graph Construction

Spatial relation: from a reference stroke to an argument stroke.

We predefine three spatial relations:
right (R), below (B), and intersection (I).

We predefine another constraint that
Directional spatial relation (R and B)
are **exclusive** with
Topological spatial relation (I).



n_{str} : the number of strokes (number of nodes).

n_r : the number of different spatial relations from a reference stroke to an argument stroke.

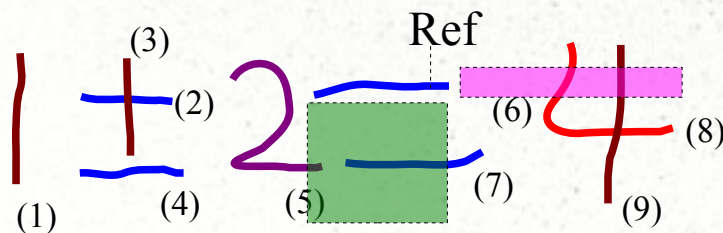
$n_r = 2$


Relational Graph Construction


Spatial relation: from a reference stroke to an argument stroke.

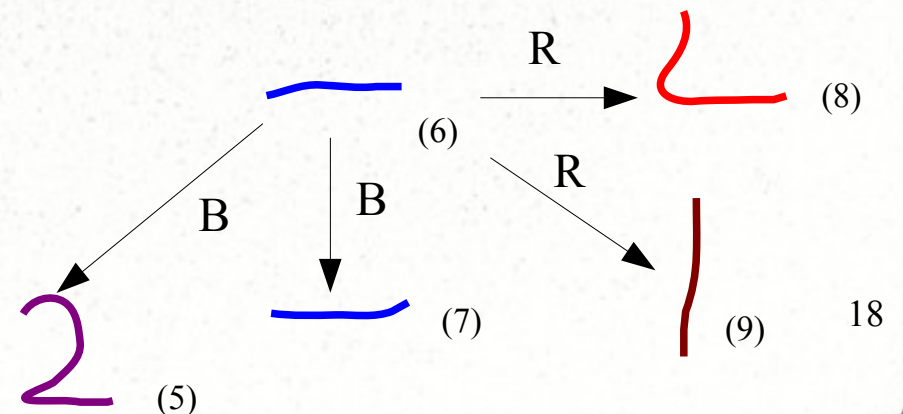
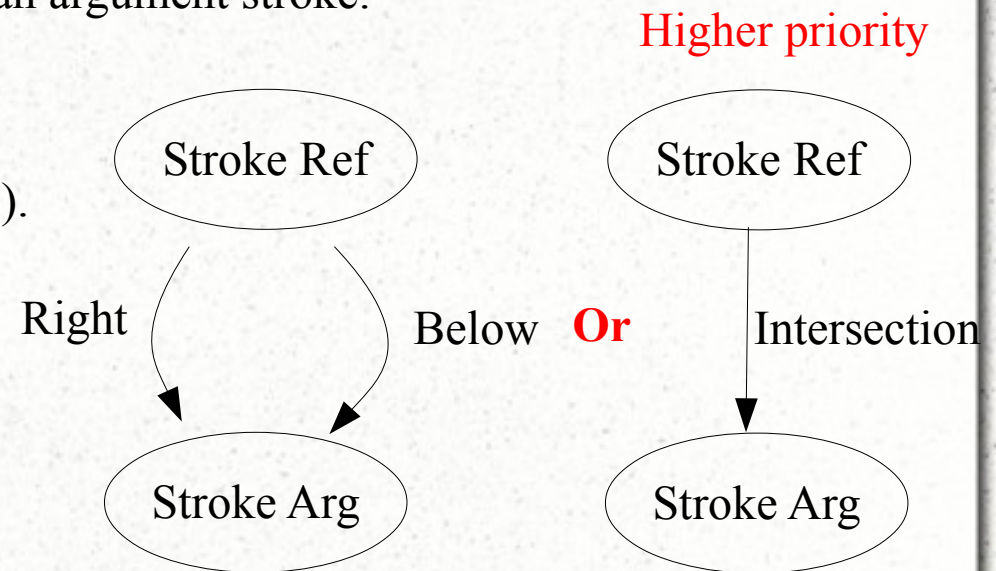
We predefine three spatial relations:
right (R), below (B), and intersection (I).

We predefine another constraint that
Directional spatial relation (R and B)
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Topological spatial relation (I).



Right: 

Below: 

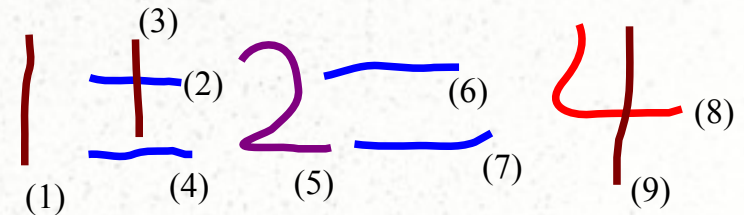


Number of edges

Spatial relation: from a reference stroke to an argument stroke.

n_r : the number of different spatial relations.

n_{str} : the number of strokes.



$$n_r = 2 \quad n_{str} = 9$$

Complete directed graph

Too many edges!

$$O(n_{str}^2) \quad n_r n_{str} (n_{str} - 1)$$

Number of edges in graph

$$= 144$$

$$n_c \leq (n_{str} - 1)$$

We prefer some symbols composed of the $n_c = 2$ closest strokes

Reduced

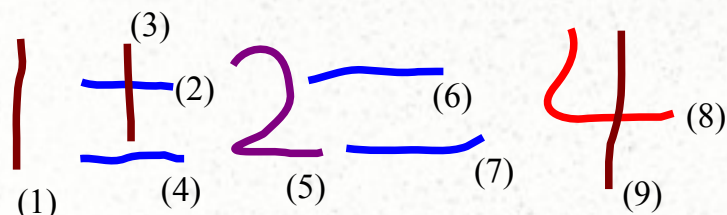
$$n_r \cdot n_{str} \cdot n_c$$

$$2 \cdot 9 \cdot 2 = 36$$

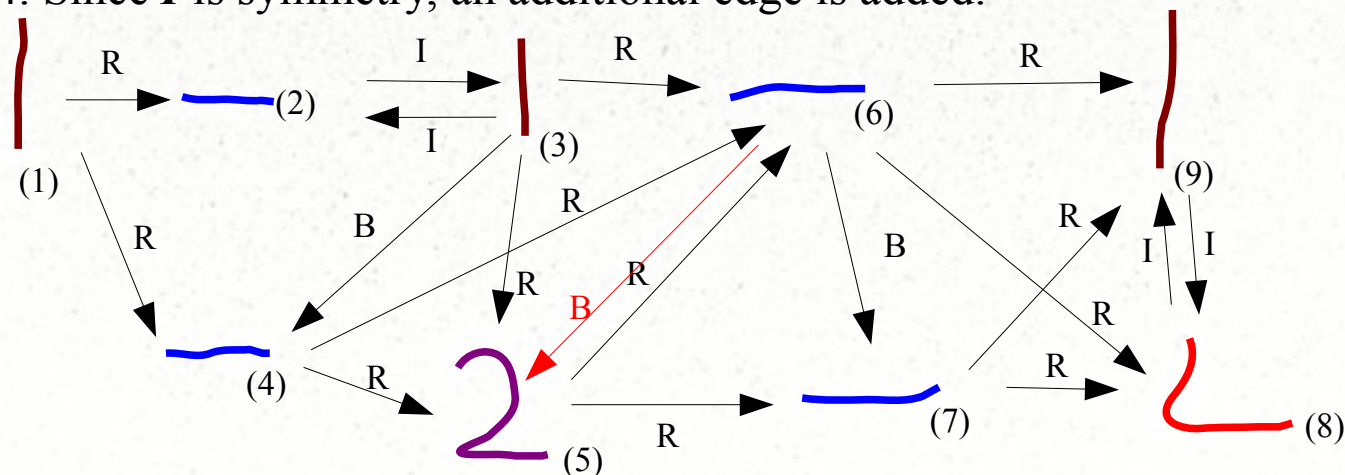
Since we, human, have a limited perceived visual angle.

Relational Graph Construction

Spatial relation: from a reference stroke to an argument stroke.



1. Start with top-left stroke.
2. Choose 2 closest strokes for each spatial relation.
3. Limit relational graph into Directed Acyclic Graph (DAG).
4. Since *I* is symmetry, an additional edge is added.

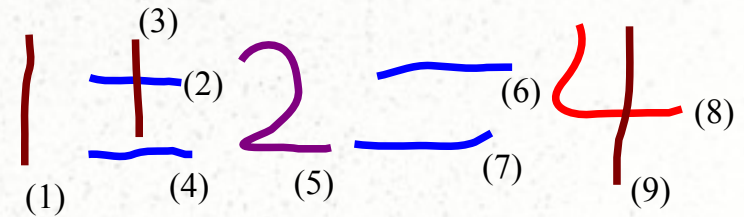


Number of edges

Spatial relation: from a reference stroke to an argument stroke.

n_r : the number of different spatial relations.

n_{str} : the number of strokes.



$$n_r = 2 \quad n_{str} = 9$$

Complete directed graph

Number of edges in graph

$$n_r n_{str} (n_{str} - 1)$$

$$= 144$$

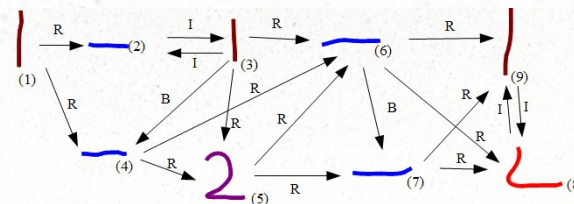
$$n_c \leq (n_{str} - 1)$$

We prefer some symbols composed of the $n_c = 2$ closest strokes

Reduced

$$n_r \cdot n_{str} \cdot n_c$$

$$2 \cdot 9 \cdot 2 = 36$$



Pruning

$$18$$

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 - **Discover Symbols (Sub-graphs)**
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Discover Symbols (Sub-graphs)

Many samples

$$4 \pm 1 = 2$$

$$4 - 2 = 1$$

$$1 \pm 2 = 4$$

$$4 - 1 = 2$$

$$2 \pm 4 = 1$$

One sample

(1) (3) (2) (5) (6) (8) (4) (7) (9)

Discover Symbols (Sub-graphs)

Many equations

$$4 \pm 1 = 2$$

$$4 - 2 = 1$$

$$1 \pm 2 = 4$$

$$4 - 1 = 2$$

$$2 \pm 4 = 1$$

1. We prefer the **frequent** patterns as the symbols.



2. Almost equally frequent pattern but with different numbers of strokes.



Three times



Three times

Which one?

Minimum Description Length principle

Minimum Description Length (**MDL**) principle is involved in searching the lexical unit that leads to the **best compression** of data.

[1] describes an unsupervised language learning method using MDL principle on text corpora.

***SUBDUE** (SUBstructure Discovery Using Examples) uses the **MDL principle** to identify **patterns** that minimize the number of bits needed to describe the **input graph** after being compressed by the pattern.[2]*

[1] Marcken, C. D., Unsupervised Language Acquisition, Massachusetts Institute of Technology, 1996

[2] Diane J. Cook and Lawrence B. Holder, <http://ailab.wsu.edu/subdue/>

Discover Symbols (Sub-graphs)

Many samples

$$4 \pm 1 = 2$$

$$4 - 2 = 1$$

$$1 \pm 2 = 4$$

$$4 - 1 = 2$$

$$2 \pm 4 = 1$$

1. We prefer the **frequent** patterns as the symbols.

$$=$$

2. Almost equally frequent pattern but with different numbers of strokes.

$$+$$

$$\pm$$

Which one?



Minimum Description Length (MDL) principle

Hierarchical structure (Iterative learning)

Many samples

$$4 + 1 = 2$$

$$4 + 2 = 1$$

$$1 \pm 2 = 4$$

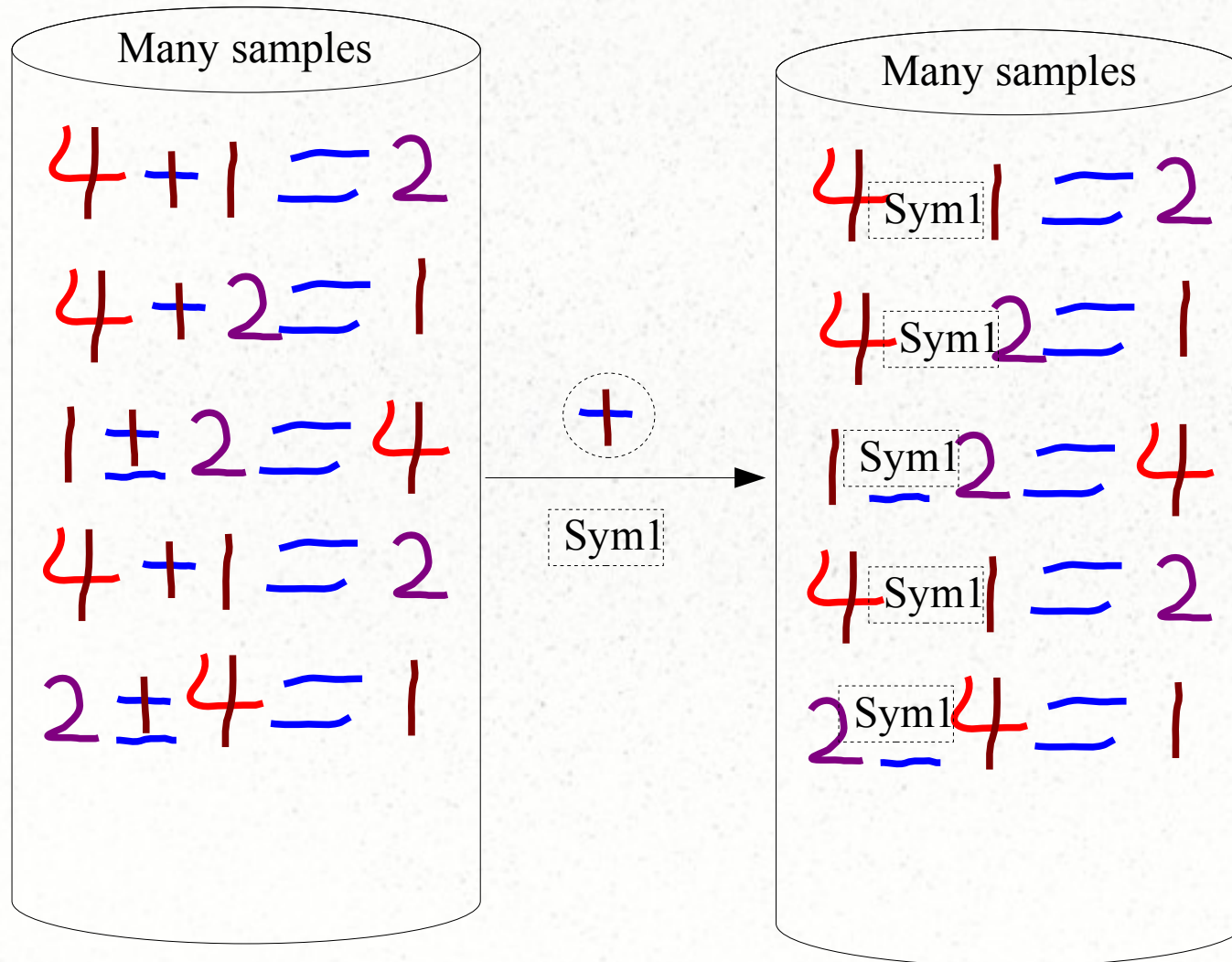
$$4 + 1 = 2$$

$$2 \pm 4 = 1$$

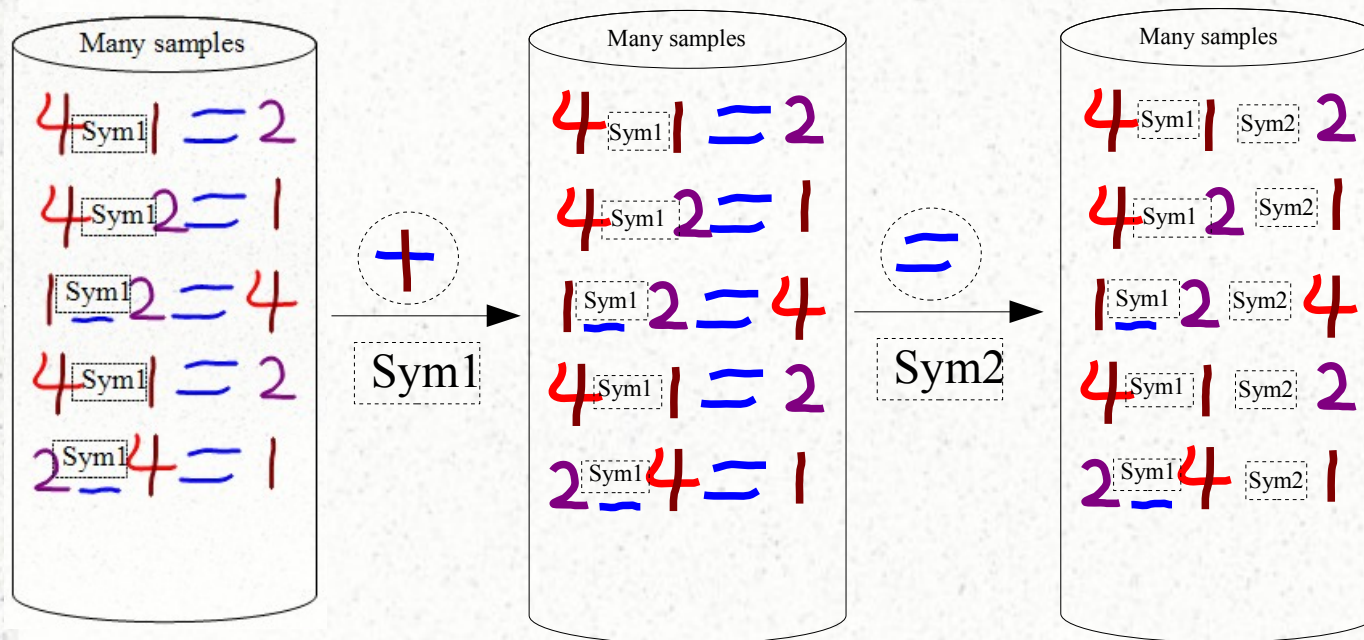
But if $+$ is much more frequent than \pm ,

we will choose the $+$ as the symbol according to MDL principle.

Hierarchical structure (Iterative learning)



Hierarchical structure (Iterative learning)



Lexicon:

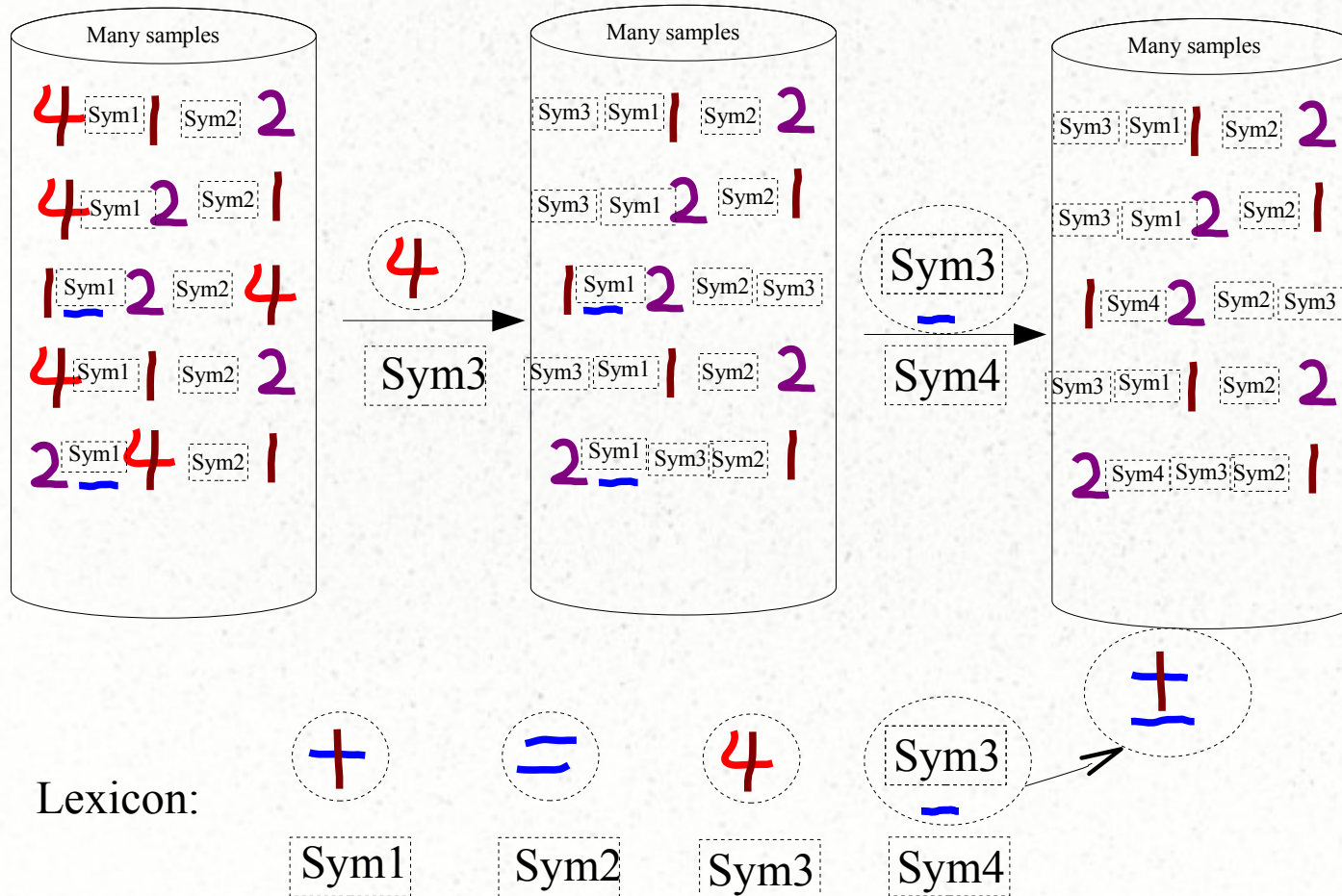


Sym1



Sym2

Hierarchical structure (Iterative learning)



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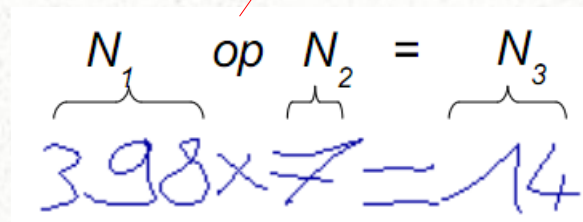
Dataset (Experiment)

A synthetic dataset from real isolated handwritten characters.

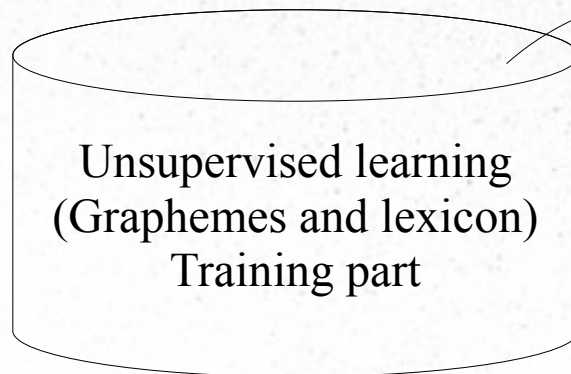
$N_{i=\{1,2,3\}}$ is 70% of 1 digit, 20% of 2 digits and 10% of 3 digits randomly.

$\{0, 1, \dots, 9\}$

$\{+, -, \times, \div\}$

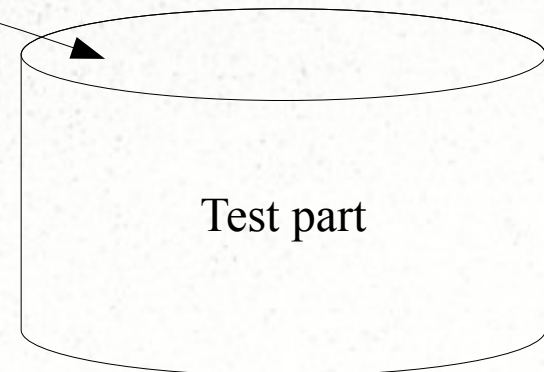


A handwritten equation $398 \times 7 = 14$ is shown. Above the equation, labels N_1 , op , N_2 , and N_3 are placed under the corresponding parts of the equation. N_1 is under '398', op is under ' \times ', N_2 is under '7', and N_3 is under '14'. A red arrow points from the set $\{+, -, \times, \div\}$ above to the ' \times ' operator in the equation.



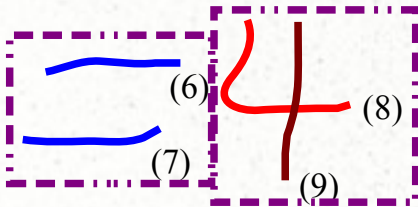
5427 symbols from 180 writers

Lexicon



3035 symbols from 100 writers

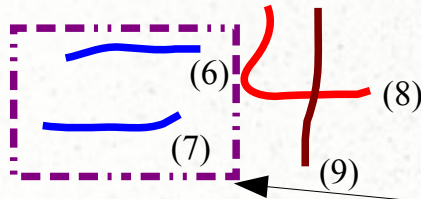
Measure (Experiment)



$S(e, G)$: ground-truth for the expression.

$$S(e, G) = \{\{(6), (7)\}, \{(8), (9)\}\}$$

Measure (Experiment)



$S(e, G)$: ground-truth for the expression.

$$S(e, G) = \{\{(6), (7)\}, \{(8), (9)\}\}$$

$S(e, L)$: hierarchical segmentation using lexicon L .

$$S(e, L) = \{\{(6)\}, \{(7)\}, \{(6), (7)\}, \{(8)\}, \{(9)\}\}$$

$$R_{\text{Recall}} = \frac{|S(e, G) \cap S(e, L)|}{|S(e, G)|} = 0.5$$

We got the recall rate of **64.3%** for **multi-stroke** symbols
(863 symbols from 1343 symbols) on the test part of our dataset.

Conclusion

- Quantization of strokes
- Construction of relational graph
- Lexicon extraction using MDL principle (SUBDUE)
- The recall rate of 64.3% (863/1343 multi-stroke symbols) is obtained.

Future work

- More complex spatial relation definition for more complex language, such as flowchart.
- Annotation assistance system for graphical symbols

Thank you for your attention.
Questions?