Unit 3: Control flow and list comprehensions

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December 3, 2020

Contents

1 Control flow and list comprehensions

In this lecture, we continue to explore basic concepts of the Python programming language such as conditional execution and loops.

1.1 Conditional execution

Frequently, we want to execute a code block only if some condition holds. We can do this using the `if` statement:

```
[1]: if 2*2 == 4:
         print('Python knows arithmetic!')
```

```
Python knows arithmetic!
```

A few observations:

- Conditional blocks are grouped using indentation (leading spaces). Remember from the previous unit that whitespace matters in Python!
- We write the equality operator using *two* equal signs, ==. This is to distinguish it from the assignment operator =.

We can also add an `else` block that will be executed whenever a condition is false:

```
[2]: if 2*2 == 3:
         # this branch will never be executed
         print('Something is fishy here')
```

1

```python
    else:
        print('Python knows arithmetic!')
```

```
Python knows arithmetic!
```

Finally, we can add more than one conditional branch using the `elif` clause:

```python
[3]: var = 1
     if var == 0:
         print('var is 0')
     elif var == 1:
         print('var is 1')
     else:
         print('var is neither 0 nor 1')
```

```
var is 1
```

1.1.1 Conditional expressions

We already encountered == to test whether two values are equal. Python offers many more operators that return either `True` or `False` and can be used as conditional expressions.

Expression	Description
==	Equal. Works for numerical values, strings, etc.
!=	Not equal. Works on numerical values, strings, etc.
>, >=, <,<=	Usual comparison of numerical values
a is b, a is not b	Test identity. a is b is True if a and b are the same object (in memory)
a in b, a not in b	Test whether a is or is not included in b where b is a collection
if obj, if not obj	Any Python object evaluates to True or False in an intuitive fashion (see below)

Additionally, there are logical operators that allow us to combine two logical values:

Expression	Description
a and b	True if both a and b are True
a or b	True if at least one of a or b is True

Examples:

```python
[4]: # list1 and list2 reference the same object
     list1 = [1,2]
     list2 = list1
     list1 is list2      # objects are identical, returns True
```

```
[4]: True
```

```python
[5]: # list1 and list2 do NOT reference the same object, but contain
     # identical elements.
     list2 = list1.copy()
     list1 is list2      # returns False
```

```
[5]:  False
```

```
[6]:  # Check if collections contain the same elements
      list1 == list2        # returns True
```

```
[6]:  True
```

```
[7]:  # Check whether element is in collection
      1 in list1            # returns True
```

```
[7]:  True
```

```
[8]:  # Combine logical expressions using 'and'
      1 in list1 and 2 in list1   # returns True
```

```
[8]:  True
```

We can also use the `in` operator to determine whether a key is contained in a dictionary:

```
[9]:  dct = {'institution': 'University of Glasgow'}
      'institution' in dct          # prints True
```

```
[9]:  True
```

```
[10]:  'course' not in dct           # prints True
```

```
[10]:  True
```

As mentioned above, any object evaluates to `True` or `False` in an `if` statement:

```
if obj:
    # do something if obj evaluates to True
```

The rules are quite intuitive: an object evaluates to `False` if

- it has a numerical type and is `0` (or `0.0`, or complex `0+0j`)
- it is an empty collection (tuple, list, dictionary, array, etc.)
- it is of logical (boolean) type and has value `False`
- it is `None`, a special built-in value used to denote that a variable does not reference anything.

In all other cases, an expression evaluates to `True`.

Examples:

```
[11]:  # evaluate numerical variable
       x = 0.0
       if x:
           print('Value is non-zero')
       else:
           print('Value is zero')
```

```
Value is zero
```

```
[12]:  # Evaluate boolean variable
       flag = True
       if flag:
           print('Flag is True')
       else:
           print('Flag is False')
```

```
Flag is True
```

1.2 Loops

Whenever we want to iterate over several items, we use the `for` loop. The `for` loop in Python is particular powerful because it can "magically" iterate over all sorts of data, not just integer ranges.

The standard use-case is to iterate over a set of integers:

```
[13]: # iterate over 0,...,3 and print each element
      for i in range(4):
          print(i)
```

```
0
1
2
3
```

We use the built-in `range` function to define the sequence of integers over which to loop. As usual in Python, the last element is *not* included. We can explicitly specify the start value and increment using the more advanced syntax `range(start,stop,step)`:

```
[14]: # iterate over 1,3
      for i in range(1,4,2):
          print(i)
```

```
1
3
```

Unlike in some other languages, we can directly iterate over elements of a collection:

```
[15]: cities = ('Glasgow', 'Edinburgh', 'St. Andrews')
      for city in cities:
          print(city)
```

```
Glasgow
Edinburgh
St. Andrews
```

We could of course alternatively iterate over indices and extract the corresponding element, but there is no need to:

```
[16]: for i in range(len(cities)):
          # print city at index i
          print(cities[i])
```

```
Glasgow
Edinburgh
St. Andrews
```

Note that when looping over dictionaries, the default is to iterate over *keys*:

```
[17]: dct = {'key1': 'value1', 'key2': 'value2'}
      for key in dct:
          print(key)
```

```
key1
key2
```

We can explicitly choose whether to iterate over keys, values, or both:

```python
[18]:  # iterate over keys, same as example above.
       for key in dct.keys():
           print(key)
```

```
key1
key2
```

```python
[19]:  # iterate over values
       for value in dct.values():
           print(value)
```

```
value1
value2
```

```python
[20]:  # iterate over keys and value as the same time
       # using the items() method
       for key, value in dct.items():
           # use format string to print key: value
           print('{}: {}'.format(key, value))
```

```
key1: value1
key2: value2
```

Note that `items()` returns the key-value pairs as `tuples`, so we need to unpack each tuple by writing `key, value` as the running variables of the `for` loop.

Sometimes the set of items over which to iterate is not known ex ante, and then we can instead use the `while` loop with a terminal condition:

```python
[21]:  z = 1.001
       i = 0

       # How many iterations will be performed? Not obvious ex ante.
       while z < 100.0:
           z = z*z + 0.234
           i = i + 1

       # print number of iterations
       print("loop terminated after {:d} iterations".format(i))
```

```
loop terminated after 5 iterations
```

1.2.1 Advanced looping

Oftentimes, we want to iterate over a list of items and at the same time keep track of an item's index. We can do this elegantly using the `enumerate()` function:

```python
[22]:  cities = ('Glasgow', 'Edinburgh', 'St. Andrews')

       # Iterate over cities, keep track of index in variable i
       for i, city in enumerate(cities):
           print('City {}: {}'.format(i, city))
```

```
City 0: Glasgow
City 1: Edinburgh
City 2: St. Andrews
```

We can skip an iteration or terminate the loop using the `continue` and `break` statements, respectively:

```
[23]: for city in cities:
          if city == 'Edinburgh':
              # skip to next iteration in case of Edinburgh
              continue
          print(city)
```

```
Glasgow
St. Andrews
```

```
[24]: for city in cities:
          if city == 'Glasgow':
              # Terminate iteration as soon as we find Glasgow
              print('Found Glasgow')
              break
```

```
Found Glasgow
```

1.3 List comprehensions

Python implements a powerful feature called "list comprehensions" that can be used to create collections such as tuples and lists without writing loop statements.

For example, imagine we want to create a list of squares of the integers $0, \ldots, 4$. We can do this using a loop and a list's `append()` method:

```
[25]: # Initialise empty list
      squares = []

      # Loop over integers 0,...,4
      for i in range(5):
          # The power operator in Python is **
          squares.append(i**2)
      squares
```

```
[25]: [0, 1, 4, 9, 16]
```

This is quite bloated and can be collapsed into a single expression using a list comprehension:

```
[26]: squares = [i**2 for i in range(5)]
      squares
```

```
[26]: [0, 1, 4, 9, 16]
```

If the desired result should be a `tuple`, we can instead write

```
[27]: squares = tuple(i**2 for i in range(5))
      squares
```

```
[27]: (0, 1, 4, 9, 16)
```

Alternatively, we can also create a dictionary using curly braces and the syntax `{key: <expression> for ...}`:

```
[28]: squares = {i: i**2 for i in range(5)}
      squares
```

```
[28]: {0: 0, 1: 1, 2: 4, 3: 9, 4: 16}
```

List comprehensions can be nested and combined with conditions to create almost arbitrarily complex expressions (this doesn't mean that you should, though!)

```
[29]: # Create incomprehensible list comprehension.
      # The modulo operator in Python is %
      items = [i*j for i in range(5) if i % 2 == 0 for j in range(i)]
      items
```

```
[29]: [0, 2, 0, 4, 8, 12]
```

Written out as two nested loops, this code is equivalent to

```
[30]: items = []
      for i in range(5):
          if i % 2 == 0:
              for j in range(i):
                  items.append(i*j)
      items
```

```
[30]: [0, 2, 0, 4, 8, 12]
```

2 Exercises

These exercises are not meant to demonstrate the most efficient use of Python, but to help you practice the material we have studied above. In fact, you'd most likely *not* want to use the solutions presented here in real code!

2.1 Exercise 1: Approximate Euler's number

Euler's number is defined as

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n} \right)^n$$

1. Create a sequence of the approximations to e for $n = 10, 20, 30, \ldots, 100$ using a list comprehension.
2. Compute the approximation error for each of the elements. *Hint:* To get the built-in value for e, use the import statement `from math import e`

2.2 Exercise 2: Approximate the sum of a geometric series

Let $\alpha \in (0, 1)$. The sum of the geometric series $(1, \alpha, \alpha^2, \ldots)$ is given by

$$\sigma = \sum_{i=0}^{\infty} \alpha^i = \frac{1}{1 - \alpha}$$

Assume that $\alpha = 0.1$. Write a loop that accumulates the values of the sequence $(1, \alpha, \alpha^2, \ldots)$ until the difference to the true value is smaller than 1×10^{-8}. How many elements does it take?

Hint: In Python (and many other languages) the floating-point value 1×10^{-8} is written as `1e-8`.

2.3 Exercise 3: Binary search (advanced)

The bisection method can be used to find to root of a function $f(x)$, ie. the point x_0 such that $f(x_0) = 0$. In this exercise, we will use the same algorithm to find the interval of a strictly monotonic sequence of numbers that brackets the value zero (this is a binary search algorithm with approximate matching).

Assume that we have an array `x` of 11 increasing real numbers given by

```python
import numpy as np
x = np.linspace(-0.5, 1.0, 11)
x
```

```
[31]: array([-0.5 , -0.35, -0.2 , -0.05,  0.1 ,  0.25,  0.4 ,  0.55,  0.7 ,
              0.85,  1.  ])
```

Write code that identifies the bracketing interval (in this case [-0.05,0.1]) using the following algorithm:

1. Initialize the index of the bracket lower bound to `lbound=0` and the index of the bracket upper bound to `ubound=len(x)-1`.

2. Compute the midpoint between these two indices (rounded to the nearest integer), `mid = (ubound + lbound) // 2`.

   Hint: The operator `//` truncates the result of a division to the nearest integer.

3. Inspect `x[mid]`, the value at index `mid`. If `x[mid]` has the same sign as `x[ubound]`, update the upper bound, `ubound=mid`. Otherwise, update the lower bound.

4. Continue until `ubound = lbound + 1`, ie. until you have found the bracket `x[lbound] <= 0 < x[ubound]`.

2.4 Exercise 4: diagonal and band matrices

Let `a` be a matrix of zeros with shape `(m, n)` and integer data type:

`a = np.zeros((m,n), dtype=int)`

1. Set `m = n = 4`. Fill up the diagonal of `a` with ones so that it becomes an identity matrix. Verify that `np.identity()` gives the same result.

2. Recreate `a` to have dimensions `m = 4` and `n = 5`. Modify the main, first lower and first upper diagonals so that the resulting matrix looks like this, where omitted elements are zeros:

$$\begin{bmatrix} 1 & 2 & & & \\ 3 & 1 & 2 & & \\ & 3 & 1 & 2 & \\ & & 3 & 1 & 2 \end{bmatrix}$$

Hint: Use nested `for` loops to set the diagonal elements.

2.5 Exercise 5: triangular matrices

Let `a` be a matrix of zeroes with shape `(m, n)` and integer data type:

`a = np.zeros((m,n), dtype=int)`

Assume that `m = n = 4`. Using loops and `if` statements, modify the elements of `a` to obtain the following upper-triangular matrix, where omitted elements are set to zero:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ & 5 & 6 & 7 \\ & & 8 & 9 \\ & & & 10 \end{bmatrix}$$

3 Solutions

3.1 Solution for exercise 1

```python
[32]: # Compute approximation for n = 10, 20,..., 100
      euler_approx = [(1.0+1.0/i)**i for i in range(10,101,10)]
      print('Approximate values')
      print(euler_approx)

      # import 'correct' value
      from math import e

      # We need to subtract e from each element to get the approximation error
      euler_error = [approx - e for approx in euler_approx]
      print('Approximation error')
      print(euler_error)
```

```
Approximate values
[2.5937424601000023, 2.653297705144422, 2.6743187758703026, 2.
 →685063838389963,
2.691588029073608, 2.6959701393302162, 2.6991163709761854, 2.
 →7014849407533275,
2.703332461058186, 2.7048138294215285]
Approximation error
[-0.12453936835904278, -0.06498412331462289, -0.043963052588742446,
-0.03321799006908188, -0.026693799385437256, -0.02231168912882886,
-0.019165457482859694, -0.016796887705717634, -0.01494936740085917,
-0.01346799903751661]
```

3.2 Solution for exercise 2

We don't know now many iterations we will need to get to the required tolerance of 1×10^{-8}, so this is a good opportunity to use a `while` loop.

```python
[33]: # Convergence tolerance
      tol = 1e-8
      alpha = 0.1
      # The correct value
      sigma_exact = 1.0/(1.0 - alpha)

      # keep track of number of iterations
      n = 0

      # Initialise approximated sum
      sigma = 0.0

      # Iterate until absolute difference is smaller than tolerance level.
      # The built-in function abs() returns the absolute value.

      while abs(sigma - sigma_exact) > tol:
          # We can combine addition and assignment into a single operator +=
          # This is equivalent to
          #    sigma = sigma + alpha**n
          sigma += alpha**n
          # Increment exponent
          n += 1
```

```
print('Number of iterations: {}, approx. sum: {:.8f}'.format(n, sigma))
```

```
Number of iterations: 9, approx. sum: 1.11111111
```

3.3 Solution for exercise 3

To complete the exercise, all you have to do is to translate the algorithm given in the exercise into code.

Since we don't know how many iterations it takes to find the bracket, we use a `while` loop that continues as long as `lbound` and `ubound` are more than 1 apart.

[34]:
```python
import numpy as np

# Given array of increasing numbers
x = np.linspace(-0.5, 1.0, 11)


lbound = 0
ubound = len(x) - 1

while ubound > (lbound + 1):
    # Index of mid point. Indices have to be integers, so
    # we need to truncate the division result to an integer.
    mid = (ubound + lbound) // 2

    if (x[mid] * x[ubound]) > 0.0:
        # x[mid] and x[ubound] have same sign!
        ubound = mid
        print('Setting upper bound index to {}'.format(ubound))
    else:
        # x[mid] and x[lbound] have the same sign
        # or at least one of them is zero
        lbound = mid
        print('Setting lower bound index to {}'.format(lbound))

print('Value at lower bound: {:.4g}'.format(x[lbound]))
print('Value at upper bound: {:.4g}'.format(x[ubound]))
```

```
Setting upper bound index to 5
Setting lower bound index to 2
Setting lower bound index to 3
Setting upper bound index to 4
Value at lower bound: -0.05
Value at upper bound: 0.1
```

In this implementation we use the fact that two non-zero real numbers a and b have the same sign whenever $a \cdot b > 0$.

3.4 Solution for exercise 4

For the first part, we just need to loop over the diagonal elements and overwrite the zeros with ones:

[35]:
```python
# Part 1: create identity matrix

import numpy as np

n = 4
a = np.zeros((n,n), dtype=int)
```

```
# loop over diagonal elements, set them to 1
for i in range(n):
    a[i,i] = 1

a               # print identity matrix
```

[35]: array([[1, 0, 0, 0],
             [0, 1, 0, 0],
             [0, 0, 1, 0],
             [0, 0, 0, 1]])

You can get the same result using NumPy's `np.identity()` function:

[36]:
```
n = 4
np.identity(n, dtype=int)
```

[36]: array([[1, 0, 0, 0],
             [0, 1, 0, 0],
             [0, 0, 1, 0],
             [0, 0, 0, 1]])

For the second part, we need nested loops over rows and columns and we check at each position (i, j) whether it is on the main diagonal, or on the first upper or lower diagonals.

[37]:
```
# Part 2: create band matrix

import numpy as np

m = 4
n = 5

a = np.zeros((m,n), dtype=int)

# loop over rows
for i in range(m):
    # loop over columns
    for j in range(n):
        if i == j:
            # main diagonal element
            a[i,j] = 1
        elif i == (j - 1):
            # upper diagonal element
            a[i,j] = 2
        elif i == (j + 1):
            # lower diagonal element
            a[i,j] = 3

a    # print final matrix
```

[37]: array([[1, 2, 0, 0, 0],
             [3, 1, 2, 0, 0],
             [0, 3, 1, 2, 0],
             [0, 0, 3, 1, 2]])

3.5 Solution for exercise 5

We solve this exercise using two nested `for` loops, the first over rows, the second over columns:

```python
[38]: import numpy as np

m = 4
n = 4

a = np.zeros((m,n), dtype=int)

# keep track of value to be inserted
value = 1

# loop over rows
for i in range(m):
    # loop over columns
    for j in range(i, n):
        a[i,j] = value
        # increment value for next matching element
        value += 1

a         # print final matrix
```

```
[38]: array([[ 1,  2,  3,  4],
             [ 0,  5,  6,  7],
             [ 0,  0,  8,  9],
             [ 0,  0,  0, 10]])
```

Note that the loop over columns uses the current row index as the lower bound, since we never need to insert anything at element (i, j) if $j < i$.