# Unit 7: Handling data with pandas

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# 1 Handling data with pandas

# 1.1 Motivation

So far, we have encountered NumPy arrays as the only way to store numerical data (we mostly ignored the built-in containers provided directly in Python). However, while NumPy arrays are great for storing homogenous data without any particular structure, they are somewhat limited when we want to use them for high-level data analysis.

For example, we usually want to process data sets with

- 1. several variables;
- 2. multiple observations, which need not be identical across variables (some values may be missing);
- 3. non-homogenous data types: for examples, names need to be stored as strings, birthdays as dates and income as a floating-point number.

While NumPy can in principle handle such situations, it puts all the burden on the user. Most users would prefer to not have to deal with such low-level details.

Imagine we want to store names, birth dates and annual income for two people:

Date of birth	Income		
1985-01-01 1997-05-12	30,000		
	1985-01-01		

No income was reported for Bob, so it's missing. With NumPy, we could do this as follows:

```
[1]: import numpy as np
from datetime import date
```

```
[2]: data.dtype # print array data type
```

[2]: dtype('0')

While we can create such arrays, they are almost useless for data analysis, in particular since everything is stored as a generic object.

• To be fair, NumPy offers an alternative array type called "record" or "structured" array which can handle fields of different data types.

However, the pandas library offers much more beyond that, so there is little reason to use structured arrays.

Pandas was created to offer more versatile data structures that are straightforward to use for storing, manipulating and analysing heterogeneous data:

- 1. Data is clearly organised in *variables* and *observations*, similar to econometrics programs such as Stata.
- 2. Each variable is permitted to have a different data type.
- 3. We can use *labels* to select observations, instead of having to use a linear numerical index as with NumPy.

We could, for example, index a data set using National Insurance Numbers.

4. Pandas offers many convenient data aggregation and reduction routines that can be applied to subsets of data.

For example, we can easily group observations by city and compute average incomes.

5. Pandas also offers many convenient data import / export functions that go beyond what's in NumPy.

Should we be using pandas at all times, then? No!

- For low-level tasks where performance is essential, use NumPy.
- For homogenous data without any particular data structure, use NumPy.
- On the other hand, if data is heterogeneous, needs to be imported from an external data source and cleaned or transformed before performing computations, use pandas.

There are numerous tutorials on pandas on the internet, so we will keep this unit short and illustrate only the main concepts. Useful references to additional material include:

- The official user guide.
- The official pandas cheat sheet which nicely illustrates the most frequently used operations.
- The official API reference with details on every pandas object and function.
- There are numerous tutorials (including videos) available on the internet. See here for a list.

# 1.2 Creating pandas data structures

Pandas has two main data structures:

- 1. Series represents observations of a single variable.
- 2. DataFrame is a container for several variables. You can think of each individual column of a DataFrame as a Series, and each row represents one observation.

The easiest way to create a Series or DataFrame is to create them from pre-existing data.

To access pandas data structures and routines, we need to import them first. The near-universal convention is to make pandas available using the name pd:

```
[3]: import pandas as pd
```

Examples:

We can create a DataFrame from a NumPy array:

```
import numpy as np
import pandas as pd  # universal convention: import using pd
from numpy.random import default_rng

# Draw normally distributed data
rng = default_rng(123)
data = rng.normal(size=(10,3))

# Define variable (or column) names
varnames = ['A', 'B', 'C']

# Create pandas DataFrame
pd.DataFrame(data, columns=varnames)
```

```
[4]: A B C
0 -0.989121 -0.367787 1.287925
1 0.193974 0.920231 0.577104
2 -0.636464 0.541952 -0.316595
3 -0.322389 0.097167 -1.525930
4 1.192166 -0.671090 1.000269
5 0.136321 1.532033 -0.659969
6 -0.311795 0.337769 -2.207471
7 0.827921 1.541630 1.126807
8 0.754770 -0.145978 1.281902
9 1.074031 0.392621 0.005114
```

This code creates a DataFrame of three variables called A, B and C with 10 observations each.

Alternatively, we can create a DataFrame from non-homogenous data as follows:

```
[5]: # Names (strings)
names = ['Alice', 'Bob']

# Birth dates (datetime objects)
bdates = pd.to_datetime(['1985-01-01', '1997-05-12'])

# Incomes (floats)
incomes = np.array([35000, np.nan]) # code missing income as NaN

# create DataFrame from dictionary
pd.DataFrame({'Name': names, 'Birthdate': bdates, 'Income': incomes})
```

```
[5]: Name Birthdate Income
0 Alice 1985-01-01 35000.0
```

```
1 Bob 1997-05-12 NaN
```

If data types differ across columns, as in the above example, it is often convenient to create the DataFrame by passing a dictionary as an argument. Each key represents a column name and each corresponding value contains the data for that variable.

# 1.3 Viewing data

With large data sets, you hardly ever want to print the entire DataFrame. Pandas by default limits the amount of data shown. You can use the head() and tail() methods to explicitly display a specific number of rows from the top or the end of a DataFrame.

To illustrate, we use a data set of 23 UK universities that contains the following variables:

- Institution: Name of the institution
- Country: Country/nation within the UK (England, Scotland, ...)
- Founded: Year in which university (or a predecessor institution) was founded
- Students: Total number of students
- Staff: Number of academic staff
- Admin: Number of administrative staff
- Budget: Budget in million pounds
- Russell: Binary indicator whether university is a member of the Russell Group, an association of the UK's top research universities.

The data was compiled based on information from Wikipedia.

Before we read in any data, it is convenient to define a variable pointing to the directory where the data resides. We can either use a relative local path ../data, which, however, will not work when running the notebook in some cloud environments such as Google Colab. Alternatively, we can use the full URL to the data file in the GitHub repository.

```
[6]: # Uncomment this to use files in the local data/ directory
# DATA_PATH = '../data'

# Load data directly from GitHub
DATA_PATH = 'https://raw.githubusercontent.com/richardfoltyn/MLFP-ECON5130/main/data'
```

We can now read in the data stored in the file universities.csv like this:

```
[7]: import pandas as pd

# URL to CSV file in GitHub repository
file = f'{DATA_PATH}/universities.csv'

# Load sample data set of UK universities. Individual fields are separated
# using; so we need to pass sep=';' as an argument.
df = pd.read_csv(file, sep=';')
```

We can now display the first and last three rows:

```
[8]: df.head(3)
                   # show first three rows
[8]:
                   Institution Country Founded Students Staff
                                                                 Admin \
          University of Glasgow Scotland
                                                30805 2942.0 4003.0
                                        1451
       University of Edinburgh Scotland
                                          1583
                                                   34275 4589.0 6107.0
     2 University of St Andrews Scotland
                                          1413
                                                  8984 1137.0 1576.0
       Budget Russell
       626.5
     1 1102.0
                    1
       251.2
```

```
[9]: df.tail(3)
                      # show last three rows
                         Institution
                                               Country
                                                         Founded Students
                                                                             Staff
[9]:
     20
              University of Stirling
                                              Scotland
                                                            1967
                                                                      9548
                                                                               NaN
     21
         Queen's University Belfast Northern Ireland
                                                            1810
                                                                     18438 2414.0
     22
                  Swansea University
                                                 Wales
                                                            1920
                                                                     20620
                                                                               NaN
                  Budget Russell
          Admin
         1872.0
                  113.3
     20
                                0
         1489.0
                   369.2
                                1
     21
         3290.0
                     NaN
                                0
```

To quickly compute some descriptive statistics for the *numerical* variables in the DataFrame, we use describe():

```
df.describe()
                                                                          Budget
                  Founded
                               Students
                                                Staff
                                                             Admin
[10]:
                23.000000
                              23.000000
                                            20.000000
                                                         19.000000
                                                                      22.000000
       count
              1745.652174 24106.782609 3664.250000
                                                       3556.736842
                                                                      768.609091
       mean
       std
               256.992149
                            9093.000735
                                         2025.638038
                                                       1550.434342
                                                                      608.234948
       min
              1096.000000
                            8984.000000
                                          1086.000000
                                                       1489.000000
                                                                      113.300000
                           18776.500000
       25%
              1589.000000
                                          2294.250000
                                                       2193.500000
                                                                      340.850000
       50%
              1826.000000
                           23247.000000
                                          3307.500000
                                                       3485.000000
                                                                      643.750000
       75%
              1941.500000
                           30801.500000
                                          4439.750000
                                                       4347.500000
                                                                     1023.500000
       max
              2004.000000
                           41180.000000
                                          7913.000000
                                                       6199.000000
                                                                    2450.000000
                Russell
       count 23.000000
       mean
               0.739130
       std
               0.448978
       min
               0.000000
       25%
               0.500000
       50%
               1.000000
       75%
               1.000000
       max
               1.000000
```

Note that this automatically ignores the columns Institution and Country as they contain strings, and computing the mean, etc. of a string variable does not make sense.

To see low-level information about the data type used in each column, we call info():

```
[11]: df.info()
```

```
RangeIndex: 23 entries, 0 to 22
Data columns (total 8 columns):
                  Non-Null Count
#
     Column
                                  Dtype
0
     Institution 23 non-null
                                   object
1
     Country
                  23 non-null
                                   object
                                   int64
     Founded
                  23 non-null
     Students
                  23 non-null
                                   int64
3
     Staff
                  20 non-null
                                   float64
 4
 5
     Admin
                  19 non-null
                                   float64
                                   float64
6
     Budget
                  22 non-null
                  23 non-null
     Russell
                                   int64
dtypes: float64(3), int64(3), object(2)
memory usage: 1.6+ KB
```

<class 'pandas.core.frame.DataFrame'>

Pandas automatically discards missing information in computations. For example, the number of academic staff is missing for several universities, so the number of *non-null* entries reported in the table above is less than 23, the overall sample size.

# 1.4 Indexing

Pandas supports two types of indexing:

- 1. Indexing by position. This is basically identical to the indexing of other Python and NumPy containers.
- 2. Indexing by label, i.e., by the values assigned to the row or column index. These labels need not be integers in increasing order, as is the case for NumPy.

We will see how to assign labels below.

Pandas indexing is performed either by using brackets [], or by using .loc[] for label indexing, or .iloc[] for positional indexing.

Indexing via [] can be somewhat confusing:

- specifying df['name'] returns the column name as a Series object.
- On the other hand, specifying a range such as df[5:10] returns the *rows* associated with the *positions* 5,...,9.

Examples:

```
[12]: import pandas as pd
       # Load sample data set of UK universities
       df = pd.read_csv(f'{DATA_PATH}/universities.csv', sep=';')
       df['Institution']
                                       # select a single column
[12]: 0
                       University of Glasgow
                     University of Edinburgh
       2
                    University of St Andrews
       3
                      University of Aberdeen
                   University of Strathclyde
       4
       5
       6
                                         UCL
                     University of Cambridge
       7
       8
                        University of Oxford
       9
                       University of Warwick
       10
                     Imperial College London
       11
                       King's College London
       12
                    University of Manchester
       13
                       University of Bristol
                    University of Birmingham
       14
             Queen Mary University of London
       15
                          University of York
       16
                    University of Nottingham
       17
                        University of Dundee
       18
                          Cardiff University
       19
                      University of Stirling
       20
                  Queen's University Belfast
       21
                          Swansea University
       Name: Institution, dtype: object
[13]: df[['Institution', 'Students']]
                                            # select multiple columns using a list
[13]:
                               Institution Students
                     University of Glasgow
                                                30805
                   University of Edinburgh
       1
                                                34275
                  University of St Andrews
       2
                                                8984
                    University of Aberdeen
       3
                                                14775
                 University of Strathclyde
                                                22640
       4
                                       LSE
                                                11850
       5
```

41180

UCL

```
University of Cambridge
7
                                         23247
8
               University of Oxford
                                         24515
              University of Warwick
9
                                         27278
10
            Imperial College London
                                         19115
11
              King's College London
                                         32895
           University of Manchester
12
                                         40250
13
              University of Bristol
                                         25955
14
           University of Birmingham
                                         35445
15
   Queen Mary University of London
                                         20560
16
                 University of York
                                         19470
           University of Nottingham
17
                                         30798
               University of Dundee
18
                                         15915
                 Cardiff University
                                         25898
19
20
             University of Stirling
                                          9548
21
         Queen's University Belfast
                                         18438
                 Swansea University
                                         20620
```

To return the rows at positions 1, 2 and 3 we use

```
[14]: df[1:4]
[14]:
                                 Country Founded Students
                                                             Staff
                     Institution
                                                                     Admin \
          University of Edinburgh Scotland
                                             1583
                                                      34275 4589.0 6107.0
      2 University of St Andrews Scotland
                                                      8984 1137.0 1576.0
                                             1413
          University of Aberdeen Scotland
                                                      14775 1086.0 1489.0
                                             1495
         Budget Russell
      1 1102.0
          251.2
                      0
      2
          219.5
```

Pandas follows the Python convention that indices are 0-based, and the endpoint of a slice is not included.

#### 1.4.1 Manipulating indices

'b',

Pandas uses *labels* to index and align data. These can be integer values starting at 0 with increments of 1 for each additional element, which is the default, but they need not be. The two main methods to manipulate indices are:

- set\_index(keys=['column1', ...]): uses the values of column1 and optionally additional columns as indices, discarding the current index.
- reset\_index(): resets the index to its default value, a sequence of increasing integers starting at 0.

Both methods return a new DataFrame and leave the original DataFrame unchanged. If we want to change the existing DataFrame, we need to pass the argument inplace=True.

For example, we can replace the row index and use the Roman lower-case characters  $a, b, c, \ldots$  as labels instead of integers:

```
[15]: import pandas as pd
    df = pd.read_csv(f'{DATA_PATH}/universities.csv', sep=';')

# Create list of lower-case letters which has same
# length as the number of observations.
index = [chr(97+i) for i in range(len(df))] # len(df) returns number of obs.
index
[15]: ['a',
```

```
'c',
'd',
'e',
'f',
'g',
'h',
'i',
'j',
'k',
Ί'
'm',
'n',
'o',
'p',
'q',
'r',
't',
'u',
'v',
'w']
```

```
[16]:
                          Institution
                                        Country Founded Students
                                                                    Staff
                                                                            Admin \
      index
                University of Glasgow Scotland
      a
                                                    1451
                                                             30805 2942.0 4003.0
      b
              University of Edinburgh Scotland
                                                    1583
                                                             34275 4589.0 6107.0
             University of St Andrews Scotland
                                                    1413
                                                             8984 1137.0 1576.0
             Budget Russell
      index
      a
              626.5
                           1
      b
             1102.0
                           1
      С
              251.2
                           0
```

To add to the confusion, note that when specifying a range in terms of labels, the last element *is* included! Hence the row with index c in the above example is shown.

We can reset the index to its default integer values using the reset\_index() method:

```
[17]: # Reset index labels to default value (integers 0, 1, 2, ...)
df_new = df.reset_index(drop=True)
df_new.head(3) # print first 3 rows of new DataFrame
```

```
Institution
                                   Country Founded Students
                                                              Staff
                                                                      Admin \
[17]:
            University of Glasgow Scotland
      0
                                              1451
                                                       30805 2942.0 4003.0
          University of Edinburgh Scotland
                                              1583
                                                       34275 4589.0 6107.0
        University of St Andrews Scotland
                                              1413
                                                       8984 1137.0 1576.0
         Budget Russell
      0
          626.5
                      1
         1102.0
                      1
      1
```

The drop=True argument tells pandas to throw away the old index values instead of storing them as a column of the resulting DataFrame.

# 1.4.2 Selecting elements

To more clearly distinguish between selection by label and by position, pandas provides the .loc[] and .iloc[] methods of indexing. To make your intention obvious, you should therefore adhere to the following rules:

- 1. Use df['name'] only to select *columns* and nothing else.
- 2. Use .loc[] to select by label.
- 3. Use .iloc[] to select by position.

#### Selection by label

To illustrate, using .loc[] unambiguously indexes by label:

With .loc[] we can even perform slicing on column names, which is not possible with the simpler df[] syntax:

```
[19]: df.loc['d':'f', 'Institution':'Founded']
[19]:
                             Institution
                                           Country
                                                     Founded
       index
       d
                 University of Aberdeen
                                          Scotland
                                                        1495
              University of Strathclyde
                                          Scotland
                                                        1964
       е
                                     LSE
                                           England
                                                        1895
```

This includes all the columns between Institution and Founded, where the latter is included since we are slicing by label.

Trying to pass in positional arguments will return an error for the given DataFrame since the index labels are a, b, c,... and not 0, 1, 2...

```
[20]: df.loc[0:4]

TypeError: cannot do slice indexing on Index with these indexers [0] of type int
```

However, we can reset the index to its default value. Then the index labels are integers and coincide with their position, so that .loc[] works:

```
[21]: df.reset_index(inplace=True, drop=True)
                                                   # reset index labels to integers,
                                                   # drop original index
      df.loc[0:4]
                       \\ Institution
                                              Founded Students
[21]:
                                     Country
                                                                  Staff
                                                                          Admin
             University of Glasgow
                                    Scotland
                                                                         4003.0
                                                 1451
                                                          30805 2942.0
      0
           University of Edinburgh
                                    Scotland
                                                 1583
                                                          34275 4589.0
                                                                         6107.0
      1
          University of St Andrews
                                    Scotland
                                                                         1576.0
      2
                                                 1413
                                                           8984
                                                                 1137.0
            University of Aberdeen
                                    Scotland
                                                 1495
                                                          14775 1086.0 1489.0
      3
         University of Strathclyde Scotland
                                                 1964
                                                          22640
                                                                    NaN 3200.0
```

```
Budget Russell
0 626.5 1
1 1102.0 1
```

```
2 251.2 0
3 219.5 0
4 304.4 0
```

Again, the end point with label 4 is included because we are selecting by label.

Somewhat surprisingly, we can also pass boolean arrays to .loc[] even though these are clearly not labels:

```
[22]: df.loc[df['Country'] == 'Scotland']
[22]:
                                               Founded Students
                                                                  Staff
                                                                          Admin \
                        Institution
                                      Country
              University of Glasgow Scotland
                                                  1451
                                                          30805 2942.0 4003.0
            University of Edinburgh Scotland
      1
                                                  1583
                                                          34275 4589.0 6107.0
                                                                 1137.0 1576.0
      2
           University of St Andrews
                                     Scotland
                                                           8984
                                                  1413
             University of Aberdeen
                                     Scotland
                                                  1495
                                                          14775
                                                                 1086.0
                                                                         1489.0
      3
          University of Strathclyde
      4
                                     Scotland
                                                  1964
                                                          22640
                                                                    NaN
                                                                         3200.0
      18
               University of Dundee Scotland
                                                  1967
                                                                 1410.0
                                                                         1805.0
                                                          15915
      20
             University of Stirling Scotland
                                                  1967
                                                           9548
                                                                    NaN 1872.0
          Budget Russell
      0
           626.5
                        1
      1
          1102.0
                        1
      2
           251.2
                        0
           219.5
      3
                        0
           304.4
                        0
      4
      18
           256.4
                        0
           113.3
```

Indexing via .loc[] supports a few more types of arguments, see the official documentation for details.

### Selection by position

Conversely, if we want to select items exclusively by their position and ignore their labels, we use .iloc[]:

Again, .iloc[] supports a multitude of other arguments, including boolean arrays. See the official documentation for details.

# 1.5 Aggregation and reduction

### 1.5.1 Working with entire DataFrames

The simplest way to perform data reduction is to invoke the desired routine on the entire DataFrame:

```
Admin 3556.736842
Budget 768.609091
Russell 0.739130
dtype: float64
```

Methods such as mean() are by default applied column-wise to each column. The numeric\_only=True argument is used to discard all non-numeric columns (depending on the version of pandas, mean() will issue a warning otherwise).

One big advantage over NumPy is that missing values (represented by np.nan) are automatically ignored:

```
[25]: # mean() automatically drops 3 missing observations df['Staff'].mean()
```

[25]: 3664.25

# 1.5.2 Splitting and grouping

Applying aggregation functions to the entire DataFrame is similar to what we can do with NumPy. The added flexibility of pandas becomes obvious once we want to apply these functions to subsets of data, i.e., groups, which we can define based on values or index labels.

For example, we can easily group our universities by country:

```
[26]: import pandas as pd

df = pd.read_csv('../data/universities.csv', sep=';')

groups = df.groupby(['Country'])
```

Here groups is a special pandas objects which can subsequently be used to process group-specific data. To compute the group-wise averages, we can simply run

```
[27]: groups.mean()
                            Founded
                                        Students
                                                        Staff
                                                                    Admin ∖
[27]:
      Country
      England
                       1745.923077 27119.846154 4336.692308 4112.0000000
      Northern Ireland 1810.000000 18438.000000 2414.000000 1489.000000
      Scotland
                       1691.428571 19563.142857 2232.800000 2864.571429
      Wales
                        1901.500000 23259.000000 3330.000000 4514.500000
                            Budget
                                    Russell
      Country
      England
                       1001.700000 1.000000
      Northern Ireland 369.200000 1.000000
      Scotland
                        410.471429 0.285714
      Wales
                        644.800000 0.500000
```

Groups support column indexing: if we want to only compute the total number of students for each country in our sample, we can do this as follows:

```
Name: Students, dtype: int64
```

There are numerous routines to aggregate grouped data, for example:

- mean(), sum(): averages and sums over numerical items within groups.
- std(), var(): within-group std. dev. and variances
- size(): group sizes
- first(), last(): first and last elements in each group
- min(), max(): minimum and maximum elements within a group

### Examples:

```
[29]: groups.size()
                         # return number of elements in each group
[29]: Country
      England
                         13
      Northern Ireland
                         1
      Scotland
                          7
      Wales
                          2
      dtype: int64
[30]: groups.first() # return first element in each group
[30]:
                                     Institution Founded Students
                                                                    Staff \
      Country
                                            LSE
      England
                                                    1895
                                                            11850 1725.0
      Northern Ireland Queen's University Belfast
                                                   1810
                                                            18438 2414.0
                           University of Glasgow
      Scotland
                                                    1451
                                                            30805 2942.0
      Wales
                              Cardiff University
                                                    1883
                                                            25898 3330.0
                        Admin Budget Russell
      Country
      England
                       2515.0
                               415.1
      Northern Ireland 1489.0
                               369.2
                                           1
      Scotland
                       4003.0
                               626.5
                                           1
      Wales
                       5739.0
                               644.8
                                           1
```

We can create custom aggregation routines by calling agg() or aggregate() on the grouped object. To illustrate, we count the number of universities in each country that have more than 20,000 students:

Note that we called agg() only on the column Students, otherwise the function would be applied to every column separately, which is not what we want.

The most flexible aggregation method is apply() which calls a given function, passing the entire group-specific subset of data (including all columns) as an argument, and glues together the results.

For example, if we want to compute the average budget per student (in pounds), we can do this as follows:

```
[32]: # Budget is in millions of pounds, rescale by 1.0e6 groups.apply(lambda x: x['Budget'].sum() / x['Students'].sum() * 1.0e6)
```

We couldn't have done this with agg(), since agg() never gets to see the entire chunk of data but only one column at a time.

This section provided only a first look at pandas's "split-apply-combine" functionality implemented via groupby. See the official documentation for more details.

### 1.6 Visualisation

We covered plotting with Matplotlib in earlier units. Pandas itself implements some convenience wrappers around Matplotlib plotting routines which allow us to quickly inspect data stored in DataFrames. Alternatively, we can extract the numerical data and pass it to Matplotlib's routines manually.

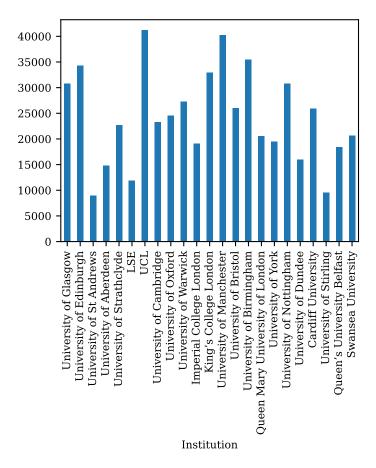
For example, to plot student numbers as a bar chart, we can directly use pandas:

```
[33]: import pandas as pd

df = pd.read_csv(f'{DATA_PATH}/universities.csv', sep=';')

# set institution as label so they automatically show up in plot
df2 = df.set_index(keys=['Institution'])
df2['Students'].plot(kind='bar') # same as df2['Students'].plot.bar()
```

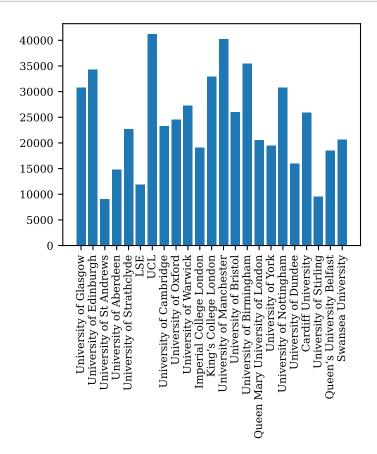
[33]: <AxesSubplot:xlabel='Institution'>



Alternatively, we can construct the graph using Matplotlib ourselves:

```
[34]: import matplotlib.pyplot as plt

labels = df['Institution'].to_list()  # labels as list
values = df['Students'].to_numpy()  # data as NumPy array
plt.bar(labels, values)
plt.tick_params(axis='x', labelrotation=90)
```



Sometimes Matplotlib's routines directly work with pandas's data structures, sometimes they don't. In cases where they don't, we can convert a DataFrame or Series object to a NumPy array using the to\_numpy() method, and convert a Series to a Python list using to\_list(), as illustrated in the example above.

To plot timeseries-like data, we can use the plot() method, which optionally accepts arguments to specify which columns should be used for the x-axis and which for the y-axis:

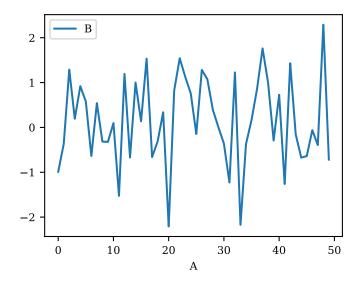
```
[35]: import numpy as np
import pandas as pd

# Instantiate RNG
rng = np.random.default_rng(123)

# Create pandas DataFrame
nobs = 50
df = pd.DataFrame({'A': np.arange(nobs), 'B': rng.normal(size=nobs)})

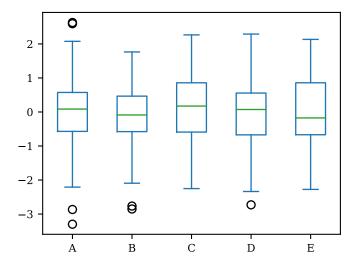
df.plot(x='A', y='B') # plot A on x-axis, B on y-axis
```

[35]: <AxesSubplot:xlabel='A'>



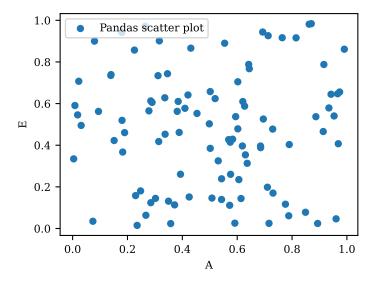
To quickly generate some descriptive statistics, we can use the built-in box plot:

# [36]: <AxesSubplot:>



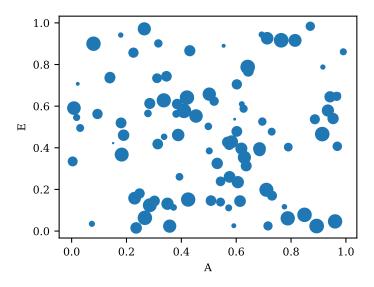
Similarly, we can generate scatter plots, plotting one column against another:

# [37]: <AxesSubplot:xlabel='A', ylabel='E'>



```
[38]: # We can even use a column to specify the dot size!
df.plot.scatter(x='A', y='E', s=df['B']*100.0)
```

[38]: <AxesSubplot:xlabel='A', ylabel='E'>



In general, the wrappers implemented in pandas are useful to get an idea how the data looks like. For reusable code or more complex graphs, we'll usually want to directly use Matplotlib and pass the data converted to NumPy arrays.

# 1.7 Optional exercises

The following exercises use data files from the data/ folder.

# 1.7.1 Exercise 1: Basic data manipulations

In this exercise, we will perform some basic data manipulation and plot the results.

- 1. Load the CSV file FRED\_QTR.csv (using sep=','). Set the columns Year and Quarter as (joint) indices.
  - *Hint:* You can do this by specifying these column names in the index\_col argument of read\_csv(). Alternatively, you can cell set\_index() once you have loaded the data.
- 2. This data comes at a quarterly frequency. Convert it to annual values by computing the average values for each year.
  - *Hint:* Group the data by Year using the groupby() function and compute the mean on the grouped data.
- 3. Compute two new variables from the annualised data and add them to the DataFrame:
  - Inflation, defined as the growth rate of CPI (consumer price index)
  - GDP\_growth, defined as the growth rate of GDP
- 4. Drop all rows with missing values (these show up as NaN).
  - *Hint:* There is no need to manually filter out NaN values, you can use the dropna() method instead.
- 5. Plot the columns GDP\_growth, Inflation, UNRATE (unemployment rate) and LFPART (labour force participation) using the pandas plotting routines. Use the option subplots=True and layout=(2,2) to create a  $2 \times 2$  grid. See the documentation for plot() for details.

# 1.7.2 Exercise 2: Decade averages

Load the FRED data from the CSV file FRED\_QTR.csv (using sep=',') and perform the following tasks:

- 1. Compute the quarterly GDP growth rate and inflation, similar to what you did in the previous exercise
- 2. Add the column Decade which contains the decade for every observation. Use 1940 to code the 40s, 1950 for the 50s, etc.
- 3. We want to retain only observations for decades for which all 40 quarters are present:
  - 1. Group the data by Decade and count the number of observations using count().
  - 2. A decade should be kept in the data set only if *all* variables have the full 40 observations.
  - 3. Drop all observations for which this is not the case.
- 4. With the remaining observations, compute the decade averages for quarterly GDP growth, inflation and the unemployment rate (UNRATE). Annualise the GDP growth and inflation figures by multiplying them by 4.
- 5. Create a bar chart that plots these three variables by decade.

#### 1.7.3 Exercise 3: Group averages

Load the universities data from the CSV file universities.csv (using sep=';') and perform the following tasks:

- 1. Group the data by Russell Group membership using the indicator variable Russell. For each group, compute the averages of the following ratios using apply():
  - The ratio of academic staff (Staff) to students (Students)
  - The ratio of administrative staff (Admin) to students.
  - The budget (Budget) per student in pounds.

Additionally, compute the number of universities is each group.

- 2. Repeat the task using a different approach:
  - 1. Compute the above ratios and add them as new columns to the initial DataFrame.
  - 2. Group the data by Russell Group membership.
  - 3. Compute the mean of each ratio using mean().

- 4. Compute the number of universities in each group using count(), and store the result in the column Count in the DataFrame you obtained in the previous step.
- 3. Create a bar chart, plotting the value for universities in and outside of the Russell Group for each of the four statistics computed above.

# 1.7.4 Exercise 4: Grouping by multiple dimensions

Load the universities data from the CSV file universities.csv (using sep=';') and perform the following tasks:

- 1. Create an indicator Pre1800 which is True for universities founded before the year 1800.
- 2. Group the data by Country and the value of Pre1800.

*Hint:* You need to pass a list of column names to groupby().

- 3. Compute the number of universities for each combination of (Country, Pre1800).
- 4. Create a bar chart showing the number of pre- and post-1800 universities by country (i.e., create four groups of bars, each group showing one bar for pre- and one for post-1800).
- 5. Create a bar chart showing the number of universities by country by pre- and post-1800 period (i.e., create two groups of bars, each group showing four bars, one for each country.)

#### 1.7.5 Exercise 5: Okun's law (advanced)

In this exercise, we will estimate Okun's law on quarterly data for each of the last eight decades.

Okun's law relates unemployment to the output gap. One version (see Jones: Macroeconomics, 2019) is stated as follows:

$$u_t - \overline{u}_t = \alpha + \beta \left( \frac{Y_t - \overline{Y}_t}{\overline{Y}_t} \right)$$

where  $u_t$  is the unemployment rate,  $\overline{u}_t$  is the natural rate of unemployment,  $Y_t$  is output (GDP) and  $\overline{Y}_t$  is potential output. We will refer to  $u_t - \overline{u}_t$  as "cyclical unemployment" and to the term in parenthesis on the right-hand side as the "output gap." Okun's law says that the coefficient  $\beta$  is negative, i.e., cyclical unemployment is higher when the output gap is low (negative) because the economy is in a recession.

Load the FRED data from the CSV file FRED\_QTR.csv (using sep=',') and perform the following tasks:

- 1. Compute the output gap and cyclical unemployment rate as defined above and add them as columns to the DataFrame.
- 2. Assign each observation to a decade as you did in previous exercises.
- 3. Write a function regress\_okun() which accepts a DataFrame containing a decade-spefic subsample as the only argument, and estimates the coefficients  $\alpha$  (the intercept) and  $\beta$  (the slope) of the above regression equation.

This function should return a DataFrame of a single row and two columns which store the intercept and slope.

*Hint:* Use NumPy's lstsq() to perform the regression. To regress the dependent variable y on regressors X, you need to call lstsq(X, y). To include the intercept, you will manually have to create X such that the first column contains only ones.

- 4. Group the data by decade and call the apply() method, passing regress\_okun you wrote as the argument.
- 5. Plot your results: for each decade, create a scatter plot of the raw data and overlay it with the regression line you estimated.

# 1.8 Solutions

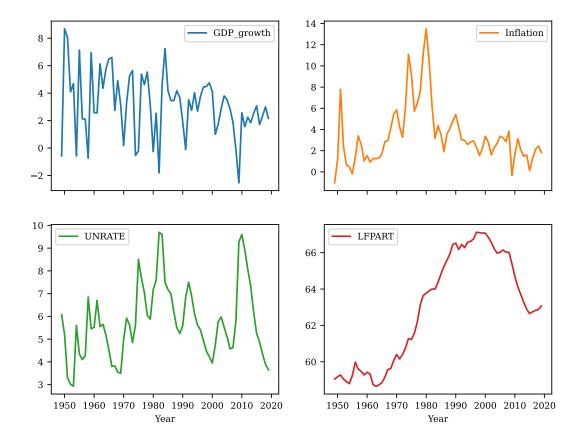
These solutions illustrate *one* possible way to solve the exercises. Pandas is extremely flexible (maybe too flexible) and allows us to perform these tasks in many different ways, so your implementation might look very different.

The solutions are also provided as Python scripts in the lectures/solutions/unit07/ folder.

#### 1.8.1 Solution for exercise 1

One possible implementation looks as follows:

```
[39]: import pandas as pd
       # Use either local or remote path to data/ directory
       # DATA_PATH = '../data'
      DATA_PATH = 'https://raw.githubusercontent.com/richardfoltyn/MLFP-ECON5130/main/data'
      filepath = f'{DATA_PATH}/FRED_QTR.csv'
      df = pd.read_csv(filepath, sep=',', index_col=['Year', 'Quarter'])
      # Alternatively, set index columns later
      # df = pd.read_csv(filepath, sep=',')
      # df.set_index(keys=['Year', 'Quarter'], inplace=True)
      # Convert to annual frequency
      # Group by year
      grp = df.groupby(['Year'])
       # Compute annual data as mean of quarterly values
      df_year = grp.mean()
      # Alternative ways to perform the same aggregation:
       # df_year = grp.agg('mean')
       # df_year = grp.agg(np.mean)
       # Compute CPI and GDP growth rates (in percent)
      df_year['Inflation'] = df_year['CPI'].diff() / df_year['CPI'].shift() * 100.0
      df_year['GDP_growth'] = df_year['GDP'].diff() / df_year['GDP'].shift() * 100.0
       # Drop all rows that contain any NaNs
      df_year = df_year.dropna(axis=0)
      # Columns to plot
      varnames = ['GDP_growth', 'Inflation', 'UNRATE', 'LFPART']
      df year.plot.line(y=varnames, subplots=True, layout=(2, 2),
                         sharex=True, figsize=(8, 6))
      # Alternatively, we can call plot() directly, which
      # defaults to generating a line plot:
      # df_year.plot(y=varnames, subplots=True, layout=(2, 2),
                      sharex=True, figsize=(10, 10))
```



#### A few comments:

1. We can set the index column when loading a CSV file by passing the column names as index\_col:

```
df = pd.read_csv(filepath, sep=',', index_col=['Year', 'Quarter'])
```

Alternatively, we can first load the CSV file and set the index later:

```
df = pd.read_csv(filepath, sep=',')
df.set_index(keys=['Year', 'Quarter'], inplace=True)
```

- 2. There are several ways to compute the means of grouped data:
  - 1. We can call mean() on the group object directly:

```
df_year = grp.mean()
```

2. Alternatively, we can call agg() and pass it the aggregation routine that should be applied:

```
df_year = grp.agg('mean')
df_year = grp.agg(np.mean)
```

Here we again have multiple options: pandas understands 'mean' if passed as a string (which might not be the case for some other functions), or we pass an actual function such as np.mean.

3. The easiest way to compute differences between adjacent rows is to use the diff() method, which returns  $x_t - x_{t-1}$ . Pandas then automatically matches the correct values and sets the first observation to NaN as there is no preceding value to compute the difference.

To compute a growth rate  $(x_t - x_{t-1})/x_{t-1}$ , we additionally need to lag a variable to get the correct period in the denominator. In pandas this is achieved using the shift() method (which defaults to shifting by 1 period).

#### 1.8.2 Solution for exercise 2

This time we do not specify index\_cols when reading in the CSV data since we need Year as a regular variable, not as the index.

We then compute the decade for each year, using the fact that // performs division with integer truncation. As an example, 1951 // 10 is 195, and (1951 // 10) \* 10 = 1950, which we use to represent the 1950s.

```
[40]: import pandas as pd
       # Use either local or remote path to data/ directory
       # DATA PATH = '../data'
       DATA_PATH = 'https://raw.githubusercontent.com/richardfoltyn/MLFP-ECON5130/main/data'
       filepath = f'{DATA_PATH}/FRED_QTR.csv'
       df = pd.read_csv(filepath, sep=',')
       # Compute GDP growth rates, inflation (in percent)
       df['GDP_growth'] = df['GDP'].diff() / df['GDP'].shift() * 100.0
       df['Inflation'] = df['CPI'].diff() / df['CPI'].shift() * 100.0
       # Assign decade using // to truncate division to
       # integer part. So we have 194x // 10 = 194 for any x.
       df['Decade'] = (df['Year'] // 10) * 10
       grp = df.groupby(['Decade'])
       # Print number of obs. by decade
       print(grp.count())
       # Create series that contains True for each
       # decade if all variables have 40 observations.
       use_decade = (grp.count() == 40).all(axis=1)
       # Convert series to DataFrame, assign column name 'Keep'
       df_decade = use_decade.to_frame('Keep')
       # merge into original DataFrame, matching rows on value
       # of column 'Decade'
       df = df.merge(df_decade, on='Decade')
       # Restrict data only to rows which are part of complete decade
       df = df.loc[df['Keep'], :].copy()
       # Drop 'Keep' column
       del df['Keep']
       # Compute average growth rates and unemployment rate by decade
       grp = df.groupby(['Decade'])
       df_avg = grp[['GDP_growth', 'Inflation', 'UNRATE']].mean()
# Convert to (approximate) annualised growth rates
       df_avg['GDP_growth'] *= 4.0
       df_avg['Inflation'] *= 4.0
```

	Year	Quarter	GDP	CPI	UNRATE	LFPART	GDPPOT	NROU	GDP_growth	\
Decade										
1940	8	8	8	8	8	8	4	4	7	
1950	40	40	40	40	40	40	40	40	40	
1960	40	40	40	40	40	40	40	40	40	
1970	40	40	40	40	40	40	40	40	40	
1980	40	40	40	40	40	40	40	40	40	
1990	40	40	40	40	40	40	40	40	40	
2000	40	40	40	40	40	40	40	40	40	
2010	40	40	40	40	40	40	40	40	40	

Inflation

Decade	
1940	7
1950	40
1960	40
1970	40
1980	40
1990	40
2000	40
2010	40

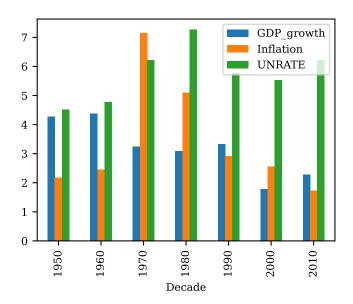
The tricky part is to keep only observations for "complete" decades that have 40 quarters of data. We see that this is not the case for the 1940s:

- 1. We group by Decade and use count() to determine the number of non-missing observations for each variable.
- 2. count() == 40 evaluates to True for some variable if it has 40 observations.
- 3. We then use all() to aggregate across all variables, i.e., we require 40 observations for every variable to keep the decade.
- 4. Finally, we merge the indicator whether a decade should be kept in the data set using merge(), where we match on the value of the column Decade. Note that the argument to merge() must be a DataFrame, so we first have to convert our indicator data.
- 5. Finally, we keep only those observations which have a flag that is True.

The rest of the exercise is straightforward as it just repeats what we have done previously. You can create the bar chart directly with pandas as follows:

```
[41]: df_avg.plot.bar(y=['GDP_growth', 'Inflation', 'UNRATE'])
```

# [41]: <AxesSubplot:xlabel='Decade'>



# 1.8.3 Solution for exercise 3

We first read in the CSV file, specifying ';' as the field separator:

```
[42]: import pandas as pd

# Use either local or remote path to data/ directory
# DATA_PATH = '../data'
DATA_PATH = 'https://raw.githubusercontent.com/richardfoltyn/MLFP-ECON5130/main/data'
```

```
# Load CSV file
filepath = f'{DATA_PATH}/universities.csv'
df = pd.read_csv(filepath, sep=';')
```

For the first task we use apply() to create a new Series object for each ratio of interest.

We compute the ratios for each institution which will result in NaNs if either the numerator of denominator is missing. We thus use np.nanmean() to compute averages, ignoring any NaNs.

Finally, we combine all Series into a DataFrame. We do this by specifying the data passed to DataFrame() as a dictionary, since then we can specify the column names as keys.

```
[43]: # Variant 1
       # Compute means using apply()
       grp = df.groupby(['Russell'])
       # Create Series objects with the desired means
       staff = grp.apply(lambda x: np.nanmean(x['Staff'] / x['Students']))
       admin = grp.apply(lambda x: np.nanmean(x['Admin'] / x['Students']))
       # Budget in millions of pounds
       budget = grp.apply(lambda x: np.nanmean(x['Budget'] / x['Students']))
       # Convert to pounds
       budget *= 1.0e6
       # Count number of institutions in each group.
       # We can accomplish this by calling size() on the group object.
       count = grp.size()
       # Create a new DataFrame. Each column is a Series object.
       df all = pd.DataFrame({'Staff Student': staff,
                               'Admin_Student': admin,
                              'Budget_Student': budget,
                              'Count': count})
       df_all
```

```
[43]: Staff_Student Admin_Student Budget_Student Count Russell 0 0.096219 0.147762 16847.834366 6 1 0.155131 0.169079 35406.453649 17
```

For the second task, we first insert additional columns which contain the ratios of interest for each university.

We then drop all unused columns, group by the Russell indicator and compute the means by directly calling mean() on the group object.

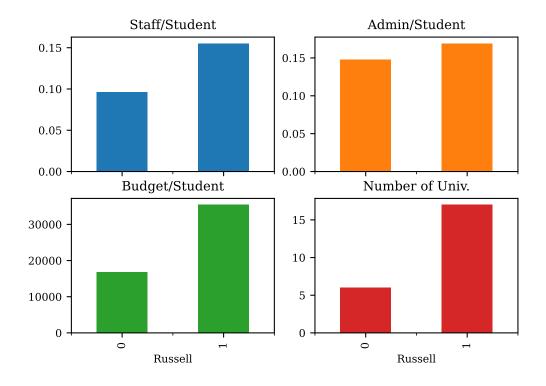
```
# Aggregate by Russell indicator
grp = df.groupby(['Russell'])
# Count number of institutions in each group.
# We can accomplish this by calling size() on the group object.
count = grp.size()

df_all = grp.mean()
# Add counter
df_all['Count'] = count

df_all
```

```
[44]: Staff_Student Admin_Student Budget_Student Count Russell
0 0.096219 0.147762 16847.834366 6
1 0.155131 0.169079 35406.453649 17
```

We plot the results using pandas's bar() function. Since the data is of vastly different magnitudes, we specify sharey=False so that each panel will have its own scaling on the y-axis.



#### 1.8.4 Solution for exercise 4

We create an indicator variable called Pre1800 which is set to True whenever the founding year in column Founded is lower than 1800.

We then group the data by Country and Pre1800 and count the number of universities in each group using count().

```
[46]: import pandas as pd
       # Use either local or remote path to data/ directory
       # DATA PATH = '../data'
       DATA_PATH = 'https://raw.githubusercontent.com/richardfoltyn/MLFP-ECON5130/main/data'
       # Load CSV file
       filepath = f'{DATA_PATH}/universities.csv'
       df = pd.read_csv(filepath, sep=';')
       # Create mask for founding period
       df['Pre1800'] = (df['Founded'] < 1800)</pre>
       # Create group by country and founding period;
       grp = df.groupby(['Country', 'Pre1800'])
       # Number of universities by country and founding period.
       # Since we are grouping by two attributes, this will create a
       # Series with a multi-level (hierarchical) index
       count = grp.size()
       count
```

```
[46]: Country
                        Pre1800
      England
                        False
                                   8
                        True
                                   5
      Northern Ireland False
                                   1
      Scotland
                        False
                                   3
                        True
                                   4
      Wales
                        False
      dtype: int64
```

The resulting Series only contains values for those combinations that are actually present in the data. For example, the combination (Wales, True) does not show up because there are no Welsh universities founded before 1800 in our sample. We will have to "complete" the data and add zero entries in all such cases.

First, we create a DataFrame with countries in rows and the number of universities for the pre- and post-1800 periods in columns. To accomplish this, we need to pivot the second row index using the unstack() method. The level=-1 argument tells it to use the last row index, and fill\_value=0 will assign zeros to all elements that were not present in the initial DataFrame, such as the combination (Wales, True).

```
[47]: # DataFrame with countries in rows, Pre-1800 indicator in columns

# Pivot inner index level to create separate columns for True/False
# values of Pre1800 indicator
df_count = count.unstack(level=-1, fill_value=0)

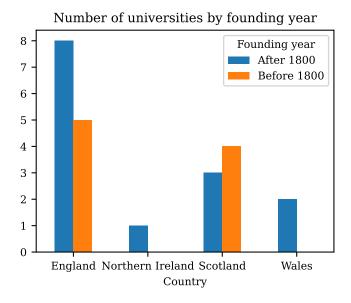
# Set name of column index to something pretty: this will
# be used as the legend title
df_count.columns.rename('Founding year', inplace=True)
# Rename columns to get pretty labels in legend
df_count.rename(columns={True: 'Before 1800', False: 'After 1800'},
```

```
inplace=True)
df_count
```

Whenever we use pandas's built-in plotting functions, these use index names and labels to automatically label the graph. We therefore first have to assign these objects "pretty" names.

We can then generate the bar chart as follows:

```
[48]: # Create bar chart by country
title = 'Number of universities by founding year'
# pass rot=0 to undo the rotation of x-tick labels
# which pandas applies by default
df_count.plot.bar(xlabel='Country', rot=0, title=title)
```



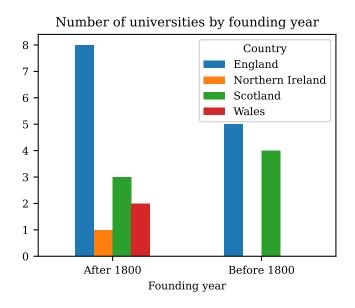
Note how the legend title is automatically set to the column index name and the legend labels use the column index labels.

We create the second DataFrame with the founding period in rows and country names in columns in exactly the same way, but now call unstack(level=0) so that the first index level will be pivoted.

```
[49]: Country England Northern Ireland Scotland Wales Founding year
After 1800 8 1 3 2
Before 1800 5 0 4 0
```

```
[50]: # Create bar chart by founding year
# pass rot=0 to undo the rotation of x-tick labels
# which pandas applies by default
df_count.plot.bar(rot=0, title=title)
```

[50]: <AxesSubplot:title={'center':'Number of universities by founding year'},
 xlabel='Founding year'>



#### 1.8.5 Solution for exercise 5

This exercise is quite involved, so we will discuss it in parts. First, we write the function that will be called by apply() to process sub-sets of the data which belong to a single decade:

```
[51]: def regress_okun(x):
           # x is a DataFrame, restricted to rows for the current decade
           # Extract dependent and regressor variables
           outcome = x['unempl_gap'].to_numpy()
           GDP_gap = x['GDP_gap'].to_numpy()
           # Regressor matrix including intercept
           regr = np.ones((len(GDP_gap), 2))
           # overwrite second column with output gap
           regr[:,1] = GDP_gap
           # Solve least-squares problem (pass rcond=None to avoid a warning)
           coefs, *rest = np.linalg.lstsq(regr, outcome, rcond=None)
           # Construct DataFrame which will be returned to apply()
           # Convert data to 1 x 2 matrix
           data = coefs[None]
           columns = ['Const', 'GDP_gap']
           df_out = pd.DataFrame(data, columns=columns)
           return df_out
```

This function is passed in a single argument which is a DataFrame restricted to the sub-sample that is currently being processed.

- Our task is to perform the required calculations and to return the result as a DataFrame. apply() then glues together all decade-specific DataFrames to form the result of the operation.
- We first extract the relevant variables as NumPy arrays, and we create a regressor matrix which has ones in the first column. This column represents the intercept.
- We invoke lstsq() to run the regression. lstsq() returns several arguments which we mop up in the tuple \*rest since we are only interested in the regression coefficients.
  - Note that we wouldn't be using lstsq() to run OLS on a regular basis, but it's sufficient for this use case.
- Finally, we build the DataFrame to be returned by this function. It has only one row (since we ran only one regression) and two columns, one for each regression coefficient.

This was the hard part. We now need to perform some standard manipulations to prepare the data:

- 1. We construct the output gap (in percent), which we store in the column GDP\_gap.
- 2. We construct the cyclical unemployment rate and store it in the column unempl\_gap.
- 3. We determine the decade each observation belongs to using the same code as in previous exercises.
- 4. We then drop all unused variables from the DataFrame and also all observations which contain missing values.

Lastly, we can call apply() to run the regression for each decade.

```
[52]: import pandas as pd
       import numpy as np
       import matplotlib.pyplot as plt
       # Use either local or remote path to data/ directory
       # DATA PATH = '../data'
      DATA_PATH = 'https://raw.githubusercontent.com/richardfoltyn/MLFP-ECON5130/main/data'
      # Load CSV file
      filepath = f'{DATA_PATH}/FRED_QTR.csv'
      df = pd.read_csv(filepath, sep=',')
      # Generate output gap (in percent)
      df['GDP gap'] = (df['GDP'] - df['GDPPOT']) / df['GDPPOT'] * 100.0
      # Generate deviations of unempl. rate from natural unempl. rate
      df['unempl_gap'] = df['UNRATE'] - df['NROU']
      # Assign decade using // to truncate division to
       # integer part. So we have 194x // 10 = 194 for any x.
      df['Decade'] = (df['Year'] // 10) * 10
      # Keep only variables of interest
      df = df[['Decade', 'GDP_gap', 'unempl_gap']]
       # Drop rows with any missing obs.
      df = df.dropna(axis=0)
      # Group by decade
      grp = df.groupby(['Decade'])
      # Apply regression routine to sub-set of data for each decade
      df_reg = grp.apply(regress_okun)
      # Get rid of second row index introduced by apply()
      df reg = df reg.reset index(level=-1, drop=True)
       # Display intercept and slope coefficients
```

```
# estimated for each decade.
df_reg
```

```
[52]:
                 Const
                         GDP_gap
      Decade
      1940 -0.259986 -0.567257
      1950
            -0.277104 -0.494637
            -0.331665 -0.467206
      1960
            -0.032063 -0.398751
      1970
      1980
             -0.178001 -0.666688
      1990
             -0.102465 -0.489427
      2000
             -0.355138 -0.723567
      2010
            -0.279333 -0.983768
```

The following code creates 8 panels of scatter plots showing the raw data and overlays a regression line for each decade.

```
[53]: # Number of plots (= number of decades)
       Nplots = len(df_reg)
       # Fix number of columns, determine rows as needed
       nrow = int(np.ceil(Nplots / ncol))
       fig, axes = plt.subplots(nrow, ncol, sharey=True, sharex=True,
                                figsize=(6, 11))
       for i, ax in enumerate(axes.flatten()):
           # decade in current iteration
           decade = df_reg.index.values[i]
           # restrict DataFrame to decade-specific data
           dfi = df.loc[df['Decade'] == decade]
           # Scatter plot of raw data
           ax.scatter(dfi['GDP_gap'], dfi['unempl_gap'], color='steelblue',
                      alpha=0.7, label='Raw data')
           # Extract regression coefficients
           const = df_reg.loc[decade, 'Const']
           slope = df_reg.loc[decade, 'GDP_gap']
           # plot regression line:
           # We need to provide one point and a slope to define the line to be plotted.
           ax.axline((0.0, const), slope=slope, color='red',
                     lw=2.0, label='Regression line')
           # Add label containing the current decade
           ax.text(0.95, 0.95, f"{decade}'s", transform=ax.transAxes,
                   va='top', ha='right')
           # Add legend in the first panel only
           if i == 0:
               ax.legend(loc='lower left', frameon=False)
           # Add x- and y-labels, but only for those panels
           # that are on the left/lower boundary of the figure
           if i >= nrow * (ncol - 1):
               ax.set_xlabel('Output gap (%)')
           if (i % ncol) == 0:
               ax.set_ylabel('Cycl. unempl. rate (%-points)')
       fig.suptitle("Okun's law")
```

[53]: Text(0.5, 0.98, "Okun's law")

