Unit 8: Handling data with pandas

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1 Handling data with pandas

1.1 Motivation

So far, we have encountered NumPy arrays as the only way to store numerical data (we mostly ignored the built-in containers provided directly in Python). However, while NumPy arrays are great for storing homogenous data without any particular structure, they are somewhat limited when we want to use them for high-level data analysis.

For example, we usually want to process data sets with

1. several variables;
2. multiple observations, which need not be identical across variables (some values may be missing);
3. non-homogenous data types: for examples, names need to be stored as strings, birthdays as dates and income as a floating-point number.

While NumPy can in principle handle such situations, it puts all the burden on the user. Most users would prefer to not have to deal with such low-level details.

Imagine we want to store names, birth dates and annual income for two people:

Name	Date of birth	Income
Alice	1985-01-01	30,000
Bob	1997-05-12	-

No income was reported for Bob, so it's missing. With NumPy, we could do this as follows:

[1]:
```
import numpy as np
from datetime import date

date1 = date(1985, 1, 1)            # birth date for Alice
date2 = date(1997, 5, 12)           # birth date for Bob

data = np.array([['Alice', date1, 30000.0],
                 ['Bob', date2, None]])

data
```

[1]:
```
array([['Alice', datetime.date(1985, 1, 1), 30000.0],
       ['Bob', datetime.date(1997, 5, 12), None]], dtype=object)
```

[2]:
```
data.dtype          # print array data type
```

[2]:
```
dtype('O')
```

While we can create such arrays, they are almost useless for data analysis, in particular since everything is stored as a generic `object`.

- To be fair, NumPy offers an alternative array type called "record" or "structured" array which can handle fields of different data types.

  However, the pandas library offers much more beyond that, so there is little reason to use structured arrays.

Pandas was created to offer more versatile data structures that are straightforward to use for storing, manipulating and analysing heterogeneous data:

1. Data is clearly organised in *variables* and *observations*, similar to econometrics programs such as Stata.
2. Each variable is permitted to have a different data type.
3. We can use *labels* to select observations, instead of having to use a linear numerical index as with NumPy.

   We could, for example, index a data set using National Insurance Numbers.
4. Pandas offers many convenient data aggregation and reduction routines that can be applied to subsets of data.

   For example, we can easily group observations by city and compute average incomes.
5. Pandas also offers many convenient data import / export functions that go beyond what's in NumPy.

Should we be using pandas at all times, then? No!

- For low-level tasks where performance is essential, use NumPy.
- For homogenous data without any particular data structure, use NumPy.
- On the other hand, if data is heterogeneous, needs to be imported from an external data source and cleaned or transformed before performing computations, use pandas.

There are numerous tutorials on pandas on the internet, so we will keep this unit short and illustrate only the main concepts. Useful references to additional material include:

- The official user guide.
- The official pandas cheat sheet which nicely illustrates the most frequently used operations.
- The official API reference with details on every pandas object and function.
- There are numerous tutorials (including videos) available on the internet. See here for a list.

1.2 Creating pandas data structures

Pandas has two main data structures:

1. `Series` represents observations of a single variable.
2. `DataFrame` is a container for several variables. You can think of each individual column of a `DataFrame` as a `Series`, and each row represents one observation.

The easiest way to create a `Series` or `DataFrame` is to create them from pre-existing data.

To access pandas data structures and routines, we need to import them first. The near-universal convention is to make pandas available using the name `pd`:

[3]:
```
import pandas as pd
```

Examples:

We can create a `DataFrame` from a NumPy array:

[4]:
```
import numpy as np
import pandas as pd                 # universal convention: import using pd
from numpy.random import default_rng

# Draw normally distributed data
rng = default_rng(123)
data = rng.normal(size=(10,3))

# Define variable (or column) names
varnames = ['A', 'B', 'C']

# Create pandas DataFrame
pd.DataFrame(data, columns=varnames)
```

[4]:
```
          A         B         C
0 -0.989121 -0.367787  1.287925
1  0.193974  0.920231  0.577104
2 -0.636464  0.541952 -0.316595
3 -0.322389  0.097167 -1.525930
4  1.192166 -0.671090  1.000269
5  0.136321  1.532033 -0.659969
6 -0.311795  0.337769 -2.207471
7  0.827921  1.541630  1.126807
8  0.754770 -0.145978  1.281902
9  1.074031  0.392621  0.005114
```

This code creates a `DataFrame` of three variables called `A`, `B` and `C` with 10 observations each.

Alternatively, we can create a `DataFrame` from non-homogenous data as follows:

[5]:
```
# Names (strings)
names = ['Alice', 'Bob']

# Birth dates (datetime objects)
bdates = pd.to_datetime(['1985-01-01', '1997-05-12'])

# Incomes (floats)
incomes = np.array([35000, np.nan])          # code missing income as NaN

# create DataFrame from dictionary
pd.DataFrame({'Name': names, 'Birthdate': bdates, 'Income': incomes})
```

[5]:
```
    Name  Birthdate   Income
0  Alice 1985-01-01  35000.0
1    Bob 1997-05-12      NaN
```

If data types differ across columns, as in the above example, it is often convenient to create the `DataFrame` by passing a dictionary as an argument. Each key represents a column name and each corresponding value contains the data for that variable.

1.3 Viewing data

With large data sets, you hardly ever want to print the entire `DataFrame`. Pandas by default limits the amount of data shown. You can use the `head()` and `tail()` methods to explicitly display a specific number of rows from the top or the end of a `DataFrame`.

To illustrate, we use a data set of 23 UK universities that contains the following variables:

- `Institution`: Name of the institution
- `Country`: Country/nation within the UK (England, Scotland, ...)
- `Founded`: Year in which university (or a predecessor institution) was founded
- `Students`: Total number of students
- `Staff`: Number of academic staff
- `Admin`: Number of administrative staff
- `Budget`: Budget in million pounds
- `Russell`: Binary indicator whether university is a member of the Russell Group, an association of the UK's top research universities.

The data was compiled based on information from Wikipedia.

We read in the data stored in the file `universities.csv` (from the `data/` folder) like this:

[6]:
```python
import pandas as pd

# relative path to CSV file
file = '../data/universities.csv'

# Load sample data set of UK universities
df = pd.read_csv(file, sep=';')
```

We can now display the first and last three rows:

[7]:
```python
df.head(3)       # show first three rows
```

[7]:
```
                  Institution   Country  Founded  Students   Staff   Admin  \
0      University of Glasgow  Scotland     1451     30805  2942.0  4003.0
1    University of Edinburgh  Scotland     1583     34275  4589.0  6107.0
2   University of St Andrews  Scotland     1413      8984  1137.0  1576.0

   Budget  Russell
0   626.5        1
1  1102.0        1
2   251.2        0
```

[8]:
```python
df.tail(3)       # show last three rows
```

[8]:
```
                     Institution           Country  Founded  Students   Staff  \
20       University of Stirling          Scotland     1967      9548     NaN
21  Queen's University Belfast  Northern Ireland     1810     18438  2414.0
22            Swansea University             Wales     1920     20620     NaN

     Admin  Budget  Russell
20  1872.0   113.3        0
21  1489.0   369.2        1
22  3290.0     NaN        0
```

To quickly compute some descriptive statistics for the *numerical* variables in the `DataFrame`, we use `describe()`:

[9]:
```
df.describe()
```

[9]:
```
           Founded      Students        Staff        Admin       Budget  \
count    23.000000     23.000000    20.000000    19.000000    22.000000
mean   1745.652174  24106.782609  3664.250000  3556.736842   768.609091
std     256.992149   9093.000735  2025.638038  1550.434342   608.234948
min    1096.000000   8984.000000  1086.000000  1489.000000   113.300000
25%    1589.000000  18776.500000  2294.250000  2193.500000   340.850000
50%    1826.000000  23247.000000  3307.500000  3485.000000   643.750000
75%    1941.500000  30801.500000  4439.750000  4347.500000  1023.500000
max    2004.000000  41180.000000  7913.000000  6199.000000  2450.000000

         Russell
count  23.000000
mean    0.739130
std     0.448978
min     0.000000
25%     0.500000
50%     1.000000
75%     1.000000
max     1.000000
```

Note that this automatically ignores the columns `Institution` and `Country` as they contain strings and computing the mean, etc. of a string variable does not make sense.

To see low-level information about the data type used in each column, we call `info()`:

[10]:
```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 23 entries, 0 to 22
Data columns (total 8 columns):
 #   Column       Non-Null Count  Dtype
---  ------       --------------  -----
 0   Institution  23 non-null     object
 1   Country      23 non-null     object
 2   Founded      23 non-null     int64
 3   Students     23 non-null     int64
 4   Staff        20 non-null     float64
 5   Admin        19 non-null     float64
 6   Budget       22 non-null     float64
 7   Russell      23 non-null     int64
dtypes: float64(3), int64(3), object(2)
memory usage: 1.6+ KB
```

Pandas automatically discards missing information in computations. For example, the number of academic staff is missing for several universities, so the number of *non-null* entries reported in the table above is less than 23, the overall sample size.

1.4 Indexing

Pandas supports two types of indexing:

1. Indexing by position. This is basically identical to the indexing of other Python and NumPy containers.
2. Indexing by label, i.e. by the values assigned to the row or column index. These labels need not be integers in increasing order, as is the case for NumPy.

We will see how to assign labels below.

Pandas indexing is performed either by using brackets `[]`, or by using `.loc[]` for label indexing, or `.iloc[]` for positional indexing.

Indexing via `[]` can be somewhat confusing:

- specifying `df['name']` returns the column `name` as a `Series` object.
- On the other hand, specifying a range such as `df[5:10]` returns the *rows* associated with the *positions* 5,...,9.

Examples:

[11]:
```
import pandas as pd

# Load sample data set of UK universities
df = pd.read_csv('../data/universities.csv', sep=';')
df['Institution']                # select a single column
```

[11]:
```
0                 University of Glasgow
1               University of Edinburgh
2              University of St Andrews
3               University of Aberdeen
4             University of Strathclyde
5                                   LSE
6                                   UCL
7               University of Cambridge
8                  University of Oxford
9                 University of Warwick
10              Imperial College London
11                King's College London
12             University of Manchester
13                University of Bristol
14             University of Birmingham
15      Queen Mary University of London
16                   University of York
17             University of Nottingham
18                 University of Dundee
19                   Cardiff University
20               University of Stirling
21           Queen's University Belfast
22                   Swansea University
Name: Institution, dtype: object
```

[12]:
```
df[['Institution', 'Students']]     # select multiple columns using a list
```

[12]:
```
                        Institution  Students
0             University of Glasgow     30805
1           University of Edinburgh     34275
2          University of St Andrews      8984
3            University of Aberdeen     14775
4         University of Strathclyde     22640
5                               LSE     11850
6                               UCL     41180
7           University of Cambridge     23247
8              University of Oxford     24515
9             University of Warwick     27278
10          Imperial College London     19115
11            King's College London     32895
12         University of Manchester     40250
13            University of Bristol     25955
14         University of Birmingham     35445
15  Queen Mary University of London     20560
16               University of York     19470
```

```
17         University of Nottingham     30798
18            University of Dundee     15915
19              Cardiff University     25898
20           University of Stirling      9548
21       Queen's University Belfast     18438
22              Swansea University     20620
```

To return the rows at positions 1, 2 and 3 we use

[13]:
```python
df[1:4]
```

[13]:
```
                 Institution   Country  Founded  Students   Staff   Admin  \
1   University of Edinburgh  Scotland     1583     34275  4589.0  6107.0
2  University of St Andrews  Scotland     1413      8984  1137.0  1576.0
3    University of Aberdeen  Scotland     1495     14775  1086.0  1489.0

   Budget  Russell
1  1102.0        1
2   251.2        0
3   219.5        0
```

Pandas follows the Python convention that indices are 0-based, and the endpoint of a slice is not included.

1.4.1 Manipulating indices

Pandas uses *labels* to index and align data. These can be integer values starting at 0 with increments of 1 for each additional element, which is the default, but they need not be. The two main methods to manipulate indices are:

- `set_index(keys=['column1', ...])`: uses the values of `column1` and optionally additional columns as indices, discarding the current index.
- `reset_index()`: resets the index to its default value, a sequence of increasing integers starting at 0.

Both methods return a new `DataFrame` and leave the original `DataFrame` unchanged. If we want to change the existing `DataFrame`, we need to pass the argument `inplace=True`.

For example, we can replace the row index and use the Roman lower-case characters `a`, `b`, `c`, ... as labels instead of integers:

[14]:
```python
import pandas as pd
df = pd.read_csv('../data/universities.csv', sep=';')

# Create list of lower-case letters which has same
# length as the number of observations.
index = [chr(97+i) for i in range(len(df))]      # len(df) returns number of obs.
index
```

[14]:
```
['a',
 'b',
 'c',
 'd',
 'e',
 'f',
 'g',
 'h',
 'i',
 'j',
 'k',
 'l',
 'm',
 'n',
```

```
 'o',
 'p',
 'q',
 'r',
 's',
 't',
 'u',
 'v',
 'w']
```

[15]:
```
df['index'] = index                                    # create new column 'index'
df.set_index(keys=['index'], inplace=True)             # set letters as index!

# print first 3 rows using labels
df['a':'c']                    # This is the same as df[:3]
```

[15]:
```
                    Institution   Country  Founded  Students   Staff   Admin  \
index
a        University of Glasgow  Scotland     1451     30805  2942.0  4003.0
b      University of Edinburgh  Scotland     1583     34275  4589.0  6107.0
c     University of St Andrews  Scotland     1413      8984  1137.0  1576.0

       Budget  Russell
index
a       626.5        1
b      1102.0        1
c       251.2        0
```

To add to the confusion, note that when specifying a range in terms of labels, the last element *is* included! Hence the row with index `c` in the above example is shown.

We can reset the index to its default integer values using the `reset_index()` method:

[16]:
```
# Reset index labels to default value (integers 0, 1, 2, ...)
df_new = df.reset_index(drop=True)
df_new.head(3)                 # print first 3 rows of new DataFrame
```

[16]:
```
                Institution   Country  Founded  Students   Staff   Admin  \
0     University of Glasgow  Scotland     1451     30805  2942.0  4003.0
1   University of Edinburgh  Scotland     1583     34275  4589.0  6107.0
2  University of St Andrews  Scotland     1413      8984  1137.0  1576.0

   Budget  Russell
0   626.5        1
1  1102.0        1
2   251.2        0
```

The `drop=True` argument tells pandas to throw away the old index values instead of storing them as a column of the resulting `DataFrame`.

1.4.2 Selecting elements

To more clearly distinguish between selection by label and by position, pandas provides the `.loc[]` and `.iloc[]` methods of indexing. To make your intention obvious, you should therefore adhere to the following rules:

1. Use `df['name']` only to select *columns* and nothing else.
2. Use `.loc[]` to select by label.
3. Use `.iloc[]` to select by position.

Selection by label

To illustrate, using `.loc[]` unambiguously indexes by label:

[17]:
```
df.loc['d':'f', ['Institution', 'Students']]
```

[17]:
```
                        Institution  Students
index
d          University of Aberdeen     14775
e       University of Strathclyde     22640
f                             LSE     11850
```

With `.loc[]` we can even perform slicing on column names, which is not possible with the simpler `df[]` syntax:

[18]:
```
df.loc['d':'f', 'Institution':'Founded']
```

[18]:
```
                        Institution   Country  Founded
index
d          University of Aberdeen  Scotland     1495
e       University of Strathclyde  Scotland     1964
f                             LSE   England     1895
```

This includes all the columns between `Institution` and `Founded`, where the latter is included since we are slicing by label.

Trying to pass in positional arguments will return an error for the given `DataFrame` since the index labels are a, b, c,... and not 0, 1, 2...

[19]:
```
df.loc[0:4]
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
<ipython-input-1-11cc54301474> in <module>
----> 1 df.loc[0:4]

~/.conda/envs/py3-default/lib/python3.8/site-packages/pandas/core/indexing.py in␣
 ↪__getitem__(self, key)
    877
    878                 maybe_callable = com.apply_if_callable(key, self.obj)
--> 879                 return self._getitem_axis(maybe_callable, axis=axis)
    880
    881     def _is_scalar_access(self, key: Tuple):

~/.conda/envs/py3-default/lib/python3.8/site-packages/pandas/core/indexing.py in␣
 ↪_getitem_axis(self, key, axis)
   1086         if isinstance(key, slice):
   1087             self._validate_key(key, axis)
-> 1088             return self._get_slice_axis(key, axis=axis)
   1089         elif com.is_bool_indexer(key):
   1090             return self._getbool_axis(key, axis=axis)

~/.conda/envs/py3-default/lib/python3.8/site-packages/pandas/core/indexing.py in␣
 ↪_get_slice_axis(self, slice_obj, axis)
   1120
   1121         labels = obj._get_axis(axis)
-> 1122         indexer = labels.slice_indexer(

   1123             slice_obj.start, slice_obj.stop, slice_obj.step, kind="loc"
   1124         )

~/.conda/envs/py3-default/lib/python3.8/site-packages/pandas/core/indexes/base.py␣
 ↪in slice_indexer(self, start, end, step, kind)
   4964         slice(1, 3, None)
   4965         """
-> 4966         start_slice, end_slice = self.slice_locs(start, end, step=step,␣
 ↪kind=kind)
```

```
   4967
   4968         # return a slice

~/.conda/envs/py3-default/lib/python3.8/site-packages/pandas/core/indexes/base.py
↪in slice_locs(self, start, end, step, kind)
   5165         start_slice = None
   5166         if start is not None:
-> 5167             start_slice = self.get_slice_bound(start, "left", kind)
   5168         if start_slice is None:
   5169             start_slice = 0

~/.conda/envs/py3-default/lib/python3.8/site-packages/pandas/core/indexes/base.py
↪in get_slice_bound(self, label, side, kind)
   5077         # For datetime indices label may be a string that has to be
↪converted
   5078         # to datetime boundary according to its resolution.
-> 5079         label = self._maybe_cast_slice_bound(label, side, kind)
   5080
   5081         # we need to look up the label

~/.conda/envs/py3-default/lib/python3.8/site-packages/pandas/core/indexes/base.py
↪in _maybe_cast_slice_bound(self, label, side, kind)
   5029         # this is rejected (generally .loc gets you here)
   5030         elif is_integer(label):
-> 5031             self._invalid_indexer("slice", label)
   5032
   5033         return label

~/.conda/envs/py3-default/lib/python3.8/site-packages/pandas/core/indexes/base.py
↪in _invalid_indexer(self, form, key)
   3265         Consistent invalid indexer message.
   3266         """
-> 3267         raise TypeError(
   3268             f"cannot do {form} indexing on {type(self).__name__} with
↪these "
   3269             f"indexers [{key}] of type {type(key).__name__}"

TypeError: cannot do slice indexing on Index with these indexers [0] of type int
```

However, we can reset the index to its default value. Then the index labels are integers and coincide with their position, so that `.loc[]` works:

[20]:
```
df.reset_index(inplace=True, drop=True)     # reset index labels to integers,
                                            # drop original index
df.loc[0:4]
```

[20]:
```
                   Institution   Country  Founded  Students   Staff   Admin  \
0       University of Glasgow  Scotland     1451     30805  2942.0  4003.0
1     University of Edinburgh  Scotland     1583     34275  4589.0  6107.0
2   University of St Andrews  Scotland     1413      8984  1137.0  1576.0
3     University of Aberdeen  Scotland     1495     14775  1086.0  1489.0
4  University of Strathclyde  Scotland     1964     22640     NaN  3200.0

   Budget  Russell
0   626.5        1
1  1102.0        1
2   251.2        0
3   219.5        0
4   304.4        0
```

Again, the end point with label `4` is included because we are selecting by label.

Somewhat surprisingly, we can also pass boolean arrays to `.loc[]` even though these are clearly not labels:

[21]:
```
df.loc[df['Country'] == 'Scotland']
```

[21]:
```
                    Institution   Country  Founded  Students   Staff   Admin  \
0        University of Glasgow  Scotland     1451     30805  2942.0  4003.0
1      University of Edinburgh  Scotland     1583     34275  4589.0  6107.0
2     University of St Andrews  Scotland     1413      8984  1137.0  1576.0
3       University of Aberdeen  Scotland     1495     14775  1086.0  1489.0
4   University of Strathclyde  Scotland     1964     22640     NaN  3200.0
18        University of Dundee  Scotland     1967     15915  1410.0  1805.0
20      University of Stirling  Scotland     1967      9548     NaN  1872.0

    Budget  Russell
0    626.5        1
1   1102.0        1
2    251.2        0
3    219.5        0
4    304.4        0
18   256.4        0
20   113.3        0
```

Indexing via `.loc[]` supports a few more types of arguments, see the official documentation for details.

Selection by position

Conversely, if we want to select items exclusively by their position and ignore their labels, we use `.iloc[]`:

[22]:
```
df.iloc[0:4, 0:2]           # select first 4 rows, first 2 columns
```

[22]:
```
                Institution   Country
0     University of Glasgow  Scotland
1   University of Edinburgh  Scotland
2  University of St Andrews  Scotland
3    University of Aberdeen  Scotland
```

Again, `.iloc[]` supports a multitude of other arguments, including boolean arrays. See the official documentation for details.

1.5 Aggregation and reduction

1.5.1 Working with entire DataFrames

The simplest way to perform data reduction is to invoke the desired routine on the entire `DataFrame`:

[23]:
```
import pandas as pd

df = pd.read_csv('../data/universities.csv', sep=';')
df.mean()
```

[23]:
```
Founded      1745.652174
Students    24106.782609
Staff        3664.250000
Admin        3556.736842
Budget        768.609091
Russell         0.739130
dtype: float64
```

Methods such as `mean()` are by default applied column-wise to each numerical column.

One big advantage over NumPy is that missing values (represented by `np.nan`) are automatically ignored:

```
[24]: # mean() automatically drops 3 missing observations
      df['Staff'].mean()
```

```
[24]: 3664.25
```

1.5.2 Splitting and grouping

Applying aggregation functions to the entire `DataFrame` is similar to what we can do with NumPy. The added flexibility of pandas becomes obvious once we want to apply these functions to subsets of data, i.e. groups, which we can define based on values or index labels.

For example, we can easily group our universities by country:

```
[25]: import pandas as pd

      df = pd.read_csv('../data/universities.csv', sep=';')

      groups = df.groupby(['Country'])
```

Here `groups` is a special pandas objects which can subsequently be used to process group-specific data. To compute the group-wise averages, we can simply run

```
[26]: groups.mean()
```

```
[26]:                      Founded      Students        Staff        Admin  \
      Country
      England          1745.923077  27119.846154  4336.692308  4112.000000
      Northern Ireland 1810.000000  18438.000000  2414.000000  1489.000000
      Scotland         1691.428571  19563.142857  2232.800000  2864.571429
      Wales            1901.500000  23259.000000  3330.000000  4514.500000

                            Budget   Russell
      Country
      England          1001.700000  1.000000
      Northern Ireland  369.200000  1.000000
      Scotland          410.471429  0.285714
      Wales             644.800000  0.500000
```

Groups support column indexing: if we want to only compute the total number of students for each country in our sample, we can do this as follows:

```
[27]: groups['Students'].sum()
```

```
[27]: Country
      England             352558
      Northern Ireland     18438
      Scotland            136942
      Wales                46518
      Name: Students, dtype: int64
```

There are numerous routines to aggregate grouped data, for example:

- `mean()`, `sum()`: averages and sums over numerical items within groups.
- `std()`, `var()`: within-group std. dev. and variances
- `size()`: group sizes
- `first()`, `last()`: first and last elements in each group
- `min()`, `max()`: minimum and maximum elements within a group

Examples:

[28]:
```
groups.size()       # return number of elements in each group
```

[28]:
```
Country
England             13
Northern Ireland     1
Scotland             7
Wales                2
dtype: int64
```

[29]:
```
groups.first()      # return first element in each group
```

[29]:
```
                                 Institution  Founded  Students   Staff  \
Country
England                                  LSE     1895     11850  1725.0
Northern Ireland  Queen’s University Belfast     1810     18438  2414.0
Scotland              University of Glasgow      1451     30805  2942.0
Wales                     Cardiff University     1883     25898  3330.0

                   Admin  Budget  Russell
Country
England           2515.0   415.1        1
Northern Ireland  1489.0   369.2        1
Scotland          4003.0   626.5        1
Wales             5739.0   644.8        1
```

We can create custom aggregation routines by calling `agg()` or `aggregate()` on the grouped object. To illustrate, we count the number of universities in each country that have more than 20,000 students:

[30]:
```
groups['Students'].agg(lambda x: np.sum(x >= 20000))
```

[30]:
```
Country
England             10
Northern Ireland     0
Scotland             3
Wales                2
Name: Students, dtype: int64
```

Note that we called `agg()` only on the column `Students`, otherwise the function would be applied to every column separately, which is not what we want.

The most flexible aggregation method is `apply()` which calls a given function, passing the entire group-specific subset of data (including all columns) as an argument, and glues together the results.

For example, if we want to compute the average budget per student (in pounds), we can do this as follows:

[31]:
```
# Budget is in millions of pounds, rescale by 1.0e6
groups.apply(lambda x: x['Budget'].sum() / x['Students'].sum() * 1.0e6)
```

[31]:
```
Country
England             36936.050239
Northern Ireland    20023.863760
Scotland            20981.875539
Wales               13861.301002
dtype: float64
```

We couldn’t have done this with `agg()`, since `agg()` never gets to see the entire chunk of data but only one column at a time.

This section provided only a first look at pandas’s “split-apply-combine” functionality implemented via `groupby`. See the official documentation for more details.

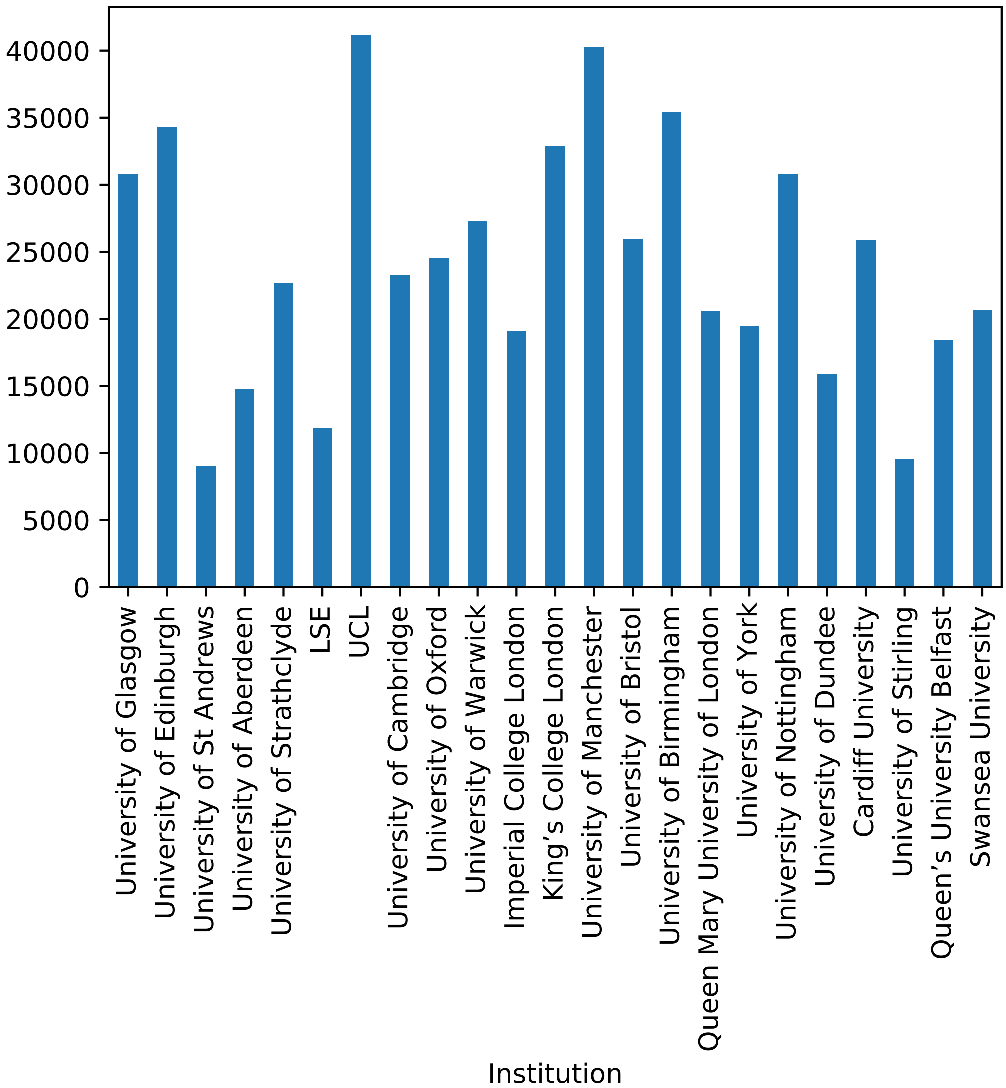
1.6 Visualisation

We covered plotting with Matplotlib in earlier units. Pandas itself implements some convenience wrappers around Matplotlib plotting routines which allow us to quickly inspect data stored in `DataFrames`. Alternatively, we can extract the numerical data and pass it to Matplotlib's routines manually.

For example, to plot student numbers as a bar chart, we can directly use pandas:

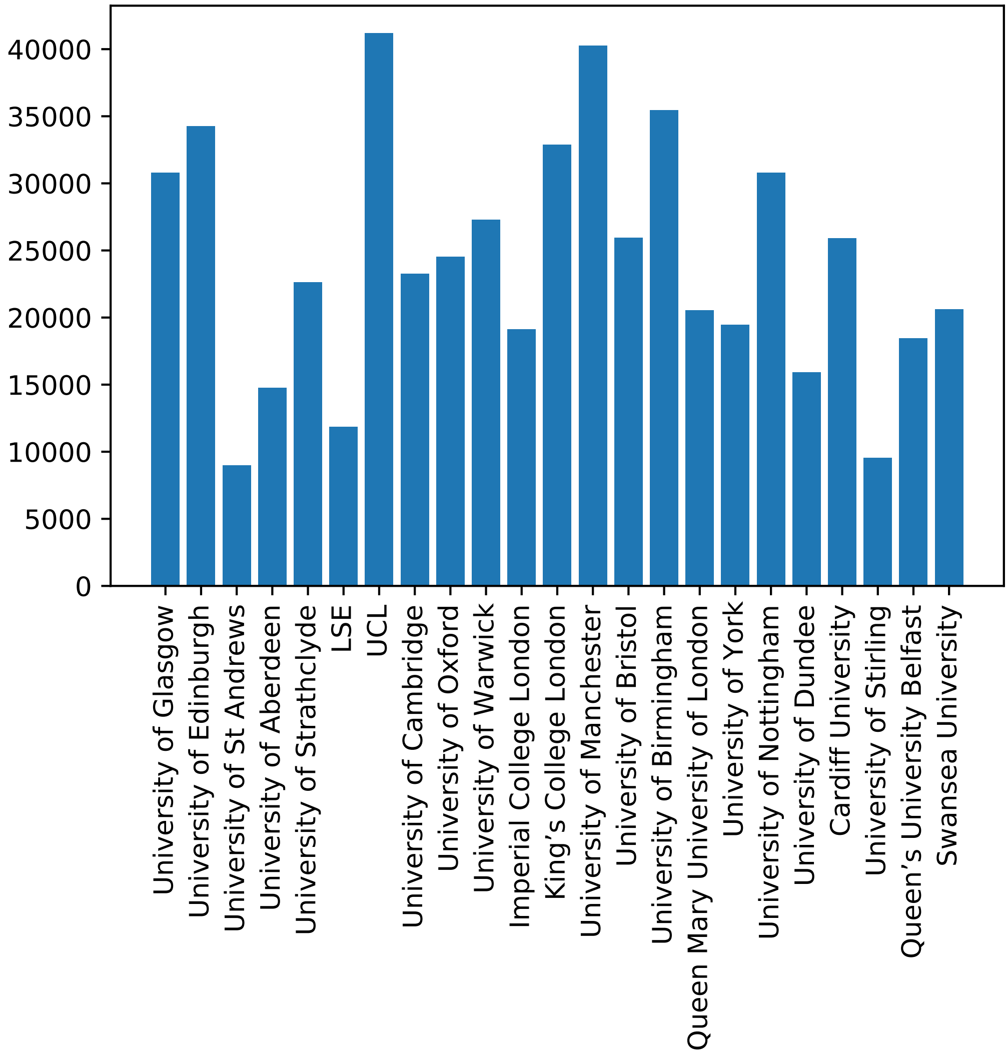
[32]:
```
import pandas as pd

df = pd.read_csv('../data/universities.csv', sep=';')

# set institution as label so they automatically show up in plot
df2 = df.set_index(keys=['Institution'])
df2['Students'].plot(kind='bar')         # same as df2['Students'].plot.bar()
```

[32]:
```
<AxesSubplot:xlabel='Institution'>
```



Alternatively, we can construct the graph using Matplotlib ourselves:

[33]:
```python
import matplotlib.pyplot as plt

labels = df['Institution'].to_list()        # labels as list
values = df['Students'].to_numpy()          # data as NumPy array
plt.bar(labels, values)
plt.tick_params(axis='x', labelrotation=90)
```



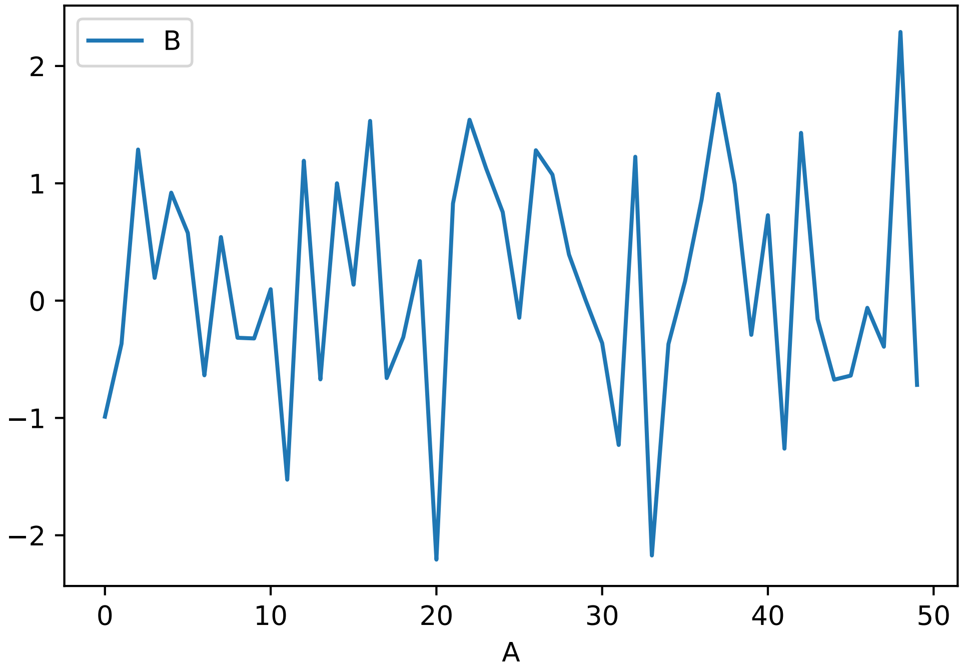
Sometimes Matplotlib's routines directly work with pandas's data structures, sometimes they don't. In cases where they don't, we can convert a `DataFrame` or `Series` object to a NumPy array using the `to_numpy()` method, and convert a `Series` to a Python list using `to_list()`, as illustrated in the example above.

To plot timeseries-like data, we can use the `plot()` method, which optionally accepts arguments to specify which columns should be used for the x-axis and which for the y-axis:

[34]:
```python
import numpy as np
import pandas as pd

# Instantiate RNG
rng = np.random.default_rng(123)

# Create pandas DataFrame
```

```
nobs = 50
df = pd.DataFrame({'A': np.arange(nobs), 'B': rng.normal(size=nobs)})

df.plot(x='A', y='B')        # plot A on x-axis, B on y-axis
```

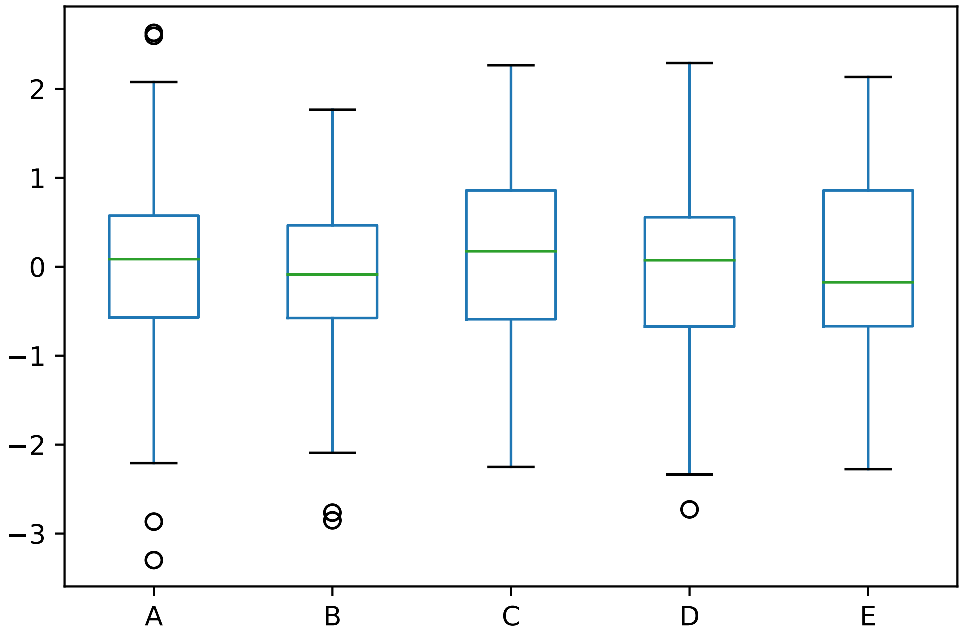
[34]: <AxesSubplot:xlabel='A'>



To quickly generate some descriptive statistics, we can use the built-in box plot:

[35]:
```
import numpy as np
import pandas as pd

# Instantiate RNG
rng = np.random.default_rng(123)

# Create pandas DataFrame
df = pd.DataFrame(rng.normal(size=(100, 5)),
                  columns=['A', 'B', 'C', 'D', 'E'])

df.plot.box()            # same as df.plot(kind='box')
```

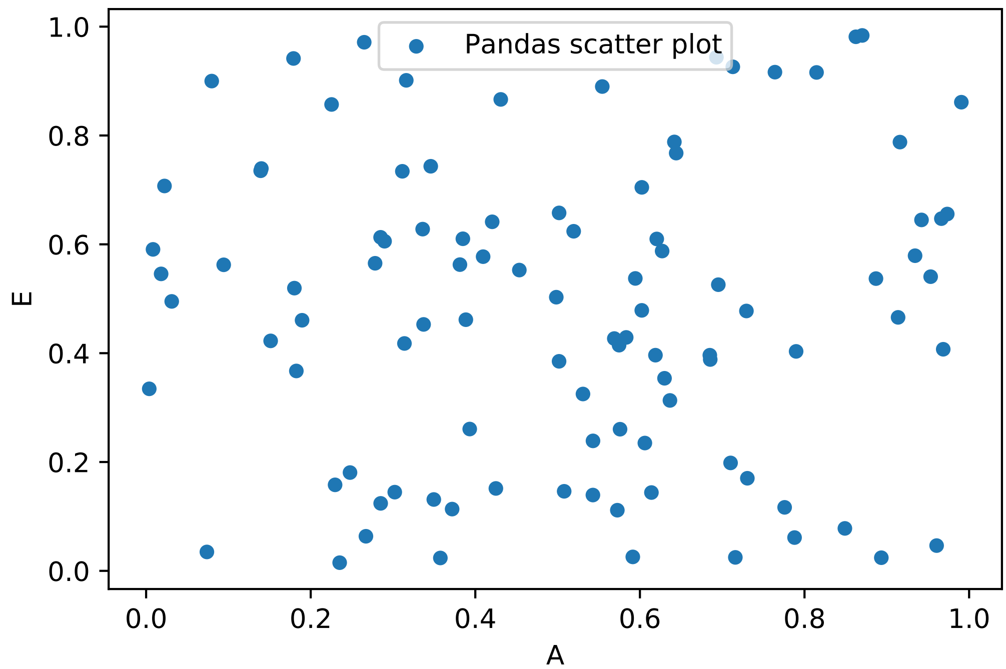
[35]: <AxesSubplot:>



Similarly, we can generate scatter plots, plotting one column against another:

[36]:
```
# Create pandas DataFrame
df = pd.DataFrame(rng.uniform(size=(100, 5)),
                  columns=['A', 'B', 'C', 'D', 'E'])

df.plot.scatter(x='A', y='E', label='Pandas scatter plot')
```

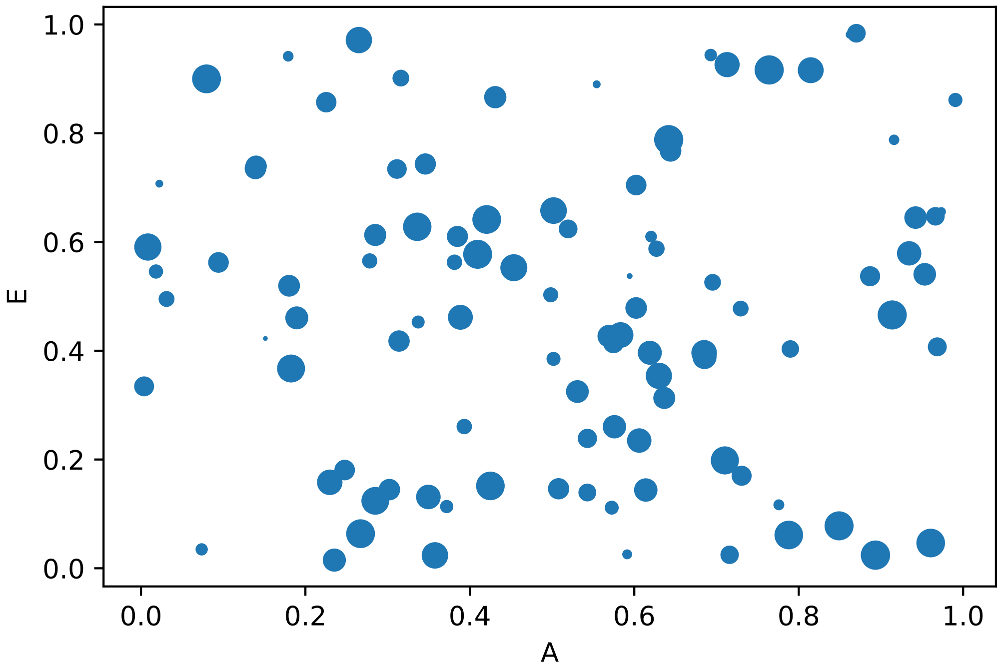
[36]: `<AxesSubplot:xlabel='A', ylabel='E'>`



[37]:

```
# We can even use a column to specify the dot size!
df.plot.scatter(x='A', y='E', s=df['B']*100.0)
```

[37]: <AxesSubplot:xlabel='A', ylabel='E'>



In general, the wrappers implemented in pandas are useful to get an idea how the data looks like. For reusable code or more complex graphs, we'll usually want to directly use Matplotlib and pass the data converted to NumPy arrays.

2 Exercises

The following exercises use data files from the `data/` folder.

2.1 Exercise 1: Basic data manipulations

In this exercise, we will perform some basic data manipulation and plot the results.

1. Load the CSV file `FRED_QTR.csv` (using `sep=','`). Set the columns `Year` and `Quarter` as (joint) indices.

   Hint: You can do this by specifying these column names in the `index_col` argument of `read_csv()`. Alternatively, you can cell `set_index()` once you have loaded the data.

2. This data comes at a quarterly frequency. Convert it to annual values by computing the average values for each year.

   Hint: Group the data by `Year` using the `groupby()` function and compute the mean on the grouped data.

3. Compute two new variables from the annualised data and add them to the `DataFrame`:

   - `Inflation`, defined as the growth rate of `CPI` (consumer price index)
   - `GDP_growth`, defined as the growth rate of `GDP`

4. Drop all rows with missing values (these show up as `NaN`).

   Hint: There is no need to manually filter out `NaN` values, you can use the `dropna()` method instead.

5. Plot the columns `GDP_growth`, `Inflation`, `UNRATE` (unemployment rate) and `LFPART` (labour force participation) using the pandas plotting routines. Use the option `subplots=True` and `layout=(2,2)` to create a 2×2 grid. See the documentation for `plot()` for details.

2.2 Exercise 2: Decade averages

Load the FRED data from the CSV file `FRED_QTR.csv` (using `sep=','`) and perform the following tasks:

1. Compute the quarterly GDP growth rate and inflation, similar to what you did in the previous exercise.
2. Add the column `Decade` which contains the decade for every observation. Use 1940 to code the 40s, 1950 for the 50s, etc.
3. We want to retain only observations for decades for which all 40 quarters are present:
   1. Group the data by `Decade` and count the number of observations using `count()`.
   2. A decade should be kept in the data set only if *all* variables have the full 40 observations.
   3. Drop all observations for which this is not the case.
4. With the remaining observations, compute the decade averages for quarterly GDP growth, inflation and the unemployment rate (`UNRATE`). Annualise the GDP growth and inflation figures by multiplying them by 4.
5. Create a bar chart that plots these three variables by decade.

2.3 Exercise 3: Group averages

Load the universities data from the CSV file `universities.csv` (using `sep=';'`) and perform the following tasks:

1. Group the data by Russell Group membership using the indicator variable `Russell`. For each group, compute the averages of the following ratios using `apply()`:

   - The ratio of academic staff (`Staff`) to students (`Students`)
   - The ratio of administrative staff (`Admin`) to students.
   - The budget (`Budget`) per student in pounds.

   Additionally, compute the number of universities is each group.

2. Repeat the task using a different approach:

   1. Compute the above ratios and add them as new columns to the initial `DataFrame`.
   2. Group the data by Russell Group membership.
   3. Compute the mean of each ratio using `mean()`.
   4. Compute the number of universities in each group using `count()`, and store the result in the column `Count` in the `DataFrame` you obtained in the previous step.

3. Create a bar chart, plotting the value for universities in and outside of the Russell Group for each of the four statistics computed above.

2.4 Exercise 4: Grouping by multiple dimensions

Load the universities data from the CSV file `universities.csv` (using `sep=';'`) and perform the following tasks:

1. Create an indicator `Pre1800` which is `True` for universities founded before the year 1800.

2. Group the data by `Country` and the value of `Pre1800`.

   Hint: You need to pass a list of column names to `groupby()`.

3. Compute the number of universities for each combination of `(Country, Pre1800)`.
4. Create a bar chart showing the number of pre- and post-1800 universities by country (i.e. create four groups of bars, each group showing one bar for pre- and one for post-1800).
5. Create a bar chart showing the number of universities by country by pre- and post-1800 period (i.e. create two groups of bars, each group showing four bars, one for each country.)

2.5 Exercise 5: Okun's law (advanced)

In this exercise, we will estimate Okun's law on quarterly data for each of the last eight decades.

Okun's law relates unemployment to the output gap. One version (see Jones: Macroeconomics, 2019) is stated as follows:

$$u_t - \overline{u}_t = \alpha + \beta \left(\frac{Y_t - \overline{Y}_t}{\overline{Y}_t} \right)$$

where u_t is the unemployment rate, $\overline{u}_t$ is the natural rate of unemployment, Y_t is output (GDP) and $\overline{Y}_t$ is potential output. We will refer to $u_t - \overline{u}_t$ as "cyclical unemployment" and to the term in parenthesis on the right-hand side as the "output gap." Okun's law says that the coefficient β is negative, i.e. cyclical unemployment is higher when the output gap is low (negative) because the economy is in a recession.

Load the FRED data from the CSV file `FRED_QTR.csv` (using `sep=','`) and perform the following tasks:

1. Compute the output gap and cyclical unemployment rate as defined above and add them as columns to the `DataFrame`.
2. Assign each observation to a decade as you did in previous exercises.
3. Write a function `regress_okun()` which accepts a `DataFrame` containing a decade-spefic sub-sample as the only argument, and estimates the coefficients α (the intercept) and β (the slope) of the above regression equation.

   This function should return a `DataFrame` of a single row and two columns which store the intercept and slope.

   Hint: Use NumPy's `lstsq()` to perform the regression. To regress the dependent variable `y` on regressors `X`, you need to call `lstsq(X, y)`. To include the intercept, you will manually have to create `X` such that the first column contains only ones.
4. Group the data by decade and call the `apply()` method, passing `regress_okun` you wrote as the argument.
5. Plot your results: for each decade, create a scatter plot of the raw data and overlay it with the regression line you estimated.

3 Solutions

These solutions illustrate *one* possible way to solve the exercises. Pandas is extremely flexible (maybe too flexible) and allows us to perform these tasks in many different ways, so your implementation might look very different.

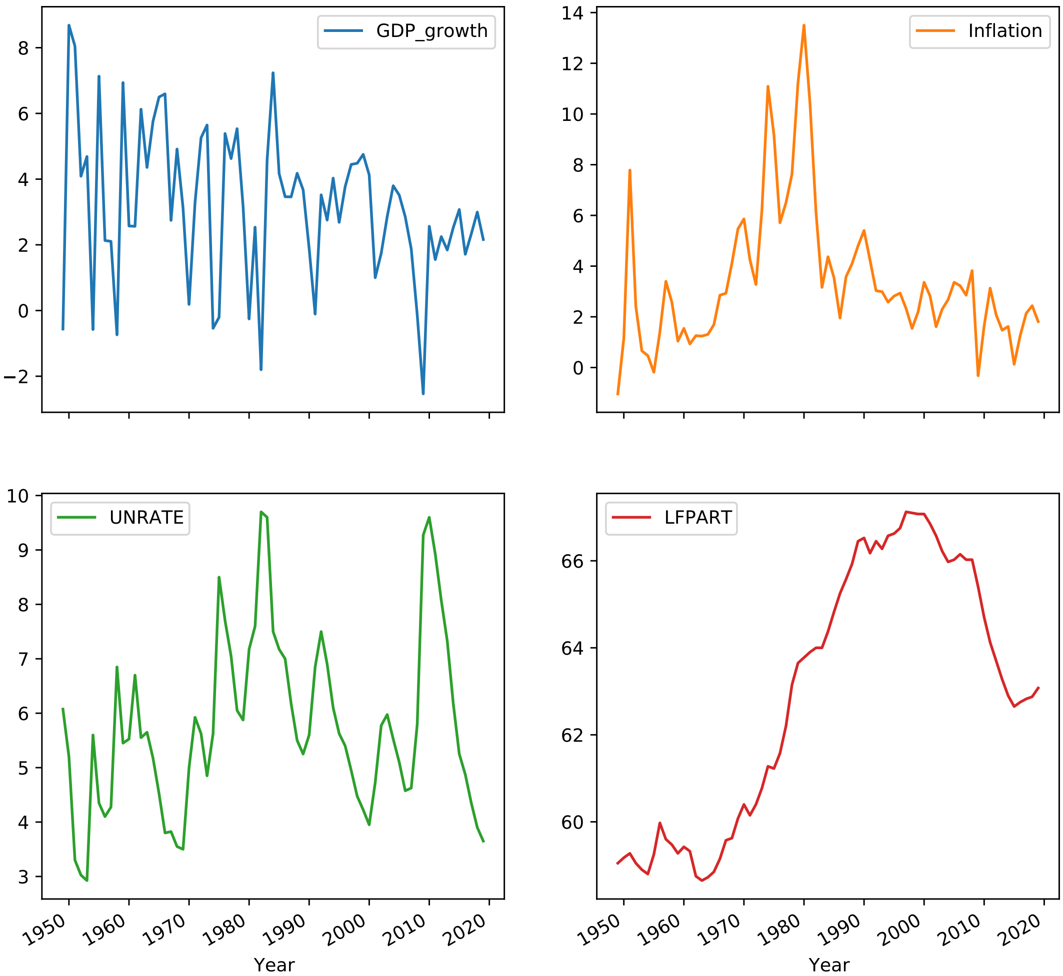
3.1 Solution for exercise 1

One possible implementation looks as follows:

[38]:
```
import pandas as pd
import numpy as np
```

```python
filepath = '../data/FRED_QTR.csv'

df = pd.read_csv(filepath, sep=',', index_col=['Year', 'Quarter'])
# Alternatively, set index columns later
# df = pd.read_csv(filepath, sep=',')
# df.set_index(keys=['Year', 'Quarter'], inplace=True)

# Convert to annual frequency
# Group by year
grp = df.groupby(['Year'])
# Compute annual data as mean of quarterly values
df_year = grp.mean()

# Alternative ways to perform the same aggregation:
# df_year = grp.agg('mean')
# df_year = grp.agg(np.mean)

# Compute CPI and GDP growth rates (in percent)
df_year['Inflation'] = df_year['CPI'].diff() / df_year['CPI'].shift() * 100.0
df_year['GDP_growth'] = df_year['GDP'].diff() / df_year['GDP'].shift() * 100.0

# Drop all rows that contain any NaNs
df_year = df_year.dropna(axis=0)

# Columns to plot
varnames = ['GDP_growth', 'Inflation', 'UNRATE', 'LFPART']
df_year.plot.line(y=varnames, subplots=True, layout=(2, 2),
                  sharex=True, figsize=(10, 10))

# Alternatively, we can call plot() directly, which
# defaults to generating a line plot:
#
# df_year.plot(y=varnames, subplots=True, layout=(2, 2),
#              sharex=True, figsize=(10, 10))
```

```
[38]: array([[<AxesSubplot:xlabel='Year'>, <AxesSubplot:xlabel='Year'>],
              [<AxesSubplot:xlabel='Year'>, <AxesSubplot:xlabel='Year'>]],
             dtype=object)
```



A few comments:

1. We can set the index column when loading a CSV file by passing the column names as `index_col`:

   ```
   df = pd.read_csv(filepath, sep=',', index_col=['Year', 'Quarter'])
   ```

   Alternatively, we can first load the CSV file and set the index later:

   ```
   df = pd.read_csv(filepath, sep=',')
   df.set_index(keys=['Year', 'Quarter'], inplace=True)
   ```

2. There are several ways to compute the means of grouped data:

   1. We can call `mean()` on the group object directly:

      ```
      df_year = grp.mean()
      ```

   2. Alternatively, we can call `agg()` and pass it the aggregation routine that should be applied:

      ```
      df_year = grp.agg('mean')
      df_year = grp.agg(np.mean)
      ```

      Here we again have multiple options: pandas understands `'mean'` if passed as a string (which might not be the case for some other functions), or we pass an actual function such as `np.mean`.

3. The easiest way to compute differences between adjacent rows is to use the `diff()` method, which returns $x_t - x_{t-1}$. Pandas then automatically matches the correct values and sets the first observation to `NaN` as there is no preceding value to compute the difference.

To compute a growth rate $(x_t - x_{t-1})/x_{t-1}$, we additionally need to lag a variable to get the correct period in the denominator. In pandas this is achieved using the `shift()` method (which defaults to shifting by 1 period).

3.2 Solution for exercise 2

This time we do not specify `index_cols` when reading in the CSV data since we need `Year` as a regular variable, not as the index.

We then compute the decade for each year, using the fact that // performs division with integer truncation. As an example, 1951 // 10 is 195, and (1951 // 10) * 10 = 1950, which we use to represent the 1950s.

[39]:
```
import pandas as pd

filepath = '../data/FRED_QTR.csv'

df = pd.read_csv(filepath, sep=',')

# Compute GDP growth rates, inflation (in percent)
df['GDP_growth'] = df['GDP'].diff() / df['GDP'].shift() * 100.0
df['Inflation'] = df['CPI'].diff() / df['CPI'].shift() * 100.0

# Assign decade using // to truncate division to
# integer part. So we have 194x // 10 = 194 for any x.
df['Decade'] = (df['Year'] // 10) * 10

grp = df.groupby(['Decade'])

# Print number of obs. by decade
print(grp.count())

# Create series that contains True for each
# decade if all variables have 40 observations.
use_decade = (grp.count() == 40).all(axis=1)
# Convert series to DataFrame, assign column name 'Keep'
df_decade = use_decade.to_frame('Keep')
# merge into original DataFrame, matching rows on value
# of column 'Decade'
df = df.merge(df_decade, on='Decade')
# Restrict data only to rows which are part of complete decade
df = df.loc[df['Keep'], :].copy()
# Drop 'Keep' column
del df['Keep']

# Compute average growth rates and unemployment rate by decade
grp = df.groupby(['Decade'])

df_avg = grp[['GDP_growth', 'Inflation', 'UNRATE']].mean()
# Convert to (approximate) annualised growth rates
df_avg['GDP_growth'] *= 4.0
df_avg['Inflation'] *= 4.0
```

```
        Year  Quarter  GDP  CPI  UNRATE  LFPART  GDPPOT  NROU  GDP_growth  \
Decade
1940       8        8    8    8       8       8       4     4           7
1950      40       40   40   40      40      40      40    40          40
1960      40       40   40   40      40      40      40    40          40
1970      40       40   40   40      40      40      40    40          40
1980      40       40   40   40      40      40      40    40          40
1990      40       40   40   40      40      40      40    40          40
2000      40       40   40   40      40      40      40    40          40
2010      40       40   40   40      40      40      40    40          40
```

```
        Inflation
Decade
1940            7
1950           40
1960           40
1970           40
1980           40
1990           40
2000           40
2010           40
```

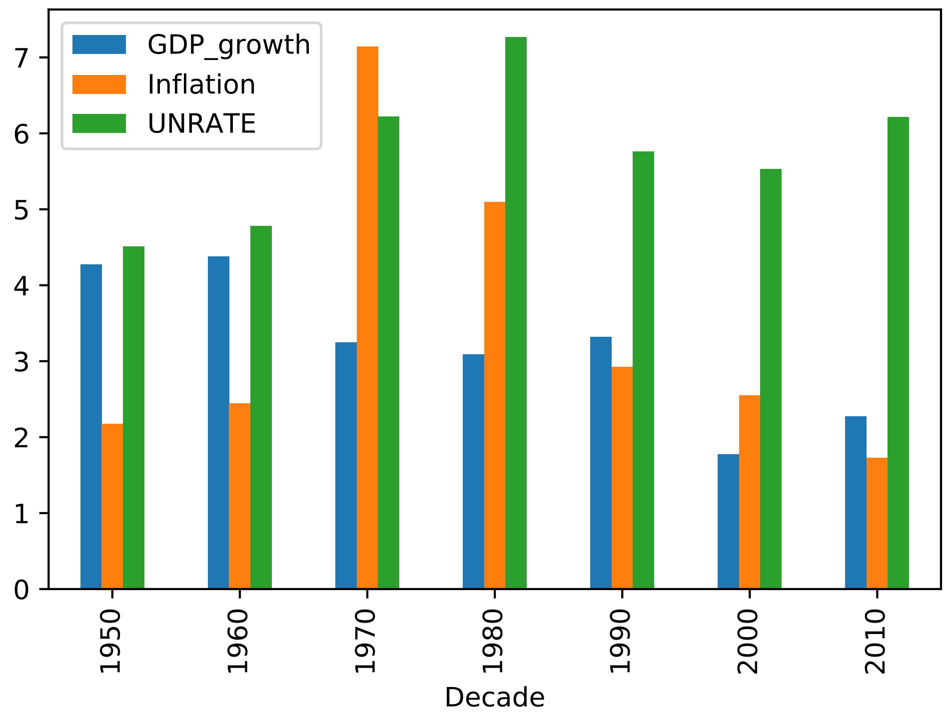
The tricky part is to keep only observations for "complete" decades that have 40 quarters of data. We see that this is not the case for the 1940s:

1. We group by `Decade` and use `count()` to determine the number of non-missing observations for each variable.
2. `count() == 40` evaluates to `True` for some variable if it has 40 observations.
3. We then use `all()` to aggregate across all variables, i.e. we require 40 observations for every variable to keep the decade.
4. Finally, we merge the indicator whether a decade should be kept in the data set using `merge()`, where we match on the value of the column `Decade`. Note that the argument to `merge()` must be a `DataFrame`, so we first have to convert our indicator data.
5. Finally, we keep only those observations which have a flag that is `True`.

The rest of the exercise is straightforward as it just repeats what we have done previously. You can create the bar chart directly with pandas as follows:

```
[40]: df_avg.plot.bar(y=['GDP_growth', 'Inflation', 'UNRATE'])
```

```
[40]: <AxesSubplot:xlabel='Decade'>
```



3.3 Solution for exercise 3

We first read in the CSV file, specifying `';'` as the field separator:

[41]:
```
import pandas as pd
import numpy as np

# Load CSV file
filepath = '../data/universities.csv'
df = pd.read_csv(filepath, sep=';')
```

For the first task we use `apply()` to create a new `Series` object for each ratio of interest.

We compute the ratios for each institution which will result in NaNs if either the numerator of denominator is missing. We thus use `np.nanmean()` to compute averages, ignoring any NaNs.

Finally, we combine all `Series` into a `DataFrame`. We do this by specifying the data passed to `DataFrame()` as a dictionary, since then we can specify the column names as keys.

[42]:
```
# Variant 1
# Compute means using apply()

grp = df.groupby(['Russell'])

# Create Series objects with the desired means
staff = grp.apply(lambda x: np.nanmean(x['Staff'] / x['Students']))
admin = grp.apply(lambda x: np.nanmean(x['Admin'] / x['Students']))
# Budget in millions of pounds
budget = grp.apply(lambda x: np.nanmean(x['Budget'] / x['Students']))
# Convert to pounds
budget *= 1.0e6
# Count number of institutions in each group.
# We can use the 'Russell' column for this because it has no
# missing data.
count = grp['Russell'].count()

# Create a new DataFrame. Each column is a Series object.
df_all = pd.DataFrame({'Staff_Student': staff,
                       'Admin_Student': admin,
                       'Budget_Student': budget,
                       'Count': count})

df_all
```

[42]:
```
         Staff_Student  Admin_Student  Budget_Student  Count
Russell
0             0.096219       0.147762    16847.834366      6
1             0.155131       0.169079    35406.453649     17
```

For the second task, we first insert additional columns which contain the ratios of interest for each university.

We then drop all unused columns, group by the `Russell` indicator and compute the means by directly calling `mean()` on the group object.

[43]:
```
# Variant 2:
# Compute ratios first, apply aggregation later

# Create new variables directly in original DataFrame
df['Staff_Student'] = df['Staff'] / df['Students']
df['Admin_Student'] = df['Admin'] / df['Students']
# Budget in pounds (original Budget is in million pounds)
df['Budget_Student'] = df['Budget'] / df['Students'] * 1.0e6
```

```python
# Keep only newly constructed ratios
columns_keep = [name for name in df.columns
                if name.endswith('_Student')]
# Also keep Russell indicator
columns_keep += ['Russell']
df = df[columns_keep].copy()

# Aggregate by Russell indicator
grp = df.groupby(['Russell'])
# Count number of institutions in each group.
# We can use the 'Russell' column for this because it has no
# missing data.
count = grp['Russell'].count()

df_all = grp.mean()
# Add counter
df_all['Count'] = count

df_all
```

[43]:

	Staff_Student	Admin_Student	Budget_Student	Count
Russell				
0	0.096219	0.147762	16847.834366	6
1	0.155131	0.169079	35406.453649	17

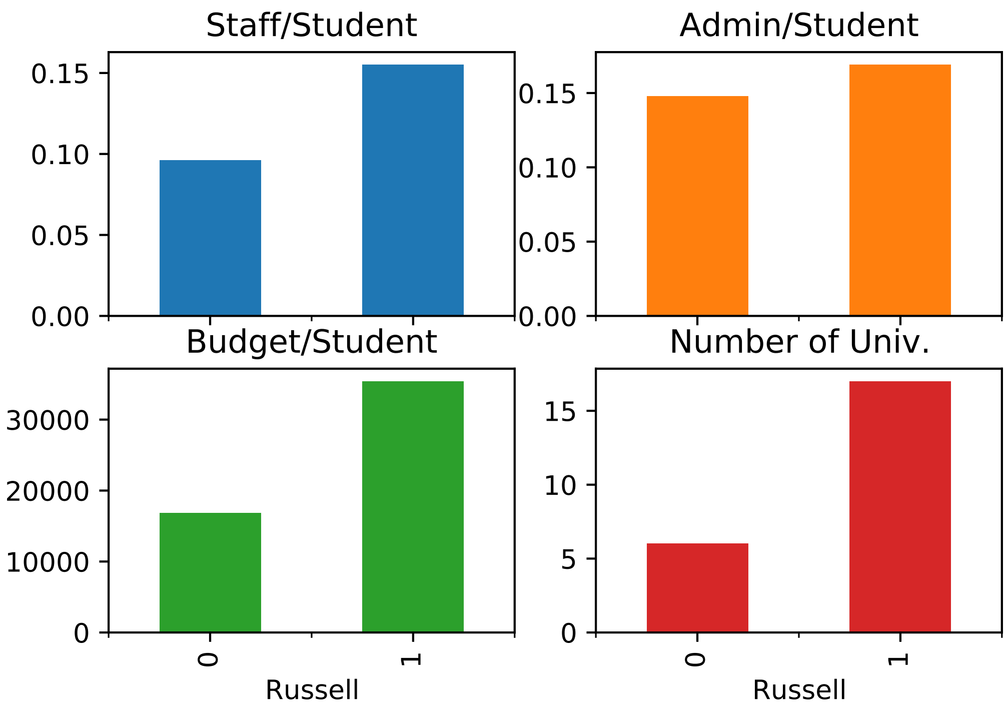
We plot the results using pandas's `bar()` function. Since the data is of vastly different magnitudes, we specify `sharey=False` so that each panel will have its own scaling on the y-axis.

[44]:

```python
# Plot results as bar charts, one panel for each variable

# Pretty titles
title = ['Staff/Student', 'Admin/Student', 'Budget/Student', 'Number of Univ.']
# Create bar chart using pandas's bar() function
df_all.plot.bar(sharey=False, subplots=True, layout=(2, 2), legend=False,
                title=title)
```

[44]:

```
array([[<AxesSubplot:title={'center':'Staff/Student'}, xlabel='Russell'>,
        <AxesSubplot:title={'center':'Admin/Student'}, xlabel='Russell'>],
       [<AxesSubplot:title={'center':'Budget/Student'}, xlabel='Russell'>,
        <AxesSubplot:title={'center':'Number of Univ.'}, xlabel='Russell'>]],
      dtype=object)
```



3.4 Solution for exercise 4

We create an indicator variable called `Pre1800` which is set to `True` whenever the founding year in column `Founded` is lower than 1800.

We then group the data by `Country` and `Pre1800` and count the number of universities in each group using `count()`.

[45]:
```python
import pandas as pd
import matplotlib.pyplot as plt

# Load CSV file
filepath = '../data/universities.csv'
df = pd.read_csv(filepath, sep=';')

# Create mask for founding period
df['Pre1800'] = (df['Founded'] < 1800)

# Create group by country and founding period;
grp = df.groupby(['Country', 'Pre1800'])

# Number of universities by country and founding period.
# Since we are grouping by two attributes, which will create a
# Series with a multi-level (hierarchical) index
count = grp['Institution'].count()

count
```

[45]:
```
Country           Pre1800
England           False      8
                  True       5
Northern Ireland  False      1
Scotland          False      3
```

```
                    True       4
Wales               False      2
Name: Institution, dtype: int64
```

The resulting `Series` only contains values for those combinations that are actually present in the data. For example, the combination `(Wales, True)` does not show up because there are no Welsh universities founded before 1800 in our sample. We will have to "complete" the data and add zero entries in all such cases.

First, we create a `DataFrame` with countries in rows and the number of universities for the pre- and post-1800 periods in columns. To accomplish this, we need to pivot the second row index using the `unstack()` method. The `level=-1` argument tells it to use the last row index, and `fill_value=0` will assign zeros to all elements that were not present in the initial `DataFrame`, such as the combination `(Wales, True)`.

[46]:
```
# DataFrame with countries in rows, Pre-1800 indicator in columns

# Pivot inner index level to create separate columns for True/False
# values of Pre1800 indicator
df_count = count.unstack(level=-1, fill_value=0)

# Set name of column index to something pretty: this will
# be used as the legend title
df_count.columns.rename('Founding year', inplace=True)
# Rename columns to get pretty labels in legend
df_count.rename(columns={True: 'Before 1800', False: 'After 1800'},
                inplace=True)

df_count
```

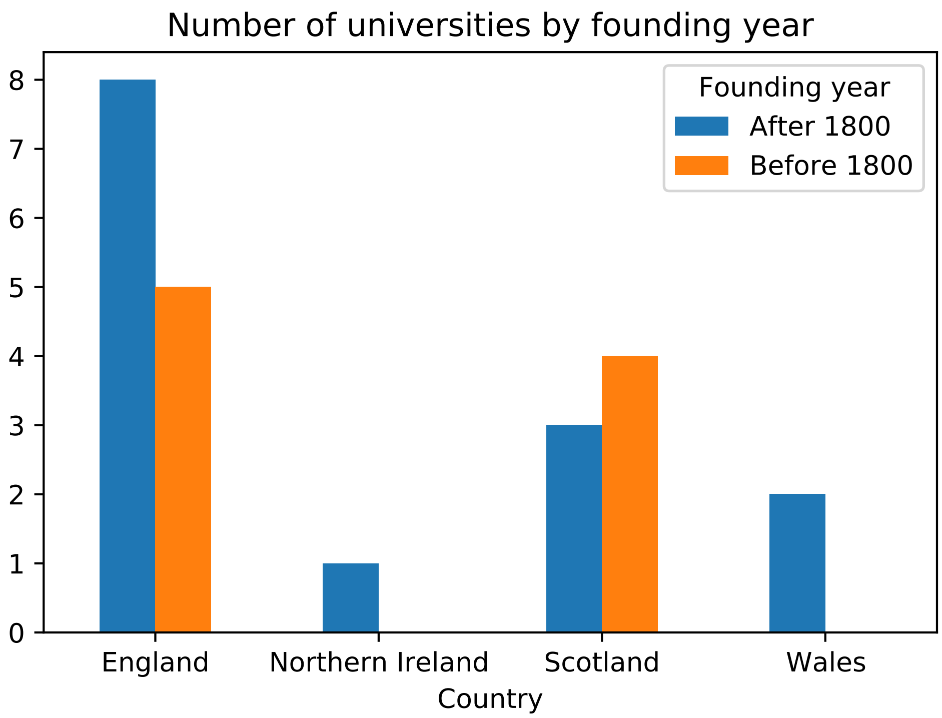
[46]:
```
Founding year     After 1800  Before 1800
Country
England                    8            5
Northern Ireland           1            0
Scotland                   3            4
Wales                      2            0
```

Whenever we use pandas's built-in plotting functions, these use index names and labels to automatically label the graph. We therefore first have to assign these objects "pretty" names.

We can then generate the bar chart as follows:

[47]:
```
# Create bar chart by country
title = 'Number of universities by founding year'
# pass rot=0 to undo the rotation of x-tick labels
# which pandas applies by default
df_count.plot.bar(xlabel='Country', rot=0, title=title)
```

[47]:
```
<AxesSubplot:title={'center':'Number of universities by founding year'},
xlabel='Country'>
```



Note how the legend title is automatically set to the column index name and the legend labels use the column index labels.

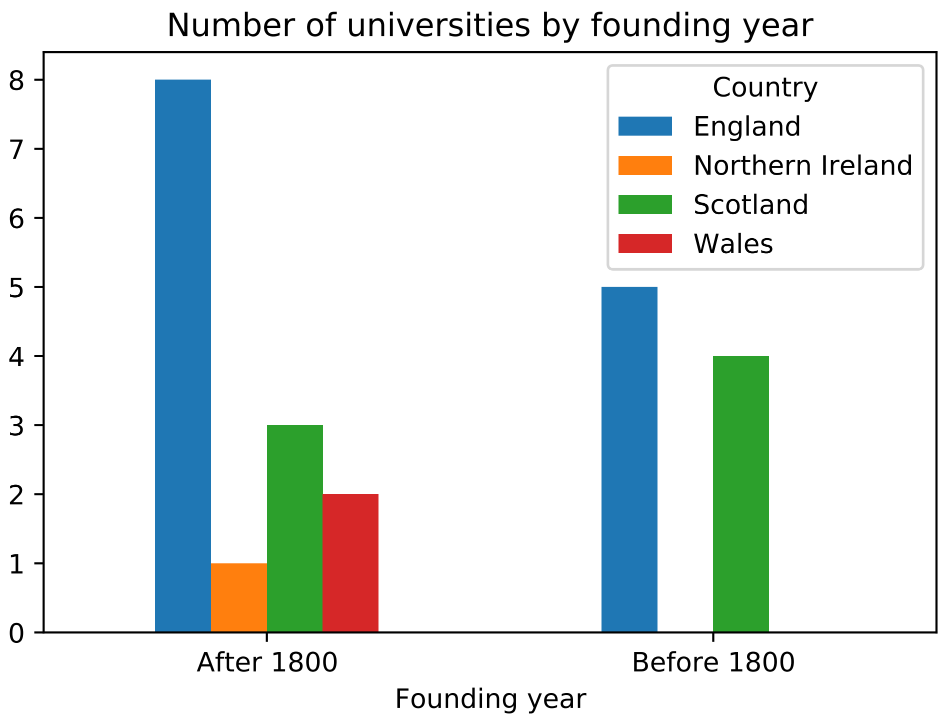
We create the second `DataFrame` with the founding period in rows and country names in columns in exactly the same way, but now call `unstack(level=0)` so that the first index level will be pivoted.

[48]:
```
# Pivot first row index level to create separate columns for each country
df_count = count.unstack(level=0, fill_value=0)

# Set index name to something pretty
df_count.index.rename('Founding year', inplace=True)
# Rename index labels to get pretty text in legend
df_count.rename(index={True: 'Before 1800', False: 'After 1800'},
                inplace=True)

df_count
```

[48]:
```
Country          England  Northern Ireland  Scotland  Wales
Founding year
After 1800             8                 1         3      2
Before 1800            5                 0         4      0
```

[49]:
```
# Create bar chart by founding year
# pass rot=0 to undo the rotation of x-tick labels
# which pandas applies by default
df_count.plot.bar(rot=0, title=title)
```

[49]:
```
<AxesSubplot:title={'center':'Number of universities by founding year'},
xlabel='Founding year'>
```



3.5 Solution for exercise 5

This exercise is quite involved, so we will discuss it in parts. First, we write the function that will be called by `apply()` to process sub-sets of the data which belong to a single decade:

[50]:
```
def regress_okun(x):
    # x is a DataFrame, restricted to rows for the current decade

    # Extract dependent and regressor variables
    outcome = x['unempl_gap'].to_numpy()
    GDP_gap = x['GDP_gap'].to_numpy()

    # Regressor matrix including intercept
    regr = np.ones((len(GDP_gap), 2))
    # overwrite second column with output gap
    regr[:,1] = GDP_gap

    # Solve least-squares problem (pass rcond=None to avoid a warning)
    coefs, *rest = np.linalg.lstsq(regr, outcome, rcond=None)

    # Construct DataFrame which will be returned to apply()
    # Convert data to 1 x 2 matrix
    data = coefs[None]
    columns = ['Const', 'GDP_gap']
    df_out = pd.DataFrame(data, columns=columns)

    return df_out
```

This function is passed in a single argument which is a `DataFrame` restricted to the sub-sample that is currently being processed.

- Our task is to perform the required calculations and to return the result as a `DataFrame`. `apply()` then glues together all decade-specific DataFrames to form the result of the operation.

- We first extract the relevant variables as NumPy arrays, and we create a regressor matrix which has ones in the first column. This column represents the intercept.
- We invoke `lstsq()` to run the regression. `lstsq()` returns several arguments which we mop up in the tuple `*rest` since we are only interested in the regression coefficients.

  Note that we wouldn't be using `lstsq()` to run OLS on a regular basis, but it's sufficient for this use case.
- Finally, we build the `DataFrame` to be returned by this function. It has only one row (since we ran only one regression) and two columns, one for each regression coefficient.

This was the hard part. We now need to perform some standard manipulations to prepare the data:

1. We construct the output gap (in percent), which we store in the column `GDP_gap`.
2. We construct the cyclical unemployment rate and store it in the column `unempl_gap`.
3. We determine the decade each observation belongs to using the same code as in previous exercises.
4. We then drop all unused variables from the `DataFrame` and also all observations which contain missing values.

Lastly, we can call `apply()` to run the regression for each decade.

[51]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# Load CSV file
filepath = '../data/FRED_QTR.csv'
df = pd.read_csv(filepath, sep=',')

# Generate output gap (in percent)
df['GDP_gap'] = (df['GDP'] - df['GDPPOT']) / df['GDPPOT'] * 100.0

# Generate deviations of unempl. rate from natural unempl. rate
df['unempl_gap'] = df['UNRATE'] - df['NROU']

# Assign decade using // to truncate division to
# integer part. So we have 194x // 10 = 194 for any x.
df['Decade'] = (df['Year'] // 10) * 10

# Keep only variables of interest
df = df[['Decade', 'GDP_gap', 'unempl_gap']]
# Drop rows with any missing obs.
df = df.dropna(axis=0)

# Group by decade
grp = df.groupby(['Decade'])

# Apply regression routine to sub-set of data for each decade
df_reg = grp.apply(regress_okun)
# Get rid of second row index introduced by apply()
df_reg = df_reg.reset_index(level=-1, drop=True)

# Display intercept and slope coefficients
# estimated for each decade.
df_reg
```

[51]:
```
           Const   GDP_gap
Decade
1940   -0.259986 -0.567257
1950   -0.277104 -0.494637
1960   -0.331665 -0.467206
1970   -0.032063 -0.398751
1980   -0.178001 -0.666688
```

```
1990   -0.102465 -0.489427
2000   -0.355138 -0.723567
2010   -0.279333 -0.983768
```

The following code creates 8 panels of scatter plots showing the raw data and overlays a regression line for each decade.

The code is somewhat more involved than usual because we have 9 panels but only 8 sets of data to be plotted, and we want to add axes labels only for those panels that are on the left and lower boundaries.

[52]:
```
# Number of plots (= number of decades)
Nplots = len(df_reg)

# Fix number of columns, determine rows as needed
ncol = 3
nrow = int(np.ceil(Nplots / ncol))

fig, axes = plt.subplots(nrow, ncol, sharey=True, sharex=True,
                         figsize=(8, 8))

for i, ax in enumerate(axes.flatten()):

    # skip if we are out of data (we have 9 panels, but only 8 decades)
    if i >= Nplots:
        # Turn off frame, axes, etc.
        ax.get_xaxis().set_visible(False)
        ax.get_yaxis().set_visible(False)
        ax.set_frame_on(False)
        break

    # decade in current iteration
    decade = df_reg.index.values[i]
    # restrict DataFrame to decade-specific data
    dfi = df.loc[df['Decade'] == decade]
    # Scatter plot of raw data
    ax.scatter(dfi['GDP_gap'], dfi['unempl_gap'], color='steelblue',
               alpha=0.7, label='Raw data')
    # Extract regression coefficients
    const = df_reg.loc[decade, 'Const']
    slope = df_reg.loc[decade, 'GDP_gap']

    # plot regression line:
    # We need to provide two points to define the line to be plotted.
    ax.axline((0.0, const), (1.0, const+slope), color='red',
              lw=2.0, label='Regression line')

    # Add label containing the current decade
    ax.text(0.95, 0.95, f"{decade}'s", transform=ax.transAxes,
            va='top', ha='right')

    # Add legend in the first panel only
    if i == 0:
        ax.legend(loc='lower left', frameon=False)

    # Add x- and y-labels, but only for those panels
    # that are on the left/lower boundary of the figure
    if i >= nrow * (ncol - 1):
        ax.set_xlabel('Output gap (%)')
    if (i % 3) == 0:
        ax.set_ylabel('Cycl. unempl. rate (%-points)')

fig.suptitle("Okun's law")
```

```
[52]: Text(0.5, 0.98, "Okun's law")
```