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FOREIGN EXCHANGE OPTIONS UNDER STOCHASTIC VOLATILITY AND STOCHASTIC INTEREST RATES

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In this paper, we present a stochastic volatility model with stochastic interest rates in a Foreign Exchange (FX) setting. The instantaneous volatility follows a mean-reverting Ornstein–Uhlenbeck process and is correlated with the exchange rate. The domestic and foreign interest rates are modeled by mean-reverting Ornstein–Uhlenbeck processes. The main result is an analytic formula for the price of a European call on the exchange rate. It is derived using martingale methods in arbitrage pricing of contingent claims and Fourier inversion techniques.

Keywords: Foreign exchange options; Ornstein–Uhlenbeck process; stochastic volatility.

1. Introduction

It is common knowledge among practitioners and academics that one of the hardest, if not the hardest, difficulty to tackle in modeling financial markets is the behavior of volatility. In the well-known Black and Scholes model [5], volatility is assumed to be constant. This hypothesis is far from being realistic, as has been pointed out in the empirical financial literature; we refer, for instance, to Bakshi *et al.* [3] and Bates [4]. Over the past decades, many authors have used different specifications to overcome Black–Scholes assumptions in order to describe the empirical leptokurtic distribution of stock returns. Successive approaches to modeling non-constant volatility led to local volatility models (see, e.g., Derman and Kani [8], Dupire [9]), jump diffusion models (Merton [19], Anderson–Andreasen [2]), and stochastic volatility models (Hull and White [17], Heston [16], Scott [22], Stein and Stein [23]). Hull and White [17] solve a special option pricing problem with stochastic volatility by using a Taylor series expansion technique. Stein and Stein assume that the volatility follows a mean-reverting Ornstein–Uhlenbeck (OU) process and obtain the analytic density function of stock returns to evaluate the option price. But, since Stein and Stein have assumed the absence of correlation between volatility and asset price

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returns, as such their solution does not capture important skewness effects that arise from such correlation. Heston [16] proposes a stochastic volatility model in which two correlated diffusions represent the dynamics of the underlying asset and its volatility. The latter is a square-root process as used, in a different context, by Cox *et al.* [7]. As noted by Heston, increasing the volatility of volatility only increases the kurtosis of spot returns and does not capture skewness. In order to affect skewness it is crucial to include correlation of volatility with spot exchange rate returns, and to choose the correlation properly.

Most academic literature on pricing of foreign exchange options can be divided into two categories. In the first category, both foreign and domestic interest rates are assumed to be constant whilst the spot exchange rate is assumed to be stochastic, possibly with a stochastic volatility. In a recent article, Hakala and Wystup [15] present Heston's stochastic volatility model in a foreign exchange setting, assuming constant domestic and foreign interest rates. Although the assumption of constant interest rates is not appealing at all, at most convenient, empirical verification of this class of models has been disappointing; see Tucker *et al.* [24], Bodurtha and Courtadon [6]. For options on bonds and other interest rate-dependent instruments such as foreign exchange options, movements in interest rates are critical and the assumption of a constant interest rate and volatility do not reflect market reality.

The second class of models for pricing foreign exchange options include stochastic volatility. Feiger and Jacquillat [13], following Merton's [20] model for pricing equity options, attempted to obtain foreign exchange option prices by first pricing currency bond options. However, they have not obtained closed form solutions. Amin and Jarrow [1] provide an alternative model for foreign exchange option pricing, incorporating stochastic interest rates. They apply martingale measures to obtain a simple closed form pricing formula for European foreign exchange options. However, the volatility of the exchange rate is a deterministic function of time.

In this paper, we develop a practically useful model of the exchange rate dynamics with stochastic interest rate and stochastic volatility which extends the model considered by Amin and Jarrow [1]. Using martingale methods, we obtain closed form pricing formula for foreign exchange options. The rest of the paper is organized as follows: The model is developed in Sec. 2 which includes the domestic and foreign term structures and describes conditions for an arbitrage free economy. The main theorem is stated in Sec. 3, and in Sec. 4 proof of the main theorem is presented.

2. The Economy

In this section, we establish the framework under which we will price options. We follow a similar line of argument as in Amin and Jarrow [1] and Frachot [12].

We adopt a "domestic" point of view, which is denoted by the subscript d, while subscript f refers to a foreign denominated variable. We assume that the market

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price of risk satisfies sufficient regularity conditions to characterize the no arbitrage assumption by the existence of a risk-neutral probability measure P.

We also assume that the domestic zero coupon bond prices discounted by the domestic spot rate are martingales under P.

2.1. *Domestic term structure*

We assume the spot rate dynamics, under P, follows the Vasicêk model [25]. That is,

$$dr_d(t) = (a_d - \lambda_d r_d(t))dt + \sigma_d dw_d(t), \tag{2.1}$$

where $w_d(t)$ is standard Brownian motion under P.

Lemma 2.1 (Domestic Bond Price Dynamics — Musiela and Rutkowski [21]). *The dynamics of the bond price $B_d(t,T)$ under the spot martingale measure P is given by*

$$dB_d(t,T) = B_d(t,T)(r_d(t)dt - b_d(t,T)dw_d(t)), \tag{2.2}$$

where $b_d(t,T)$ is given by

$$b_d(t,T) = \frac{\sigma_d}{\lambda_d}(1 - e^{-\lambda_d(T-t)}). \tag{2.3}$$

2.2. *Foreign term structure*

Since the probability measure P has been defined in the domestic economy, there is no reason for the foreign zero coupon bond price (discounted by the foreign spot rate) to be a martingale under P. In order to prevent arbitrage between investments in domestic and foreign bonds, following Frachot [12] and Musiela and Rutkowski [21], the dynamics of the exchange rate under the domestic martingale measure P follow the stochastic differential equation:

$$dQ(t) = Q(t)[(r_d(t) - r_f(t))dt + v(t)dw_d(t)]. \tag{2.4}$$

Here, we specify that the volatility $v(t)$ is given by the mean-reverting Ornstein–Uhlenbeck process:

$$dv(t) = \kappa(\theta - v(t))dt + \sigma_v dw_v(t), \tag{2.5}$$

where κ, θ and σ_v are constants and $w_v(t)$ is Brownian motion under the domestic martingale measure P. In order to capture the skewness of spot returns, we allow $w_v(t)$ and $w_d(t)$ to be correlated. Specifically, we set

$$w_d(t) = \rho w_v(t) + \sqrt{1-\rho^2}\, w(t), \tag{2.6}$$

where $w_v(t)$ and $w(t)$ are independent Brownian motion under the domestic martingale measure P.

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Let us now define a new equivalent risk-neutral probability measure P^f by its Radon–Nikodym derivative:

$$\frac{dP^f}{dP} = \exp\left[-\frac{1}{2}\int_0^T v^2(u)du + \int_0^T v(u)dw_d(u)\right] \tag{2.7}$$

and the process $w_f(t)$ given by $w_f(t) = w_d(t) - \int_0^t v(u)d(u)$ is a standard Brownian motion under P^f. We assume the foreign spot rate dynamics under the risk-neutral measure P^f follow Vasicêk's [25] model for interest rates

$$dr_f(t) = (a_f - \lambda_d r_f(t))dt + \sigma_f dw_f(t), \tag{2.8}$$

where $w_f(t)$ is a standard Brownian motion under P^f.

Since the domestic-denominated foreign zero-coupon bond (whose time t price is $Q(t)B_f(t,T)$) can also be considered as a domestic asset, we have:

$$Q(t)B_f(t,T) = \mathbb{E}_t^P\left[Q(T)B_f(T,T)\exp\left\{-\int_t^T r_d(u)du\right\}\right], \tag{2.9}$$

or by definition of P^f:

$$B_f(t,T) = \mathbb{E}_t^{P_f}\left[\exp\left\{-\int_t^T r_f(u)du\right\}\right], \tag{2.10}$$

where $\mathbb{E}_t[\,\cdot\,]$ is the conditional expectation given the information to time t.

Lemma 2.2 (Foreign Bond Price Dynamics — Musiela and Rutkowski [21]). *The dynamics of the bond price $B_f(t,T)$ under the spot martingale measure P^f is given by*

$$dB_f(t,T) = B_f(t,T)(r_f(t)dt - b_f(t,T)dw_f(t)), \tag{2.11}$$

where $b_f(t,T)$ is given by

$$b_f(t,T) = \frac{\sigma_f}{\lambda_f}(1 - e^{-\lambda_f(T-t)}). \tag{2.12}$$

We use the result in Lemma 2.2 and the fact $dw_f(t) = dw_d(t) - v(t)dt$ to obtain the following lemma.

Lemma 2.3 (Foreign Bond Price Dynamics under Domestic Measure). *The dynamics of the bond price $B_f(t,T)$ under the domestic martingale measure P are given by*

$$dB_f(t,T) = B_f(t,T)[(r_f(t) + v(t)b_f(t,T))dt - b_f(t,T)dw_d(t)]. \tag{2.13}$$

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3. Foreign Exchange Option Pricing

Let us first consider the general assumption of a European currency call option with maturity T and strike price K. The value of a European call option is the expected terminal value of the currency option relative to the domestic money market account:

$$\begin{aligned} C(t,Q;T,K) &= \mathbb{E}_t^P\left[\exp\left(-\int_t^T r_d(u)du\right)(Q(T)-K)\cdot \mathbf{1}_{[Q(T)>K]}\right] \\ &= \mathbb{E}_t^P\left[\exp\left(-\int_t^T r_d(u)du\right)Q(T)\cdot \mathbf{1}_{[Q(T)>K]}\right] \\ &\quad - K\mathbb{E}_t^P\left[\exp\left(-\int_t^T r_d(u)du\right)\cdot \mathbf{1}_{[Q(T)>K]}\right], \end{aligned} \tag{3.1}$$

where P is the domestic risk-neutral probability measure. The next result is well-known.

Proposition 3.1 (Musiela and Rutkowski [21]). *The forward exchange rate $F(t,T)$ at time t for settlement date T is given by the following formula*:

$$F(t,T) = \frac{B_f(t,T)}{B_d(t,T)}Q(t). \tag{3.2}$$

Since $Q(T) = F(T,T)$, the payoff $C(T,Q;T,K)$ can be expressed in the following way:

$$\begin{aligned} C(T,Q;T,K) &= C(T,F;T,K) \\ &= F(T,T)\cdot \mathbf{1}_{[F(T,T)>K]} - K\cdot \mathbf{1}_{[F(T,T)>K]}. \end{aligned}$$

Hence, (3.1) can be rewritten as follows:

$$\begin{aligned} C(t,Q;T,K) &= E_t^P\left[\exp\left\{-\int_t^T r_d(u)du\right\}F(T,T)\cdot \mathbf{1}_{[F(T,T)>K]}\right] \\ &\quad - KE_t^P\left[\exp\left\{-\int_t^T r_d(u)du\right\}\cdot \mathbf{1}_{[F(T,T)>K]}\right]. \end{aligned} \tag{3.3}$$

We are in the position to state the main result of the paper.

Theorem 3.1 (Foreign Exchange Currency Option). *Let* $0 \le t \le T$. *Assuming* (2.4), (2.5), *and* (2.6), *the time t price of a European currency option is given by*

$$\begin{aligned} C(t,Q(t),v(t),K) &= B_f(t,T)Q(t)P_1(v(t),x(t),K) \\ &\quad - KB_d(t,T)P_2(v(t),x(t),K), \end{aligned} \tag{3.4}$$

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and the functions P_1 and P_2 are given by

$$P_j(v(t), x(t), K) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left(f_j(\phi)\,\frac{\exp\{-i\phi\,\ln K\}}{i\phi}\right) d\phi, \quad j = 1, 2, \tag{3.5}$$

where $x(t) = \ln\left[Q(t)\frac{B_f(t,T)}{B_d(t,T)}\right]$ and f_1 and f_2 are the corresponding characteristic functions given by

$$\begin{aligned} f_1(\phi) = \exp\Bigg\{ & i\phi\left(x(t) + \frac{1}{2}\int_t^T [b_d^2(u,T) - b_f^2(u,T)]du\right) - \frac{1}{2}\int_t^T b_f^2(u,T)du \\ & + \frac{(1-\rho^2)}{2}\int_t^T ((1+i\phi)b_f(u,T) - i\phi b_d(u,T))^2 du \\ & + \frac{\kappa\rho\theta}{\sigma_v}\int_t^T ((1+i\phi)b_f(u,T) - i\phi b_d(u,T))du \\ & + \frac{\rho}{\sigma_v}((1+i\phi)b_f(t,T) - i\phi b_d(t,T)) - \frac{\rho(1+i\phi)}{2\sigma_v}\left(v^2(t) + \sigma_v^2(T-t)\right)\Bigg\} \\ & \times \exp\bigg[\frac{1}{2}A(t,T,s_1,s_5)v^2(t) + B(t,T,s_1,s_2,s_3,s_4,s_5)v(t) \\ & + C(t,T,s_1,s_2,s_3,s_4,s_5)\bigg], \end{aligned} \tag{3.6}$$

with

$$\begin{aligned} s_1 &= -\frac{1}{2}(1+i\phi)\left(\frac{(1+i\phi)(1-\rho^2)}{2} + \frac{2\kappa\rho}{\sigma_v} - \frac{1}{2}\right), \\ s_2 &= -\Bigg\{ i\phi\frac{\sigma_d}{\lambda_d}\left((1-\rho^2)(1+i\phi) + \frac{\kappa\rho}{\sigma_v}\right) \\ &\quad + (1+i\phi)\frac{\sigma_f}{\lambda_f}\left(1 - \frac{\kappa\rho}{\sigma_v} - (1-\rho^2)(1+i\phi)\right) - \frac{\rho(1+i\phi)\kappa\theta}{\sigma_v}\Bigg\}, \\ s_3 &= -i\phi\frac{\sigma_d}{\lambda_d}\left\{\frac{\rho\lambda_d}{\sigma_v} - (1-\rho^2)(1+i\phi) - \frac{\kappa\rho}{\sigma_v}\right\}, \\ s_4 &= -(i\phi+1)\frac{\sigma_f}{\lambda_f}\left\{(1-\rho^2)(1+i\phi) + \frac{\kappa\rho}{\sigma_v} - \frac{\rho\lambda_f}{\sigma_v} - 1\right\} \quad \textit{and} \\ s_5 &= \frac{\rho(1+i\phi)}{2\sigma_v}. \end{aligned}$$

Similarly,

$$\begin{aligned} f_2(\phi) = \exp\Bigg\{ & i\phi\left(x(t) + \frac{1}{2}\int_t^T [b_d^2(u,T) - b_f^2(u,T)]du\right) - \frac{1}{2}\int_t^T b_f^2(u,T)du \\ & + \frac{(1-\rho^2)}{2}\int_t^T ((i\phi-1)b_d(u,T) - i\phi b_f(u,T))^2 du \end{aligned}$$

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$$
\begin{aligned}
&+ \frac{\kappa\rho\theta}{\sigma_v} \int_t^T (i\phi b_f(u,T) - (i\phi - 1) b_d(u,T)) du \\
&+ \frac{\rho}{\sigma_v}(i\phi b_f(t,T) - (i\phi - 1) b_d(t,T)) - \frac{i\rho\phi}{2\sigma_v}\left(v^2(t) + \sigma_v^2(T-t)\right) \Bigg\} \\
&\times \exp\bigg[\frac{1}{2}E(t,T,q_1,q_5)v^2(t) + F(t,T,q_1,q_2,q_3,q_4,q_5)v(t) \\
&+ G(t,T,q_1,q_2,q_3,q_4,q_5)\bigg],
\end{aligned} \tag{3.7}
$$

with

$$
\begin{aligned}
q_1 &= -\frac{i\phi}{2}\left(i\phi(1-\rho^2) + \frac{2\kappa\rho}{\sigma_v} - 1\right), \\
q_2 &= -\left\{\frac{\sigma_d}{\lambda_d}(i\phi - 1)\left(i\phi(1-\rho^2) + \frac{\kappa\rho}{\sigma_v}\right)\right. \\
&\qquad \left. + i\phi\frac{\sigma_f}{\lambda_f}\left(1 - \frac{\kappa\rho}{\sigma_v} - (1-\rho^2)(i\phi)\right) - i\phi\frac{\rho\kappa\theta}{\sigma_v}\right\}, \\
q_3 &= -(i\phi - 1)\frac{\sigma_d}{\lambda_d}\left\{\frac{\kappa\rho}{\sigma_v} + i\phi(1-\rho^2) - \frac{\rho\lambda_d}{\sigma_v}\right\}, \\
q_4 &= -i\phi\frac{\sigma_f}{\lambda_f}\left\{i\phi(1-\rho^2) + \frac{\kappa\rho}{\sigma_v} - \frac{\rho\lambda_f}{\sigma_v} - 1\right\} \quad \textit{and} \\
q_5 &= \frac{i\phi\rho}{2\sigma_v}.
\end{aligned}
$$

The functions $A(t,T)$, $B(t,T)$, and $C(t,T)$ are derived in Sec. 4. The functions $E(t,T)$, $F(t,T)$, $G(t,T)$ are also given in Sec. 4.

4. Proof of the Main Theorem

The proof will be developed by utilizing a number of lemmas, each of which will be stated carefully.

Definition 4.1. Let T be a fixed maturity date. We define the T-forward measure P_T by:

$$
\left.\frac{dP_T}{dP}\right|_{\mathcal{F}_T} = \exp\left[-\frac{1}{2}\int_0^T b_d^2(u,T)du - \int_0^T b_d(u,T)\, dw_d(u)\right].
$$

Then, by Girsanov's Theorem, the process $w_T(t) = w_d(t) + \int_0^t b_d(u,T)du$ is standard Brownian motion under P_T. Under the forward probability measure P_T, the time t price of European call foreign exchange option is given by (cf. (3.3)):

$$
\begin{aligned}
C(t,Q,T,K) = B_d(t,T)\, \mathbb{E}_t^{P_T}\left[F(T,T)\cdot \mathbf{1}_{[F(T,T)>K]}\right] \\
- KB_d(t,T)\, \mathbb{E}_t^{P_T}\left[\mathbf{1}_{[F(T,T)>K]}\right].
\end{aligned} \tag{4.1}
$$

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Lemma 4.1 (Forward Rate Dynamics under the Domestic Measure). *The dynamics of the forward exchange rate $F(t,T)$ under the domestic martingale measure P is given by*

$$dF(t,T) = \sigma_Q(t,T)F(t,T)(b_d(t,T)dt + dw_d(t)), \tag{4.2}$$

where

$$\sigma_Q(t,T) = b_d(t,T) + v(t) - b_f(t,T). \tag{4.3}$$

Proof. Applying Ito's product rule to Eq. (3.2), we obtain

$$dF(t,T) = F(t,T)\,(v(t) + b_d(t,T) - b_f(t,T))\,(b_d(t,T)dt + dw_d(t))$$

and let

$$\sigma_Q(t,T) = b_d(t,T) + v(t) - b_f(t,T). \tag{4.4}$$

□

Lemma 4.2. *The forward exchange rate $F(t,T)$ admits the following representation under P_T:*

$$F(T,T) = F(t,T)\exp\left[\int_t^T \sigma_Q(u,T)dw_T(u) - \frac{1}{2}\int_t^T \sigma_Q^2(u,T)du\right].$$

Proof. From Definition 4.1 and Eq. (4.2), we get

$$\begin{aligned} dF(t,T) &= \sigma_Q(t,T)F(t,T)(b_d(t,T)dt + dw_d(t)) \\ &= \sigma_Q(t,T)F(t,T)dw_T(t). \end{aligned}$$

This is a simple stochastic differential equation with the well-known solution:

$$F(T,T) = F(t,T)\exp\left[\int_t^T \sigma_Q(u,T)dw_T(u) - \frac{1}{2}\int_t^T \sigma_Q^2(u,T)du\right]. \tag{4.5}$$

□

From Lemma 4.2 we have

$$\begin{aligned} &B_d(t,T)\,\mathbb{E}_t^{P_T}[F(T,T)\cdot \mathbf{1}_{[F(T,T)>K]}] \\ &\quad= B_d(t,T)\,\mathbb{E}_t^{P_T}\left[F(t,T)\exp\left[\int_t^T \sigma_Q(u,T)dw_T(u)\right.\right. \\ &\qquad \left.\left.- \frac{1}{2}\int_t^T \sigma_Q^2(u,T)du\right]\mathbf{1}_{[F(T,T)>K]}\right] \\ &\quad= B_f(t,T)Q(t)\,\mathbb{E}_t^{P_T}\left[\exp\left[\int_t^T \sigma_Q(u,T)dw_T(u)\right.\right. \\ &\qquad \left.\left.- \frac{1}{2}\int_t^T \sigma_Q^2(u,T)du\right]\mathbf{1}_{[F(T,T)>K]}\right]. \end{aligned}$$

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Definition 4.2. Let T be a fixed maturity date. We define the probability measure $\widetilde{P}_T$ by:

$$\eta_T = \frac{d\widetilde{P}_T}{dP_T}\bigg|_{\mathcal{F}_T} = \exp\left[\int_0^T \sigma_Q(u,T)dw_T(u) - \frac{1}{2}\int_0^T \sigma_Q^2(u,T)du\right].$$

From Definition 4.2 and the discussion above, it follows from Bayes' rule that

$$\begin{aligned} B_d(t,T)\,\mathbb{E}_t^{P_T}[F(T,T)\cdot \mathbf{l}_{[F(T,T)>K]}] &= B_f(t,T)Q(t)\,\frac{\mathbb{E}_t^{P_T}[\mathbf{l}_{[F(T,T)>K]}\eta_T]}{\mathbb{E}_t^{P_T}[\eta_T]} \\ &= Q(t)B_f(t,T)\mathbb{E}_t^{\widetilde{P}_T}[\mathbf{l}_{[F(T,T)>K]}]. \end{aligned}$$

Hence,

$$C(t,Q,T,K) = Q(t)B_f(t,T)\,\mathbb{E}_t^{\widetilde{P}_T}[\mathbf{l}_{[F(T,T)>K]}] - KB_d(t,T)\,\mathbb{E}_t^{P_T}[\mathbf{l}_{[F(T,T)>K]}],$$

or in terms of the logarithm of the forward exchange rate $x(t) = \ln F(t,T)$,

$$C(t,Q,T,K) = Q(t)B_f(t,T)\,P_1(v(t),x(t),K) - KB_d(t,T)P_2(v(t),x(t),K), \quad (4.6)$$

and

$$\begin{aligned} P_1(v(t),x(t),K) &\overset{\text{def.}}{=} \widetilde{P}_T(x(T) > \ln K | v(t),x(t)), \\ P_2(v(t),x(t),K) &\overset{\text{def.}}{=} P_T(x(T) > \ln K | v(t),x(t)), \end{aligned}$$

where we use the Markov property of (v,x) under $\widetilde{P_T}$ and P_T.

An effective way to obtain closed form solutions for the probabilities P_1 and P_2 is to derive their corresponding conditional characteristic functions, which are defined by

$$\begin{aligned} f_1(\phi) &\equiv \mathbb{E}_t^{\widetilde{P}_T}[\exp\{i\phi x(T)\}], \quad \text{and} \\ f_2(\phi) &\equiv \mathbb{E}_t^{P_T}[\exp\{i\phi x(T)\}]. \end{aligned}$$

We now use the Radon–Nikodym derivatives to obtain expressions for the characteristic functions under the original martingale measure P.

Lemma 4.3.

$$\frac{d\widetilde{P}_T}{dP}\bigg|_{\mathcal{F}_t} = \exp\left[\int_t^T (v(u) - b_f(u,T))\,dw_d(u) - \frac{1}{2}\int_t^T (v(u) - b_f(u,T))^2 du\right].$$

Proof.

$$\begin{aligned} \frac{d\widetilde{P}_T}{dP}\bigg|_{\mathcal{F}_t} &= \frac{d\widetilde{P}_T}{dP_T}\bigg|_{\mathcal{F}_t} \times \frac{dP_T}{dP}\bigg|_{\mathcal{F}_t} \\ &= \exp\left[\int_t^T \sigma_Q(u,T)dw_T(u) - \frac{1}{2}\int_t^T \sigma_Q^2(u,T)du\right] \end{aligned}$$

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$$\times \exp\left[-\int_t^T b_d(u,T)dw_d(u) - \frac{1}{2}\int_t^T b_d^2(u,T)du\right]$$

$$= \exp\left[\int_t^T (b_d(u,T) + v(u) - b_f(u,T))(b_d(u,T) + dw_d(u)) - \frac{1}{2}\int_0^T (b_d(u,T) + v(u) - b_f(u,T))^2 du\right]$$

$$\times \exp\left[-\int_t^T b_d(u,T)dw_d(u) - \frac{1}{2}\int_t^T b_d^2(u,T)du\right]$$

$$= \exp\left[\int_t^T (v(u) - b_f(u,T))dw_d(u) - \frac{1}{2}\int_t^T (v(u) - b_f(u,T))^2 du\right].$$

□

Following Lemma 4.3, $f_1(\phi)$ takes the following form:

$$f_1(\phi) = \mathbb{E}_t^P\left[\exp\{i\phi x(T)\}\exp\left[\int_t^T (v(u) - b_f(u,T))\,dw_d(u) - \frac{1}{2}\int_t^T (v(u) - b_f(u,T))^2 du\right]\right], \tag{4.7}$$

and following Definition 4.1, $f_2(\phi)$ takes the following form:

$$f_2(\phi) = \mathbb{E}_t^P\left[\exp\{i\phi x(T)\}\exp\left[-\int_t^T b_d(u,T)dw_d(u) - \frac{1}{2}\int_t^T b_d^2(u,T)du\right]\right], \tag{4.8}$$

under the martingale measure P.

We now derive a closed form expression for the characteristic function $f_1(\phi)$. The derivation of a closed form for $f_2(\phi)$ follows similar in analogy to the one of $f_1(\phi)$.

Lemma 4.4. *The forward exchange rate $F(t,T)$ admits the following representation under P:*

$$F(T,T) = F(t,T)\exp\left[\int_t^T \sigma_Q(u,T)\,dw_d(u) + \int_t^T \sigma_Q(u,T)\left[b_d(u,T) - \frac{1}{2}\sigma_Q(u,T)\right]du\right].$$

The logarithm of the forward exchange rate $x(t) = \ln F(t,T)$ is given by

$$x(T) = x(t) + \int_t^T \sigma_Q(u,T)\left[b_d(u,T) - \frac{1}{2}\sigma_Q(u,T)\right]du + \int_t^T \sigma_Q(u,T)dw_d(u). \tag{4.9}$$

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From Eq. (4.9), notations (2.6) and (4.4) and standard computations,

$$\begin{aligned}
f_1(\phi) &= \mathbb{E}_t^P\Bigg[\exp\{i\phi x(T)\}\exp\Bigg[\int_t^T(v(u)-b_f(u,T))\,dw_d(u)\\
&\qquad -\frac{1}{2}\int_t^T(v(u)-b_f(u,T))^2du\Bigg]\Bigg]\\
&= \exp\Bigg[i\phi\Bigg[x(t)+\frac{1}{2}\int_t^T(b_d^2(u,T)-b_f^2(u,T))du\Bigg]-\frac{1}{2}\int_t^T b_f^2(u,T)du\Bigg]\\
&\quad\times\mathbb{E}_t^P\Bigg[\exp\Bigg[\rho(1+i\phi)\int_t^T v(u)dw_v(u)+(1+i)\phi\int_t^T v(u)b_f(u,T)du\\
&\qquad -\frac{(i+1)\phi}{2}\int_t^T v^2(u)du+\rho\int_t^T(i\phi b_d(u,T)-(1+i\phi)b_f(u,T))dw_v(u)\\
&\qquad +\sqrt{1-\rho^2}\int_t^T((1+i\phi)(v(u)-b_f(u,T))+i\phi b_d(u,T))dw(u)\Bigg]\Bigg].
\end{aligned}$$

Lemma 4.5. *Given the dynamics* (2.5) *of* $v(t)$, *we obtain*

$$\begin{aligned}
&\int_t^T(i\phi b_d(u,T)-(1+i\phi)b_f(u,T))\,dw_v(u)\\
&\quad=\frac{1}{\sigma_v}\Bigg[((i\phi+1)b_f(t,T)-i\phi b_d(t,T))v(t)+\kappa\theta\int_t^T((i\phi+1)b_f(u,T)\\
&\qquad -i\phi b_d(u,T))du+\int_t^T(i\phi(\sigma_d e^{-\lambda_d(T-u)}+\kappa b_d(u,T)\\
&\qquad -(1+i\phi)(\sigma_f e^{-\lambda_f(T-u)}+\kappa b_f(u,T)))v(u)du\Bigg],
\end{aligned}\tag{4.10}$$

and

$$\begin{aligned}
\int_t^T v(u)dw_v(u) &= \frac{1}{2\sigma_v}\Bigg[v^2(T)-v^2(t)-2\kappa\theta\int_t^T v(u)du\\
&\qquad +2\kappa\int_t^T v^2(u)du-\sigma_v^2(T-t)\Bigg].
\end{aligned}\tag{4.11}$$

Using (4.11) of Lemma 4.5, noting that $w_v(t)$ and $w(t)$ are uncorrelated and recalling that if φ is a standard normal random variable, then $\mathbb{E}[e^{\lambda\varphi}]=e^{\lambda^2/2}$. Thus,

$$\begin{aligned}
f_1(\phi) &= \exp\Bigg[i\phi\Bigg[x(t)+\frac{1}{2}\int_t^T(b_d^2(u,T)-b_f^2(u,T))du\Bigg]-\frac{1}{2}\int_t^T b_f^2(u,T)du\\
&\qquad +\frac{1-\rho^2}{2}\int_t^T((1+i\phi)(b_f(u,T)-i\phi b_d(u,T)))^2du
\end{aligned}$$

$$
\begin{aligned}
&- \frac{\rho(1+i\phi)}{2\sigma_v}(v^2(t) + \sigma_v^2(T-t))\Bigg] \\
&\times \mathbb{E}_t^P\Bigg[\exp\Bigg[\frac{\rho(1+i\phi)}{2\sigma_v}\left(v^2(T) - 2\kappa\theta\int_t^T v(u)du + 2\kappa\int_t^T v^2(u)du\right) \\
&+ i\phi\int_t^T v(u)b_f(u,T)du - \frac{(1+i)\phi}{2}\int_t^T v^2(u)du \\
&+ \rho\int_t^T (i\phi b_d(u,T) - (1+i\phi)b_f(u,T))dw_v(u) \\
&+ \int_t^T b_f(u,T)v(u)du + \frac{(1-\rho^2)(1+i\phi)^2}{2}\int_t^T v^2(u)du \\
&+ (1-\rho^2)(1+i\phi)\int_t^T (i\phi b_d(u,T) - (1+i\phi)b_f(u,T))v(u)du\Bigg]\Bigg].
\end{aligned}
$$

Note: $w_v(t)$ and $v(t)$ are independent of $w(t)$.

Following (4.10) of Lemma 4.5, we obtain

$$
\begin{aligned}
f_1(\phi) = \exp\Bigg[&i\phi\left[x(t) + \frac{1}{2}\int_t^T (b_d^2(u,T) - b_f^2(u,T))du\right] - \frac{1}{2}\int_t^T b_f^2(u,T)du \\
&+ \frac{1-\rho^2}{2}\int_t^T ((1+i\phi)b_f(u,T) - i\phi b_d(u,T))^2 du \\
&+ \frac{\kappa\theta\rho}{\sigma_v}\int_t^T ((1+i\phi)b_f(u,T) - i\phi b_d(u,T))du \\
&+ \frac{\rho}{\sigma_v}((1+i\phi)b_f(t,T) - i\phi b_d(t,T))v(t) - \frac{\rho(1+i\phi)}{2\sigma_v}(v^2(t) + \sigma_v^2(T-t))\Bigg] \\
&\times \mathbb{E}_t^P\Bigg[\exp\Bigg[\left(\frac{(1-\rho^2)(1+i\phi)^2}{2} + \frac{\kappa\rho(1+i\phi)}{\sigma_v} - \frac{(1+i\phi)}{2}\right)\int_t^T v^2(u)du \\
&+ \int_t^T\Bigg((1-\rho^2)(1+i\phi)(i\phi b_d(u,T) - (1+i\phi)b_f(u,T)) \\
&+ \frac{\rho}{\sigma_v}(i\phi(\sigma_d e^{-\lambda_d(T-u)} + \kappa b_d(u,T)) \\
&- (1+i\phi)(\sigma_f e^{-\lambda_f(T-u)} + \kappa b_f(u,T))) \\
&- \frac{\rho\kappa\theta(1+i\phi)}{\sigma_v} + (1+i\phi)b_f(u,T)\Bigg)v(u)du + \frac{\rho(1+i\phi)}{2\sigma_v}v^2(T)\Bigg]\Bigg].
\end{aligned}
$$

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Further simplification results in

$$
\begin{aligned}
f_1(\phi) = \exp\Bigg[& i\phi\left[x(t) + \frac{1}{2}\int_t^T (b_d^2(u,T) - b_f^2(u,T))du\right] - \frac{1}{2}\int_t^T b_f^2(u,T)du \\
& + \frac{(1-\rho^2)}{2}\int_t^T ((1+i\phi)b_f(u,T) - i\phi b_d(u,T))^2 du \\
& + \frac{\kappa\theta\rho}{\sigma_v}\int_t^T ((1+i\phi)b_f(u,T) - i\phi b_d(u,T))du \\
& + \frac{\rho}{\sigma_v}((1+i\phi)b_f(t,T) - i\phi b_d(t,T))v(t) - \frac{\rho(1+i\phi)}{2\sigma_v}(v^2(t) + \sigma_v^2(T-t))\Bigg] \\
& \times \mathbb{E}_t^P\Bigg[\exp\Bigg\{ - s_1\int_t^T v^2(u)du - s_2\int_t^T v(u)du - s_3\int_t^T e^{-\lambda_d(T-u)}v(u)du \\
& - s_4\int_t^T e^{-\lambda_f(T-u)}v(u)du + s_5 v^2(T)\Bigg\}\Bigg],
\end{aligned}
$$

where

$$
\begin{aligned}
s_1 &= -(1+i\phi)\left[\frac{(1-\rho^2)(1+i\phi)}{2} + \frac{\kappa\rho}{\sigma_v} - \frac{1}{2}\right], \\
s_2 &= \Bigg[i\phi\frac{\sigma_d}{\lambda_d}\left((1-\rho^2)(1+i\phi) + \frac{\kappa\rho}{\sigma_v}\right) \\
&\quad + (1+i\phi)\frac{\sigma_f}{\lambda_f}\left(1 - \frac{\kappa\rho}{\sigma_v} - (1-\rho^2)(1+i\phi)\right) - \frac{\rho\kappa\theta(1+i\phi)}{\sigma_v}\Bigg], \\
s_3 &= -i\phi\,\frac{\sigma_d}{\lambda_v}\left[\frac{\rho\lambda_d}{\sigma_v} - (1-\rho^2)(1+i\phi) - \frac{\kappa\rho}{\sigma_v}\right], \\
s_4 &= -(1+i\phi)\frac{\sigma_f}{\lambda_f}\left[(1-\rho^2)(1+i\phi) + \frac{\kappa\rho}{\sigma_v} - \frac{\rho\lambda_f}{\sigma_v} - 1\right] \quad \text{and} \\
s_5 &= \frac{\rho(1+i\phi)}{2\sigma_v}.
\end{aligned}
$$

We need to calculate the expectation (we note that v is Markov under P):

$$
\begin{aligned}
F(v(t),t,T) = \mathbb{E}_t^P\Bigg[\exp\Bigg\{ & - s_1\int_t^T v^2(u)du - s_2\int_t^T v(u)du - s_3\int_t^T e^{-\lambda_d(T-u)}v(u)du \\
& - s_4\int_t^T e^{-\lambda_f(T-u)}v(u)du + s_5 v^2(T)\Bigg\}\Bigg].
\end{aligned}
$$

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Here we follow [10, pp. 273–277]. Let us define

$$\begin{aligned} y_t = F(v(t), t, T) \exp\bigg[&- s_1 \int_0^t v^2(u)du - s_2 \int_0^t v(u)du \\ &- s_3 \int_0^t e^{-\lambda_d(T-u)} v(u)du - s_4 \int_0^t e^{-\lambda_f(T-u)} v(u)du \bigg]. \end{aligned} \tag{4.12}$$

Then,

$$\begin{aligned} y_t = \mathbb{E}_t^P \bigg[\exp\bigg\{ &- s_1 \int_0^T v^2(u)du - s_2 \int_0^T v(u)du - s_3 \int_0^T e^{-\lambda_d(T-u)} v(u)du \\ &- s_4 \int_0^T e^{-\lambda_f(T-u)} v(u)du + s_5 v^2(T) \bigg\} \bigg], \end{aligned}$$

and so is a martingale.

Applying the Ito differentiation rule to y_t given by (4.12) and setting the drift term equal to zero, we obtain the backward partial differential equation for $F(v, t, T)$ given by

$$\begin{aligned} &\frac{\partial F}{\partial t} + \frac{1}{2}\sigma_v^2 \frac{\partial^2 F}{\partial v^2} + \kappa(\theta - v(t)) \frac{\partial F}{\partial v} \\ &\quad - F(v, t)(s_1 v^2 + v(t)(s_2 + s_3 e^{-\lambda_d(T-t)} + s_4 e^{-\lambda_f(T-t)})) = 0, \end{aligned}$$

with terminal condition $F(v, T, T) = \exp(s_5 v^2(T))$. We look for solutions of the above differential equation in the form:

$$F(v, t, T) = \exp\left[\frac{1}{2} A(t, T) v^2 + B(t, T) v + C(t, T) \right],$$

with $A(T, T) = 2s_5$. Substituting the expression for $F(v, t, T)$ in the partial differential equation, we obtain the following system of ordinary differential equations that determine $A(t, T)$, $B(t, T)$ and $C(t, T)$:

$$\begin{aligned} \frac{\partial A}{\partial t} &= -\sigma_v^2 A^2(t, T) + 2\kappa A(t, T) + 2s_1, \\ \frac{\partial B}{\partial t} &= \big(\kappa - \sigma_v^2 A(t, T)\big) B(t, T) - \kappa\theta A(t, T) \\ &\quad + s_2 + s_3 e^{-\lambda_d(T-t)} + s_4 e^{-\lambda_f(T-t)}, \\ \frac{\partial C}{\partial t} &= -\frac{1}{2}\sigma_v^2 B^2(t, T) - \frac{1}{2}\sigma_v^2 A(t, T) - \kappa\theta B(t, T), \end{aligned}$$

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where $A(T,T) = 2s_5$ and $B(T,T) = C(T,T) = 0$. Solving the above system, we obtain:

$$A(t,T) = \frac{1}{\sigma_v^2}\left(\kappa - \gamma_1 \frac{\sinh\{\gamma_1(T-t)\} + \gamma_2\cosh\{\gamma_1(T-t)\}}{\cosh\{\gamma_1(T-t)\} + \gamma_2\sinh\{\gamma_1(T-t)\}}\right),$$

$$B(t,T) = \frac{1}{\sigma_v^2\gamma_1}\left(\frac{(\kappa\theta\gamma_1 - \gamma_2\gamma_3) + \gamma_3(\sinh\{\gamma_1(T-t)\} + \gamma_2\cosh\{\gamma_1(T-t)\})}{\cosh\{\gamma_1(T-t)\} + \gamma_2\sinh\{\gamma_1(T-t)\}} - \kappa\theta\gamma_1\right),$$

$$\begin{aligned} C(t,T) &= -\frac{1}{2}\ln(\cosh\{\gamma_1(T-t)\} + \gamma_2\sinh\{\gamma_1(T-t)\}) + \frac{\kappa(T-t)}{2} \\ &\quad + \frac{\kappa^2\theta^2\gamma_1^2 - \gamma_3^2}{2\sigma_v^2\gamma_1^3}\left(\frac{\sinh\{\gamma_1(T-t)\}}{\cosh\{\gamma_1(T-t)\} + \gamma_2\sinh\{\gamma_1(T-t)\}} - \gamma_1(T-t)\right) \\ &\quad + \frac{(\kappa\theta\gamma_1 - \gamma_2\gamma_3)\gamma_3}{\sigma_v^2\gamma_1^3}\left(\frac{\cosh\{\gamma_1(T-t)\} - 1}{\cosh\{\gamma_1(T-t)\} + \gamma_2\sinh\{\gamma_1(T-t)\}}\right), \end{aligned}$$

where

$$\gamma_1 = \sqrt{2\sigma_v^2 s_1 + \kappa^2}, \quad \gamma_2 = \frac{\kappa - 2\sigma_v^2 s_5}{\gamma_1}, \quad \gamma_3 = \kappa^2\theta - s_6\sigma_v^2,$$

with

$$s_6 = s_2 + s_3 e^{-\lambda_d(T-t)} + s_4 e^{-\lambda_f(T-t)}.$$

The functions $A(t,T)$, $B(t,T)$ and $C(t,T)$ are substituted into (3.6) of Theorem 3.1 to provide a closed form solution of the characteristic function $f_1(\phi)$.

The functions $E(t,T)$, $F(t,T)$ and $G(t,T)$ are obtained using a similar procedure and are given by:

$$E(t,T) = \frac{1}{\sigma_v^2}\left(\kappa - \beta_1 \frac{\sinh\{\beta_1(T-t)\} + \beta_2\cosh\{\beta_1(T-t)\}}{\cosh\{\beta_1(T-t)\} + \beta_2\sinh\{\beta_1(T-t)\}}\right),$$

$$F(t,T) = \frac{1}{\sigma_v^2\beta_1}\left(\frac{(\kappa\theta\beta_1 - \beta_2\beta_3) + \beta_3(\sinh\{\beta_1(T-t)\} + \beta_2\cosh\{\beta_1(T-t)\})}{\cosh\{\beta_1(T-t)\} + \beta_2\sinh\{\beta_1(T-t)\}} - \kappa\theta\beta_1\right),$$

$$\begin{aligned} G(t,T) &= -\frac{1}{2}\ln(\cosh\{\beta_1(T-t)\} + \beta_2\sinh\{\beta_1(T-t)\}) + \frac{\kappa(T-t)}{2} \\ &\quad + \frac{\kappa^2\theta^2\beta_1^2 - \beta_3^2}{2\sigma_v^2\beta_1^3}\left(\frac{\sinh\{\gamma_1(T-t)\}}{\cosh\{\beta_1(T-t)\} + \beta_2\sinh\{\beta_1(T-t)\}} - \beta_1(T-t)\right) \\ &\quad + \frac{(\kappa\theta\beta_1 - \beta_2\beta_3)\beta_3}{\sigma_v^2\beta_1^3}\left(\frac{\cosh\{\beta_1(T-t)\} - 1}{\cosh\{\beta_1(T-t)\} + \beta_2\sinh\{\beta_1(T-t)\}}\right), \end{aligned}$$

where

$$\beta_1 = \sqrt{2\sigma_v^2 q_1 + \kappa^2}, \quad \beta_2 = \frac{\kappa - 2\sigma_v^2 q_5}{\beta_1}, \quad \beta_3 = \kappa^2\theta - q_6\sigma_v^2,$$

with

$$q_6 = q_2 + q_3 e^{-\lambda_d(T-t)} + q_4 e^{-\lambda_f(T-t)}.$$

The functions $E(t,T)$, $F(t,T)$ and $G(t,T)$ are substituted into (3.7) of Theorem 3.1 to provide a closed form solution of the characteristic function $f_2(\phi)$.

Given the characteristic functions $f_1(\phi)$ and $f_2(\phi)$, the probability functions $P_j(v(t), x(t), K)$ are obtained using the Fourier inversion formula:

$$P_j(v(t), x(t), K) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left(f_j(\phi)\, \frac{\exp\{-i\phi \ln K\}}{i\phi} \right) d\phi, \quad j = 1, 2.$$

□

5. Conclusions

In this paper, we have developed a practically useful model of the exchange rate dynamics with stochastic interest rates and stochastic volatility and provide a Heston's type closed form solution. The model is based on the Amin and Jarrow [1] framework, but includes stochastic volatility. Such a work has been already considered in a deterministic context by many researchers including Hakala and Wystup [15], Amin and Jarrow [1], Feiger and Jacquillat [13], Jamshidian [18] and Hilliard *et al.* [14]. Unfortunately, there are compelling reasons to support a stochastic volatility model, since the entire autoregressive heteroskedasticity (ARCH) literature is based on empirical evidence that conditional variances depend on past values of not only interest rates but also exchange rates [12]. Since our focus is also to obtain closed form formulae for option prices, the choice of useful models is rather limited as shown by Filipovic [11]. The one chosen in this paper is one of the best models that is still tractable, that is the case with Gaussian interest rate and Gaussian volatilities. Despite the deficiencies (negative values), it is very popular in many sectors of Derivatives industry.

The final result for the price of a FX call option has been derived and is in closed form, but the result is complex. Because of this, the numerical calculations to which it gives rise are lengthy and involve issues more specific to the discipline of numerical methods. These considerations will be addressed in a paper currently in preparation.

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