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Auto-tuning of model-based feedforward controller by feedback control signal in ultraprecision motion systems

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A B S T R A C T

In this paper, an auto-tuning algorithm of model-based feedforward controller by feedback control signal in ultraprecision motion systems is proposed. The algorithm is predicated on the simple fact that for a control system with well-tuned feedback controller, at low frequency range, feedback control signal is a good approximation of the ideal feedforward signal required to achieve perfect tracking control. Based on this mechanism, model-based feedforward such as acceleration, jerk (third derivative of reference trajectory) and snap (fourth derivative of reference trajectory) feedforward can be tuned by least squares algorithm with feedback control signal. Simulation and experiment both well validate the proposed algorithm.

1. Introduction

Ultraprecision motion systems are widely used in state-of-the-art manufacturing equipment such as lithography machine [1]. The operation circle of these systems usually consists of acceleration, constant velocity and deceleration phase. The motion control strategy of these systems is usually the so-called two degree-of-freedom (DOF) control, where feedback and feedforward control are utilized at the same time. While feedback control is mainly used for system stabilization and disturbance rejection, feedforward control is for enhancement of dynamic response and attenuation of settling time, the time cost it takes for tracking error to converge within an acceptable level after acceleration and thus greatly influences system throughput. Therefore, feedforward control is indispensable and extensively utilized in motion control industry.

The most classical feedforward control strategy is model-based feedforward. The essence of model-based feedforward control is to approximate the inversion of plant model by feedforward controller. In [2–4], a model-based feedforward controller using acceleration and snap (fourth derivative of reference trajectory) feedforward is proposed, where acceleration is for compensation of rigid-body dynamics and snap feedforward for partial compensation of resonant dynamics. However, model-based feedforward may well suffer from model inaccuracy and one of the most distinctive characteristics for ultraprecision motion systems is the extremely stringent performance requirements that tolerance for model inaccuracy is low. Therefore, data-based methods are extensively studied. In [5–8], finite impulse response (FIR) filter tuned by data-based method is used in cooperation with acceleration and snap feedforward. In [9–11], iterative learning control (ILC), one of the most powerful feedforward algorithms in practice, is used for further compensation.

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1.1. Motivation

In engineering practices, the design and tuning of feedforward controller in ultraprecision motion systems usually consist of two steps [1,5,7–9,12]. In step one, classical model-based feedforward such as acceleration feedforward is utilized and tuned. If further performance improvement is required, in step two, advanced data-based techniques abovementioned such as ILC and FIR feedforward are utilized in cooperation with classical model-based feedforward. Although approaches used in step two are powerful and popular, classical model-based feedforward still deserves further exploration. Instead of competing with the approaches in step two on tracking performance, this paper mainly seeks to exploit the potential of classical model-based feedforward. The main motivation is that in many applications, well-tuned classical model-based feedforward controller is able to provide enough tracking performance, in applications where more advanced techniques are required, well-tuned classical model-based feedforward controller can largely ease the burden of the advanced methods in step two and thus accelerate the whole controller design process.

1.2. Contribution

The performance of model-based feedforward is mainly limited by the following two problems:

(i) Tuning. Model-based feedforward is highly sensitive to model inaccuracy. Even slight gap between tuned feedforward coefficients and the ideal values may lead to large performance deterioration in ultraprecision motion systems. Therefore tuning is of significant importance for model-based feedforward.
(ii) Structure complexity. The capability to approximate plant inversion of mode-based feedforward controller is restricted by the controller structure. Approximating plant inversion at a larger frequency range requires more complicated controller structure.

This paper mainly focuses on the first problem, namely, tuning of model-based feedforward controller. The biggest obstacle for model-based feedforward tuning is that it is hard to tell whether the feedforward coefficients are accurate enough and how much is the gap between these coefficients and the ideal values. This problem was solved in [13], where research on the generation mechanism of tracking error shows that the shape of tracking error is a good criterion to judge the accuracy of feedforward coefficients and thus can be used for feedforward tuning. However, the feedforward tuning approach proposed in [13] is a manual tuning one due to reliance on subjective criterion such as the similarity between the shape of tracking error and derivatives of reference trajectory. From industrial perspective, an auto-tuning algorithm is preferred.

The main contributions of this paper can be stated as follow:

(i) The idea in [13] is extended to attain a feedforward tuning algorithm by feedback control signal rather than tracking error. The algorithm is based on the simple fact that for a control system with well-tuned feedback controller, at low frequency range, the feedback control signal is a good approximation of the ideal feedforward signal required to achieve perfect tracking control.
(ii) The proposed tuning algorithm is an auto-tuning rather than manual tuning one. Classical model-based feedforward such as acceleration, jerk and snap feedforward can be effectively and handily tuned by least squares algorithm. Simulation and experiment both well validate the proposed algorithm.

2. Plant dynamics and control architecture

2.1. Plant dynamics

Ultraprecision motion system like the motion stage of lithography machine is usually actuated by air bearings or magnetic-levitated planar motors, rendering the system friction-free. The dynamics of this kind of system can then be modeled by the multi-mass-block model shown in Fig. 1. The transfer function of the plant model is

$$G_{p_0}(s) = \frac{1}{ms^2} + \frac{1}{m}\sum_{i=1}^{N-1}\frac{\alpha_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \tag{1}$$

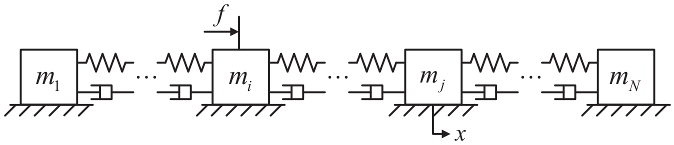


Fig. 1. Multi-mass-block model.

m is the total mass of system, N is the number of mass blocks, ζ_i and ω_i are damping ratio and resonant frequency of the i-th resonant mode, α_i is model parameter determined by eigenmodes and positions of system input f and output x, see [14,15] for detailed discussions on the dynamics of multi-mass-block model. Time delay is ubiquitous in control systems in practice due to mechanical or electrical factors. For ultraprecision motion systems, even time delay of several sampling periods (several hundred microseconds) will significantly influence tracking error. Therefore, time delay must be considered into system dynamics. Aside from time delay, for digital control systems in practice, zero-order holder exists between controller and plant model, at low frequency range, zero-order hold effect is approximately equal to a half sampling period time delay, which will also impact tracking error in ultraprecision motion systems a lot. Taking time delay and zero-order hold effect into consideration, the equivalent plant model is revised as

$$G_p(s) = e^{-\tau s} G_{\mathrm{zoh}}(s) G_{p_0}(s) \tag{2}$$

τ is time delay, $G_{\mathrm{zoh}}(s)$ is the transfer function of zero-order holder

$$G_{\mathrm{zoh}}(s) = \frac{1 - e^{-T_s s}}{T_s s} \tag{3}$$

T_s is sampling period. If zero-order hold effect is approximated by a half sampling period time delay, then the equivalent plant model can be simplified as

$$G_p(s) \approx e^{-\left(\tau + \frac{T_s}{2}\right)s} G_{p_0}(s) \tag{4}$$

In the remainder of this paper, we will use (4) to describe the system dynamics. Then the inversion of plant model is

$$G_p^{-1}(s) = e^{\left(\tau + \frac{T_s}{2}\right)s} G_{p_0}^{-1}(s) \tag{5}$$

By Taylor expansion at $s = 0$, we can get

$$e^{\left(\tau + \frac{T_s}{2}\right)s} = 1 + \left(\tau + \frac{T_s}{2}\right)s + \cdots \tag{6}$$

Similarly, with (1), we can get the expansion of $G_{p_0}^{-1}$ at $s = 0$

$$G_{p_0}^{-1}(s) = m\left(s^2 + a_1 s^4 + a_2 s^5 + a_3 s^6 + \cdots\right) \tag{7}$$

a_i is model parameter determined by plant dynamics, detailed deductions to get (7) can be found in [13]. Substitute (6) and (7) into (5), we can get

$$\begin{aligned} G_p^{-1}(s) &= \left(1 + \left(\tau + \tfrac{T_s}{2}\right)s + \cdots\right)\left(m\left(s^2 + a_1 s^4 + \cdots\right)\right) \\ &= m\left(s^2 + \left(\tau + \tfrac{T_s}{2}\right)s^3 + \hat{a}_1 s^4 + \cdots\right) \end{aligned} \tag{8}$$

$\hat{a}_i$ is model parameter determined by plant dynamics. Truncation of series (8) can give good approximation of plant model inversion at low frequency range.

2.2. Control Architecture

Two degree-of-freedom (DOF) control shown in Fig. 2 is one of the most frequently used control architectures in motion control industry. G_c is feedback controller, F is feedforward controller, G_p is plant model, r is reference trajectory, y is system output, e is tracking error, u_{fb} is feedback control signal, u_{ff} is feedforward control signal, u is total control signal, d_1 is force disturbance, d_2 is position disturbance, d_3 is sensor noise. Disturbance and noise are out of consideration here for the moment and then tracking error can be expressed as

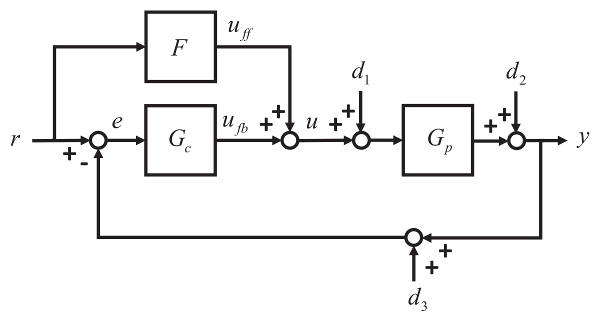


Fig. 2. Two degree-of-freedom control architecture.

$$e = \frac{1 - FG_p}{1 + G_c G_p} r = S(1 - FG_p) r \tag{9}$$

S is sensitivity function. It shows that the design principle of feedforward controller is to approximate the plant inversion G_p^{-1}. Instead of approximating plant inversion at all frequencies, approximation of plant inversion at low frequency range is sufficient for applications where reference trajectory mainly contains low frequency components. Therefore model-based feedforward like acceleration, jerk and snap feedforward is able to provide good tracking performance in many applications as long as feedforward coefficients are well tuned. Feedforward controller can then be expressed as

$$F = \underbrace{m_a s^2}_{\text{acc ff}} + \underbrace{m_j s^3}_{\text{jerk ff}} + \underbrace{m_s s^4}_{\text{snap ff}} \tag{10}$$

According to (8) and (10), ideal acceleration, jerk and snap feedforward coefficients are $m, m(\tau + \frac{T_s}{2})$ and $m\hat{a}_1$, respectively. The potential to approximate plant inversion grows with the order of feedforward controller. However the order of feedforward controller is limited by the derivability of reference trajectory. The order of trajectory used in the industry is usually equal or lower than four in consideration of throughput and thus feedforward term higher than fourth order is out of consideration in this paper.

3. Auto-tuning of model-based feedforward

3.1. The connection between feedback control signal and ideal feedforward signal

According to control architecture in Fig. 2, if disturbance and noise are out of consideration, the transfer function from reference trajectory r to feedback control signal u_{fb} is

$$u_{fb} = G_c \frac{1 - FG_p}{1 + G_c G_p} r = T\left(G_p^{-1} - F\right) r \tag{11}$$

T is complementary sensitivity function

$$T = \frac{G_c G_p}{1 + G_c G_p} \tag{12}$$

F is feedforward controller in use and G_p^{-1} is plant model inversion, i.e., the ideal feedforward controller required to achieve perfect tracking control, then $G_p^{-1} - F$ represents the feedforward compensation required to achieve perfect tracking control after use of F. Therefore, (11) shows that feedback control signal is the ideal feedforward signal required to achieve perfect tracking filtered by complementary sensitivity function. For most control systems, complementary sensitivity function is a low-pass filter whose gain approximates one and phase lag approximates zero at low frequency range, which means feedback control signal is a good approximation of ideal feedforward control signal required to realize perfect tracking at low frequency range, i.e.,

$$u_{fb} \approx \left(G_p^{-1} - F\right) r \tag{13}$$

Another simple way to get the same result is that for most control systems, the gain of open loop transfer function $L = G_c G_p$ is usually high at low frequency range, i.e., $|L| \gg 1$, then we can get

$$\begin{aligned} u_{fb} &= G_c \tfrac{1-FG_p}{1+G_cG_p} r \approx G_c \tfrac{1-FG_p}{G_cG_p} r \\ &= \left(G_p^{-1} - F\right) r \end{aligned} \tag{14}$$

High performance motion systems are usually controlled by well-tuned feedback controller with high control bandwidth and thus high gain at low frequency range, feedback control signal is then a rather good approximation of ideal feedforward control signal.

Since information on ideal feedforward signal is contained in feedback control signal and the structure of feedforward controller is already known through prior knowledge on plant dynamics, an auto-tuning algorithm of model-based feedforward controller can be attained.

3.2. Simulation Parameters

The feedforward tuning algorithm will be introduced with simulation in next subsection, and simulation parameters will be given here first.

The plant model used in simulation is the double-mass-block model in non-collocated form shown in Fig. 3, the transfer function is

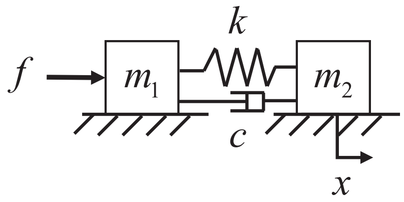


Fig. 3. Non-collocated double-mass-block system.

$$G_{p_0}(s) = \frac{1}{(m_1+m_2)}\left(\frac{1}{s^2} - \frac{1}{s^2 + 2\zeta_n\omega_n s + \omega_n^2}\right) \tag{15}$$

ζ_n and ω_n are damping ratio and resonant frequency, respectively. Model parameters are listed in Table 1. Time delay is $\tau = T_s$, and T_s = 200 μs, then the plant used in simulation is

$$G_p(s) = e^{-(1.5T_s)s}G_{p_0}(s) = e^{-\hat{\tau}s}G_{p_0}(s) \tag{16}$$

Where $\hat{\tau} = 1.5T_s$. The Taylor expansion of plant inversion at $s = 0$ is

$$G_p^{-1}(s) = (m_1+m_2)\left(s^2 + \hat{\tau}s^3 + \left(\frac{1}{\omega_n^2} + \frac{\hat{\tau}^2}{2}\right)s^4 + \cdots\right) \tag{17}$$

Thus the ideal acceleration, jerk and snap feedforward coefficients are $m_1 + m_2 = 25$ kg, $(m_1+m_2)\hat{\tau} = 0.0075$ kg s, $(m_1+m_2)\left(\frac{1}{\omega_n^2} + \frac{\hat{\tau}^2}{2}\right) = 2.4174 \times 10^{-6}$ kg s^2, respectively. PID with notch filter is used as feedback controller

$$G_c(s) = \left(k_p + k_i\frac{1}{s} + k_d\frac{s}{T_f s + 1}\right)\frac{s^2 + 2\zeta_1\omega_N s + \omega_N^2}{s^2 + 2\zeta_2\omega_N s + \omega_N^2} \tag{18}$$

Control bandwidth (the crossover frequency at 0dB in the Bode diagram of open loop transfer function) is 180 Hz. Notch filter is used to suppress resonant dynamics to achieve high control bandwidth with guaranteed stability. Fourth order trajectory shown in Fig. 4 is used. dis stands for displacement and derivatives from first to fourth order are called vel, acc, jerk, snap, respectively. Detailed trajectory planning algorithm can be found in [4].

3.3. Feedforward tuning algorithm

3.3.1. Sequential tuning of acceleration and jerk feedforward

Based on the analysis in previous sections, an auto-tuning algorithm of model-based feedforward controller can be attained. The first phase is sequential tuning of acceleration and jerk feedforward. Acceleration feedforward is tuned first, at this stage, feedforward controller is

Table 1
model parameters used in simulation

m_1	m_2	ω_n	ζ_n
5kg	20kg	700Hz	0.03

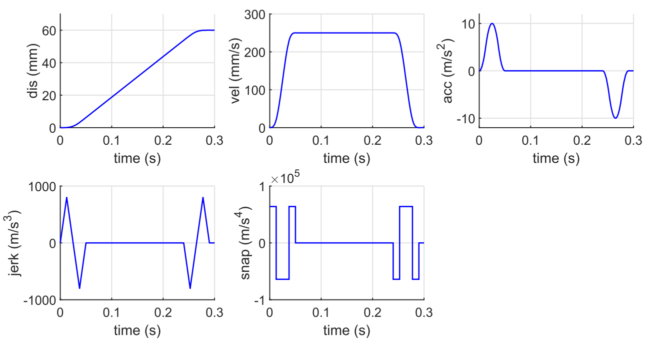


Fig. 4. Reference trajectory used in simulation, dis bound 60 mm, vel bound 250 mm/s, acc bound 10 m/s^2, jerk bound 800 m/s^3, snap bound 64000 m/s^4.

$$F = m_a s^2 \tag{19}$$

Substitute (8) and (19) into (11), we can get

$$\begin{aligned} u_{fb}(s) &= T(s)\left((m-m_a)r^{(2)}(s) + m\left(\frac{T_s}{2}+\tau\right)r^{(3)}(s) + m\hat{a}_1 r^{(4)}(s) + \cdots\right) \\ &\approx \left((m-m_a)r^{(2)}(s) + m\left(\frac{T_s}{2}+\tau\right)r^{(3)}(s) + m\hat{a}_1 r^{(4)}(s) + \cdots\right) \end{aligned} \tag{20}$$

At low frequency range, low order terms overwhelm high order terms in magnitude, then we have

$$u_{fb}(s) \approx (m-m_a)r^{(2)}(s) \tag{21}$$

It shows that when acceleration feedforward coefficient is not accurate enough, feedback control signal has the shape of acceleration and is an approximation of the extra acceleration feedforward part required for further performance improvement. Convert (21) back into time domain, we have

$$u_{fb}(t) \approx (m-m_a)\, r^{(2)}(t) \tag{22}$$

m_a and $r^{(2)}(t)$ are already known and $u_{fb}(t)$ can be attained after executing tracking control task once. Then we have the following least squares problem

$$\min_{\Delta m_a} \quad \|\mathbf{A}\Delta m_a - \mathbf{b}\|_2 \tag{23}$$

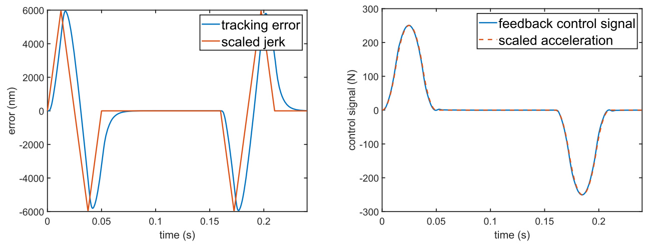
$\mathbf{A} = \left[r^{(2)}(t_1) \quad \cdots \quad r^{(2)}(t_N)\right]^{\mathrm{T}}$ is acceleration data matrix, $\mathbf{b} = \left[u_{fb}(t_1) \quad \cdots \quad u_{fb}(t_N)\right]^{\mathrm{T}}$ is feedback control signal data vector, N is number of time instants selected to estimate feedforward coefficient, Δm_a is the gap between the current acceleration feedforward coefficient and the ideal value. It is easy to get the solution of optimization problem (23)

$$\Delta m_a = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b} \tag{24}$$

Then acceleration feedforward coefficient can be updated as

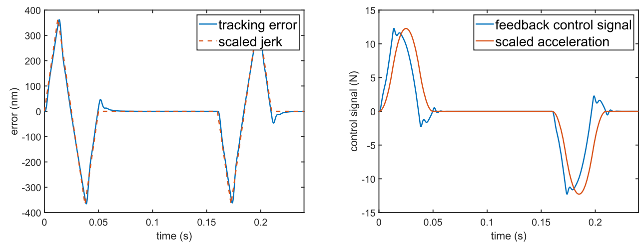
$$m_a := m_a + \Delta m_a \tag{25}$$

symbol $:=$ in (25) stands for assignment operator. Figs. 5 and 6 shows tracking error and feedback control signal with no feedforward and inaccurate acceleration feedforward, respectively. Quantitative analysis on tracking error has been given in [13] and for feedback control signal, as theoretical analysis predicted, feedback control signal has the shape of acceleration and is a good approximation of the ideal feedforward signal which is dominated by the acceleration feedforward part. Use



(a) (b)

Fig. 5. Tracking performance with no feedforward. (a) tracking error. (b) feedback control signal.



(a) (b)

Fig. 6. Tracking performance with inaccurate acceleration feedforward $m_a = 24$ kg. (a) tracking error. (b) feedback control signal.

the data in Fig. 5 where the acceleration is larger than 2 m/s^2 (The subsequent jerk and snap feedforward tuning also use the data from the same time interval.), by (24) and (25), we can get acceleration feedforward coefficient estimation $m_a = 25.0744$ kg , which is close to ideal value 25 kg (the total mass of the system).

The next step is tuning of jerk feedforward if further improvement of tracking performance is required. Feedforward controller now becomes

$$F = m_a s^2 + m_j s^3 \tag{26}$$

The feedback control signal now becomes

$$u_{fb}(s) \approx (m - m_a) r^{(2)}(t) + \left(m\left(\frac{T_s}{2} + \tau\right) - m_j \right) r^{(3)}(s) \approx \left(m\left(\frac{T_s}{2} + \tau\right) - m_j \right) r^{(3)}(s) \tag{27}$$

Fig. 7 gives the tracking performance with tuned acceleration feedforward. As theoretical analysis predicted, when acceleration feedforward coefficient is close to ideal value, feedback control signal is dominated by the jerk feedforward part and thus has the shape of jerk. Similarly, jerk feedforward coefficient can be tuned by least squares, in the simulation example here, the estimation result is 0.0076 kg s, which is very close to ideal value 0.0075 kg s.

3.3.2. Joint tuning of acceleration and jerk feedforward

The second phase is joint tuning of acceleration and jerk feedforward if tracking performance requires further improvement. The main motivation to do so is that the existence of jerk feedforward will to some extent attenuate the tuning accuracy of acceleration feedforward in the first phase, this effect becomes more marked if the reference trajectory has a large jerk. In turn, the inaccuracy of acceleration feedforward will decrease the tuning accuracy of jerk feedforward. This problem can be solved by joint tuning of acceleration and jerk feedforward, i.e., tune acceleration and jerk feedforward at the same time. The tuning formula now becomes

$$\mathbf{a} := \mathbf{a} + \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b} \tag{28}$$

$\mathbf{a} = (\, m_a \quad m_j \,)^{\mathrm{T}}$ is feedforward coefficient vector, $\mathbf{A}$ is trajectory data matrix

$$\mathbf{A} = \begin{bmatrix} r^{(2)}(t_1) & r^{(3)}(t_1) \\ \vdots & \vdots \\ r^{(2)}(t_N) & r^{(3)}(t_N) \end{bmatrix} \tag{29}$$

and $\mathbf{b} = [\, u_{fb}(t_1) \quad \cdots \quad u_{fb}(t_N) \,]^{\mathrm{T}}$ is feedback control signal data vector. Fig. 8 gives the tracking performance with previously tuned acceleration and jerk feedforward. Tune acceleration and jerk feedforward jointly, we get updated feedforward

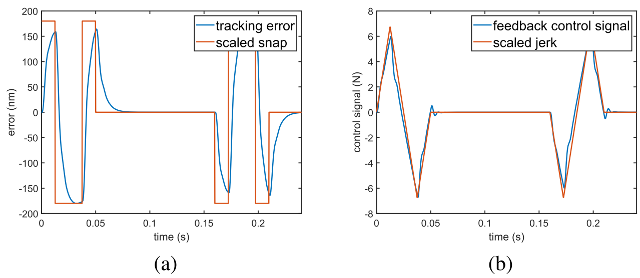


Fig. 7. Tracking performance with tuned acceleration feedforward. (a) tracking error. (b) feedback control signal.

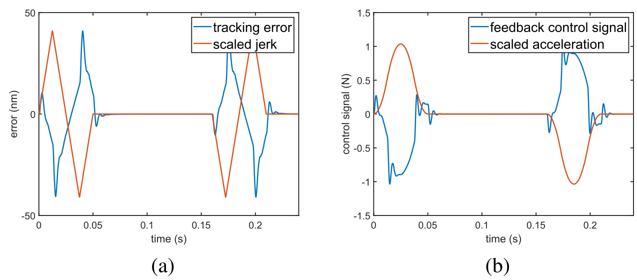
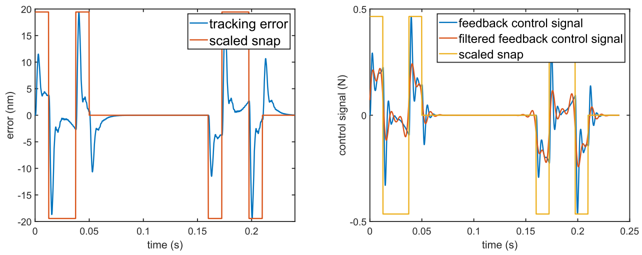
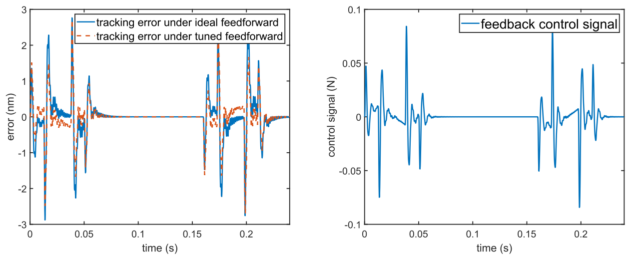


Fig. 8. Tracking performance with tuned acceleration and jerk feedforward. (a) tracking error. (b) feedback control signal.



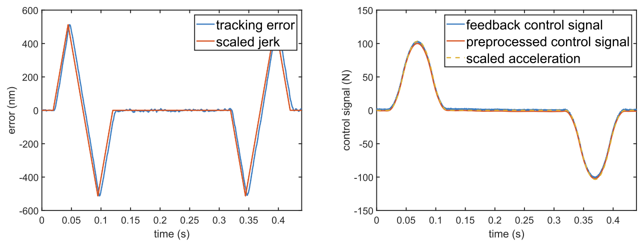
(a) (b)

Fig. 9. Tracking performance with jointly tuned acceleration and jerk feedforward. (a) tracking error. (b) feedback control signal.



(a) (b)

Fig. 10. Tracking performance with jointly tuned acceleration, jerk and snap feedforward. (a) tracking error. (b) feedback control signal.



(a) (b)

Fig. 11. Tracking performance with no feedforward. (a) tracking error. (b) feedback control signal.

coefficients $m_a = 24.9853$ kg, $m_j = 0.0075$ kg s. It can be seen that tuning accuracy is markedly improved and Fig. 9 gives tracking performance with jointly tuned acceleration and jerk feedforward.

3.3.3. *Joint tuning of acceleration, jerk and snap feedforward*

If further tracking performance enhancement is required, tuning of snap feedforward is needed. Feedforward controller now becomes

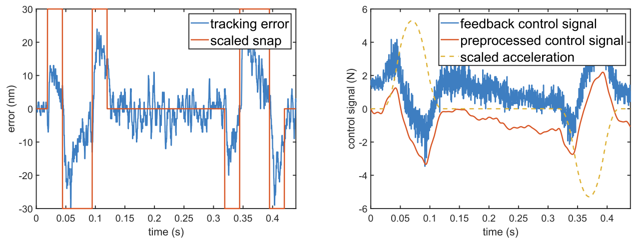
$$F = m_a s^2 + m_j s^3 + m_s s^4 \tag{30}$$

Instead of tuning snap feedforward alone, joint tuning strategy is used again to guarantee tuning accuracy. Feedforward coefficient vector now becomes $\mathbf{a} = \begin{pmatrix} m_a & m_j & m_s \end{pmatrix}^{\mathrm{T}}$, trajectory data matrix now becomes

$$\mathbf{A} = \begin{bmatrix} r^{(2)}(t_1) & r^{(3)}(t_1) & r^{(4)}(t_1) \\ \vdots & \vdots & \vdots \\ r^{(2)}(t_N) & r^{(3)}(t_N) & r^{(4)}(t_N) \end{bmatrix} \tag{31}$$

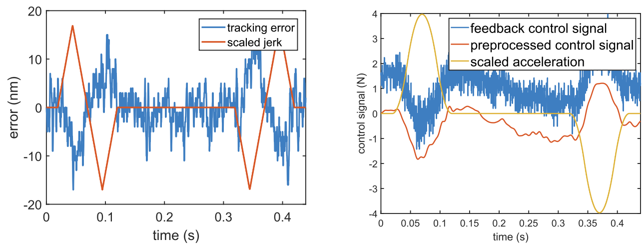
The feedback control signal now becomes

$$u_{fb}(s) \approx m(\hat{a}_1 - m_s) r^{(4)}(s) + \underbrace{m\hat{a}_2 r^{(5)}(s) + \cdots}_{\text{high frequency components}} \tag{32}$$



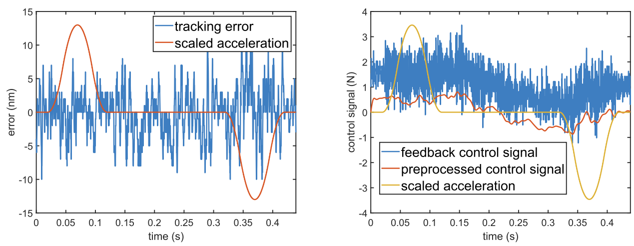
(a) (b)

Fig. 12. Tracking performance with tuned acceleration feedforward. (a) tracking error. (b) feedback control signal.



(a) (b)

Fig. 13. Tracking performance with tuned acceleration and jerk feedforward. (a) tracking error. (b) feedback control signal.



(a) (b)

Fig. 14. Tracking performance with jointly tuned acceleration and jerk feedforward. (a) tracking error. (b) feedback control signal.

Before use (28) to update feedforward coefficients, feedback control signal control has to be preprocessed since terms higher than fourth order, which mainly contain high frequency components, may well deteriorate tuning accuracy. Preprocessing can be done by low-pass zero phase error filtering. Zero phase error filtering introduces no phase lag in frequency domain and thus no time delay in time domain. 80 Hz low-pass zero phase error filtering is used here for preprocessing. Feedback control signal before and after preprocessing is shown in Fig. 9 (b). With unfiltered feedback control signal, the feedforward coefficients are updated as $m_a = 24.9125$ kg, $m_j = 0.0074$ kg s, $m_s = 2.7455 \times 10^{-6}$ kg s^2, the tuning accuracy of acceleration and jerk feedforward is not enhanced but deteriorated. With the filtered feedback control signal, feedforward coefficients are updated as $m_a = 25.0002$ kg, $m_j = 0.0075$ kg s, $m_s = 2.4856 \times 10^{-6}$ kg s^2, the tuning accuracy is markedly improved. Fig. 10 gives tracking performance with tuned and ideal feedforward, it can be seen that tracking performance is significantly improved and tracking performance with tuned feedforward parallels that with ideal feedforward, tuning finished.

3.4. Implementation details

Although the tuning algorithm is given in details with simulation example, some critical implementation details need to be discussed.

3.4.1. The effect of disturbance and noise

Disturbance and noise are out of consideration in previous sections. However, disturbance and noise are inevitable and must be carefully handled in ultraprecision motion systems. In practice, high frequency components in feedback control signal caused by noise and external disturbance can be handled by low-pass zero phase error filtering. DC component usually exists in feedback control signal to guarantee zero steady state error under external disturbance, the DC part has to be contracted from feedback control signal before feedforward tuning. The handling of disturbance and noise will be introduced in more detail in subSection 4.2 with experiment.

3.4.2. Feedforward signal

In many applications, reference trajectory is planned in advance, acceleration, jerk and snap signal required by feedforward controller can be directly attained from planed trajectory. Otherwise, on-line difference has to be executed to attain such signals. For indifferentiable position commands, the proposed approach fails, since derivatives of reference trajectory are required as input of feedforward controller.

3.4.3. Use of zero phase error filtering

Zero phase error filtering is mainly used for the following reasons: (i) attenuating high frequency components in feedback control signal caused by plant dynamics, disturbance and noise; (ii) compared with real-time filtering, zero phase error filtering introduces no phase lag and thus no time delay, which is very critical to guarantee tuning accuracy. In simulation, zero phase error filtering is only used during the joint tuning of acceleration, jerk and snap feedforward, in practice, zero phase error filtering is recommended to use at every tuning phase due to external disturbance and noise and this will be introduced in more detail in subSection 4.2 with experiment.

4. Experiment

4.1. Experiment platform

Experiment is carried on the ultraprecision motion system shown in Fig. 15, the system consists of a long-stroke stage and a short-stroke stage. The long-stroke stage is driven by moving coils actuator while the short-stroke stage is driven by moving magnets actuators. Both stages are magnetic levitated and have six DOFs. The long-stroke stage is used for long-stroke motion with micron accuracy and the short one on it is used for short-stroke motion with nanometer accuracy.

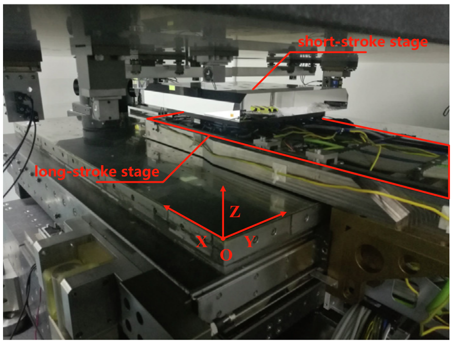


Fig. 15. Ultraprecision motion system actuated by six DOFs magnetic-levitated planar motor.

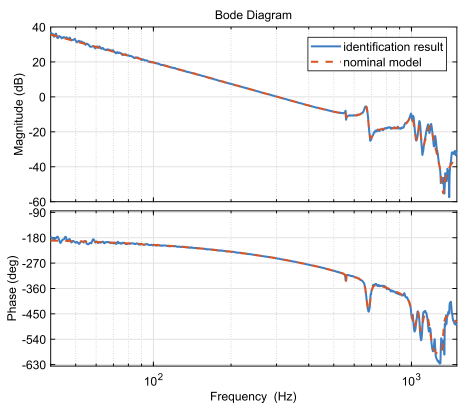


Fig. 16. System identification of axis *x*.

The displacement of the short-stroke stage is measured by laser interferometer with 0.15 nm resolution. The sampling period of the system is 200 μs. Experiment is carried on axis x. The mover of short-stroke stage and the mirror block on it constitute a double-mass-block system. The mass of the mirror block is around 17 kg and the mass of the mover is around 8 kg, the stiffness between the mirror block and the mover is around 9.6×10^7 N/m. Fig. 16 shows the system identification result of axis x. The -40 dB/dec slop at low frequency range stands for rigid-body dynamics, the primary resonant mode occurs at around 670 Hz. The phase lag at low frequency range shows the existence of time delay, time delay is around 3.5 sampling periods, namely, 700 μs. To achieve high control bandwidth with sufficient stability margin under the phase lag caused by time delay, PI + double lead controller instead of PID is used as feedback controller

$$G_c(z) = K\frac{z+z_1}{z-1}\left(\frac{z+z_2}{z+p_2}\right)^2 \tag{33}$$

The control bandwidth is 240Hz. The Bode and Nyquist diagrams of open loop transfer function are shown in Figs. 17 and 18, respectively. The bounds of trajectory used in experiment are 60 mm, 200 mm/s, 4 m/s^2, 157 m/s^3 and 6250 m/s^4, respectively.

4.2. Experiment results

The first tuning phase is sequential tuning of acceleration and jerk feedforward and the first step is acceleration feedforward tuning. Fig. 11 shows the tracking performance with no feedforward, namely, $m_a = 0$. As theoretical analysis predicted, feedback control signal has the shape of acceleration and is an approximation of acceleration feedforward signal required. Feedback control signal is filtered by 80 Hz low-pass zero phase error filtering, filtered feedback control signal is then subtracted by its DC component to get preprocessed control signal, see the red line in Fig. 11 (b). Use (24), (25) and the data where acceleration is larger than 1 m/s^2 during acceleration phase to tune acceleration feedforward (The subsequent jerk feedforward tuning also use the data from the same time interval.). Fig. 12 gives the tracking performance with tuned

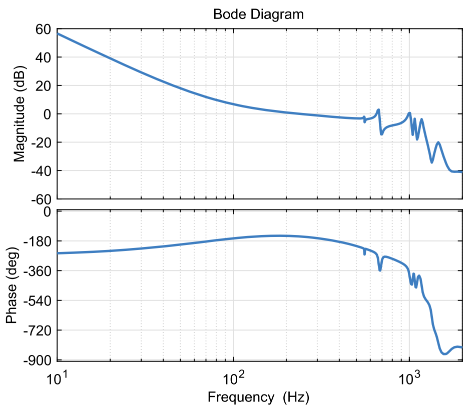


Fig. 17. Bode diagram of open loop transfer function.

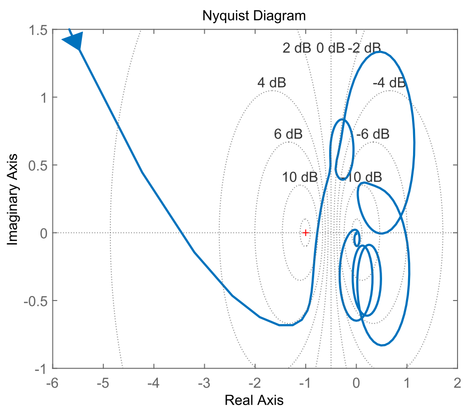


Fig. 18. Nyquist diagram of open loop transfer function.

acceleration feedforward. As theoretical analysis predicted, feedback control signal is now dominated by jerk feedforward part and thus has the shape of jerk when acceleration feedforward is initially tuned. The second step is jerk feedforward tuning. Similarly, after preprocessing of feedback control signal (the red line in Fig. 12 (b)), jerk feedforward can be tuned by least squares algorithm. Fig. 13 gives the tracking performance with tuned acceleration and jerk feedforward. The attenuation of tracking error shows the effectiveness of feedforward tuning. Judging by the shape of feedback control signal which is similar to the shape of acceleration but with opposite sign, the inaccuracy of acceleration feedforward is now the dominant factor influencing the residual tracking error and current acceleration feedforward is a little larger than its ideal value. The inaccuracy of acceleration feedforward is mainly caused by the existence of jerk feedforward during acceleration feedforward tuning. Instead of tuning acceleration feedforward alone again, joint tuning of acceleration and jerk feedforward is required, since the previous tuning of jerk feedforward may also be influenced by inaccurate acceleration feedforward in turn.

The second phase is joint tuning of acceleration and jerk feedforward. With (28) and preprocessed feedback control signal (the red line in Fig. 13 (b)), tracking performance can be further improved by joint tuning of acceleration and jerk feedforward, see Fig. 14. Since the tracking performance during acceleration phase parallels that of constant velocity phase, there is no need for use of snap feedforward and tuning procedure is finished.

5. Further discussions

5.1. Connections with iterative learning control

The proposed algorithm has very close connections with iterative learning control (ILC). In fact, the feedback control signal used in the proposed algorithm can be directly regarded as ILC signal where the learning law of ILC is just right the feedback controller. To some extent, the proposed approach is to fit ILC signal with fixed structure model-based feedforward controller. Similar idea has already been proposed in [12], where FIR filter is used to fit ILC signal and the ILC algorithm is from [9]. What distinguish the proposed algorithm from that in [12] can be stated as follow:

(i) In [12], the structure of FIR filter is attained by trial, how to choose the structure of FIR filter is to some extent empirical and unguided. The proposed approach extensively utilizes prior information on plant dynamics to determine the structure of feedforward controller (namely, acceleration + jerk + snap feedforward). From this perspective, the proposed approach enjoys a better understanding of physics and thus convenience of implementation.
(ii) Theoretically, the capability to approximate plant inversion can theoretically grows with the order of FIR filter, however, the capability to approximate plant inversion of the proposed approach is limited by its structure unless higher order trajectory is used at the expense of throughput to enable higher order feedforward.

5.2. Comparisons with stable-inversion model-based feedforward

Aside from the proposed approach, other model-based feedforward methods such as the famous zero-phase-error tracking control (ZPETC), zero-magnitude-error tracking control (ZMETC) and non-minimum-phase-zero-ignored tracking control (NMPZITC) are also widely studied [16–18]. These methods are proposed to handle non-minimum phase dynamics caused by zero-order hold sampling and time delay which make the direct inversion of the plant model infeasible [19,18,20]. The difference between the proposed method and these stable-inversion model-based feedforward can be stated as follow:

(i) Model-based feedforward is very sensitive to model inaccuracy. What matters most for both kinds of methods is tuning, which is of significant importance for ultraprecision motion systems. For the proposed approach, with prior information on plant dynamics, the accuracy of feedforward coefficients can be judged by the shape of feedback control signal, and feedforward controller can be tuned by least squares easily and effectively. For stable-inversion feedforward, data-based or adaptive techniques have to be used for tuning, see [21,22].
(ii) the proposed approach applies for motion systems with rigid-body dynamics and the generality is correspondingly limited. Stable-inversion feedforward does not rely on prior information on plant dynamics and is thus more general.

5.3. Compensation of time delay

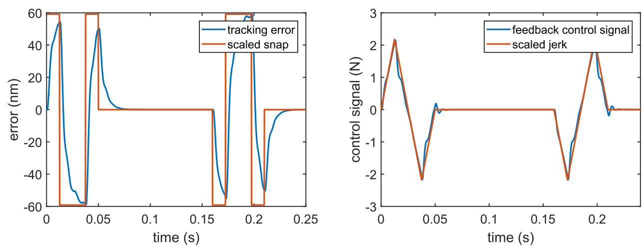
In this paper, time delay is compensated by jerk feedforward, while in many other works, time delay is handled by shifting acceleration signal forward, in other words, changing the delay parameter between position and acceleration signal. It is just like solving the same problem from different perspectives. However, what matters most for both methods is tuning. For shifting acceleration signal, the delay parameter has to be very accurate to achieve required tracking performance. Fig. 19 gives the simulation result where acceleration feedforward coefficient is accurate but the delay parameter is not. The time delay of the system is 1.5 sampling period. The actual delay parameter used is 1 sampling period, namely, acceleration signal is shifted forward for one sampling period with regard to position signal. Other simulation parameters are all the same with those in subSection 3.2. This situation is equivalent to the situation where the delay of the plant is $1.5 - 1 = 0.5$ sampling period. From Fig. 19, it can be seen that even slight gap between actual delay parameter and the ideal value will lead to large tracking error and makes feedback control signal have the shape of jerk. From this perspective, it does not matter what kind of strategy is taken to handle time delay but how to tune corresponding parameters. According to [13] and this paper, the information about whether the current parameters are accurate enough is contained in the shape of tracking error or the shape of feedback control signal. And this information can be used to tune corresponding parameters.

5.4. Approximation of ideal feedforward signal

Instead of directly using feedback control signal u_{fb} , better approximation of ideal feedforward signal can be attained by using $T^{-1}u_{fb}$. However, non-minimum phase dynamics will make this choice more complicated than it seems to be. Though the multi-mass-block model is a minimum phase system in continuous time domain, non-minimum phase zeros may occur due to zero-order hold sampling and time delay, see [17–20]. Fig. 20 gives the pole-zero map of T of the nominal model, non-minimum phase dynamics can be clearly observed. The existence of non-minimum phase zeros makes the direct inverse of T infeasible. More techniques are required to handle this problem, see [17]. Though u_{fb} is not the best approximation of ideal feedforward signal, approximation accuracy of u_{fb} is usually qualified for feedforward tuning, since ultraprecision motion systems are with high control bandwidth and thus high loop gain at low frequency range. Moreover, u_{fb} is easy to attain and use of u_{fb} can largely simplify the implementation of the proposed approach, which is very beneficial for industrial use.

5.5. Choice between sequential and joint tuning

In fact, joint tuning can be used at the very beginning, since considered basis functions (derivatives of reference trajectory) are in general not orthogonal to each other and thus joint optimization of all considered feedforward parameters may yield better results. However, sequential tuning is still of its value, the main reason is that the required order of feedforward controller varies from applications to applications. In some applications, a well-tuned acceleration feedforward is just enough, in some applications, e.g., the experimental example in this paper, acceleration and jerk feedforward are required, in applications where derivatives of reference trajectory are very large, tuning of snap feedforward is also required, see [7,23]. Tuning of unnecessary feedforward terms may not elevate but attenuate tracking performance. By combining sequential tuning with joint tuning, such problem can be solved.



(a) (b)

Fig. 19. Tracking performance with inaccurate delay parameters. (a) tracking error. (b) feedback control signal.

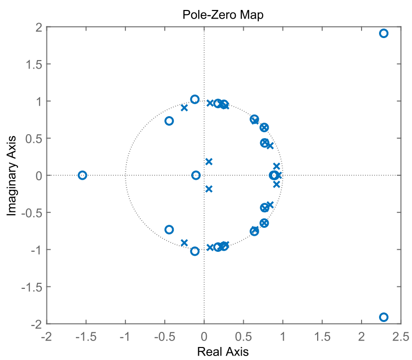


Fig. 20. Pole and zero map of complementary sensitivity function T.

5.6. Parameter bias

Since approximation is used in the proposed approach, parameter bias is inevitable. In general, the parameter bias mainly comes from the following aspects:

(i) Approximation of ideal feedforward signal. Feedback control signal $u_{fb} = T\left(G_p^{-1} - F\right)r$ is just an approximation of ideal feedforward signal $\left(G_p^{-1} - F\right)r$. Fitting feedback control signal rather than ideal feedforward signal will inevitably introduce bias. Fortunately, the control bandwidth in high precision motion systems is usually much higher than the frequency range where most energy of reference trajectory distributes that the bias caused by use of feedback control signal is nearly negligible.

(ii) The influence of higher order terms. The existence of higher order terms in ideal feedforward controller will introduce bias. For example, when it comes to the tuning of acceleration feedforward, (22) directly neglects the effect of terms higher than second order. Such treatment will also introduce bias. Since lower terms overwhelm higher order terms in magnitude, the bias is limited. Furthermore, the subsequent joint tuning will further decrease the bias. Therefore, the bias introduced by higher order terms is negligible.

(iii) Low-pass filtering. The use of low-pass filtering will undoubtedly introduce bias, Fortunately, since the bandwidth of low-pass filter is higher than the frequency range where most energy of reference trajectory distributes, the bias introduced by low-pass filtering is limited. Aside from this, low-pass filtering may also elevate tuning accuracy to some extent. For example, the tuning accuracy of snap feedforward can be improved by suppressing high frequency components with low-pass filtering, see the simulation example in subSection 3.3.3.

(iv) Disturbance. Disturbance is out of consideration in the theoretical analysis and simulation, however, disturbance is inevitable in practice. The components in feedback control signal caused by disturbance will also introduce bias. During acceleration phase, reference-induced error is much larger than disturbance-induced error, then reference-induced feedback control signal is much larger than the part induced by disturbance and thus the bias introduced by disturbance is limited.

(v) Noise. The bias introduced by noise is limited for the following reasons: 1) low-pass filtering of feedback control signal; 2) the proposed method utilizes least squares algorithm and least squares can effectively attenuate the effect of noise. On the whole, bias inherently exists in the proposed approach due to the abovementioned reasons, but the effect of the bias is limited that classical model-based feedforward can still be effectively tuned by the proposed algorithm according to the simulation and experiment.

5.7. Choice of time interval for tuning

In simulation, the data of time interval where acceleration is larger than 2 m/s^2 is used for tuning while the data of time interval where acceleration is larger than 1 m/s^2 is used for tuning in experiment. The main reason for the choice of threshold is persistence of excitation. In fact, the choice is, to some extent, arbitrary as long as corresponding feedforward terms can be sufficiently excited. According to the authors' experience, it is recommended to set the threshold larger than 20% of peak value.

Another important aspect has to be mentioned is that tuning of jerk and snap feedforward utilized the data from the same time interval as tuning of acceleration feedforward in this paper. However, this choice may be inappropriate under other trajectories. Jerk and snap of the trajectories used in this paper are nonzero during acceleration phase. If the bounds of derivatives of reference trajectory are changed, there may be time interval where jerk and snap are zero during acceleration phase, see [4]. If so, it is recommended to use nonzero interval for tuning of jerk and snap feedforward to get better performance.

5.8. Scope of applications and limitations

The proposed algorithm provides a tuning procedure for classical model-based feedforward such as acceleration, jerk and snap feedforward. The feedforward controller, if tuned appropriately, can compensate for rigid-body dynamics, time delay at low frequency range, and part of resonant dynamics. In applications where structural flexibility is marked or derivatives of trajectory are very large that trajectory contains non-negligible high frequency components, more advanced techniques are required for further compensation. However, the proposed algorithm is still of explicit advantages which can be stated as follows:

(i) The proposed method can fully exploit the potential of classical mode-based feedforward such as acceleration and jerk feedforward. Even in applications where more advanced techniques are required, a well-tuned classical model-based feedforward controller can largely ease the burden of the subsequent data-based techniques and thus accelerate the whole controller design procedure.

(ii) Compared with the method in [13], the proposed method is an auto-tuning one and is thus more practical for industrial use.
(iii) The proposed method extensively utilizes the prior knowledge on plant dynamics and feedback control signal, thus enjoys better understanding of physics and is very easy to implement, which is very important for industrial use.

6. Conclusions

In this paper, an algorithm for tuning of classical model-based feedforward controller such as acceleration, jerk and snap feedforward is proposed. The prior knowledge on plant dynamics is first extensively utilized to analysis the components of feedback control signal. Then a feedforward tuning method by feedback control signal with least squares algorithm is proposed. The proposed method predicates on the simple fact that feedback control signal is a good approximation of ideal feedforward control signal at low frequency range. The tuning procedure combines sequential tuning and joint tuning to achieve high tuning accuracy. Simulation and experiment on an ultraprecision motion system both well validate the proposed approach. Problems such as tuning bias, connections with other feedforward strategies, approximation accuracy, limitations and advantages are further discussed. The proposed method is an auto-tuning one and easy to implement, and thus is of much practical value for industrial use.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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